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The MegaMenger Sponge Project at MoMath

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Here is a look at what the Menger Sponge is and how to create it (1; 2). A fractal is generated by a seed (geometric, algebraic), and an algorithm that provides the rule for growing or shrinking the seed by some fraction. The rule is then applied to each new part of the structure to generate more, smaller copies of the original. At each stage, if we zoom in to the structure, it appears to look like a previous stage. This self-similarity can be exact or statistical.

Karl Menger produced the 3-D fractal that bears his name in 1926 in his research in topological dimension (2). It is a 3-D version of the 2-D Sierpinski Carpet, and the 1-D *Cantor Set*, created by a subtractive process. Start with a cube (stage 0, or the seed); by subdividing each edge into three equal parts you will divide each face into nine equal squares, and thus divide the cube into 27 equal cubelets. Push out the central cubelets on each face and the interior center cubelet to obtain the stage 1 structure, like a Rubik's Cube with centers drilled out. Repeat the process with each of the 20 small cubes left, producing 20 x 20 = 400 smaller cubes, for stage 2. Repeat ad infinitum, to approach an object with "infinite surface area and zero volume" (2). In practice, we build by additive modules, rather than trying to carve out cubes from a given large cube!

But how many of us have actually tried building stages of the Sponge in real 3-D space? This year there was an international group-building effort organized, called the MegaMenger Sponge project (3). The goal was to try to build a stage 4 Menger Sponge, by having groups around the world build stage 3 sponges, and link them at least virtually via the internet to create a stage 4, as a mathematical crowdsourcing event.

The project was conceived and implemented by Laura Taalman (James Madison University, VA and Mathematical Association of America Ambassador) and Matt Parker (Queen Mary University, London UK and Think Maths) and circulated via the website megamenger.com (5). Cubes would be built using folded business cards, and linked together to form higher stages. It was timed to coincide with the Celebration of the Mind gatherings in honor of Martin Gardner, held in late October.

I first found out about the MegaMenger Sponge project (megamenger.com) from a distribution list email, but had been part of building one on prior occasions. The use of business cards to create cubes and attach the cubes together has been known since before YouTube (where you can get various video tutorials).

The first time was many years ago when Robert Messer at Albion College in Albion, MI had his students build one; they got halfway to stage 3. Some years later, Ada Dong did the same, getting to stage 2 with her class in a summer program for high school students at Lawrence Technological University. This effort was recreated by my Lawrence Technological University Math Club students, and is an ongoing project.

I joined the group at the Museum of Mathematics (MoMath) in New York City, while on a weekend visit to see my sister. Knowing I was visiting for the weekend, I had checked the MoMath website for activities (4). I discovered their *Menger Sponge* project and knew I had to take my cousin, with whom I was staying. She is my arts guide, and I am her math guide.

A diverse group of people, from 9 year olds to senior citizens, gathered on October 25th on the 0th floor of MoMath (that's the lower level) to assemble the different stages. People could wander in and out all day to look or participate. There were directions and there were helpful human guides. Directing it all, refreshing supplies, and caring for all aspects of the project, was Laura Taalman, this year's Mathematician in Residence at MoMath!

It was well organized. There were tables for assembling each stage, from 0 to 3. Directions were on each table, and whiteboards gave the sequence: fill the basket at the end, move the stage 0's to the stage 1 table, and iteratively continue the process (as shown in the accompanying Figures). Plain business cards were supplied, as well as printed cover cards. The cover cards were to mask the tabs from construction and make a smooth surface; they were printed with a Sierpinski carpet on one side. A nice touch.

By late morning when we arrived, many people were busy at work. We sat at the beginner's table (fold cards, build stage 0 cubes, Figures 1a and 1b). My cousin caught on quickly, and we both enjoyed the conversation with others at the table. And that provided some unexpected, gratifying networking, another benefit of gathering together people with mathematical curiosity.

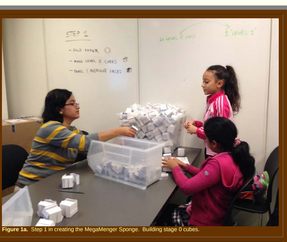

Figure 1a. Step 1 in creating the MegaMenger Sponge. Building stage 0 cubes.

Figure 1b. Step 1 in creating the MegaMenger Sponge. Building stage 0 cubes.

Figure 2. Step 2 in creating the MegaMenger Sponge. Building stage 1.

Figure 3. More stage 1 blocks.

Figure 4. Step 3 in creating the MegaMenger Sponge. Attaching stage 1 blocks.

Figure 5. Step 4 in creating the MegaMenger Sponge. Cover cards on stage 2.

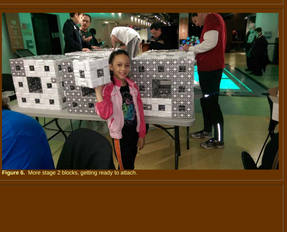

Figure 6. More stage 2 blocks, getting ready to attach.

Figure 7. Step 5 in creating the MegaMenger Sponge. Attaching stage 2 blocks.

Figure 8. Step 6 in creating the MegaMenger Sponge. Attaching stage 2 blocks. Whoopee!

I gave information about the AMC8 contest to a woman whose 9-year-old daughter and friend were busy making cubes. A high school teacher and I talked about books on origami, and also information on fractals. (In a nice coincidence, we crossed paths later in the afternoon while walking the High Line, a former elevated railway now a walking trail and park by the Hudson River—we had geographic as well as mathematical curiosity.) One of the other participants mentioned that he had used a stage 2 Sponge as a coffee table for a while. I did not ask what happened to it.

The day produced several stage 2 sponges, as well as much fun and learning experience for the participants. Laura Taalman sent participants an email, calling for those who could come back to continue to build during the week, and also on the next weekend. Photos show some of the results.

Here is a sample quiz.

Given:

Stage 0 takes 6 business cards to make (without the cover cards for a smooth surface); stage 1 takes 20 of the stage 0 cubes; stage 2 takes 20 stage 1's; stage 3 takes 20 stage 2's; and stage 4 takes 20 stage 3's.

Find:

a) How many business cards are needed for each of stages 1 – 4? (Ignore the covers that make the finished product look smooth.)

b) How much does each stage weigh (approximately)?

c) How many cards are needed for covering the outer surface (to smooth the surface) of each stage 1 - 4?

d) How much weight has been added to the object by the covering cards at each stage?

REFERENCES

1. http://en.wikipedia.org/wiki/Fractal

2. http://en.wikipedia.org/wiki/Menger_sponge

3. www.megamenger.com

4. www.momath.org

5. Sponsors include Queen Mary University, Museum of Math, and the Manchester (UK) Science Festival.

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