Web-based Supplementary Material for "Joint Modeling of Cross-Sectional Health Outcomes and Longitudinal Predictors via Mixtures of Means and Variances" by

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Web Appendix A

Posterior computations for the joint LC model

(1) update for longitudinal submodel

- **update** the mean profile class memberships D_i, $i = 1, ..., n$: the full conditional posterior distribution $[D_i|\cdot] \sim \text{Multinomial}(\tilde{\pi}^D_{i1}, ..., \tilde{\pi}^D_{iK_D})$, where for $d = 1, \cdots, K_D$,

$$\tilde{\pi}^D_{id} = \Pr(D_i = d|.) = \frac{\pi^D_d |\boldsymbol{\Sigma}_d|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(\boldsymbol{b}_i - \boldsymbol{\beta}_d)'\Sigma_d^{-1}(\boldsymbol{b}_i - \boldsymbol{\beta}_d) - \frac{1}{2}(W_i - \theta_{C_i,d})^2\right\}}{\sum_{d=1}^{K_D} \pi^D_d |\boldsymbol{\Sigma}_d|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(\boldsymbol{b}_i - \boldsymbol{\beta}_d)'\Sigma_d^{-1}(\boldsymbol{b}_i - \boldsymbol{\beta}_d) - \frac{1}{2}(W_i - \theta_{C_i,d})^2\right\}}.$$

  $\theta_{C_i,d} = \boldsymbol{Z}_i^T \boldsymbol{\eta}$ in the latent class probit submodel given $D_i = d$ and C_i as well as other covariates.

- **update** the mean profile class parameters:

  – **update** $\boldsymbol{\beta}_d$: Assuming the prior for $\boldsymbol{\beta}_d \overset{ind}{\sim} \text{MVN}(\boldsymbol{\nu}, \boldsymbol{V})$, then the full conditional posterior density for $\boldsymbol{\beta}_d$ for $d = 1, ..., K_D$ is $[\boldsymbol{\beta}_d|.] \sim \text{MVN}\left(\tilde{\boldsymbol{\nu}}_d, \tilde{\boldsymbol{V}}_d\right)$ where

$$\tilde{\boldsymbol{\nu}}_d = \left\{\boldsymbol{V}^{-1} + \boldsymbol{\Sigma}_d^{-1}\sum_{i=1}^n \text{I}(D_i = d)\right\}^{-1}\left\{\boldsymbol{V}^{-1}\boldsymbol{\nu} + \boldsymbol{\Sigma}_d^{-1}\sum_{i=1}^n \text{I}(D_i = d)\boldsymbol{b}_i\right\}$$
$$\tilde{\boldsymbol{V}}_d = \left\{\boldsymbol{V}^{-1} + \boldsymbol{\Sigma}_d^{-1}\sum_{i=1}^n \text{I}(D_i = d)\right\}^{-1}$$

  – **update** $\boldsymbol{\Sigma}_d$: Assuming the prior for $\boldsymbol{\Sigma}_d \overset{ind}{\sim} \text{Inverse-Wishart}(m, \boldsymbol{\Lambda})$, then the full conditional posterior density is, $[\boldsymbol{\Sigma}_d|\cdot] \sim \text{Inverse-Wishart}\left(\tilde{m}_d, \tilde{\boldsymbol{\Lambda}}_d\right)$ where

$$\tilde{m}_d = m + \sum_{i=1}^n \text{I}(D_i = d)$$
$$\tilde{\boldsymbol{\Lambda}}_d = \left\{\Lambda^{-1} + \sum_{i=1}^n \text{I}(D_i = d)\left(\boldsymbol{b}_i - \boldsymbol{\beta}_d\right)\left(\boldsymbol{b}_i - \boldsymbol{\beta}_d\right)'\right\}^{-1}$$

- **update** the mixing proportion $\{\pi^D_d\}_d$: assuming $\{\pi^D_d\}_d \sim \text{Dirichlet}(e^D_1, ..., e^D_{K_D})$ then the full conditional posterior distribution is $\left[\{\pi^D_d\}_d|\cdot\right] \sim \text{Dirichlet}(\{e^D_d + \sum_{i=1}^n \text{I}(D_i = d)\}_d)$.

- **update** the variance class memberships C_i, $i = 1, ..., n$: the full conditional posterior distribution $[C_i|\cdot] \sim \text{Multinomial}(\tilde{\pi}_{i1}^C, ..., \tilde{\pi}_{iK_C}^C)$ where for $c = 1, \cdots, K_C$,

$$\tilde{\pi}_{ic}^C = \Pr(C_i = c|.) = \frac{\pi_c^C \exp\left\{-\frac{1}{2\tau^2}\left(\log\sigma_i^2 - \mu_c\right)^2 - \frac{1}{2}(W_i - \theta_{c,D_i})^2\right\}}{\sum_{c=1}^{K_C} \pi_c^C \exp\left\{-\frac{1}{2\tau^2}\left(\log\sigma_i^2 - \mu_c\right)^2 - \frac{1}{2}(W_i - \theta_{c,D_i})^2\right\}}$$

$\theta_{c,D_i} = \boldsymbol{Z}_i^T \boldsymbol{\eta}$ in the probit latent class submodel given $C_i = c$ and D_i as well as other covariates.

- **update** the variance class parameters:
  - **update** μ_c: assuming the prior for $\mu_c \overset{ind}{\sim} \text{N}(a, b)$, then the full conditional posterior distribution is, $[\mu_c|\cdot] \sim \text{N}\left(\tilde{a}, \tilde{b}\right)$ where

$$\tilde{a} = \frac{\sum_{i=1}^{n} \text{I}(C_i = c)\log\sigma_i^2/\tau^2 + a/b}{1/b + \sum_{i=1}^{n} \text{I}(C_i = c)/\tau^2}$$

$$\tilde{b} = \left\{1/b + \sum_{i=1}^{n} \text{I}(C_i = c)/\tau^2\right\}^{-1}$$

  - **update** τ^2: assuming $\tau^2 \sim \text{Inverse-Gamma}(v, e)$, then the full conditional posterior distribution is

$$\left[\tau^2|\cdot\right] \sim \text{Inverse-Gamma}\left\{v + \frac{n}{2}, e + \sum_{i=1}^{n}\sum_{c=1}^{K_C} \frac{1}{2}\text{I}(C_i = c)\left(\log\sigma_i^2 - \mu_c\right)^2\right\}.$$

- **update** the mixing proportions $\{\pi_c^C\}_c$: assuming $\{\pi_c^C\}_c \sim \text{Dirichlet}(e_1^C, ..., e_{K_C}^C)$ then the full conditional posterior distribution is

$$\left[\{\pi_c^C\}_c|\cdot\right] \sim \text{Dirichlet}\left(\{e_c^C + \sum_{i=1}^{n} \text{I}(C_i = c)\}_c\right).$$

- **update** the random effects $\boldsymbol{b}_i$, $i = 1, ..., n$ the full conditional posterior distribution is $\boldsymbol{b}_i$ $[\boldsymbol{b}_i|.] \sim \text{MVN}\left(\tilde{\boldsymbol{\beta}}_i, \tilde{\boldsymbol{\Sigma}}_i\right)$, where

$$\tilde{\boldsymbol{\beta}}_i = \left(\boldsymbol{\Sigma}_{D_i}^{-1} + \frac{1}{\sigma_i^2}\sum_{j=1}^{n_i} \boldsymbol{t}_{ij}\boldsymbol{t}_{ij}'\right)^{-1}\left(\Sigma_{D_i}^{-1}\boldsymbol{\beta}_{D_i} + \frac{1}{\sigma_i^2}\sum_{j=1}^{n_i} y_{ij}\boldsymbol{t}_{ij}\right)$$

$$\tilde{\boldsymbol{\Sigma}}_{id} = \left(\boldsymbol{\Sigma}_{D_i}^{-1} + \frac{1}{\sigma_i^2}\sum_{j=1}^{n_i} \boldsymbol{t}_{ij}\boldsymbol{t}_{ij}'\right)^{-1}$$

$\boldsymbol{t}_{ij}$ is a vector of functional forms of the observation time t_{ij} for the longitudinal measurement y_{ij} such that $y_{ij} \sim \mathrm{N}\left\{f(\boldsymbol{b}_i;t_{ij}), \sigma_i^2\right\}$ with $f(\boldsymbol{b}_i;t_{ij}) = \boldsymbol{b}_i^T \boldsymbol{t}_{ij}$.

- **update** the variances σ_i^2, $i = 1, ..., n$

$$\pi(\sigma_i^2|.) \propto \prod_{c=1}^{K_C} \mathrm{LN}\left(\sigma_i^2; \mu_c, \tau^2\right)^{\mathrm{I}(C_i=c)} \prod_{j=1}^{n_i} \mathrm{N}\left\{y_{ij}; f(\boldsymbol{b}_i;t_{ij}), \sigma_i^2\right\}$$
$$\propto (\sigma_i^2)^{-\frac{n_i}{2}-1} \exp\left[-\sum_{c=1}^{K_C} \mathrm{I}(C_i = c)\frac{(\log\sigma_i^2 - \mu_c)^2}{2\tau^2} - \frac{1}{2\sigma_i^2}\sum_{j=1}^{n_i}\left\{y_{ij} - f(\boldsymbol{b}_i;t_{ij})\right\}^2\right]$$

$\mathrm{LN}\left(\sigma_i^2; \mu_c, \tau^2\right)$ represents the density of log normal disribution with mean μ_c and variance τ^2 evaluated at σ_i^2 and $\mathrm{N}\left\{y_{ij}; f(\boldsymbol{b}_i;t_{ij}), \sigma_i^2\right\}$ represents the density of normal distribution with mean $f(\boldsymbol{b}_i;t_{ij})$ and variance σ_i^2 evaluated at y_{ij}. Since there is no closed form of the full conditional posterior density, the draws for σ_i^2, $i = 1, ..., n$ at each iteration of the Gibbs sampling are obtained using the inverse cumulative distribution sampling method.

(2) update for LC probit model:

- **update** W_i, $i = 1, ..., n$

$$[W_i|o_i = 1, \cdot] \sim \mathrm{N}(\theta_{C_i,D_i}, 1)I_{(0,\infty)}(\cdot)$$
$$[W_i|o_i = 0, \cdot] \sim \mathrm{N}(\theta_{C_i,D_i}, 1)I_{(-\infty,0)}(\cdot)$$

where, $\theta_{C_i,D_i} = \boldsymbol{Z}_i^T\boldsymbol{\eta}$ in the latent class probit submodel given C_i and D_i.

- **update** $\boldsymbol{\eta}$: Assuming the prior for $\boldsymbol{\eta} \sim \mathrm{MVN}(\boldsymbol{\nu}_{\boldsymbol{\eta}}, \boldsymbol{V}_{\boldsymbol{\eta}})$, then the full conditional posterior density for $\boldsymbol{\eta}$ is $[\boldsymbol{\eta}|.] \sim \mathrm{MVN}\left(\tilde{\boldsymbol{\nu}}_{\boldsymbol{\eta}}, \tilde{\boldsymbol{V}}_{\boldsymbol{\eta}}\right)$ where

$$\tilde{\boldsymbol{\nu}}_{\boldsymbol{\eta}} = \left(\boldsymbol{V}_{\boldsymbol{\eta}}^{-1} + \sum_{i=1}^{n} \boldsymbol{Z}_i\boldsymbol{Z}_i'\right)^{-1}\left(\boldsymbol{V}_{\boldsymbol{\eta}}^{-1}\boldsymbol{\nu}_{\boldsymbol{\eta}} + \sum_{i=1}^{n} W_i\boldsymbol{Z}_i\right)$$
$$\tilde{\boldsymbol{V}}_{\boldsymbol{\eta}} = \left(\boldsymbol{V}_{\boldsymbol{\eta}}^{-1} + \sum_{i=1}^{n} \boldsymbol{Z}_i\boldsymbol{Z}_i'\right)^{-1}$$

$\boldsymbol{Z}_i$ is the i^{th} row of the design matrix in the probit submodel given D_i and C_i as well as other covariates.

Posterior computations for the joint MSRE model

(1) update for longitudinal submodel

- **update** the mean profile class memberships D_i, $i = 1, ..., n$: the full conditional posterior distribution $[D_i|\cdot] \sim \text{Multinomial}(\tilde{\pi}_{i1}^D, ..., \tilde{\pi}_{iK_D}^D)$, where for $d = 1, \cdots, K_D$,

$$\tilde{\pi}_{id}^D = \Pr(D_i = d|.) = \frac{\pi_d^D |\mathbf{\Sigma}_d|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(\boldsymbol{b}_i - \boldsymbol{\beta}_d)' \Sigma_d^{-1} (\boldsymbol{b}_i - \boldsymbol{\beta}_d)\right\}}{\sum_{d=1}^{K_D} \pi_d^D |\mathbf{\Sigma}_d|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(\boldsymbol{b}_i - \boldsymbol{\beta}_d)' \Sigma_d^{-1} (\boldsymbol{b}_i - \boldsymbol{\beta}_d)\right\}}$$

- **update** the mean profile class parameters:
  - **update** $\boldsymbol{\beta}_d$: Assuming the prior for $\boldsymbol{\beta}_d \overset{ind}{\sim} \text{MVN}(\boldsymbol{\nu}, \boldsymbol{V})$, then the full conditional posterior density for $\boldsymbol{\beta}_d$ for $d = 1, ..., K_D$ is $[\boldsymbol{\beta}_d|.] \sim \text{MVN}\left(\tilde{\boldsymbol{\nu}}_d, \tilde{\boldsymbol{V}}_d\right)$ where

$$\tilde{\boldsymbol{\nu}}_d = \left\{\boldsymbol{V}^{-1} + \mathbf{\Sigma}_d^{-1} \sum_{i=1}^{n} \text{I}(D_i = d)\right\}^{-1} \left\{\boldsymbol{V}^{-1}\boldsymbol{\nu} + \mathbf{\Sigma}_d^{-1} \sum_{i=1}^{n} \text{I}(D_i = d)\boldsymbol{b}_i\right\}$$
$$\tilde{\boldsymbol{V}}_d = \left\{\boldsymbol{V}^{-1} + \mathbf{\Sigma}_d^{-1} \sum_{i=1}^{n} \text{I}(D_i = d)\right\}^{-1}$$

  - **update** $\mathbf{\Sigma}_d$: Assuming the prior for $\mathbf{\Sigma}_d \overset{ind}{\sim} \text{Inverse-Wishart}(m, \mathbf{\Lambda})$, then the full conditional posterior density is, $[\mathbf{\Sigma}_d|\cdot] \sim \text{Inverse-Wishart}\left(\tilde{m}_d, \tilde{\mathbf{\Lambda}}_d\right)$ where

$$\tilde{m}_d = m + \sum_{i=1}^{n} \text{I}(D_i = d)$$
$$\tilde{\mathbf{\Lambda}}_d = \left\{\Lambda^{-1} + \sum_{i=1}^{n} \text{I}(D_i = d)\left(\boldsymbol{b}_i - \boldsymbol{\beta}_d\right)\left(\boldsymbol{b}_i - \boldsymbol{\beta}_d\right)'\right\}^{-1}$$

- **update** the mixing proportion $\{\pi_d^D\}_d$: assuming $\{\pi_d^D\}_d \sim \text{Dirichlet}(e_1^D, ..., e_{K_D}^D)$ then the full conditional posterior distribution is $\left[\{\pi_d^D\}_d|\cdot\right] \sim \text{Dirichlet}(\{e_d^D + \sum_{i=1}^n \text{I}(D_i = d)\}_d)$.

