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STRONG, R.

BIBLIOGRAPHY ON LAKE ICE

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of the requirements for the degree

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CONTENTS

FORMATION AND STRUCTURE OF LAKE ICE

Formation of Lake Ice

STRENGTH OF ICE

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VISCOSITY AND PLASTICITY OF ICE

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ABSTRACT

A bibliography on lake ice is compiled on the basis of a comprehensive search of the literature. The subject matter of the paper is presented in two parts: first, a complete alphabetical bibliography, and second, a series of individual bibliographies with descriptions and tabulations of data for the following subjects:

General
Formation and structure of lake ice
Strength of ice
Ice friction
Hardness of ice
Elastic constants
Viscosity and plasticity of ice
Ice breakers and ice breaking
Sources of bibliographic material

BIBLIOGRAPHY ON LAKE ICE

BY: Richard Strong

INTRODUCTION

The compilation of this bibliography was undertaken at the request of the United States Coast Guard and was financed by them. An intensive search of all pertinent literature was made using as guides the comprehensive bibliographies of Barnes (1928) and Dorsey (1940). Few titles are added to theirs and those which have been added were omitted by Barnes and Dorsey only because of later dates of publication. On the other hand, many of their references are omitted from this bibliography because they do not pertain to lake ice. In general, it is found that very few papers deal directly with lake ice. In many cases it is found difficult to draw a line between articles indirectly concerning lake ice and those concerning borderline subjects such as river, glacier, and sea ice. Often, the exclusion or inclusion of a title is purely arbitrary, although the objective throughout is to include any work which concerns itself directly or indirectly with lake ice in any aspect. Thus, much of H. T. Barnes' work on river ice is omitted. The early controversies of Tyndall, Forbes, Faraday, J. Thomson and W. Thomson (Lord Kelvin) concerning the structure and movement of glaciers are largely ignored. Similarly, later work on glacier ice is included only where it deals with such problems as viscosity and plasticity of ice in general. The problems of sea ice are completely omitted. It is hoped, however, that where borderline fields are somewhat slighted, sufficient references are included to serve as guides to more comprehensive bibliographies.

In the organization of this paper, the bibliographic treatment is emphasized. The text serves only to relate and evaluate the references and to present the more important data. Not all references were thought important enough to mention by name. However, these are included in the alphabetical bibliography for the sake of completeness.

The writer wishes to acknowledge an indebtedness to Dr. James T. Wilson of the Department of Geology, University of Michigan, for his assistance and encouragement during the preparation of this manuscript.

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This book contains no bibliography but a list of authors to whom reference is made is appended.

Contents

Chapter I: Physical laws governing the transfer of heat: Radiation. Surface emissivity and solar radiation. Conduction. Convection.

Chapter II: Physical constants of ice: Density. Maximum density of water. Heat of fusion or latent heat. Specific heat of ice. Specific heat of water and the thermal unit. Thermal conductivity of ice and snow. Thermal conductivity of water. Coefficient of expansion. Relative hardness or penetrability. Plasticity. Elasticity. Viscosity. Vapor pressure of ice and supercooled water. Electric properties.

Chapter III: Formation and structure of ice: Crystalline structure. Snow crystals. Structure of solid ice. Quincke's theory. Regelation. Spontaneous crystallization. Glacier motion. Modifications of ice. Supercooling of water.

Chapter IV: Sheet, frazil and anchor ice: Sheet ice. Rate of growth of surface ice. Frazil-ice. Artificial production. Anchor-ice. Historical. Distinguished from frazil. Radiation as the cause of anchor-ice formation. Ice floods in the St. Lawrence.

Chapter V: Precise temperature measurements: The platinum resistance thermometer. Mercury thermometer compared. Method of measurement. Refinement of apparatus. Sample results showing accuracy attained.

Chapter VI: River temperatures: Instruments used in investigations of river temperatures. First series of results under surface ice. Second series of tests at the Lachine Rapids. Details of the observations. General considerations. Freezing point diagram. Conditions which govern the formation of frazil and anchor-ice.

Chapter VII: Theories to account for frazil and anchor-ice: Early theories of Arago, Eisdale, and Farquharson. Later theories of Francis, Bell, Hunt, Henshaw and Lord Kelvin. Discussion of Henshaw's paper. Views of Keefer. Extract from article on anchor-ice from the Montreal Flood Commission Report. Depth of formation of anchor-ice in fresh and salt waters.