- **update** the variance class memberships C_i, $i = 1, ..., n$: the full conditional posterior distribution $[C_i|\cdot] \sim \text{Multinomial}(\tilde{\pi}_{i1}^C, ..., \tilde{\pi}_{iK_C}^C)$ where

$$\tilde{\pi}_{i1}^C = \Pr(C_i = c|.) = \frac{\pi_c^C \exp\left\{-\frac{1}{2\tau^2}\left(\log\sigma_i^2 - \mu_c\right)^2\right\}}{\sum_{c=1}^{K_C} \pi_c^C \exp\left\{-\frac{1}{2\tau^2}\left(\log\sigma_i^2 - \mu_c\right)^2\right\}}$$

- **update** the variance class parameters:
  - **update** μ_c: assuming the prior for $\mu_c \overset{ind}{\sim} \text{N}(a, b)$, then the full conditional posterior

distribution is, $[\mu_c|\cdot] \sim \mathrm{N}\left(\tilde{a}, \tilde{b}\right)$ where

$$\tilde{a} = \frac{\sum_{i=1}^{n} \mathrm{I}(C_i = c)\log\sigma_i^2/\tau^2 + a/b}{1/b + \sum_{i=1}^{n} \mathrm{I}(C_i = c)/\tau^2}$$

$$\tilde{b} = \left\{1/b + \sum_{i=1}^{n} \mathrm{I}(C_i = c)/\tau^2\right\}^{-1}$$

– **update** τ^2: assuming $\tau^2 \sim$ Inverse-Gamma(v, e), then the full conditional posterior distribution is

$$\left[\tau^2|\cdot\right] \sim \text{Inverse-Gamma}\left\{v + \frac{n}{2}, e + \sum_{i=1}^{n}\sum_{c=1}^{K_C} \frac{1}{2}\mathrm{I}(C_i = c)\left(\log\sigma_i^2 - \mu_c\right)^2\right\}.$$

- **update** the mixing proportions $\{\pi_c^C\}_c$: assuming $\{\pi_c^C\}_c \sim$ Dirichlet$(e_1^C, ..., e_{K_C}^C)$ then the full conditional posterior distribution is

$$\left[\{\pi_c^C\}_c|\cdot\right] \sim \text{Dirichlet}\left(\{e_c^C + \sum_{i=1}^{n} \mathrm{I}(C_i = c)\}_c\right).$$

- **update** the random effects $\boldsymbol{b}_i$, $i = 1, ..., n$ the full conditional posterior distribution is $\boldsymbol{b}_i$ $[\boldsymbol{b}_i|.] \sim \mathrm{MVN}\left(\tilde{\boldsymbol{\beta}}_i, \tilde{\boldsymbol{\Sigma}}_i\right)$, where

$$\tilde{\boldsymbol{\beta}}_i = \tilde{\boldsymbol{\Sigma}}_{id}\left\{\boldsymbol{\Sigma}_{D_i}^{-1}\boldsymbol{\beta}_{D_i} + \frac{1}{\sigma_i^2}\sum_{j=1}^{n_i} y_{ij}\boldsymbol{t}_{ij} + (\boldsymbol{Z}_i^T\boldsymbol{\eta} - \tilde{g}(\boldsymbol{\eta}, \sigma_i^2))\boldsymbol{g}(\boldsymbol{\eta}, \sigma_i^2)\right\}$$
$$\tilde{\boldsymbol{\Sigma}}_{id} = \left\{\boldsymbol{\Sigma}_{D_i}^{-1} + \frac{1}{\sigma_i^2}\sum_{j=1}^{n_i} \boldsymbol{t}_{ij}\boldsymbol{t}_{ij}^T + \boldsymbol{g}(\boldsymbol{\eta}, \sigma_i^2)\boldsymbol{g}(\boldsymbol{\eta}, \sigma_i^2)^T\right\}^{-1}$$

$\boldsymbol{t}_{ij}$ is a functional form of the time t_{ij} for the longitudinal measurement y_{ij} such that $y_{ij} \sim \mathrm{N}\left\{f(\boldsymbol{b}_i; t_{ij}), \sigma_i^2\right\}$ with $f(\boldsymbol{b}_i; t_{ij}) = \boldsymbol{b}_i^T\boldsymbol{t}_{ij}$. $\boldsymbol{g}(\boldsymbol{\eta}, \sigma_i^2)$ is a vector such that $\boldsymbol{Z}_i^T\boldsymbol{\eta} = \boldsymbol{g}(\boldsymbol{\eta}, \sigma_i^2)'\boldsymbol{b}_i + \tilde{g}(\boldsymbol{\eta}, \sigma_i^2)$ in the shared random effects and variances model.

- **update** the variances σ_i^2, $i = 1, ..., n$

$$\begin{aligned}\pi(\sigma_i^2|.) \propto & \prod_{c=1}^{K_C} \mathrm{LN}\left(\sigma_i^2; \mu_c, \tau^2\right)^{\mathrm{I}(C_i=c)} \prod_{j=1}^{n_i} \mathrm{N}\left\{y_{ij}; f(\boldsymbol{b}_i; t_{ij}), \sigma_i^2\right\} \mathrm{N}(W_i; \boldsymbol{Z}_i^T \boldsymbol{\eta}, 1) \\ & \propto (\sigma_i^2)^{-\frac{n_i}{2}-1} \exp\left\{-\sum_{c=1}^{K_C} \mathrm{I}(C_i = c) \frac{(\log\sigma_i^2 - \mu_c)^2}{2\tau^2}\right\} \\ & \times \exp\left[-\frac{1}{2\sigma_i^2} \sum_{j=1}^{n_i} \{y_{ij} - f(\boldsymbol{b}_i; t_{ij})\}^2 - \frac{1}{2}(W_i - \boldsymbol{Z}_i^T \boldsymbol{\eta})^2\right]\end{aligned}$$

  $\mathrm{LN}\,(\sigma_i^2; \mu_c, \tau^2)$ represents the density of log normal disribution with mean μ_c and variance τ^2 evaluated at σ_i^2; $\mathrm{N}\left\{y_{ij}; f(\boldsymbol{b}_i; t_{ij}), \sigma_i^2\right\}$ represents the density of normal distribution with mean $f(\boldsymbol{b}_i; t_{ij})$ and variance σ_i^2 evaluated at y_{ij} and similarly $\mathrm{N}(W_i; \boldsymbol{Z}_i^T \boldsymbol{\eta}, 1)$ represents the density of normal distribution with mean $\boldsymbol{Z}_i^T \boldsymbol{\eta}$ and variance 1 evaluated at W_i. Since there is no closed form of the full conditional posterior density, the draws for σ_i^2, $i = 1, ..., n$ at each iteration of the Gibbs sampling are obtained using the inverse cumulative distribution sampling method.

(2) update for MSRE probit model:

- **update** W_i, $i = 1, ..., m$

$$[W_i | o_i = 1, \cdot] \sim \mathrm{N}(\boldsymbol{Z}_i^T \boldsymbol{\eta}, 1) I_{(0,\infty)}(\cdot)$$

$$[W_i | o_i = 0, \cdot] \sim \mathrm{N}(\boldsymbol{Z}_i^T \boldsymbol{\eta}, 1) I_{(-\infty,0)}(\cdot)$$

  where, $\boldsymbol{Z}_i$ is the i^{th} row of the design matrix in the MSRE probit submodel.

- **update** $\boldsymbol{\eta}$: Assuming the prior for $\boldsymbol{\eta} \sim \mathrm{MVN}(\boldsymbol{\nu}_{\boldsymbol{\eta}}, \boldsymbol{V}_{\boldsymbol{\eta}})$, then the full conditional posterior density for $\boldsymbol{\eta}$ is $[\boldsymbol{\eta}|.] \sim \mathrm{MVN}\left(\tilde{\boldsymbol{\nu}}_{\boldsymbol{\eta}}, \tilde{\boldsymbol{V}}_{\boldsymbol{\eta}}\right)$ where

$$\begin{aligned}\tilde{\boldsymbol{\nu}}_{\boldsymbol{\eta}} &= \left(\boldsymbol{V}_{\boldsymbol{\eta}}^{-1} + \sum_{i=1}^{n} \boldsymbol{Z}_i \boldsymbol{Z}_i'\right)^{-1} \left(\boldsymbol{V}_{\boldsymbol{\eta}}^{-1} \boldsymbol{\nu}_{\boldsymbol{\eta}} + \sum_{i=1}^{n} W_i \boldsymbol{Z}_i\right) \\ \tilde{\boldsymbol{V}}_{\boldsymbol{\eta}} &= \left(\boldsymbol{V}_{\boldsymbol{\eta}}^{-1} + \sum_{i=1}^{n} \boldsymbol{Z}_i \boldsymbol{Z}_i'\right)^{-1}\end{aligned}$$

  $\boldsymbol{Z}_i$ is the i^{th} row of the design matrix in the MSRE probit submodel.

Web Appendix B

Computation of DIC

DIC is given by

$$\text{DIC}(\boldsymbol{x}) = -4\text{E}_{\boldsymbol{\phi}}\left\{\log f(\boldsymbol{x} \mid \boldsymbol{\phi}) \mid \boldsymbol{x}\right\} + 2\log f\left\{\boldsymbol{x} \mid \text{E}_{\boldsymbol{\phi}}(\boldsymbol{\phi} \mid \boldsymbol{x})\right\}$$

Celeux et al. (2006) suggests that, when the model has missing data or latent variables, the appropriate DIC measure is obtained by first considering the DIC measure in the "complete data" setting, where $\boldsymbol{x}$ indicates the fully observed data, and $\boldsymbol{z}$ the unobserved (typically latent) data:

$$\begin{aligned}\text{DIC}(\boldsymbol{x},\boldsymbol{z}) =& -4\text{E}_{\boldsymbol{\phi}}\left\{\log f(\boldsymbol{x},\boldsymbol{z} \mid \boldsymbol{\phi}) \mid \boldsymbol{x},\boldsymbol{z}\right\}\\ &+ 2\log f\left\{\boldsymbol{x},\boldsymbol{z} \mid \text{E}_{\boldsymbol{\phi}}(\boldsymbol{\phi} \mid \boldsymbol{x},\boldsymbol{z})\right\}\end{aligned}$$

Integrating out over the unobserved data yields

$$\begin{aligned}\text{DIC}(\boldsymbol{x}) =& \text{E}_{\boldsymbol{z}}\left[-4\text{E}_{\boldsymbol{\phi}}\left\{\log f(\boldsymbol{x},\boldsymbol{z} \mid \boldsymbol{\phi}) \mid \boldsymbol{x}\right\}\right.\\ &\left.+\ 2\log f\left\{\boldsymbol{x},\boldsymbol{z} \mid\ \text{E}_{\boldsymbol{\phi}}(\boldsymbol{\phi} \mid \boldsymbol{x},\boldsymbol{z})\right\}\right]\\ =& -4\text{E}_{\boldsymbol{z},\boldsymbol{\phi}}\left\{\log f(\boldsymbol{x},\boldsymbol{z} \mid \boldsymbol{\phi}) \mid \boldsymbol{x}\right\}\\ &+ 2\text{E}_{\boldsymbol{z}}\left[\log f\left\{\boldsymbol{x},\boldsymbol{z} \mid E_{\boldsymbol{\phi}}(\boldsymbol{\phi} \mid \boldsymbol{x},\boldsymbol{z})\right\} \mid \boldsymbol{x}\right]\end{aligned}$$

To obtain DIC for our MSRE model, let $\boldsymbol{\phi}$ denote the vector of all model parameters and $\boldsymbol{z}_i$ the latent variables $(D_i, C_i, \boldsymbol{b}_i, \sigma_i^2, W_i)'$ for the ith subject. The data $\boldsymbol{x}_i'$, $i = 1, ..., n$ correspond to the longitudinal data $(y_{i1}, ..., y_{in_i})'$ and the outcome o_i.

Dividing $\boldsymbol{z}_i$ into $\boldsymbol{z}_{i1} = (D_i, C_i)$ and $\boldsymbol{z}_{i2} = (\boldsymbol{b}_i, \sigma_i^2, W_i)$, then for the complete data log-likelihood (ignoring normalizing constants), we have

$$\begin{aligned}&\text{E}_{\boldsymbol{z},\boldsymbol{\phi}}\left\{\log f(\boldsymbol{x},\boldsymbol{z} \mid \boldsymbol{\phi}) \mid \boldsymbol{x}\right\}\\ =&\text{E}_{\boldsymbol{z}_2,\boldsymbol{\phi}}\left[\text{E}_{\boldsymbol{z}_1}\left\{\log f(\boldsymbol{x},\boldsymbol{z} \mid \boldsymbol{\phi}) \mid \boldsymbol{x},\boldsymbol{\phi},\boldsymbol{z}_2\right\}\right]\end{aligned}$$

where

$$
\begin{aligned}
&\mathrm{E}_{\boldsymbol{z}_1}\left\{\log f(\boldsymbol{x},\boldsymbol{z}\mid\boldsymbol{\phi})\mid\boldsymbol{x},\boldsymbol{\phi},\boldsymbol{z}_2\right\}\\
&=\sum_{i=1}^{n}\left[\sum_{d}\tilde{\pi}_d\left\{\log\pi_d-\frac{1}{2}\log|\boldsymbol{\Sigma_d}|-\frac{1}{2}(\boldsymbol{b}_i-\boldsymbol{\beta}_d)'\Sigma_d^{-1}(\boldsymbol{b}_i-\boldsymbol{\beta}_d)\right\}\right.\\
&+\sum_{c}\tilde{\pi}_c\left\{\log\pi_c-\log\tau-\log\sigma_i^2-\frac{1}{2\tau^2}(\log\sigma_i^2-\mu_c)^2\right\}\\
&+\sum_{j=1}^{n_i}\left\{\log\sigma_i-\frac{(y_{ij}-f(\boldsymbol{b}_i;t_{ij}))^2}{2\sigma_i^2}\right\}\\
&\left.+\,o_i\log\Phi(\boldsymbol{Z}_i'\boldsymbol{\eta})+(1-o_i)\log\left\{1-\Phi(\boldsymbol{Z}_i'\boldsymbol{\eta})\right\}\right],\\
&\tilde{\pi}_d=\mathrm{P}(D_i=d\mid\boldsymbol{x},\boldsymbol{\phi},\boldsymbol{z}_2)=\frac{\pi_d^D|\boldsymbol{\Sigma_d}|^{-\frac{1}{2}}\exp\left[-\frac{1}{2}(\boldsymbol{b}_i-\boldsymbol{\beta}_d)'\Sigma_d^{-1}(\boldsymbol{b}_i-\boldsymbol{\beta}_d)\right]}{\sum_{d=1}^{K_D}\pi_d^D|\boldsymbol{\Sigma_d}|^{-\frac{1}{2}}\exp\left[-\frac{1}{2}(\boldsymbol{b}_i-\boldsymbol{\beta}_d)'\Sigma_d^{-1}(\boldsymbol{b}_i-\boldsymbol{\beta}_d)\right]},\\
&\tilde{\pi}_c=\mathrm{P}(C_i=c\mid\boldsymbol{x},\boldsymbol{\phi},\boldsymbol{z}_2)=\frac{\pi_c^C\exp\left[-\frac{1}{2}\left(\log\sigma_i^2-\mu_c\right)^2/\tau^2\right]}{\sum_{c=1}^{K_C}\pi_c^C\exp\left[-\frac{1}{2}\left(\log\sigma_i^2-\mu_c\right)^2/\tau^2\right]}
\end{aligned}
$$

The expectation $\mathrm{E}_{\boldsymbol{z},\boldsymbol{\phi}}\left\{\log f(\boldsymbol{x},\boldsymbol{z}\mid\boldsymbol{\phi})\mid\boldsymbol{x}\right\}$ can then be approximated from M MCMC draws:

$$
\begin{aligned}
&\mathrm{E}_{\boldsymbol{z},\boldsymbol{\phi}}\left[\left\{\log f(\boldsymbol{x}\mid\boldsymbol{\phi})\mid\boldsymbol{x},\boldsymbol{z}\right\}\mid\boldsymbol{x}\right]\\
&\approx M^{-1}\sum_{m=1}^{M}\left[\sum_{i=1}^{n}\left[\sum_{d}\tilde{\pi}_d^{(m)}\left\{\log\pi_d^{(m)}-\frac{1}{2}\log|\boldsymbol{\Sigma}_d^{(m)}|\right.\right.\right.\\
&\left.-\frac{1}{2}(\boldsymbol{b}_i^{(m)}-\boldsymbol{\beta}_d^{(m)})'(\Sigma_d^{(m)})^{-1}(b_i^{(m)}-\boldsymbol{\beta}_d^{(m)})\ \right\}\\
&+\sum_{c}\tilde{\pi}_c^{(m)}\left\{\log\pi_c^{(m)}-\log\tau^{(m)}-\log(\sigma_i^2)^{(m)}\right.\\
&\left.-\frac{1}{2(\tau^2)^{(m)}}(\log(\sigma_i^2)^{(m)}-\mu_c^{(m)})^2\right\}\\
&+\sum_{j=1}^{n_i}\left\{\log\sigma_i^{(m)}-\frac{(y_{ij}-f(\boldsymbol{b}_i^{(m)};t_{ij}))^2}{2(\sigma_i^2)^{(m)}}\right\}\\
&\left.\left.+\,o_i\log\Phi(\boldsymbol{Z}_i'\boldsymbol{\eta}^{(m)})+(1-o_i)\log\left\{1-\Phi(\boldsymbol{Z}_i'\boldsymbol{\eta}^{(m)})\right\}\right]\right]
\end{aligned}
$$

Obtaining $E_{\boldsymbol{z}}\left[\log f\left\{\boldsymbol{x},\boldsymbol{z}\mid\ E_{\boldsymbol{\phi}}(\boldsymbol{\phi}\mid\boldsymbol{x},\boldsymbol{z})\right\}\mid\boldsymbol{x}\right]$ requires a bit more effort. We can broadly use the same approach of averaging over the MCMC draws to integrate with respect to $\boldsymbol{z}$, but instead of using the draws of the model parameters directly, we need to obtain their expectation conditional on the current draw of $\boldsymbol{z}$. So

$$
\begin{aligned}
&\mathrm{E}_{\boldsymbol{z}}\left[\log f\left\{\boldsymbol{x},\boldsymbol{z}\mid \mathrm{E}_{\boldsymbol{\phi}}(\boldsymbol{\phi}\mid\boldsymbol{x},\boldsymbol{z})\right\}\mid\boldsymbol{x}\right]\\
\approx &M^{-1}\sum_{m=1}^{M}\left[\sum_{i=1}^{n}\left[\sum_{d} I(D_i^{(m)}=d)\left\{\log\hat{\pi}_d^{(m)}-\frac{1}{2}\log|\hat{\boldsymbol{\Sigma}}_d^{(m)}|\right.\right.\right.\\
&\left.-\frac{1}{2}(\boldsymbol{b}_i^{(m)}-\hat{\boldsymbol{\beta}}_d^{(m)})'(\hat{\boldsymbol{\Sigma}}_d^{(m)})^{-1}(b_i^{(m)}-\hat{\boldsymbol{\beta}}_d^{(m)})\right\}\\
&+\sum_{c} I(C_i^{(m)}=c)\left\{\log\hat{\pi}_c^{(m)}-\log\hat{\tau}^{(m)}-\log(\sigma_i^2)^{(m)}-\frac{1}{2(\hat{\tau}^2)^{(m)}}(\log(\sigma_i^2)^{(m)}-\hat{\mu}_c^{(m)})^2\right\}\\
&+\sum_{j=1}^{n_i}\left\{\log\sigma_i^{(m)}-\frac{(y_{ij}-f(\boldsymbol{b}_i^{(m)};t_{ij}))^2}{2(\hat{\sigma}_i^2)^{(m)}}\right\}\\
&\left.\left.+o_i\log\Phi(\boldsymbol{Z}_i'\boldsymbol{\eta}^{(m)})+(1-o_i)\log\left\{1-\Phi(\boldsymbol{Z}_i'\boldsymbol{\eta}^{(m)})\right\}\right]\right]
\end{aligned}
$$

where $\hat{\boldsymbol{\phi}}^{(m)}=E_{\boldsymbol{\phi}}(\boldsymbol{\phi}\mid\boldsymbol{x},\boldsymbol{z}^{(m)})$.