Chapter VIII: Methods of overcoming the ice problem in engineering work: Construction and situation of power works. When a surface sheet is valuable and when not. Vulnerable spots in a power house. Artificial heating and steam injection. Electric heating of racks. Effect of head on water. Effect of rapid fall in temperature conditions. Volume of ice formed by radiation. Erosive velocity of water and its probable effect on anchor-ice formation.

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This book is a revision of Barnes' earlier work. It contains a comprehensive bibliography, arranged by subjects, of about three-hundred seventy-five entries.

Contents

Chapter I: Equilibrium of the ice-water system. Colloidal ice. The crystal structure of ice.

Chapter II: Physical constants of ice: density, specific heat, hardness, elastic constants, heat of fusion, evaporation, sublimation, color and bacteriology.

Chapter III: Rate of growth and melting of surface-formed ice, infra-red absorption of ice, various kinds of ice.

Chapter IV: Theories of formation of anchor-ice, effect of light and heat on its disintegration.

Chapter V: Frazil ice. Winter ice floods.

Chapter VI: Ice remedial work. Use of steam, thermit, hot water, calcium chloride and other chemicals.

Chapter VII: Ice pressure and expansion.

Chapter VIII: Ice navigation and ice breaking.

Chapter IX: Conservation of heat in lakes and rivers for ice prevention, evaporation, power houses and power canals.

Chapter X: Glacier ice and icebergs.

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Due to the comparative obscurity of this comprehensive book, it seems advisable to include here the introduction and table of contents. The author appended a translation in French of the introduction and table of contents which was translated into English by the present writer.

INTRODUCTION

Ice is certainly one of the most important solid constituents of the earth's surface. Glaciers alone occupy an area of more than fifteen million square kilometers, which is three per cent of the total surface of the earth and ten per cent of the continental area. An entire continent, Antarctica, is covered with this rock-like substance which is both crystalline and sedimentary in nature. The frigid zones are covered with snow during the winter and snow forms a sort of ephemeral rock in the temperate zones and in the extensive polar regions which are not covered by glaciers. The polar expanses, on the other hand, remain frozen to a certain depth during the entire year, forming a conglomerate cemented by ice. Another sort of ice rock, comparable to the igneous rocks, forms the polar ice pack and chokes the rivers and lakes of the continents. The rivers of the temperate zone and especially those of the subpolar regions which remain unfrozen in winter due to turbulence and rapids transport detached masses of sandy-appearing ice and form spongy and often large deposits of anchor ice. Large expanses of white frost appear on the plains due to radiational cooling. Under certain meteorological conditions, objects may be coated with glaze or rime ice. Rime ice is developed in mountainous regions in large, rock-like masses which cover the sharp peaks and sometimes form small glaciers as the result of avalanches. Finally the atmosphere, within the limits of the troposphere, is always filled with finely powdered ice or with ice condensed in the form of crystalline fogs or in clouds of snow, sleet and hail.

It can be seen, then, that ice forms a sort of envelope around the earth, a cryosphere, which is very homogeneous as to composition but extremely varied in its aspect. This envelope is intimately related to the lithosphere, the hydrosphere and the atmosphere. It has upper and lower limits, the latter being very high over tropical regions but lowering gradually toward the polar regions where it penetrates below the surface. It varies in general with the seasons.

In contrast to the other solid constituents of the earth, ice is a separate entity, having exceptional thermodynamic and other properties. First, ice has an exceptionally low melting point which results in a variability in state unknown in the other solid constituents of the globe; it appears and disappears, continually increasing or diminishing in amount by absorbing energy or by producing it, by assimilating or producing water vapor or liquid water. Furthermore, in the change from the solid to the liquid state (or vice versa) the volume change is in reverse of the ordinary. Consequently, an increase in pressure lowers the melting point instead of raising it.

For these two reasons - wide distribution and singularity of behavior, ice plays a very important role in the economy of nature. It is the basis of a group of physical, mechanical and morphological phenomena, as well as the cause of a whole chain of variations, both megascopic and microscopic, in the atmosphere, in the water and in the soil.

In my opinion, therefore, the study of the cryosphere could and ought to form a separate science, a special branch of physical geography, to be known as cryography, of which glaciology would constitute a special phase concerned with the geology and history of the globe.

The varied mode of existence of ice; the influence that it exerts on meteorological, geological, oceanographical, hydrographical and biological phenomena; finally its specific properties as a rock and as a mineral have long since lead to studies of this substance by the most diverse specialists. Work has been done by mathematicians and astronomers, physicists and chemists, crystallographers and mineralogists, geographers and geologists, meteorologists and climatologists, biologists and also engineers and explorers. The results have been published in the reviews dedicated to the various specialties - in the publications of the academies or in reports on expeditions. Articles on ice are found in publications of all sorts. This makes it difficult to orient oneself, not only in the general field of ice research but also in regard to specific problems. It is often easy to ignore the general accomplishments and problems in the field of ice, and it is also easy to ignore the history of a specific problem. This aggravating state of affairs is disappearing as the study of ice continues and the number of published works increases.

In my opinion, two things would be of great usefulness to the various specialists concerned with ice and the phenomena which depend on it. First would be a monograph, as complete as possible, presenting a view of the whole subject by means of a detailed and systematic discussion of all problems relating to ice, with regard to their evolution as well as to their present state of development, and containing a bibliography and a complete iconography. Second would be an international publication which would group (systematically) the workers and their works, and by that means would give to the whole of the research a rational direction, and to the investigators, the possibility of reading the works of others and of being read by others.*

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*For this purpose, the Review of Glaciology, the publication of the International commission for the study of Glaciers, could be revived in expanded form.

The Historja Naturalna Lodu is an attempt to consolidate the research done in the field and in the laboratory on ice of all types and of all origins. It is a sort of index to the problems of the important yet little known subject of ice. It includes a history of each of the problems, the results of investigation, and the problems which are not yet solved or are questionably solved.

The book is divided into fifteen chapters. The first three discuss the conditions of formation and growth of ice; Chapter IV studies the crystallography of ice (symmetry, combinations, principal aspects, twinning, geometric constants, polymorphism); Chapters V, VI, and VII, atmospheric ice; Chapters VIII and IX, the freezing of calm water (with the formation of an ice sheet) and running water (anchor ice); Chapter X, ice in the soil; Chapters XI and XII, the snow cover; Chapters XIII and XIV, the glaciers; finally Chapter XV, halo phenomena. The thermodynamic constants are treated in Chapters I and II; the electrical and magnetic properties as well as the acoustical peculiarities are contained in Chapter XI; the optical properties in Chapters XI and XV; and the mechanical properties in Chapter XIV.

I have not treated all questions with the same importance. I have not emphasized those which depend only in an indirect manner on the specific properties of ice nor those which already have been treated in excellent detailed and well-known monographs. Therefore, in trying to discuss completely the conditions of formation of atmospheric ice (Ch. III), I have restricted the discussion of hail to the essential points of the problem, to methods of study and above all to the most recent theories, especially that of A. Wegener; I have substituted as complete a bibliography and iconography as possible in place of a discussion of the more obsolete theories. I have also cited the works containing an historical review of the problem. Similarly, in the discussion of the snow cover, I have treated the physical and morphological properties in a very detailed fashion, but I have not dwelt at all on the times of accumulation and melting of snow since the study of these phenomena falls within the domain of the climatologist; I have treated similarly and for analogous reasons the layer of ice covering the water; I have omitted the special subject of icebergs, which is very important to the polar scientist but which is not of particular interest in regard to the properties of ice. Furthermore, (a discussion of) the interesting properties of icebergs will be found in various chapters of the book in the form of examples and illustrations pertinent to the general discussion. Concerning glaciers, I have ignored the climatological and geological problems by omitting the glacial epochs and by limiting myself to a citation of the classical works in which the student will find complete discussions of these problems together with their bibliographies; on the other hand, I have considered at length the structure and movement of glaciers since these phenomena are a direct expression, on a large scale, of the mechanical, physical and structural properties of ice in general and of glacier ice in particular. In this discussion I have tried to analyze the basic premises of the various theories of recrystallization and movement of glaciers. I have excluded from Chapter XV treating the halo phenomena, the fundamental mathematical deductions as being merely applications of the general formulae of reflection of light to the special problem of ice crystals.