Some components of $\hat{\boldsymbol{\phi}}^{(m)}$ have closed form solutions:

$$
\begin{aligned}
\hat{\pi}_d^{(m)} &= \frac{e_d^D+\sum_{i=1}^n \mathrm{I}(D_i^{(m)}=d)}{\sum_d e_d^D+n}\\
\hat{\pi}_c^{(m)} &= \frac{e_c^C+\sum_{i=1}^n \mathrm{I}(C_i^{(m)}=c)}{\sum_c e_c^C+n}\\
\hat{\boldsymbol{\eta}}^{(m)} &= \left(V_{\boldsymbol{\eta}}^{-1}+\sum_{i=1}^{n}\boldsymbol{Z}_i^{(m)}\boldsymbol{Z}_i^{(m)'}\right)^{-1}\left(V_{\boldsymbol{\eta}}^{-1}\nu_{\boldsymbol{\eta}}+\sum_{i=1}^{n}W_i^{(m)}\boldsymbol{Z}_i^{(m)}\right)
\end{aligned}
$$

where $\boldsymbol{Z}_i^{(m)}$ is the i^{th} row of the design matrix in the probit submodel for the mth MCMC draw and e_d^D, e_c^C, $V_{\boldsymbol{\eta}}$, and $\boldsymbol{\nu}_{\boldsymbol{\eta}}$ are specified hyperprior values.

The other components of $\hat{\boldsymbol{\phi}}^{(m)}$ will have to be obtaining by running small MCMC chains *for each of the main MCMC interations* to get the marginal expectations: $\hat{\boldsymbol{\beta}}_d^{(m)}=(M^*)^{-1}\sum_{m*}\boldsymbol{\beta}_d^{(m,m^*)}$ and $\hat{\boldsymbol{\Sigma}}_d^{(m)}=(M^*)^{-1}\sum_{m*}\Sigma_d^{(m,m^*)}$, where $\boldsymbol{\beta}_d^{(m,m^*)}$ and $\boldsymbol{\Sigma}_d^{(m,m^*)}$ are obtained by alternating draws from the following distributions with known hyperparameters V, ν, m, and $\boldsymbol{\Lambda}$:

$$\boldsymbol{\beta}_d^{(m,m^*)}\sim\mathrm{MVN}\left(\tilde{\boldsymbol{\nu}}_d^{(m,m^*)},\tilde{\boldsymbol{V}}_d^{(m,m^*)}\right),\ \text{where}$$

$$\tilde{\boldsymbol{V}}_d^{(m,m^*)} = \left\{ \boldsymbol{V}^{-1} + (\boldsymbol{\Sigma}_d^{(m,m^*)})^{-1} \sum_{i=1}^{n} \mathrm{I}(D_i = d)^{(m)} \right\}^{-1}$$
$$\tilde{\boldsymbol{\nu}}_d^{(m,m^*)} = \tilde{\boldsymbol{V}}_d^{(m,m^*)} \left\{ \boldsymbol{V}^{-1}\boldsymbol{\nu} + (\boldsymbol{\Sigma}_d^{(m,m^*)})^{-1} \sum_{i=1}^{n} \mathrm{I}(D_i = d)^{(m)} \boldsymbol{b}_i^{(m)} \right\}.$$

$\boldsymbol{\Sigma}_d^{(m,m^*)} \sim$ Inverse-Wishart $\left(\tilde{m}_d^{(m)}, \tilde{\boldsymbol{\Lambda}}_d^{(m,m^*)}\right)$, where

$$\tilde{m}_d^{(m)} = m + \sum_{i=1}^{n} \mathrm{I}(D_i^{(m)} = d),$$
$$\tilde{\boldsymbol{\Lambda}}_d^{(m,m^*)} = \left\{ \Lambda^{-1} + \sum_{i=1}^{n} \mathrm{I}(D_i^{(m)} = d) \left(\boldsymbol{b}_i^{(m)} - \boldsymbol{\beta}_d^{(m,m^*)}\right) \left(\boldsymbol{b}_i^{(m)} - \boldsymbol{\beta}_d^{(m,m^*)}\right)' \right\}^{-1}.$$

Similarly, $\hat{\mu}_c^{(m)} = (M^*)^{-1} \sum_{m*} \mu_c^{(m,m^*)}$ and $(\hat{\tau}^2)^{(m)} = (M^*)^{-1} \sum_{m*} (\tau^2)^{(m,m^*)}$, where $\mu_c^{(m,m^*)}$ and $(\tau^2)^{(m,m^*)}$ are obtained by alternating draws from the following distributions with known hyperparameters a, b, e, and f:

$\mu_c^{(m,m^*)} \sim \mathrm{N}(\tilde{a}^{(m,m^*)}, \tilde{b}^{(m,m^*)})$, where

$$\tilde{a}^{(m,m^*)} = \frac{\sum_{i=1}^{n} \mathrm{I}(C_i = c)^{(m)} \log(\sigma_i^2)^{(m)} / (\tau^2)^{(m,m^*)} + a/b}{1/b + \sum_{i=1}^{n} \mathrm{I}(C_i = c)^{(m)} / (\tau^2)^{(m,m^*)}}$$
$$\tilde{b}^{(m,m^*)} = \left\{ 1/b + \sum_{i=1}^{n} \mathrm{I}(C_i = c)^{(m)} / (\tau^2)^{(m,m^*)} \right\}^{-1}$$

$(\tau^2)^{(m,m^*)} \sim \mathrm{IG}(\tilde{v}, \tilde{e}^{(m,m^*)})$, where

$$\tilde{v} = v + \frac{n}{2}$$
$$\tilde{e}^{(m,m^*)} = e + \sum_{i=1}^{n} \sum_{c=1}^{K_C} \frac{1}{2} \mathrm{I}(C_i = c)^{(m)} \left\{ \log(\sigma_i^2)^{(m)} - \mu_c^{(m,m^*)} \right\}^2$$

Because we are conditioning on $\boldsymbol{z}$ and only need the posterior expectation, a modest number of drawn (here we use $M^* = 250$) is found to be sufficient to obtain an accurate approximation.

Similarly, we can obtain DIC for our LC model.

Computation of LPML

For our model, we have

$$
\begin{aligned}
\text{CPO}_i^{-1} &= \frac{f(\boldsymbol{y}_{(-i)}, \boldsymbol{o}_{(-i)}|\boldsymbol{v})}{f(\boldsymbol{y}, \boldsymbol{o}|\boldsymbol{v})} \\
&= \int \frac{f(\boldsymbol{y}_{(-i)}, \boldsymbol{o}_{(-i)}|\boldsymbol{C}, \boldsymbol{D}, \boldsymbol{b}, \boldsymbol{\sigma}^2, \boldsymbol{\phi}, \boldsymbol{v}) f(\boldsymbol{C}, \boldsymbol{D}, \boldsymbol{b}, \boldsymbol{\sigma}^2, \boldsymbol{\phi})}{f(\boldsymbol{y}, \boldsymbol{o}|\boldsymbol{v})} d\boldsymbol{b} d\boldsymbol{\sigma}^2 d\boldsymbol{C} d\boldsymbol{D} d\boldsymbol{\phi} \\
&= \int \frac{f(\boldsymbol{y}, \boldsymbol{o}|\boldsymbol{C}, \boldsymbol{D}, \boldsymbol{b}, \boldsymbol{\sigma}^2, \boldsymbol{\phi}, \boldsymbol{v}) f(\boldsymbol{C}, \boldsymbol{D}, \boldsymbol{b}, \boldsymbol{\sigma}^2, \boldsymbol{\phi})}{f(\boldsymbol{y}, \boldsymbol{o}|\boldsymbol{v}_i) f(\boldsymbol{y}_i, o_i|\boldsymbol{C}, \boldsymbol{D}, \boldsymbol{b}, \boldsymbol{\sigma}^2, \boldsymbol{\phi}, \boldsymbol{v})} d\boldsymbol{b} d\boldsymbol{\sigma}^2 d\boldsymbol{C} d\boldsymbol{D} d\boldsymbol{\phi} \qquad (1) \\
&= \int \frac{1}{f(\boldsymbol{y}_i, o_i|\boldsymbol{C}, \boldsymbol{D}, \boldsymbol{b}, \boldsymbol{\sigma}^2, \boldsymbol{\phi}, \boldsymbol{v}_i)} f(\boldsymbol{C}, \boldsymbol{D}, \boldsymbol{b}, \boldsymbol{\sigma}^2, \boldsymbol{\phi}|\boldsymbol{y}, \boldsymbol{o}, \boldsymbol{v}) d\boldsymbol{b} d\boldsymbol{\sigma}^2 d\boldsymbol{C} d\boldsymbol{D} d\boldsymbol{\phi} \\
&= \int \frac{f(\boldsymbol{C}, \boldsymbol{D}, \boldsymbol{b}, \boldsymbol{\sigma}^2, \boldsymbol{\phi}|\boldsymbol{y}, \boldsymbol{o}, \boldsymbol{v})}{f(\boldsymbol{y}_i|\boldsymbol{b}_i, \sigma_i^2, \boldsymbol{\phi}, \boldsymbol{v}_i) f(o_i|C_i, D_i, \boldsymbol{b}_i, \sigma_i^2, \boldsymbol{\phi}, \boldsymbol{v}_i)} d\boldsymbol{b} d\boldsymbol{\sigma}^2 d\boldsymbol{C} d\boldsymbol{D} d\boldsymbol{\phi}
\end{aligned}
$$

where $\boldsymbol{\phi}$ is the vector of model parameters which does not include the unobserved random effects and unknown residual variances. $\boldsymbol{v} = (\boldsymbol{v}_1, ..., \boldsymbol{v}_n)^T$ include all observed variables including obervation time t_{ij}, $i = 1, ..., n_i$, $j = 1, ..., n_i$ and baseline covariates of interest. $f(o_i|C_i, D_i, \boldsymbol{b}_i, \sigma_i^2, \boldsymbol{\phi}, \boldsymbol{v}_i)$ can be reduced to $f(o_i|C_i, D_i, \boldsymbol{\phi}, \boldsymbol{v}_i)$ in the case of LC probit submodel and $f(o_i|\boldsymbol{b}_i, \sigma_i^2, \boldsymbol{\phi}, \boldsymbol{v}_i)$ in the case of MSRE probit submodel. Using the MCMC posterior draws, we estimate CPO_i^{-1} by

$$
\frac{1}{B} \sum_{s=1}^{B} \frac{1}{f(\boldsymbol{y}_i|\boldsymbol{b}_i, \sigma_i^2, \boldsymbol{\phi}^{(s)}, \boldsymbol{v}_i) f(o_i|C_i, D_i, \boldsymbol{b}_i, \sigma_i^2, \boldsymbol{\phi}^{(s)}, \boldsymbol{v}_i)}
$$

where B is the number of MCMC posterior draws and $\boldsymbol{\phi}^{(s)}$ is the vector of the posterior draws of all model parameters at the s^{th} iteration. We have,

$$
f(\boldsymbol{y}_i|\boldsymbol{b}_i, \sigma_i^2, \boldsymbol{\phi}, \boldsymbol{v}_i) = \prod_{j=1}^{n_i} \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left[-\frac{\{y_{ij} - f(\boldsymbol{b}_i; t_{ij})\}^2}{2\sigma_i^2}\right]
$$

$$
f(o_i|C_i, D_i, \boldsymbol{\phi}, \boldsymbol{v}_i) = \Phi(\boldsymbol{Z}_i^T\boldsymbol{\theta})^{o_i} \left[1 - \Phi(\boldsymbol{Z}_i^T\boldsymbol{\theta})\right]^{1-o_i} \text{ for LC probit submodel}
$$

$$
f(o_i|\boldsymbol{b}_i, \sigma_i^2, \boldsymbol{\phi}, \boldsymbol{v}_i) = \Phi(\boldsymbol{Z}_i^T\boldsymbol{\gamma})^{o_i} \left[1 - \Phi(\boldsymbol{Z}_i^T\boldsymbol{\gamma})\right]^{1-o_i} \text{ for MSRE probit submodel}
$$

where, $\Phi(\cdot)$ is the cdf for standard normal distribution. $\boldsymbol{Z}_i$ denotes the corresponding i^{th} row in the design matrix for either LC or MSRE probit submodel given $\boldsymbol{v}_i$.

To obtain stable LPML measures, our calculations were based on 20 independent posterior simulation runs by retaining every 5^{th} of the 10000 posterior draws after the chains converge.

Web Appendix C

Details of the simulation study results

[Figure A. 1 about here.]

[Table A. 1 about here.]

[Table A. 2 about here.]

[Table A. 3 about here.]

[Table A. 4 about here.]

[Table A. 5 about here.]

[Table A. 6 about here.]

[Table A. 7 about here.]

[Table A. 8 about here.]

[Table A. 9 about here.]

[Table A. 10 about here.]

[Table A. 11 about here.]

Web Appendix D

Plots for Model Checking and Model Fit for the Analysis of Penn Ovarian Aging Data

[Figure A. 2 about here.]

[Figure A. 3 about here.]

[Figure A. 4 about here.]

[Figure A. 5 about here.]

[Figure A. 6 about here.]

[Figure A. 7 about here.]

Model Comparison Statistics for the Analysis of Penn Ovarian Aging Data

[Table A. 12 about here.]

Posterior Estimates of the Model Parameters under the Joint MSRE and LC Models Assuming Interactions for the Analysis of Penn Ovarian Aging Data

[Table A. 13 about here.]

[Table A. 14 about here.]

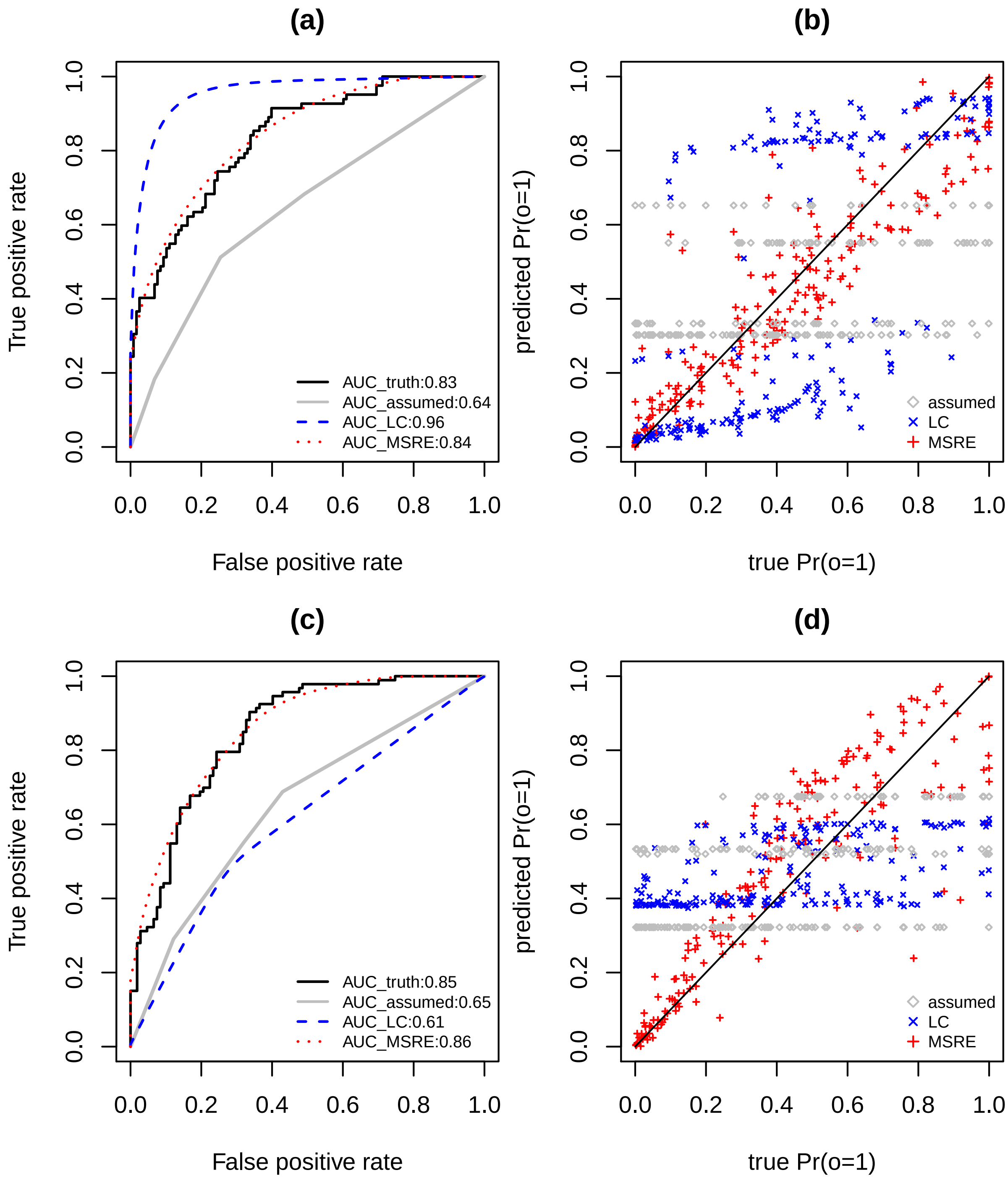


Figure A.1. Two typical ROC's under scenario #1 when the truth is joint MSRE model : (a) and (b) are from the data set where "outcome-informed mean clusters" are created by joint LC model; (c) and (d) are from the data set when an almost empty mean cluster is created by joint LC model. Note: "assumed" refers to the ML estimates of LC probit submodel given known class memberships.