This theoretical background is discussed in several excellent monographs. On the other hand, I have tried to give a detailed statement and an exhaustive discussion of the physical bases of the halo theory as it is related to the physical properties of ice and to the form and structure of atmospheric crystals. Finally, I have mentioned only occasionally the morphology of clouds since this depends very little on the specific properties of ice.

Nearly all the questions relative to ice are treated in conjunction with the more general problems of which they constitute particular cases. For example, the subject of the supercooling of water is preceded by a resume of the problem of supercooling in general (as investigated by Tammann); the theory of formation of dunes par obstacle is accompanied by a resume of modern hydrodynamical theory regarding an obstacle in a current (Karman, Prandtl); etc. These various additions have strongly augmented the scope of the present work, and in my opinion have added to its value: considered in the light of the more general problems, questions relating to ice have acquired the space and importance which is due them and thus have been clarified.

During the writing of Historja Naturalna Lodu, I have carried on some original research, the results of which appear in the present book. The problems investigated were: (1) the origin of the structure of 'heavy' and 'light' sleet (Frostgraupeln and Reifgraupeln of A. Wegener) Chapters III and VII; (2) the influence of a surface of refrigeration on the orientation of ice crystals forming on that surface, Chapters VII and VIII; (3) the general crystallography of ice (symmetry, crystal habit, twinning), Chapter IV; (4) certain questions of the special crystallography (unequal growth in the direction of the principal axis, systems of internal cavities, radial aggregations of crystals in the form of rods and plates), Chapters V and VII; (5) the classification of ice rocks, Chapter XI; (6) the recrystallization of a layer of ice formed on a water surface, Chapter VIII; (7) the mechanical theories of dunes par obstacle, Chapter XII; (8) the fundamental bases of the various theories of the glacier grain. Chapter XIII; (9) a critical review of modern theories of glacier movement, Chapter XIV; (10) the physical basis of the halo theories, Chapter XV.

In order that the present work, which is to be published in Polish, be of value to foreign students, I have appended a table of contents in French which, in my opinion, is sufficiently detailed to serve. The important thing is the understanding of the different problems which relate to ice and a knowledge of the authors and their works. The table of contents comprises a detailed index to these problems. With the aid of this table, it is easy to find the pages on which the problems of particular interest are treated. Printed there in bold-faced type are the names of the authors who have investigated each special problem and whose works appear in the bibliography at the end of each chapter. These bibliographies include, in general, only works on ice and on associated phenomena. Works cited in the text having only an indirect connection with ice are mentioned on the spot in the form of a reference or in parentheses following the name of the author. In those cases where it would have been necessary to cite a large number of such works, I have inserted them in a special section of the bibliography of the chapter. I have done the same for those problems which possess a very important literature. Hence the bibliography added to Chapter III contains two separate sections: Grad (hail) and Pieczary lodowe (ice grottoes), and that of Chapter IV is divided into two sections: Bibljografja dolyczaca lodu (bibliography relating to ice) and Bibljografja bezposrednia lodu nie dotyczaca (bibliography not relating to ice). On the other hand, for the two chapters on glaciers (XIII and XIV), I have given only a single bibliography which is placed at the end of Chapter XIV.

The manuscript of the present work was completed in 1916 but could not be published until May 1923. In the intervening period I have been occupied with other work and was able to proofread and revise the text and bibliography during the printing of the book, which was completed in May 1923. For this reason certain works appearing after 1916 have been omitted.

Table of Contents

Dorsey, N. E. (1940), Properties of ordinary water-substance in all its phases: water vapor, water and all the ices, Reinhold Publishing Corp., New York: 673 pp., 13 figs., 289 tables.

Over four-hundred references are given as footnotes. The following abstract of the table of contents includes only those sections of the book dealing with ice.

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Section IIC. Ice.

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    - Crystalloluminescence.

98. Transition of ice to ice.

FORMATION AND STRUCTURE OF LAKE ICE

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Forms of Ice

Eight forms of ice have been described, including the most common, ice-I, a low-temperature vitreous form (see Dorsey, 1940: p. 395) and six other varieties which have been reported by Bridgman (1912). Dorsey describes ice-I as crystalline, having a conchoidal fracture and vitreous luster. Barnes (1928) states that the color of natural ice is a vivid blue, the depth of blueness being a measure of the purity. The blue color is probably caused by scattering by ice molecules which are large with respect to the wavelength of blue light.