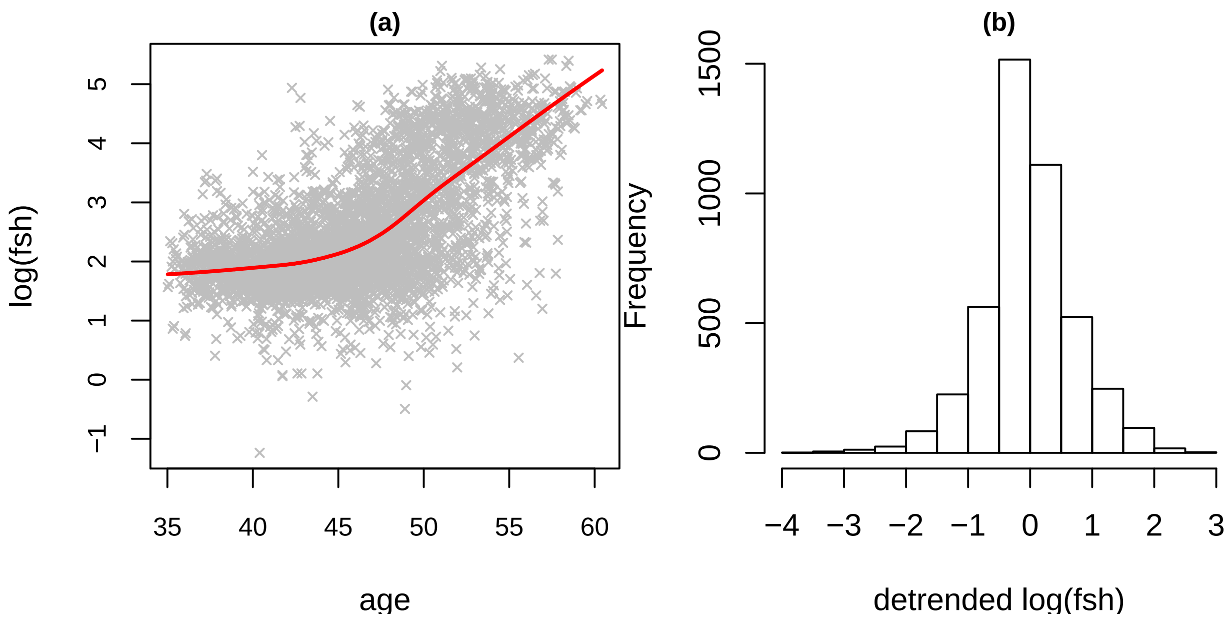


Figure A.2. (a) Estimated population longitudinal trend by Lowess method; (b) histogram of detrended log(FSH) in the analysis of Penn Ovarian Aging data.

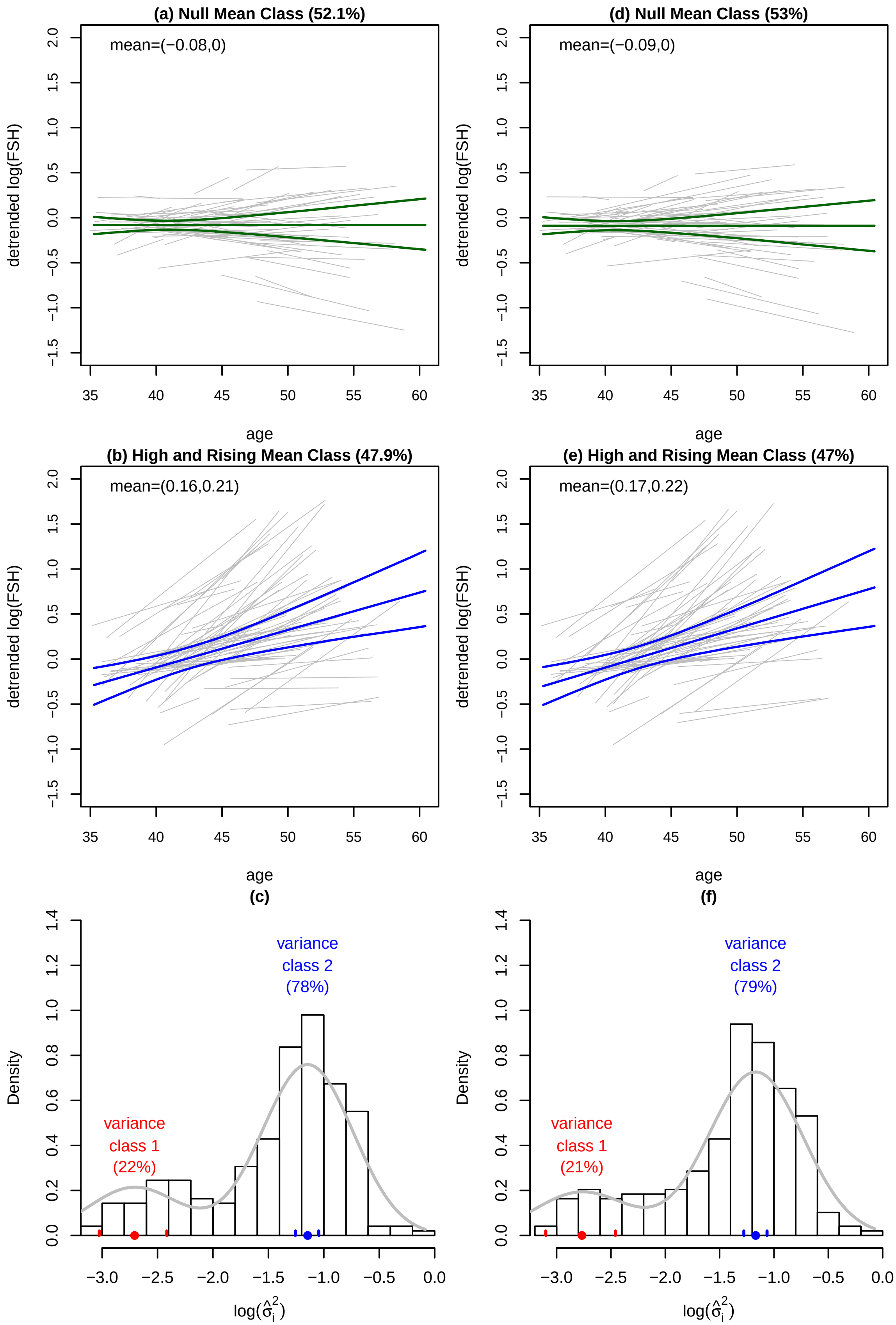
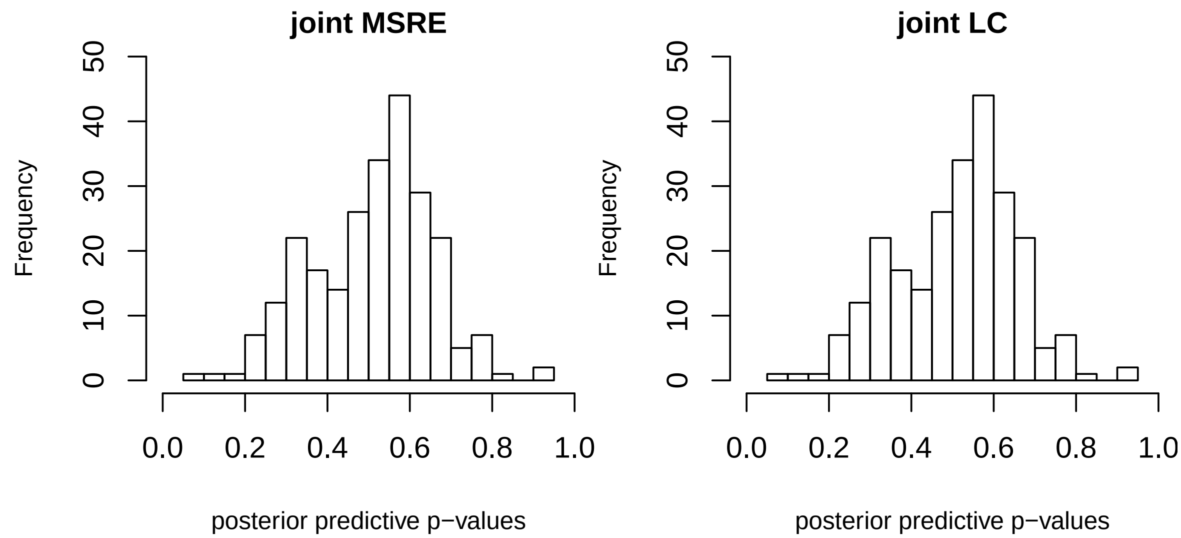
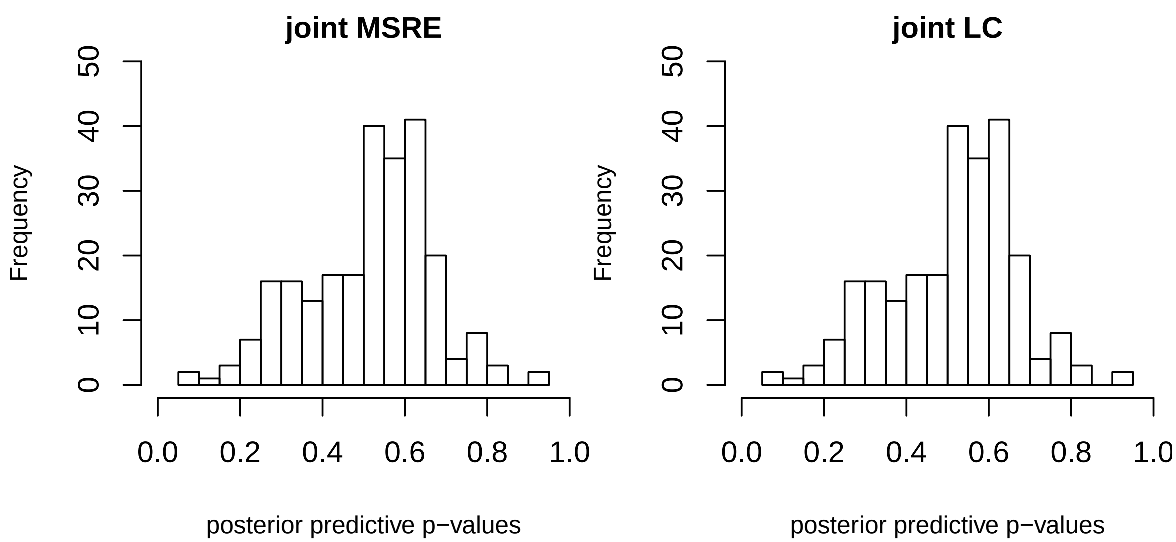


Figure A.3. Posterior pointwise 95% credible intervals for the mean profile classes and the histograms of log-variances in the analysis of Penn Ovarian Aging data with $K_D = K_C = 2$: (a), (b) and (c): under joint MSRE model; and (c), (d) and (e): under joint LC model.

Figure A.4. Posterior predictive p values under joint LC and MSRE models for the analysis of Penn Ovarian Aging data.



(a) $K_D = 1, K_C = 2$



(b) $K_D = 2, K_C = 2$

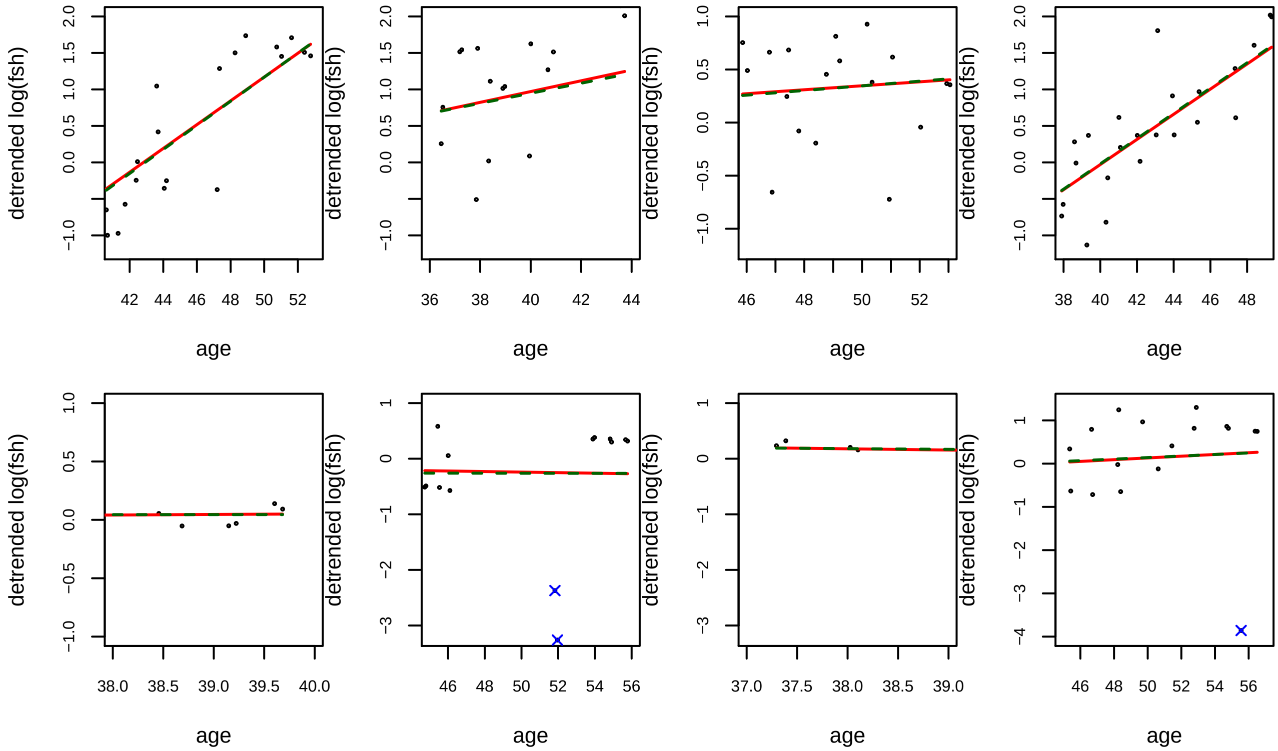


Figure A.5. The individual fits in the analysis of Penn Ovarian Aging data with $K_D = 1, K_C = 2$. Solid line: under joint MSRE model and dashed line: under joint LC model; top row: 4 randomly selected individual fits with PPD p values between 0.1 or great than 0.9 and bottom row: individuals with PPD p values less than 0.1 or great than 0.9.

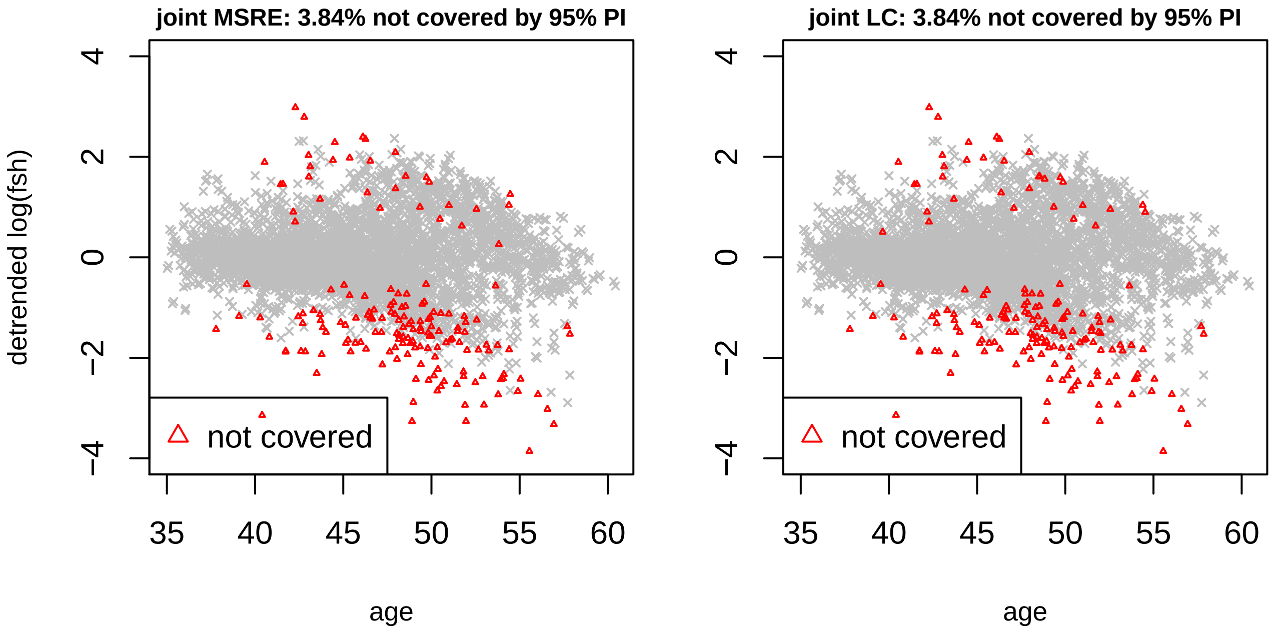


Figure A.6. Scatter plot of detrended log(FSH) versus age with red points not being covered by subject-specific 95% posterior predictive intervals assuming $K_D = 1, K_C = 2$ in the models for the analysis of Penn Ovarian Aging data: left: joint MSRE models and right: joint LC models.

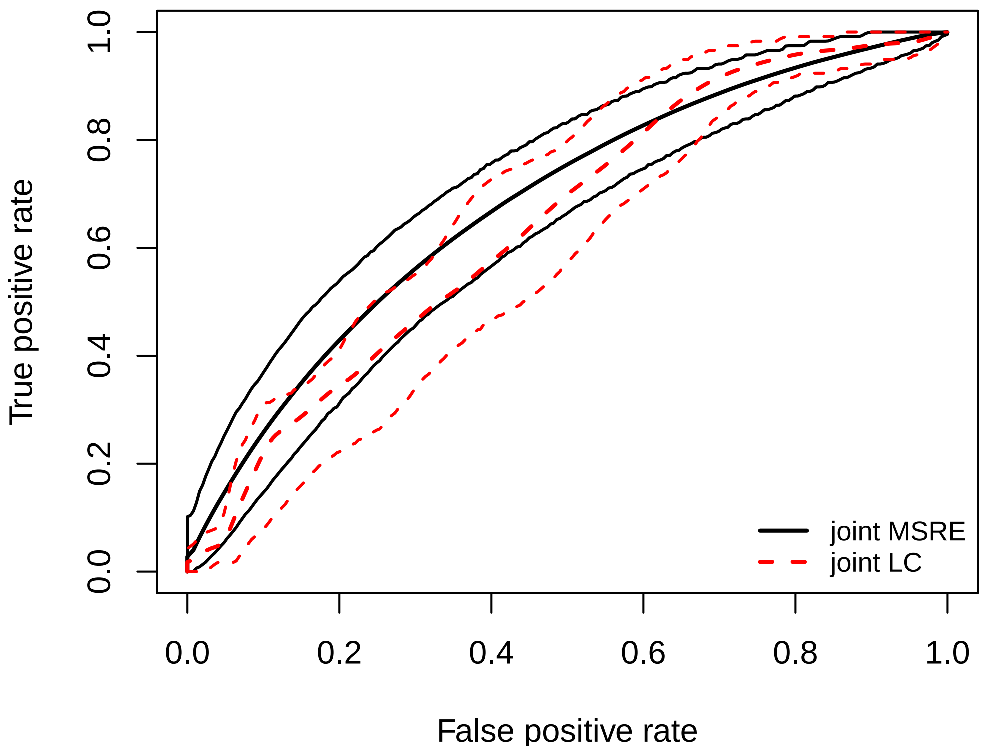


Figure A.7. Posterior average of the receiver operating characteristic (ROC) curves under joint MSRE model (average AUC=0.682 with 95% CI (0.629, 0.730)) and joint LC model (average AUC=0.645 with 95% CI (0.587, 0.698)) assuming $K_D = 1, K_C = 2$ in the analysis of Penn Ovarian Aging data.

Table A.1

Simulation results from 100 datasets of size, $n = 200$, generated from longitudinal scenario # 1 and the primary probit (a) LC, (b) MSRE models. Left columns: fitted assuming the LC model; right column: fitted assuming the MSRE model.

(a) TRUE: joint LC model

		Assumed LC structure					**Assumed MSRE structure**				
	TRUE	MEAN	BIAS	SD	RMSE	95% COV	MEAN	BIAS	SD	RMSE	95% COV
β_{11}	0.00	−0.19	−0.19	0.34	0.39	0.91	−0.53	−0.53	0.95	1.08	0.87
β_{12}	0.00	−0.04	−0.04	0.22	0.23	0.96	0.06	0.06	0.36	0.37	0.92
β_{21}	2.83	2.39	−0.44	0.23	0.49	0.40	1.79	−1.04	0.18	1.06	0.00
β_{22}	2.83	2.31	−0.52	0.21	0.56	0.18	1.73	−1.10	0.15	1.11	0.00
ω^2_{11}	2.00	1.83	−0.17	0.62	0.64	0.90	0.60	−1.40	0.63	1.54	0.57
ω^2_{12}	2.00	1.90	−0.10	0.67	0.68	0.89	0.44	−1.56	1.03	1.87	0.19
ω^2_{21}	2.00	2.56	0.56	0.48	0.74	0.74	3.66	1.66	0.38	1.71	0.03
ω^2_{22}	2.00	2.65	0.65	0.52	0.83	0.65	3.75	1.75	0.40	1.79	0.04
ρ_1	0.00	−0.12	−0.12	0.22	0.26	0.94	−0.72	−0.72	0.20	0.75	0.80
ρ_2	0.60	0.66	0.06	0.06	0.09	0.77	0.69	0.09	0.04	0.10	0.44
π_d	0.35	0.27	−0.08	0.07	0.11	0.78	0.05	−0.30	0.05	0.30	0.03
μ_1	−2.00	−1.95	0.05	0.11	0.12	0.98	−1.98	0.02	0.09	0.09	0.99
μ_2	−0.50	−0.61	−0.11	0.22	0.25	0.87	−0.58	−0.08	0.19	0.20	0.90
τ^2	0.25	0.33	0.08	0.12	0.14	0.90	0.30	0.05	0.10	0.11	0.95
π_c	0.65	0.63	−0.02	0.05	0.06	0.97	0.63	−0.02	0.04	0.05	0.98
θ_0	−0.80	−1.45	−0.65	0.50	0.82	0.89					
θ_1	1.80	2.41	0.61	0.66	0.90	0.88					
θ_2	−0.20	0.00	0.20	0.69	0.72	0.98					
θ_3	−0.30	−0.58	−0.28	0.82	0.87	0.97					
γ_0	−0.32						−0.32	0.00	0.21	0.21	0.95
γ_1	0.19						0.20	0.01	0.11	0.11	0.96
γ_2	0.18						0.17	−0.01	0.11	0.11	0.96
γ_3	−0.22						−0.38	−0.15	0.60	0.62	0.92
γ_4	−0.04						−0.02	0.01	0.32	0.32	0.93
γ_5	−0.04						0.02	0.06	0.30	0.30	0.94

(b) TRUE: joint MSRE model

		Assumed LC structure					**Assumed MSRE structure**				
	TRUE	MEAN	BIAS	SD	RMSE	95% COV	MEAN	BIAS	SD	RMSE	95% COV
β_{11}	0.00	0.85	0.85	0.91	1.25	0.33	−0.53	−0.53	1.03	1.16	0.85
β_{12}	0.00	1.27	1.27	0.95	1.59	0.36	0.13	0.13	0.45	0.47	0.87
β_{21}	2.83	2.02	−0.81	0.29	0.86	0.14	1.81	−1.02	0.25	1.05	0.02
β_{22}	2.83	1.05	−1.78	0.57	1.87	0.00	1.69	−1.14	0.33	1.18	0.00
ω^2_{11}	2.00	2.78	0.78	1.69	1.87	0.24	0.70	−1.30	0.87	1.56	0.57
ω^2_{12}	2.00	2.28	0.28	1.43	1.46	0.16	0.49	−1.51	1.08	1.86	0.18
ω^2_{21}	2.00	3.26	1.26	0.58	1.39	0.38	3.60	1.60	0.54	1.69	0.06
ω^2_{22}	2.00	4.07	2.07	0.63	2.16	0.06	3.70	1.70	0.55	1.79	0.06
ρ_1	0.00	0.25	0.25	0.71	0.75	0.32	−0.71	−0.71	0.27	0.76	0.74
ρ_2	0.60	0.77	0.17	0.08	0.19	0.29	0.68	0.08	0.13	0.15	0.46
π_d	0.35	0.46	0.11	0.31	0.33	0.03	0.07	−0.28	0.13	0.31	0.04
μ_1	−2.00	−1.93	0.07	0.08	0.10	0.86	−1.97	0.03	0.08	0.09	1.00
μ_2	−0.50	−0.47	0.03	0.15	0.16	0.90	−0.56	−0.06	0.18	0.19	0.91
τ^2	0.25	0.30	0.05	0.08	0.10	0.93	0.30	0.05	0.09	0.10	0.93
π_c	0.65	0.67	0.02	0.04	0.05	0.92	0.63	−0.02	0.04	0.05	0.98
θ_0	−0.40	−1.40	−1.00	1.13	1.51	0.35					
θ_1	−0.11	2.29	2.40	2.34	3.35	0.35					
θ_2	0.53	2.17	1.64	1.76	2.41	0.36					
θ_3	0.16	−3.53	−3.69	3.73	5.24	0.35					
γ_0	−1.00						−0.81	0.19	0.24	0.30	0.92
γ_1	1.00						0.91	−0.09	0.16	0.18	0.95
γ_2	−1.00						−0.96	0.04	0.17	0.18	0.96
γ_3	2.00						1.48	−0.52	0.58	0.78	0.87
γ_4	−2.00						−1.71	0.29	0.36	0.46	0.90
γ_5	2.00						1.86	−0.14	0.38	0.41	0.95

Table A.2

Simulation results from 100 datasets of size, $n = 200$, generated from longitudinal scenario # 2 and the primary probit (a) LC, (b) MSRE models. Left columns: fitted assuming the LC model; right column: fitted assuming the MSRE model.

(a) TRUE: joint LC model

		Assumed LC structure					**Assumed MSRE structure**				
	TRUE	MEAN	BIAS	SD	RMSE	95% COV	MEAN	BIAS	SD	RMSE	95% COV
β_{11}	0.00	0.01	0.01	0.13	0.13	0.93	0.01	0.01	0.13	0.13	0.94
β_{12}	0.00	−0.02	−0.02	0.11	0.11	0.98	−0.02	−0.02	0.11	0.11	0.99
β_{21}	2.83	2.91	0.08	0.09	0.12	0.89	2.91	0.08	0.09	0.12	0.89
β_{22}	2.83	2.69	−0.14	0.09	0.17	0.64	2.69	−0.14	0.09	0.17	0.65
ω_{11}^2	1.00	0.99	−0.01	0.16	0.16	0.98	0.99	−0.01	0.16	0.16	0.97
ω_{12}^2	1.00	1.03	0.03	0.19	0.19	0.93	1.03	0.03	0.20	0.20	0.93
ω_{21}^2	1.00	0.99	−0.01	0.14	0.14	0.92	0.99	−0.01	0.15	0.15	0.89
ω_{22}^2	1.00	1.01	0.01	0.12	0.12	0.97	1.01	0.01	0.12	0.12	0.96
ρ_1	0.00	0.00	0.00	0.12	0.12	0.98	0.00	0.00	0.12	0.12	0.98
ρ_2	−0.60	−0.59	0.01	0.07	0.07	0.93	−0.59	0.01	0.07	0.07	0.93
π_d	0.35	0.35	0.00	0.03	0.03	0.94	0.35	0.00	0.03	0.03	0.93
μ_1	−2.00	−1.95	0.05	0.11	0.12	0.97	−1.99	0.01	0.08	0.08	0.98
μ_2	−0.50	−0.62	−0.12	0.21	0.24	0.93	−0.56	−0.06	0.15	0.17	0.93
τ^2	0.25	0.33	0.08	0.13	0.15	0.91	0.29	0.04	0.09	0.10	0.92
π_c	0.65	0.63	−0.02	0.05	0.05	0.97	0.63	−0.02	0.05	0.05	0.97
θ_0	−0.80	−0.86	−0.06	0.25	0.25	0.98					
θ_1	1.80	1.93	0.13	0.37	0.39	0.98					
θ_2	−0.20	−0.27	−0.07	0.52	0.52	0.96					
θ_3	−0.30	−0.35	−0.05	0.69	0.69	0.96					
γ_0	−0.66						−0.65	0.01	0.24	0.24	0.98
γ_1	0.28						0.26	−0.02	0.12	0.12	0.96
γ_2	0.28						0.30	0.02	0.11	0.11	0.94
γ_3	−0.22						−0.49	−0.28	0.56	0.62	0.97
γ_4	−0.05						0.07	0.12	0.33	0.35	0.89
γ_5	−0.05						−0.07	−0.02	0.29	0.30	0.94

(b) TRUE: joint MSRE model

		Assumed LC structure					**Assumed MSRE structure**				
	TRUE	MEAN	BIAS	SD	RMSE	95% COV	MEAN	BIAS	SD	RMSE	95% COV
β_{11}	0.00	0.02	0.02	0.17	0.17	0.93	0.01	0.01	0.13	0.13	0.94
β_{12}	0.00	−0.01	−0.01	0.18	0.18	0.98	−0.02	−0.02	0.11	0.11	0.99
β_{21}	2.83	2.91	0.08	0.12	0.14	0.88	2.91	0.08	0.09	0.12	0.89
β_{22}	2.83	2.68	−0.14	0.12	0.19	0.65	2.69	−0.14	0.09	0.17	0.65
ω_{11}^2	1.00	1.00	0.00	0.18	0.18	0.96	0.99	−0.01	0.16	0.16	0.97
ω_{12}^2	1.00	1.06	0.06	0.34	0.35	0.91	1.03	0.03	0.20	0.20	0.92
ω_{21}^2	1.00	0.98	−0.02	0.16	0.16	0.89	0.99	−0.01	0.15	0.15	0.91
ω_{22}^2	1.00	1.00	0.00	0.14	0.14	0.94	1.01	0.01	0.12	0.12	0.95
ρ_1	0.00	0.01	0.01	0.13	0.13	0.97	0.01	0.01	0.12	0.12	0.97
ρ_2	−0.60	−0.59	0.01	0.07	0.07	0.93	−0.59	0.01	0.07	0.07	0.95
π_d	0.35	0.36	0.01	0.05	0.05	0.92	0.35	0.00	0.03	0.03	0.93
μ_1	−2.00	−1.94	0.06	0.10	0.11	0.95	−1.99	0.01	0.08	0.08	0.97
μ_2	−0.50	−0.55	−0.05	0.17	0.18	0.92	−0.54	−0.04	0.15	0.16	0.93
τ^2	0.25	0.33	0.08	0.11	0.14	0.87	0.29	0.04	0.09	0.10	0.92
π_c	0.65	0.65	0.00	0.05	0.05	0.96	0.63	−0.02	0.05	0.05	0.98
θ_0	−0.48	−0.68	−0.19	0.30	0.36	0.89					
θ_1	0.06	0.09	0.03	0.45	0.45	0.92					
θ_2	0.65	1.17	0.52	0.66	0.84	0.84					
θ_3	−0.08	−0.18	−0.09	0.91	0.92	0.87					
γ_0	−1.00						−0.93	0.07	0.24	0.25	0.97
γ_1	1.00						0.95	−0.05	0.17	0.17	0.94
γ_2	−1.00						−0.98	0.02	0.17	0.17	0.93
γ_3	2.00						1.67	−0.33	0.59	0.68	0.96
γ_4	−2.00						−1.83	0.17	0.37	0.40	0.95
γ_5	2.00						1.94	−0.06	0.39	0.39	0.97

Table A.3

Simulation results from 100 datasets of size, $n = 200$, generated from longitudinal scenario # 3 and the primary probit (a) LC, (b) MSRE models. Left columns: fitted assuming the LC model; right column: fitted assuming the MSRE model.