Crystallographic Data

Ice is commonly referred to the hexagonal system (Hess, 1904; Tarr and Rich, 1912; Barnes, 1928: etc.) However, many different interpretations have been placed on the results of x-ray analyses of ice. Seljakov (1936A, 1936B, 1937) has reported two forms of ice-I, which may resolve some of the differences observed. He describes α ice as a hexagonal form which is stable from 0° to -183°C (see W. H. Barnes, 1929) and β ice as a rhombohedral form which develops from supercooled water. W. H. Barnes (1929) has obtained dimensions for the unit cell of ice-I which he refers to the dihexagonal bipyramidal class or the ditrigonal bipyramidal class (probably the former): $a = 4.53_5$ Å; $c = 7.41$ Å; $c/a = 1.634$; cell contents - 4 H_2O molecules. Dorsey states that these dimensions are believed to be correct within a few parts in a thousand.

Optical Data

Ordinary ice is uniaxial and positive. The indices of refraction for sodium light (5893 Å) are $\omega = 1.3090$ and $\epsilon = 1.3104$. (See Dorsey, 1940, pp. 484-485.)

The Structure of Lake Ice

Lake ice commonly exhibits a banded or stratified structure in cross-section, with alternate bands of clear glassy ice and of more or less opaque ice containing numerous air bubbles. (See Silliman, 1821; Faraday, 1860; Tyndall, 1858B; Bentley, 1907; Barnes, 1906; Tammann and Dreyer, 1934; etc.) The stratification is expressed in various ways and is thought by Bentley (1907) to be caused by variations in the rate of freezing. He describes the bubbles as being tubular and giving a milky white appearance to the ice. According to Bentley the ice which freezes most rapidly contains the greatest number of air tubes and the layers containing the greatest concentration of tubes usually lie in the upper half of a given section of ice. Dieke (1864) and Koch (1913) state that the air tubes are always normal to the freezing surface. Silliman (1821) describes a 15 inch layer of pond ice which contained

21 layers of decreasing thickness downward. He states that the air bubbles were concentrated at the top of each layer. His explanation is that nocturnal cooling causes dissolved air to be expelled from the water and that the resulting air bubbles accumulate at the bottom of the previous layer. The water then freezes, forming a more or less clear layer below the air bubbles. The pond ice used by Tarr and Rich (1912) in their experiments was 30 cm thick with the top 10 cm composed of finely granular ice, the next 15 cm composed of coarsely crystalline ice with prismatic crystals standing perpendicular to the water surface, and the bottom 5 cm composed of finely granular ice with diversely oriented cyrstals. Wilson and Horeth (1948) state that ice from Lake Michigan usually had a cloudy zone as much as 2 inches thick at the top of a layer 1 foot thick. Koch (1913) describes lake ice used by him in Labrador for elasticity experiments. One piece of ice was 76 cm thick. The upper 36 cm was composed of alternate thin layers of ice with air bubbles and tubes, and of clear ice. The lower 40 cm was completely air-free. Koch also describes ice from Lago della Crocetta which was 85 cm thick and of which only the lower 10 cm was free of air bubbles. Vitman and Shandrikov (1938) imply in the manner of tabulation of their data that the lower layer of the ice used by them was usually porous, white and opaque and that the upper layer was devoid of air bubbles.

Except for the statements of Silliman and Bentley, few investigators have attempted to explain the stratification of lake ice. The association of high rates of freezing with concentration of air bubbles in the uppermost layers is consistent with the theory, to be described in more detail, that freezing proceeds more rapidly below a thin layer of ice than below a thick layer. The writer suggests that there exists a critical diurnal range of temperature (in the case of the stratified ice) such that the flow of heat outward from the water (or ice) surface occurs only at night or during alternate time periods of any (fairly short) length so that freezing occurs intermittently. Under such conditions, if Silliman's idea concerning the expulsion of dissolved water is correct, a laminated structure could result. In any case, it is difficult to explain the laminated structure on the basis of continuous freezing.

The presence of air bubbles in ice probably affects its physical properties but no systematic investigation of this effect was found in the literature.