(a) TRUE: joint LC model

		Assumed LC structure					**Assumed MSRE structure**				
	TRUE	MEAN	BIAS	SD	RMSE	95% COV	MEAN	BIAS	SD	RMSE	95% COV
β_{11}	0.00	−0.14	−0.14	0.27	0.31	0.91	−0.49	−0.49	1.11	1.21	0.80
β_{12}	0.00	−0.04	−0.04	0.20	0.20	0.97	0.12	0.12	0.54	0.55	0.81
β_{21}	2.83	2.44	−0.39	0.20	0.44	0.47	1.81	−1.02	0.28	1.06	0.06
β_{22}	2.83	2.34	−0.49	0.19	0.53	0.22	1.66	−1.17	0.43	1.25	0.01
ω_{11}^2	2.00	1.88	−0.12	0.54	0.56	0.96	0.68	−1.32	0.91	1.61	0.48
ω_{12}^2	2.00	1.93	−0.07	0.55	0.55	0.95	0.38	−1.62	0.94	1.87	0.16
ω_{21}^2	2.00	2.49	0.49	0.48	0.68	0.80	3.62	1.62	0.61	1.73	0.08
ω_{22}^2	2.00	2.61	0.61	0.51	0.79	0.71	3.69	1.69	0.68	1.82	0.02
ρ_1	0.00	−0.10	−0.10	0.23	0.25	0.93	−0.71	−0.71	0.30	0.77	0.68
ρ_2	0.60	0.65	0.05	0.06	0.08	0.86	0.66	0.06	0.20	0.21	0.43
π_d	0.35	0.28	−0.07	0.07	0.09	0.76	0.07	−0.28	0.15	0.32	0.02
μ_1	−2.00	−2.00	0.00	0.04	0.04	0.95	−2.00	0.00	0.04	0.04	0.96
μ_2	−0.50	−0.50	0.00	0.06	0.06	0.95	−0.51	−0.01	0.06	0.06	0.91
τ^2	0.06	0.06	0.00	0.02	0.02	0.89	0.06	0.00	0.02	0.02	0.94
π_c	0.65	0.65	0.00	0.03	0.03	0.96	0.65	0.00	0.03	0.03	0.97
θ_0	−0.80	−1.40	−0.60	0.42	0.74	0.87					
θ_1	1.80	2.33	0.53	0.55	0.77	0.91					
θ_2	−0.20	−0.10	0.10	0.62	0.63	1.00					
θ_3	−0.30	−0.41	−0.11	0.77	0.77	1.00					
γ_0	−0.28						−0.30	−0.01	0.20	0.20	0.96
γ_1	0.19						0.22	0.02	0.13	0.13	0.89
γ_2	0.20						0.20	0.00	0.14	0.14	0.93
γ_3	−0.37						−0.49	−0.12	0.47	0.49	0.98
γ_4	−0.07						−0.10	−0.03	0.34	0.34	0.92
γ_5	−0.07						−0.04	0.04	0.36	0.36	0.93

(b) TRUE: joint MSRE model

		Assumed LC structure					**Assumed MSRE structure**				
	TRUE	MEAN	BIAS	SD	RMSE	95% COV	MEAN	BIAS	SD	RMSE	95% COV
β_{11}	0.00	0.77	0.77	1.15	1.39	0.26	−0.50	−0.50	1.01	1.12	0.79
β_{12}	0.00	1.26	1.26	0.93	1.57	0.30	0.14	0.14	0.52	0.54	0.80
β_{21}	2.83	2.02	−0.80	0.27	0.85	0.13	1.81	−1.02	0.27	1.06	0.04
β_{22}	2.83	1.03	−1.80	0.62	1.90	0.01	1.65	−1.18	0.47	1.27	0.01
ω_{11}^2	2.00	2.85	0.85	1.67	1.88	0.24	0.68	−1.32	0.81	1.55	0.46
ω_{12}^2	2.00	2.39	0.39	1.44	1.49	0.13	0.42	−1.58	1.04	1.89	0.15
ω_{21}^2	2.00	3.14	1.14	0.82	1.40	0.40	3.61	1.61	0.71	1.76	0.08
ω_{22}^2	2.00	3.98	1.98	0.91	2.18	0.12	3.66	1.66	0.77	1.83	0.03
ρ_1	0.00	0.31	0.31	0.68	0.75	0.29	−0.70	−0.70	0.30	0.76	0.67
ρ_2	0.60	0.73	0.13	0.24	0.27	0.26	0.65	0.05	0.24	0.25	0.41
π_d	0.35	0.48	0.13	0.31	0.34	0.05	0.08	−0.27	0.16	0.32	0.02
μ_1	−2.00	−1.99	0.01	0.04	0.04	0.96	−2.00	0.00	0.04	0.04	0.95
μ_2	−0.50	−0.49	0.01	0.06	0.06	0.95	−0.49	0.01	0.06	0.06	0.93
τ^2	0.06	0.06	0.00	0.03	0.03	0.91	0.06	0.00	0.02	0.02	0.90
π_c	0.65	0.66	0.01	0.03	0.03	0.96	0.65	0.00	0.03	0.03	0.94
θ_0	−0.41	−1.43	−1.02	1.15	1.54	0.34					
θ_1	−0.12	2.40	2.52	2.28	3.40	0.31					
θ_2	0.57	1.87	1.30	1.50	1.98	0.34					
θ_3	0.15	−3.27	−3.42	3.15	4.65	0.36					
γ_0	−1.00						−0.84	0.16	0.22	0.28	0.96
γ_1	1.00						0.91	−0.09	0.15	0.18	0.92
γ_2	−1.00						−0.96	0.04	0.16	0.17	0.92
γ_3	2.00						1.58	−0.42	0.53	0.68	0.94
γ_4	−2.00						−1.75	0.25	0.35	0.43	0.94
γ_5	2.00						1.90	−0.10	0.35	0.37	0.96

Table A.4

Simulation results from 100 datasets of size, $n = 200$, generated from longitudinal scenario # 4 and the primary probit (a) LC, (b) MSRE models. Left columns: fitted assuming the LC model; right column: fitted assuming the MSRE model.

(a) TRUE: joint LC model

		Assumed LC structure					**Assumed MSRE structure**				
	TRUE	MEAN	BIAS	SD	RMSE	95% COV	MEAN	BIAS	SD	RMSE	95% COV
β_{11}	0.00	0.00	0.00	0.12	0.12	0.97	−0.01	−0.01	0.12	0.12	0.98
β_{12}	0.00	0.00	0.00	0.11	0.11	0.98	0.00	0.00	0.11	0.11	0.97
β_{21}	2.83	2.90	0.08	0.09	0.12	0.87	2.90	0.07	0.10	0.12	0.85
β_{22}	2.83	2.67	−0.16	0.09	0.18	0.61	2.67	−0.16	0.09	0.18	0.61
ω^2_{11}	1.00	0.98	−0.02	0.19	0.19	0.94	0.98	−0.02	0.19	0.20	0.92
ω^2_{12}	1.00	0.99	−0.01	0.18	0.18	0.92	0.99	−0.01	0.19	0.19	0.92
ω^2_{21}	1.00	1.00	0.00	0.12	0.12	0.95	1.00	0.00	0.12	0.12	0.94
ω^2_{22}	1.00	1.01	0.01	0.13	0.13	0.95	1.01	0.01	0.13	0.13	0.97
ρ_1	0.00	0.00	0.00	0.13	0.13	0.97	0.00	0.00	0.13	0.13	0.96
ρ_2	−0.60	−0.59	0.01	0.07	0.07	0.92	−0.59	0.01	0.07	0.07	0.94
π_d	0.35	0.36	0.01	0.03	0.04	0.96	0.36	0.01	0.03	0.04	0.96
μ_1	−2.00	−2.00	0.00	0.04	0.04	0.96	−2.00	0.00	0.04	0.04	0.96
μ_2	−0.50	−0.50	0.00	0.06	0.06	0.93	−0.50	0.00	0.07	0.07	0.91
τ^2	0.06	0.06	0.00	0.02	0.02	0.93	0.06	0.00	0.02	0.02	0.91
π_c	0.65	0.64	−0.01	0.04	0.04	0.94	0.64	−0.01	0.04	0.04	0.92
θ_0	−0.80	−0.81	−0.01	0.19	0.19	0.97					
θ_1	1.80	1.80	0.00	0.25	0.25	0.99					
θ_2	−0.20	−0.28	−0.08	0.51	0.52	0.95					
θ_3	−0.30	−0.21	0.09	0.57	0.58	0.95					
γ_0	−0.62						−0.64	−0.01	0.23	0.23	0.98
γ_1	0.29						0.28	−0.01	0.12	0.12	0.97
γ_2	0.29						0.31	0.03	0.13	0.13	0.94
γ_3	−0.36						−0.52	−0.16	0.69	0.71	0.95
γ_4	−0.09						−0.02	0.07	0.35	0.35	0.95
γ_5	−0.08						−0.10	−0.02	0.30	0.30	0.97

(b) TRUE: joint MSRE model

		Assumed LC structure					**Assumed MSRE structure**				
	TRUE	MEAN	BIAS	SD	RMSE	95% COV	MEAN	BIAS	SD	RMSE	95% COV
β_{11}	0.00	0.00	0.00	0.12	0.12	0.96	−0.01	−0.01	0.12	0.12	0.96
β_{12}	0.00	0.00	0.00	0.11	0.11	0.97	0.00	0.00	0.11	0.11	0.97
β_{21}	2.83	2.90	0.07	0.10	0.12	0.84	2.90	0.07	0.10	0.12	0.85
β_{22}	2.83	2.67	−0.16	0.09	0.18	0.59	2.67	−0.16	0.09	0.18	0.60
ω^2_{11}	1.00	0.98	−0.02	0.20	0.20	0.93	0.98	−0.02	0.19	0.20	0.94
ω^2_{12}	1.00	0.99	−0.01	0.19	0.19	0.92	0.99	−0.01	0.19	0.19	0.92
ω^2_{21}	1.00	1.00	0.00	0.13	0.13	0.95	1.00	0.00	0.12	0.12	0.95
ω^2_{22}	1.00	1.01	0.01	0.13	0.14	0.95	1.01	0.01	0.13	0.13	0.97
ρ_1	0.00	0.00	0.00	0.13	0.13	0.98	0.00	0.00	0.13	0.13	0.96
ρ_2	−0.60	−0.59	0.01	0.07	0.07	0.94	−0.59	0.01	0.07	0.07	0.94
π_d	0.35	0.36	0.01	0.03	0.04	0.96	0.36	0.01	0.03	0.04	0.96
μ_1	−2.00	−1.99	0.01	0.04	0.04	0.96	−1.99	0.01	0.04	0.04	0.96
μ_2	−0.50	−0.49	0.01	0.06	0.06	0.94	−0.49	0.01	0.06	0.06	0.94
τ^2	0.06	0.06	0.00	0.02	0.02	0.94	0.07	0.01	0.02	0.02	0.91
π_c	0.65	0.65	0.00	0.04	0.04	0.92	0.65	0.00	0.04	0.04	0.94
θ_0	−0.50	−0.55	−0.05	0.24	0.25	0.91					
θ_1	0.06	0.11	0.05	0.30	0.30	0.90					
θ_2	0.69	0.79	0.10	0.38	0.40	0.94					
θ_3	−0.08	−0.16	−0.09	0.49	0.49	0.95					
γ_0	−1.00						−0.86	0.14	0.25	0.29	0.93
γ_1	1.00						0.92	−0.08	0.15	0.17	0.90
γ_2	−1.00						−0.95	0.05	0.17	0.18	0.93
γ_3	2.00						1.56	−0.44	0.58	0.73	0.93
γ_4	−2.00						−1.78	0.22	0.36	0.43	0.90
γ_5	2.00						1.90	−0.10	0.42	0.43	0.94

Table A.5

Simulation results from 100 datasets of size, $n = 500$, generated from longitudinal scenario # 1 and the primary probit (a) LC, (b) MSRE models. Left columns: fitted assuming the LC model; right column: fitted assuming the MSRE model.

(a) TRUE: joint LC model

		Assumed LC structure					**Assumed MSRE structure**				
	TRUE	MEAN	BIAS	SD	RMSE	95% COV	MEAN	BIAS	SD	RMSE	95% COV
β_{11}	0.00	−0.09	−0.09	0.15	0.17	0.91	−0.30	−0.30	0.41	0.51	0.77
β_{12}	0.00	−0.05	−0.05	0.13	0.14	0.95	−0.10	−0.10	0.29	0.31	0.92
β_{21}	2.83	2.64	−0.19	0.12	0.22	0.64	2.37	−0.46	0.24	0.52	0.36
β_{22}	2.83	2.61	−0.22	0.11	0.24	0.60	2.29	−0.54	0.35	0.64	0.27
ω^2_{11}	2.00	1.92	−0.08	0.28	0.29	0.95	1.73	−0.27	0.55	0.61	0.87
ω^2_{12}	2.00	2.02	0.02	0.25	0.25	0.96	1.92	−0.08	0.61	0.62	0.91
ω^2_{21}	2.00	2.24	0.24	0.26	0.36	0.83	2.69	0.69	0.55	0.88	0.55
ω^2_{22}	2.00	2.29	0.29	0.31	0.42	0.84	2.85	0.85	0.54	1.00	0.49
ρ_1	0.00	−0.06	−0.06	0.10	0.12	0.94	−0.28	−0.28	0.26	0.38	0.70
ρ_2	0.60	0.63	0.03	0.05	0.06	0.86	0.68	0.08	0.04	0.09	0.54
π_d	0.35	0.32	−0.03	0.03	0.05	0.85	0.23	−0.12	0.11	0.16	0.43
μ_1	−2.00	−2.00	0.00	0.05	0.05	0.98	−2.00	0.00	0.05	0.05	0.98
μ_2	−0.50	−0.53	−0.03	0.08	0.08	0.94	−0.53	−0.03	0.08	0.08	0.94
τ^2	0.25	0.26	0.01	0.04	0.04	0.96	0.26	0.01	0.04	0.04	0.97
π_c	0.65	0.64	−0.01	0.03	0.03	0.99	0.64	−0.01	0.03	0.03	1.00
θ_0	−0.80	−1.09	−0.29	0.34	0.45	0.89					
θ_1	1.80	2.07	0.27	0.36	0.45	0.92					
θ_2	−0.20	−0.32	−0.12	0.51	0.52	0.98					
θ_3	−0.30	−0.18	0.12	0.58	0.59	0.98					
γ_0	−0.32						−0.30	0.01	0.14	0.14	0.96
γ_1	0.19						0.19	0.00	0.08	0.08	0.92
γ_2	0.18						0.19	0.00	0.07	0.07	0.89
γ_3	−0.22						−0.33	−0.11	0.31	0.32	0.91
γ_4	−0.04						−0.02	0.01	0.18	0.18	0.93
γ_5	−0.04						−0.04	0.00	0.16	0.16	0.97

(b) TRUE: joint MSRE model

		Assumed LC structure					**Assumed MSRE structure**				
	TRUE	MEAN	BIAS	SD	RMSE	95% COV	MEAN	BIAS	SD	RMSE	95% COV
β_{11}	0.00	1.49	1.49	0.21	1.50	0.01	−0.31	−0.31	0.44	0.54	0.76
β_{12}	0.00	2.08	2.08	0.21	2.09	0.01	−0.11	−0.11	0.29	0.31	0.90
β_{21}	2.83	2.27	−0.56	0.17	0.59	0.07	2.36	−0.47	0.24	0.53	0.35
β_{22}	2.83	0.86	−1.97	0.25	1.99	0.01	2.29	−0.54	0.34	0.63	0.28
ω^2_{11}	2.00	3.99	1.99	0.33	2.02	0.01	1.74	−0.26	0.53	0.59	0.88
ω^2_{12}	2.00	3.22	1.22	0.26	1.25	0.01	1.93	−0.07	0.58	0.59	0.93
ω^2_{21}	2.00	2.99	0.99	0.35	1.05	0.19	2.71	0.71	0.52	0.88	0.53
ω^2_{22}	2.00	4.24	2.24	0.46	2.29	0.01	2.84	0.84	0.52	0.99	0.50
ρ_1	0.00	0.76	0.76	0.10	0.77	0.01	−0.27	−0.27	0.26	0.38	0.72
ρ_2	0.60	0.80	0.20	0.04	0.21	0.01	0.67	0.07	0.07	0.10	0.53
π_d	0.35	0.69	0.34	0.05	0.35	0.01	0.23	−0.12	0.10	0.16	0.43
μ_1	−2.00	−1.93	0.07	0.05	0.09	0.75	−2.00	0.00	0.05	0.05	0.98
μ_2	−0.50	−0.43	0.07	0.06	0.09	0.84	−0.52	−0.02	0.07	0.08	0.95
τ^2	0.25	0.29	0.04	0.04	0.06	0.86	0.26	0.01	0.04	0.04	0.97
π_c	0.65	0.70	0.05	0.03	0.05	0.68	0.65	0.00	0.03	0.03	1.00
θ_0	−0.40	−2.57	−2.18	0.27	2.20	0.00					
θ_1	−0.11	4.57	4.68	0.59	4.72	0.01					
θ_2	0.53	3.92	3.39	0.57	3.44	0.00					
θ_3	0.16	−7.17	−7.32	1.16	7.42	0.01					
γ_0	−1.00						−0.96	0.04	0.17	0.18	0.95
γ_1	1.00						0.96	−0.04	0.11	0.11	0.95
γ_2	−1.00						−0.98	0.02	0.12	0.12	0.93
γ_3	2.00						1.85	−0.15	0.40	0.42	0.96
γ_4	−2.00						−1.89	0.11	0.27	0.29	0.97
γ_5	2.00						1.95	−0.05	0.29	0.29	0.94

Table A.6

Simulation results from 100 datasets of size, $n = 500$, generated from longitudinal scenario # 2 and the primary probit (a) LC, (b) MSRE models. Left columns: fitted assuming the LC model; right column: fitted assuming the MSRE model.