The orientation of ice crystals in lake ice, or in any ice formed on unagitated water, is easily determined. There is almost complete agreement on a uniformly vertical orientation of ice crystals in sheet ice. (See Brewster, 1834; Trouton, 1899; Barnes, 1906, 1910, 1928; McConnel, 1889; Yoshida and Tsuboi, 1929; Megaw, 1934; Tammann and Dreyer, 1934; Dorsey, 1940; and many others.) The orientation can be determined by means of a petrographic microscope or by means of a pair of polarizing prisms. A phenomenon first described by Tyndall (1858A) and named "Tyndall's flowers of ice" after him is also readily employed for the same purpose. Tyndall's flowers are described by Barnes (1906, 1928) and Dorsey (1940). When a beam of light is passed through a piece of ice, water bubbles appear which have the form of flowers with six coplanar petals. Their appearance is accompanied by a distinct clicking noise. The petals lie in a plane which is normal to the optic axis of the associated crystal and each of the petals corresponds to a secondary hexagonal axis. Hawkes (1930) suggested that the nucleus of each flower is a dust grain which is able to absorb heat and melt the surrounding ice when the beam of light is passed through the ice. The clicking noise was explained by Tyndall on the basis of the sudden contraction of the water as the ice melts. In a given sample of lake ice, the flowers lie in parallel horizontal planes, thus indicating the uniformly vertical orientation of the crystals.

The unique orientation of ice crystals described above is ascribed by Barnes (1910) and Trouton (1899) to the fact that the thermal conductivity of ice is greater along the major axis than in directions perpendicular to it. According to Trouton the ratio of the conductivities parallel to and normal to the major axis is 22 to 21. This is apparently sufficient so that the crystals orient themselves with the long axis parallel to the direction of flow of heat. Yoshida and Tsuboi (1929) conclude their paper by saying that "all directions parallel to the basal pinacoid of hexagonal ice crystals are equally suited for growth, which occurs more readily in these directions than perpendicular to the basal pinacoid. Ice formed on the surface of calm water exposed to cold air on a clear night is usually composed of monocrystals of considerable size with basal planes nearly parallel to the surface." Thus the direction of greatest thermal conductivity appears not to be related to the directions of most rapid growth.

Bentley (1907) describes six types of ice crystals. His classification is based largely on shape. The types are the lanceolate, the discoidal, the solid hexagonal, the flower-like, the spandrelliform, and the coralline. The needle-like lanceolate crystals are invariably the first to form when water begins to freeze. All six forms can coexist and most forms pass through scallop and branch-like stages before assuming their characteristic shapes. In thick masses, ice seldom exhibits perfectly hexagonal crystals. Instead, the crystals are rounded or of irregular shape.

The size of the component crystals has not been investigated thoroughly. Tarr and Rich (1912) describe pond ice crystals which are from 1 to 1-1/2 cm in diameter and from 5 to 15 cm in length. Wilson and Horeth (1948) noted few crystals larger than 1 inch in diameter in Lake Michigan ice but the crystal diameter was always largest near the bottom of the layer. McConnel and Kidd (1888) describe ice from St. Moritz lake which was built up of vertical columns. The columns were a centimeter or less in diameter and a foot or more in length (equal to the thickness of the ice). They state that each column was a single crystal and that the optic axis was horizontal. The latter condition seems unlikely.

Ice crystal diameters can easily be measured under polarized light. Further information concerning crystal size can be obtained by means of an etching process. Ice surfaces can be etched by application of infra-red (or ordinary) light so that the crystals are outlined and stand out from the surface in low relief. The phenomenon was first described by Hugi (1830-1) and was subsequently recognized and explained by Buchanan (1877 and quoted by Hawkes, 1930) on the basis of differential absorption of heat by intercrystalline liquid brine. The presence of this intercrystallic material has been demonstrated for sea ice and also for ice formed from very dilute salt solutions. Data are shown for water originally containing 7 parts per million of Cl (as NaCl). Under these conditions, liquid brine is still present between the ice crystals at temperatures as low as -19°C. The ultimate freezing point of such brine is -21.72°C. Tarr and Rich (1912) state that eutectic proportions of 23.5 percent NaCl and 76.5 percent H_2O are reached at approximately -22°C (the cryohydric point). The remaining liquid crystallizes in the eutectic proportions and the resulting mixture of salt and ice crystals is termed the cryohydrate. The ice which forms initially is formed from pure water. Hence, the residual solution is enriched in dissolved material. This intercrystallic brine becomes more and more concentrated until the eutectic proportions are reached. By these arguments, ice near the freezing point must contain some liquid brine in equilibrium with the ice. Buchanan explains the etching of ice crystals on the basis of the greater absorptive power of the brine, which results in melting at the crystal boundaries. Plyler (1925-1926) subsequently found that the intercrystalline layers have strong absorption bands in the infra-red spectrum, which substantiates Buchanan's theory. In these same arguments also lies the explanation for the formation of "rotten" ice in the spring. Rotten or "candled" ice is observed on any body of water covered with ice as soon as the ice is exposed to the sun at temperatures near the freezing point. Such ice appears solid but actually is not because the intercrystalline eutectic mixture, which serves as cement, is melted.