(a) TRUE: joint LC model

		Assumed LC structure					**Assumed MSRE structure**				
	TRUE	MEAN	BIAS	SD	RMSE	95% COV	MEAN	BIAS	SD	RMSE	95% COV
β_{11}	0.00	0.02	0.02	0.07	0.07	0.98	0.01	0.01	0.07	0.07	0.99
β_{12}	0.00	0.01	0.01	0.07	0.07	0.98	0.01	0.01	0.07	0.07	0.96
β_{21}	2.83	2.86	0.03	0.05	0.06	0.94	2.86	0.03	0.05	0.06	0.95
β_{22}	2.83	2.77	−0.06	0.06	0.09	0.81	2.77	−0.06	0.06	0.09	0.82
ω^2_{11}	1.00	0.99	−0.01	0.13	0.13	0.93	0.98	−0.02	0.13	0.13	0.91
ω^2_{12}	1.00	1.01	0.01	0.11	0.11	0.96	1.02	0.02	0.11	0.11	0.94
ω^2_{21}	1.00	1.01	0.01	0.08	0.08	0.95	1.01	0.01	0.08	0.08	0.95
ω^2_{22}	1.00	1.00	0.00	0.08	0.08	0.92	1.00	0.00	0.08	0.08	0.92
ρ_1	0.00	−0.01	−0.01	0.08	0.08	0.94	−0.01	−0.01	0.08	0.08	0.95
ρ_2	−0.60	−0.60	0.00	0.04	0.04	0.91	−0.60	0.00	0.04	0.04	0.92
π_d	0.35	0.35	0.00	0.02	0.02	0.95	0.35	0.00	0.02	0.02	0.92
μ_1	−2.00	−1.99	0.01	0.06	0.06	0.95	−2.00	0.00	0.05	0.05	0.97
μ_2	−0.50	−0.51	−0.01	0.11	0.11	0.89	−0.51	−0.01	0.09	0.09	0.89
τ^2	0.25	0.28	0.03	0.05	0.06	0.94	0.27	0.02	0.04	0.05	0.97
π_c	0.65	0.64	−0.01	0.04	0.04	0.96	0.64	−0.01	0.03	0.04	0.95
θ_0	−0.80	−0.81	−0.01	0.16	0.16	0.93					
θ_1	1.80	1.82	0.02	0.20	0.20	0.94					
θ_2	−0.20	−0.25	−0.05	0.38	0.39	0.94					
θ_3	−0.30	−0.26	0.04	0.40	0.40	0.97					
γ_0	−0.66						−0.63	0.02	0.18	0.19	0.92
γ_1	0.28						0.28	0.01	0.07	0.07	0.96
γ_2	0.28						0.27	−0.01	0.07	0.07	0.94
γ_3	−0.22						−0.41	−0.19	0.45	0.48	0.91
γ_4	−0.05						−0.01	0.04	0.17	0.18	0.95
γ_5	−0.05						−0.02	0.02	0.18	0.18	0.91

(b) TRUE: joint MSRE model

		Assumed LC structure					**Assumed MSRE structure**				
	TRUE	MEAN	BIAS	SD	RMSE	95% COV	MEAN	BIAS	SD	RMSE	95% COV
β_{11}	0.00	0.19	0.19	0.47	0.50	0.86	0.01	0.01	0.07	0.07	0.99
β_{12}	0.00	0.27	0.27	0.68	0.74	0.83	0.01	0.01	0.07	0.07	0.96
β_{21}	2.83	2.80	−0.02	0.16	0.17	0.89	2.86	0.03	0.05	0.06	0.95
β_{22}	2.83	2.53	−0.30	0.62	0.69	0.70	2.77	−0.06	0.06	0.09	0.82
ω^2_{11}	1.00	1.18	0.18	0.52	0.55	0.80	0.98	−0.02	0.13	0.13	0.93
ω^2_{12}	1.00	1.25	0.25	0.61	0.66	0.83	1.02	0.02	0.11	0.11	0.93
ω^2_{21}	1.00	1.25	0.25	0.65	0.69	0.83	1.01	0.01	0.08	0.08	0.95
ω^2_{22}	1.00	1.16	0.16	0.42	0.45	0.81	1.00	0.00	0.08	0.08	0.93
ρ_1	0.00	0.07	0.07	0.23	0.25	0.84	−0.01	−0.01	0.08	0.08	0.98
ρ_2	−0.60	−0.43	0.17	0.45	0.48	0.79	−0.60	0.00	0.04	0.04	0.92
π_d	0.35	0.39	0.04	0.11	0.12	0.81	0.35	0.00	0.02	0.02	0.91
μ_1	−2.00	−1.96	0.04	0.06	0.07	0.89	−2.00	0.00	0.05	0.05	0.98
μ_2	−0.50	−0.48	0.02	0.09	0.09	0.92	−0.50	0.00	0.09	0.09	0.92
τ^2	0.25	0.29	0.04	0.06	0.07	0.90	0.27	0.02	0.04	0.05	0.96
π_c	0.65	0.67	0.02	0.04	0.04	0.91	0.64	−0.01	0.03	0.03	0.99
θ_0	−0.48	−0.83	−0.34	0.72	0.80	0.81					
θ_1	0.06	0.67	0.61	1.63	1.74	0.82					
θ_2	0.65	1.35	0.70	1.14	1.34	0.77					
θ_3	−0.08	−1.09	−1.01	2.63	2.82	0.83					
γ_0	−1.00						−0.92	0.08	0.15	0.17	0.95
γ_1	1.00						0.97	−0.03	0.11	0.12	0.94
γ_2	−1.00						−1.00	0.00	0.13	0.13	0.92
γ_3	2.00						1.81	−0.19	0.41	0.45	0.94
γ_4	−2.00						−1.92	0.08	0.30	0.31	0.90
γ_5	2.00						1.99	−0.01	0.30	0.31	0.91

Table A.7

Simulation results from 100 datasets of size, $n = 500$, generated from longitudinal scenario # 3 and the primary probit (a) LC, (b) MSRE models. Left columns: fitted assuming the LC model; right column: fitted assuming the MSRE model.

(a) TRUE: joint LC model

		Assumed LC structure					**Assumed MSRE structure**				
	TRUE	MEAN	BIAS	SD	RMSE	95% COV	MEAN	BIAS	SD	RMSE	95% COV
β_{11}	0.00	−0.09	−0.09	0.13	0.15	0.91	−0.32	−0.32	0.32	0.45	0.74
β_{12}	0.00	−0.06	−0.06	0.13	0.15	0.93	−0.15	−0.15	0.18	0.23	0.90
β_{21}	2.83	2.65	−0.18	0.12	0.22	0.69	2.40	−0.43	0.26	0.50	0.44
β_{22}	2.83	2.60	−0.23	0.12	0.26	0.50	2.34	−0.49	0.25	0.55	0.28
ω^2_{11}	2.00	1.99	−0.01	0.30	0.30	0.93	1.77	−0.23	0.59	0.63	0.87
ω^2_{12}	2.00	1.98	−0.02	0.28	0.28	0.92	1.83	−0.17	0.64	0.66	0.87
ω^2_{21}	2.00	2.22	0.22	0.26	0.34	0.85	2.61	0.61	0.52	0.80	0.66
ω^2_{22}	2.00	2.25	0.25	0.24	0.35	0.83	2.72	0.72	0.48	0.87	0.59
ρ_1	0.00	−0.06	−0.06	0.09	0.11	0.94	−0.28	−0.28	0.22	0.35	0.77
ρ_2	0.60	0.63	0.03	0.04	0.05	0.88	0.66	0.06	0.04	0.07	0.66
π_d	0.35	0.32	−0.03	0.03	0.04	0.89	0.23	−0.12	0.08	0.15	0.52
μ_1	−2.00	−2.00	0.00	0.03	0.03	0.96	−2.00	0.00	0.03	0.03	0.95
μ_2	−0.50	−0.50	0.00	0.04	0.04	0.93	−0.50	0.00	0.04	0.04	0.96
τ^2	0.06	0.06	0.00	0.01	0.01	0.95	0.06	0.00	0.01	0.01	0.95
π_c	0.65	0.64	−0.01	0.02	0.02	0.94	0.64	−0.01	0.02	0.02	0.93
θ_0	−0.80	−1.09	−0.29	0.35	0.45	0.87					
θ_1	1.80	2.03	0.23	0.38	0.44	0.91					
θ_2	−0.20	−0.28	−0.08	0.53	0.54	0.95					
θ_3	−0.30	−0.22	0.08	0.59	0.60	0.97					
γ_0	−0.28						−0.29	−0.01	0.15	0.15	0.95
γ_1	0.19						0.20	0.00	0.08	0.08	0.93
γ_2	0.20						0.20	0.01	0.07	0.07	0.94
γ_3	−0.37						−0.44	−0.08	0.35	0.36	0.96
γ_4	−0.07						−0.07	0.00	0.20	0.20	0.95
γ_5	−0.07						−0.08	−0.01	0.19	0.19	0.97

(b) TRUE: joint MSRE model

		Assumed LC structure					**Assumed MSRE structure**				
	TRUE	MEAN	BIAS	SD	RMSE	95% COV	MEAN	BIAS	SD	RMSE	95% COV
β_{11}	0.00	1.45	1.45	0.28	1.48	0.00	−0.33	−0.33	0.50	0.60	0.75
β_{12}	0.00	2.04	2.04	0.21	2.05	0.01	−0.17	−0.17	0.19	0.25	0.87
β_{21}	2.83	2.30	−0.53	0.12	0.54	0.05	2.40	−0.43	0.25	0.50	0.44
β_{22}	2.83	0.94	−1.89	0.19	1.90	0.00	2.34	−0.49	0.25	0.55	0.29
ω^2_{11}	2.00	3.96	1.96	0.43	2.00	0.00	1.78	−0.22	0.54	0.58	0.91
ω^2_{12}	2.00	3.21	1.21	0.30	1.24	0.00	1.81	−0.19	0.63	0.65	0.86
ω^2_{21}	2.00	3.00	1.00	0.37	1.06	0.16	2.62	0.62	0.51	0.81	0.65
ω^2_{22}	2.00	4.30	2.30	0.41	2.34	0.00	2.73	0.73	0.49	0.88	0.61
ρ_1	0.00	0.76	0.76	0.07	0.76	0.01	−0.29	−0.29	0.21	0.36	0.78
ρ_2	0.60	0.80	0.20	0.03	0.20	0.01	0.66	0.06	0.05	0.07	0.66
π_d	0.35	0.68	0.33	0.06	0.33	0.00	0.23	−0.12	0.08	0.15	0.54
μ_1	−2.00	−1.99	0.01	0.03	0.03	0.93	−2.00	0.00	0.03	0.03	0.95
μ_2	−0.50	−0.49	0.01	0.04	0.04	0.92	−0.49	0.01	0.04	0.04	0.93
τ^2	0.06	0.06	0.00	0.01	0.01	0.94	0.06	0.00	0.01	0.01	0.93
π_c	0.65	0.65	0.00	0.02	0.02	0.95	0.64	−0.01	0.02	0.02	0.94
θ_0	−0.41	−2.42	−2.01	0.32	2.03	0.00					
θ_1	−0.12	4.47	4.60	0.55	4.63	0.00					
θ_2	0.57	2.96	2.39	0.40	2.42	0.00					
θ_3	0.15	−5.66	−5.81	0.81	5.87	0.00					
γ_0	−1.00						−0.93	0.07	0.16	0.17	0.94
γ_1	1.00						0.95	−0.05	0.09	0.10	0.96
γ_2	−1.00						−0.96	0.04	0.10	0.11	0.94
γ_3	2.00						1.77	−0.23	0.41	0.47	0.94
γ_4	−2.00						−1.84	0.16	0.23	0.28	0.92
γ_5	2.00						1.88	−0.12	0.24	0.27	0.92

Table A.8

Simulation results from 100 datasets of size, $n = 500$, generated from longitudinal scenario # 4 and the primary probit (a) LC, (b) MSRE models. Left columns: fitted assuming the LC model; right column: fitted assuming the MSRE model.

(a) TRUE: joint LC model

		Assumed LC structure					**Assumed MSRE structure**				
	TRUE	MEAN	BIAS	SD	RMSE	95% COV	MEAN	BIAS	SD	RMSE	95% COV
β_{11}	0.00	0.01	0.01	0.08	0.08	0.95	0.00	0.00	0.08	0.08	0.96
β_{12}	0.00	0.01	0.01	0.07	0.07	0.96	0.01	0.01	0.07	0.07	0.97
β_{21}	2.83	2.86	0.03	0.05	0.06	0.95	2.86	0.03	0.06	0.06	0.95
β_{22}	2.83	2.78	−0.05	0.05	0.07	0.87	2.78	−0.05	0.05	0.07	0.86
ω_{11}^2	1.00	1.02	0.02	0.13	0.13	0.95	1.01	0.01	0.13	0.13	0.97
ω_{12}^2	1.00	1.00	0.00	0.14	0.14	0.89	1.00	0.00	0.14	0.14	0.91
ω_{21}^2	1.00	0.99	−0.01	0.09	0.09	0.93	0.99	−0.01	0.09	0.09	0.94
ω_{22}^2	1.00	1.00	0.00	0.08	0.08	0.94	1.00	0.00	0.08	0.08	0.95
ρ_1	0.00	0.02	0.02	0.08	0.09	0.95	0.01	0.01	0.08	0.08	0.95
ρ_2	−0.60	−0.60	0.00	0.04	0.04	0.91	−0.60	0.00	0.04	0.04	0.93
π_d	0.35	0.35	0.00	0.02	0.02	0.97	0.35	0.00	0.02	0.02	0.95
μ_1	−2.00	−2.00	0.00	0.03	0.03	0.96	−2.00	0.00	0.03	0.03	0.96
μ_2	−0.50	−0.50	0.00	0.04	0.04	0.98	−0.50	0.00	0.04	0.04	0.99
τ^2	0.06	0.06	0.00	0.01	0.01	0.93	0.06	0.00	0.01	0.01	0.93
π_c	0.65	0.65	0.00	0.03	0.03	0.92	0.65	0.00	0.02	0.02	0.93
θ_0	−0.80	−0.80	0.00	0.14	0.14	0.95					
θ_1	1.80	1.79	−0.01	0.16	0.16	0.96					
θ_2	−0.20	−0.22	−0.02	0.27	0.27	0.98					
θ_3	−0.30	−0.29	0.01	0.30	0.31	0.97					
γ_0	−0.62						−0.61	0.02	0.17	0.17	0.96
γ_1	0.29						0.30	0.01	0.08	0.08	0.97
γ_2	0.29						0.28	−0.01	0.08	0.08	0.94
γ_3	−0.36						−0.48	−0.12	0.49	0.51	0.95
γ_4	−0.09						−0.07	0.02	0.21	0.21	0.94
γ_5	−0.08						−0.08	0.00	0.20	0.20	0.97

(b) TRUE: joint MSRE model

		Assumed LC structure					**Assumed MSRE structure**				
	TRUE	MEAN	BIAS	SD	RMSE	95% COV	MEAN	BIAS	SD	RMSE	95% COV
β_{11}	0.00	0.14	0.14	0.42	0.45	0.86	0.00	0.00	0.08	0.08	0.96
β_{12}	0.00	0.21	0.21	0.63	0.67	0.88	0.01	0.01	0.07	0.07	0.97
β_{21}	2.83	2.82	−0.01	0.14	0.14	0.88	2.86	0.03	0.06	0.06	0.95
β_{22}	2.83	2.60	−0.23	0.55	0.60	0.77	2.78	−0.05	0.05	0.07	0.87
ω_{11}^2	1.00	1.16	0.16	0.46	0.48	0.87	1.02	0.02	0.13	0.13	0.97
ω_{12}^2	1.00	1.16	0.16	0.53	0.56	0.83	1.00	0.00	0.14	0.14	0.91
ω_{21}^2	1.00	1.17	0.17	0.56	0.59	0.84	0.99	−0.01	0.09	0.09	0.94
ω_{22}^2	1.00	1.13	0.13	0.40	0.42	0.86	1.00	0.00	0.08	0.08	0.96
ρ_1	0.00	0.08	0.08	0.20	0.22	0.85	0.02	0.02	0.08	0.08	0.94
ρ_2	−0.60	−0.46	0.14	0.39	0.41	0.84	−0.59	0.01	0.04	0.04	0.93
π_d	0.35	0.39	0.04	0.10	0.11	0.86	0.35	0.00	0.02	0.02	0.96
μ_1	−2.00	−2.00	0.00	0.03	0.03	0.97	−2.00	0.00	0.03	0.03	0.98
μ_2	−0.50	−0.50	0.00	0.03	0.03	0.98	−0.50	0.00	0.04	0.04	0.97
τ^2	0.06	0.06	0.00	0.01	0.01	0.94	0.06	0.00	0.01	0.01	0.91
π_c	0.65	0.65	0.00	0.03	0.03	0.92	0.65	0.00	0.03	0.03	0.91
θ_0	−0.50	−0.73	−0.23	0.61	0.65	0.86					
θ_1	0.06	0.53	0.47	1.43	1.50	0.86					
θ_2	0.69	0.96	0.27	0.76	0.80	0.90					
θ_3	−0.08	−0.65	−0.57	1.82	1.90	0.86					
γ_0	−1.00						−0.94	0.06	0.17	0.18	0.96
γ_1	1.00						0.98	−0.02	0.11	0.11	0.94
γ_2	−1.00						−1.01	−0.01	0.12	0.12	0.95
γ_3	2.00						1.78	−0.22	0.40	0.46	0.96
γ_4	−2.00						−1.94	0.06	0.25	0.26	0.97
γ_5	2.00						2.03	0.03	0.28	0.28	0.96

Table A.9

Misclassification rates (%) for (a) mean profile class and (b) variance class from the simulation study when sample size $n = 500$. Left columns: data generated from the LC model; right columns: data generated from the MSRE model.

	True model: LC				**True model: MSRE**			
	Scenario				**Scenario**			
	# 1	# 2	# 3	# 4	# 1	# 2	# 3	# 4
(a) Mean profile class								
LC	8	0	8	0	60	8	60	7
MSRE	17	1	16	0	18	1	17	0
(b) Variance class								
LC	10	10	3	3	12	11	3	3
MSRE	11	10	3	3	10	10	3	3

Table A.10

(a) Mean Area under the ROC curves and (b) Brier score for the prediction of outcome from the simulation study based on 100 datasets of size, $n = 500$. Left columns: data generated from the LC model; right columns: data generated from the MSRE model. LC-assumed refers to AUC/Brier score results obtained by fitting a probit model using the known latent classes as predictors when the data are generated under the MSRE model, and equivalently defined are MSRE-assumed under model-misspecification, by using the known random effects and variances as predictors. "Percentile" refers to the 2.5 and 97.5 percentiles of the results computed under the true parameters across the simulations; "95% CI" refers to mean of the lower and upper 95% credible intervals across simulations. LC/MSRE-testing refers to results obtained for the validation sample of size $\tilde{n} = 125$, while LC/MSRE-training gives within-sample prediction outcomes.

(a) Area under the ROC curves

	TRUE: joint LC model				**TRUE: joint MSRE model**			
	Scenario				**Scenario**			
	# 1	# 2	# 3	# 4	# 1	# 2	# 3	# 4
Truth								
mean	0.81	0.81	0.81	0.81	0.84	0.85	0.83	0.84
percentile	(0.78, 0.85)	(0.76, 0.85)	(0.77, 0.85)	(0.78, 0.84)	(0.8, 0.87)	(0.82, 0.88)	(0.8, 0.85)	(0.81, 0.87)
LC-training								
mean	0.82	0.81	0.81	0.81	0.96	0.7	0.93	0.67
95% CI	(0.77, 0.87)	(0.76, 0.85)	(0.75, 0.86)	(0.78, 0.84)	(0.92, 0.99)	(0.61, 0.98)	(0.9, 0.96)	(0.59, 0.95)
LC-testing								
mean	0.7	0.79	0.71	0.8	0.66	0.6	0.67	0.61
95% CI	(0.63, 0.76)	(0.71, 0.85)	(0.64, 0.77)	(0.73, 0.85)	(0.6, 0.71)	(0.53, 0.67)	(0.62, 0.72)	(0.54, 0.69)
MSRE-training								
mean	0.77	0.8	0.77	0.8	0.84	0.85	0.82	0.84
95% CI	(0.72, 0.81)	(0.76, 0.84)	(0.72, 0.81)	(0.78, 0.85)	(0.8, 0.88)	(0.81, 0.89)	(0.79, 0.86)	(0.81, 0.87)
MSRE-testing								
mean	0.75	0.78	0.76	0.79	0.79	0.8	0.78	0.8
95% CI	(0.66, 0.83)	(0.71, 0.85)	(0.68, 0.84)	(0.71, 0.86)	(0.72, 0.85)	(0.73, 0.87)	(0.71, 0.85)	(0.73, 0.86)

(b) Brier score

	TRUE: joint LC model				**TRUE: joint MSRE model**			
	Scenario				**Scenario**			
	# 1	# 2	# 3	# 4	# 1	# 2	# 3	# 4
Truth								
mean	0.16	0.16	0.16	0.16	0.16	0.15	0.17	0.16
percentile	(0.14, 0.18)	(0.14, 0.18)	(0.14, 0.18)	(0.14, 0.18)	(0.14, 0.17)	(0.14, 0.17)	(0.15, 0.18)	(0.14, 0.17)
LC-training								
mean	0.15	0.16	0.16	0.16	0.05	0.19	0.09	0.21
95% CI	(0.12, 0.18)	(0.14, 0.18)	(0.13, 0.19)	(0.14, 0.18)	(0.02, 0.09)	(0.03, 0.23)	(0.06, 0.12)	(0.08, 0.24)
LC-testing								
mean	0.22	0.17	0.22	0.17	0.3	0.24	0.28	0.24
95% CI	(0.19, 0.26)	(0.14, 0.21)	(0.18, 0.26)	(0.14, 0.2)	(0.26, 0.35)	(0.21, 0.31)	(0.22, 0.32)	(0.21, 0.27)
MSRE-training								
mean	0.19	0.17	0.19	0.17	0.16	0.15	0.17	0.16
95% CI	(0.17, 0.21)	(0.15, 0.19)	(0.17, 0.21)	(0.15, 0.19)	(0.14, 0.18)	(0.14, 0.17)	(0.15, 0.18)	(0.14, 0.17)
MSRE-testing								
mean	0.2	0.18	0.2	0.17	0.19	0.18	0.19	0.18
95% CI	(0.17, 0.23)	(0.15, 0.22)	(0.17, 0.23)	(0.14, 0.21)	(0.16, 0.22)	(0.14, 0.21)	(0.16, 0.22)	(0.15, 0.22)

Table A.11

(a) Mean Area under the ROC curves and (b) Brier score for the prediction of outcome from the simulation study based on 100 datasets of size, $n = 200$. Left columns: data generated from the LC model; right columns: data generated from the MSRE model. LC-assumed refers to AUC/Brier score results obtained by fitting a probit model using the known latent classes as predictors when the data are generated under the MSRE model, and equivalently defined are MSRE-assumed under model-misspecification, by using the known random effects and variances as predictors. "Percentile" refers to the 2.5 and 97.5 percentiles of the results computed under the true parameters across simulations; "95% CI" refers to mean of the lower and upper 95% credible intervals across simulations. LC/MSRE-testing refers to results obtained for the validation sample of size $\tilde{n} = 50$, while LC/MSRE-training gives within-sample prediction outcomes.

(a) Area under the ROC curves

	TRUE: joint LC model				**TRUE: joint MSRE model**			
	Scenario				**Scenario**			
	# 1	# 2	# 3	# 4	# 1	# 2	# 3	# 4
Truth								
mean	0.80	0.81	0.81	0.81	0.84	0.85	0.83	0.84
Percentile	(0.75, 0.86)	(0.75, 0.86)	(0.75, 0.87)	(0.75, 0.86)	(0.79, 0.89)	(0.80, 0.90)	(0.77, 0.88)	(0.78, 0.89)
LC-training								
mean	0.80	0.82	0.80	0.81	0.85	0.69	0.83	0.64
95% CI	(0.58, 0.91)	(0.75, 0.88)	(0.63, 0.92)	(0.75, 0.86)	(0.63, 0.97)	(0.58, 0.82)	(0.60, 0.96)	(0.56, 0.72)
LC-testing								
mean	0.67	0.79	0.68	0.79	0.64	0.59	0.66	0.61
95% CI	(0.54, 0.79)	(0.69, 0.9)	(0.59, 0.78)	(0.67, 0.89)	(0.53, 0.73)	(0.49, 0.7)	(0.58, 0.74)	(0.49, 0.77)
LC-assumed								
mean	—	—	—	—	0.64	0.63	0.65	0.64
95% CI					(0.58, 0.70)	(0.56, 0.70)	(0.58, 0.70)	(0.58, 0.72)
MSRE-training								
mean	0.76	0.80	0.77	0.81	0.84	0.85	0.83	0.83
95% CI	(0.69, 0.83)	(0.73, 0.85)	(0.71, 0.85)	(0.74, 0.86)	(0.79, 0.89)	(0.79, 0.90)	(0.76, 0.88)	(0.77, 0.89)
MSRE-testing								
mean	0.74	0.78	0.75	0.79	0.78	0.8	0.78	0.79
95% CI	(0.59, 0.89)	(0.65, 0.9)	(0.61, 0.88)	(0.67, 0.89)	(0.66, 0.88)	(0.68, 0.89)	(0.64, 0.89)	(0.65, 0.9)
MSRE-assumed								
mean	0.77	0.80	0.78	0.81	—	—	—	—
95% CI	(0.69, 0.83)	(0.74, 0.85)	(0.72, 0.85)	(0.75, 0.87)				

(b) Brier score

	TRUE: joint LC model				**TRUE: joint MSRE model**			
	Scenario				**Scenario**			
	# 1	# 2	# 3	# 4	# 1	# 2	# 3	# 4
Truth								
mean	0.16	0.16	0.16	0.16	0.16	0.15	0.16	0.16
percentile	(0.13, 0.2)	(0.13, 0.19)	(0.13, 0.19)	(0.13, 0.19)	(0.13, 0.18)	(0.13, 0.18)	(0.14, 0.19)	(0.13, 0.19)
LC-training								
mean	0.15	0.16	0.15	0.16	0.12	0.2	0.14	0.22
95% CI	(0.1, 0.23)	(0.13, 0.19)	(0.09, 0.19)	(0.13, 0.19)	(0.04, 0.23)	(0.15, 0.24)	(0.05, 0.23)	(0.2, 0.24)
LC-testing								
mean	0.26	0.27	0.25	0.26	0.26	0.27	0.25	0.25
95% CI	(0.23, 0.33)	(0.22, 0.34)	(0.23, 0.28)	(0.2, 0.31)	(0.22, 0.31)	(0.22, 0.32)	(0.22, 0.29)	(0.22, 0.28)
LC-assumed								
mean	—	—	—	—	0.22	0.22	0.22	0.22
95% CI					(0.2, 0.24)	(0.2, 0.24)	(0.2, 0.24)	(0.2, 0.24)
MSRE-training								
mean	0.19	0.17	0.19	0.17	0.16	0.15	0.16	0.16
95% CI	(0.16, 0.22)	(0.14, 0.2)	(0.16, 0.21)	(0.14, 0.2)	(0.14, 0.18)	(0.12, 0.18)	(0.14, 0.2)	(0.14, 0.19)
MSRE-testing								
mean	0.26	0.26	0.26	0.26	0.25	0.26	0.26	0.25
95% CI	(0.22, 0.3)	(0.21, 0.31)	(0.22, 0.31)	(0.22, 0.31)	(0.21, 0.32)	(0.21, 0.32)	(0.21, 0.31)	(0.22, 0.29)
MSRE-assumed								
mean	0.19	0.17	0.19	0.16	—	—	—	—
95% CI	(0.16, 0.21)	(0.14, 0.19)	(0.15, 0.21)	(0.14, 0.19)				

Table A.12

Model comparison statistics from different joint models for the analysis of Penn Ovarian Aging data.

(a) Joint MSRE Model

Number of Mean Classes	**Number of Variance Classes** **DIC**			**LPML**		
	1	2	3	1	2	3
1	6908.0	**6854.1**	7049.0	-3786.4	-3781.5	-3780.9
2	6912.5	6862.1	7061.8	-3766.9	-3766.9	-3769.2
3	6990.3	6942.9	7134.6	-3770.5	-3766.5	**-3763.7**

(b) Joint LC Model

Number of Mean Classes	**Number of Variance Classes** **DIC**			**LPML**		
	1	2	3	1	2	3
1	6930.5	**6860.6**	7044.6	-3792.7	-3781.3	-3774.9
2	6925.9	6867.1	7015.6	-3777.1	-3763.4	-3752.0
3	6985.2	6939.0	7076.1	-3785.7	-3765.1	**-3750.5**

Table A.13

Posterior estimates of the model parameters under joint MSRE and LC models with $K_D = K_C = 2$ for the analysis of Penn Ovarian Aging data.

	MSRE Model			LC Model		
	mean	se	95% CI	mean	se	95% CI
β_{11}	-0.08	0.045	(-0.169, 0.009)	-0.087	0.046	(-0.179, 0)
β_{12}	0.003	0.037	(-0.07, 0.075)	0	0.036	(-0.073, 0.072)
β_{21}	0.159	0.072	(0.024, 0.306)	0.169	0.074	(0.029, 0.322)
β_{22}	0.213	0.058	(0.105, 0.332)	0.215	0.06	(0.104, 0.336)
ω^2_{11}	0.077	0.023	(0.036, 0.126)	0.08	0.023	(0.038, 0.128)
ω^2_{12}	0.045	0.019	(0.012, 0.085)	0.045	0.018	(0.014, 0.084)
ω^2_{21}	0.331	0.067	(0.225, 0.486)	0.329	0.069	(0.22, 0.488)
ω^2_{22}	0.155	0.037	(0.093, 0.237)	0.159	0.038	(0.096, 0.244)
ρ_1	0.934	0.047	(0.822, 0.983)	0.933	0.044	(0.828, 0.983)
ρ_2	0.46	0.127	(0.176, 0.671)	0.437	0.133	(0.14, 0.658)
π^D_1	0.521	0.086	(0.343, 0.682)	0.53	0.085	(0.35, 0.687)
μ_1	-2.707	0.155	(-3.026, -2.417)	-2.767	0.163	(-3.1, -2.459)
μ_2	-1.146	0.054	(-1.256, -1.046)	-1.168	0.054	(-1.277, -1.064)
τ^2	0.17	0.04	(0.104, 0.262)	0.191	0.043	(0.118, 0.287)
π^C_1	0.223	0.04	(0.146, 0.304)	0.21	0.039	(0.138, 0.291)
γ_0 (intercept)	-0.5	0.941	(-2.349, 1.353)			
γ_1 (log(BMI))	-0.051	0.28	(-0.603, 0.501)			
γ_2 (smoking)	0.375	0.185	(0.009, 0.741)			
$\gamma_3(b_{0i})$	-0.773	0.301	(-1.392, -0.211)			
$\gamma_4(b_{1i})$	0.611	0.435	(-0.205, 1.515)			
$\gamma_5(\sigma^2_i)$	1.679	0.603	(0.536, 2.897)			
θ_0 (intercept)				-1.227	1.016	(-3.26, 0.717)
θ_1 (log(BMI))				0.064	0.283	(-0.483, 0.62)
θ_2 (smoking)				0.346	0.19	(-0.027, 0.72)
θ_3 (D=2)				-0.094	0.344	(-0.777, 0.578)
θ_4 (C=2)				1.103	0.385	(0.479, 1.972)

Table A.14

Posterior estimates of the model parameters under joint MSRE and LC models assuming interaction effects with $K_D = 1, 2$ and $K_C = 1, 2$ for the analysis of Penn Ovarian Aging data.

	MSRE Model						LC Model					
	$K_D = 1, K_C = 2$			$K_D = 2, K_C = 2$			$K_D = 1, K_C = 2$			$K_D = 2, K_C = 2$		
	mean	se	95% CI	mean	se	95% CI	mean	se	95% CI	mean	se	95% CI
β_{11}	0.041	0.031	(-0.02, 0.102)	-0.078	0.045	(-0.167, 0.01)	0.039	0.031	(-0.022, 0.101)	-0.075	0.051	(-0.165, 0.019)
β_{12}	0.111	0.025	(0.061, 0.16)	0.006	0.037	(-0.068, 0.079)	0.109	0.025	(0.06, 0.158)	0.009	0.04	(-0.062, 0.084)
β_{21}				0.158	0.071	(0.025, 0.299)				0.156	0.075	(0.006, 0.309)
β_{22}				0.212	0.058	(0.102, 0.329)				0.201	0.061	(0.083, 0.324)
ω_{11}^2	0.2	0.023	(0.158, 0.249)	0.078	0.022	(0.037, 0.125)	0.201	0.023	(0.159, 0.249)	0.08	0.041	(0.039, 0.132)
ω_{12}^2	0.102	0.013	(0.079, 0.13)	0.045	0.018	(0.013, 0.085)	0.103	0.013	(0.08, 0.132)	0.039	0.021	(0.011, 0.081)
ω_{21}^2				0.334	0.067	(0.226, 0.492)				0.335	0.074	(0.215, 0.497)
ω_{22}^2				0.155	0.036	(0.094, 0.237)				0.171	0.04	(0.102, 0.259)
ρ_1	0.667	0.056	(0.549, 0.765)	0.935	0.041	(0.831, 0.983)	0.668	0.057	(0.549, 0.769)	0.913	0.08	(0.759, 0.979)
ρ_2				0.46	0.125	(0.185, 0.665)				0.473	0.135	(0.189, 0.693)
π_1^D				0.524	0.083	(0.352, 0.677)				0.525	0.086	(0.346, 0.687)
μ_1	-2.696	0.145	(-2.989, -2.422)	-2.696	0.149	(-2.999, -2.411)	-2.766	0.158	(-3.086, -2.467)	-2.772	0.165	(-3.1, -2.461)
μ_2	-1.134	0.051	(-1.239, -1.037)	-1.141	0.053	(-1.249, -1.038)	-1.158	0.055	(-1.269, -1.053)	-1.168	0.06	(-1.271, -1.064)
τ^2	0.16	0.037	(0.098, 0.24)	0.161	0.038	(0.098, 0.245)	0.185	0.042	(0.114, 0.278)	0.186	0.041	(0.117, 0.274)
π_1^C	0.227	0.039	(0.153, 0.307)	0.226	0.04	(0.152, 0.306)	0.212	0.039	(0.138, 0.291)	0.209	0.04	(0.138, 0.29)
γ_0 (intercept)	-0.471	0.945	(-2.331, 1.372)	-0.546	0.937	(-2.368, 1.284)						
γ_1 (log(BMI))	-0.064	0.282	(-0.611, 0.488)	-0.041	0.279	(-0.59, 0.506)						
γ_2 (smoking)	0.374	0.189	(0.004, 0.745)	0.377	0.186	(0.011, 0.744)						
$\gamma_3 (b_{i0})$	-0.949	0.492	(-1.946, -0.012)	-0.809	0.47	(-1.75, 0.072)						
$\gamma_4 (b_{i1})$	0.589	0.581	(-0.546, 1.71)	0.523	0.562	(-0.565, 1.656)						
$\gamma_5 (\sigma_i^2)$	1.644	0.596	(0.504, 2.857)	1.708	0.605	(0.548, 2.922)						
$\gamma_6 (b_{i0} \times \sigma_i^2)$	0.138	1.17	(-2.104, 2.442)	0.063	1.146	(-2.107, 2.338)						
$\gamma_7 (b_{i1} \times \sigma_i^2)$	0.644	1.329	(-1.925, 3.209)	0.361	1.324	(-2.21, 2.898)						
θ_0 (intercept)							-0.911	0.829	(-2.518, 0.679)	-1.429	0.999	(-3.548, 0.396)
θ_1 (log(BMI))							-0.026	0.239	(-0.487, 0.441)	-0.088	0.204	(-0.485, 0.306)
θ_2 (smoking)							0.334	0.184	(-0.022, 0.694)	0.383	0.204	(-0.009, 0.793)
θ_3 (D=2)										1.273	1.369	(-1.442, 4.093)
θ_4 (C=2)							1.038	0.362	(0.463, 1.805)	1.928	0.941	(0.527, 4.063)
θ_5 (D=2,C=2)										-1.618	1.484	(-4.607, 1.341)