Formation of Lake Ice

Growth of an ice sheet:-

A quiet body of water, such as a lake or pond, when exposed to cold air will be cooled gradually at its surface, and convection currents will distribute the chilled water throughout the lake or pond until a uniform temperature of 4°C is reached. At this temperature water has its maximum density; further cooling is not accompanied by convection. Dorsey (1940) describes a convective cell in the atmosphere directly above the water surface, which is the mechanism for the transport of heat away from the water. The heat conductivity of soil and rock is so small that it is a minor or even negligible factor in the cooling process, except perhaps in very shallow water. Hence, loss of heat is more or less restricted to conduction and radiation to the overlying atmosphere. The importance of atmospheric convection is emphasized by Dorsey (1940) who states that air moving across a body of water gains heat and tends to rise. Thus a convective cell is initiated and a continual supply of cool air moves across the water. The temperature gradient and therefore the flow of heat are at all times directed upward. However, the normal air masses present during the time when freezing ? occurs are relatively dry. Water vapor is necessary for the absorption of radiant energy. It would seem, therefore, that convection currents would have to be maintained by conduction from the water surface to the air. The insulating properties of air are well known and the writer doubts that conduction plays a major role in the cooling process. Devik (1944) has shown the relative importance of the various cooling processes. Assuming an air temperature of -10°C and a water temperature of 0°C, the heat loss by infra-red radiation is 13.8 cal/cm^2/hour with clear sky and 4.9 cal/cm^2/hour with cloudy sky. Under the same conditions, convection (caused by conduction from water to air) will result in a heat loss from the water of 2.8 cal/cm^2/hour with no wind, and 11.5 cal/cm^2/hour with a wind of 5 m/sec. Consideration of the heat loss due to evaporation assumes that the latent heat of vaporization is furnished entirely by the water. This is approximately true since the specific heat of water is approximately 4 times that of air and the thermal conductivity of water is approximately 20 times that of air. Devik states that the loss of heat due to evaporation, if the vapor pressure is 2.6 mm Hg and other conditions are as above, is 1.7 cal/cm^2/hour if the air is calm and 7.7 cal/cm^2/hour if the wind speed is 5 m/sec.

The formation of an ice sheet is described by Barnes (1928), Hess (1904), Tammann and Dreyer (1934), Dorsey (1940), Parsons (1940), Devik (1944) and others. The shallow water at the edge of a lake is cooled to the temperature of maximum density before the deeper water near the center of the lake. Therefore, ice formation begins at the shore. Needles of ice (lanceolate form) shoot out from the shore and gradually enlarge until the surface is covered. From this time on, growth is accomplished by thickening. The extent of the initial ice sheet is controlled by the amount of water surface which has been cooled. If the entire surface of the lake is supercooled more or less simultaneously, the sheet of ice extends rapidly over the surface. In large bodies of water, such as the Great Lakes, it is doubted that supercooling reaches the center of the surface during a normal winter. The only record of complete freezing of one of the Great Lakes was given by Root (1944). However, no records are kept of the extent of the ice cover of the Great Lakes from year to year so that complete information on this subject is lacking. (See Kerry, 1947.)

The orientation of the initial ice needles is in question. If, as Yoshida and Tsuboi (1929) suggest, directions parallel to the basal pinacoid are most favorable for growth, detached crystals will become plate-like and will float with their optic axes vertical. However, Klocke (1879) has observed that with rapid and severe chilling the first needles are formed with optic axes horizontal. Subsequent growth occurs in the normal manner with the formation of crystals with vertical optic axes. When the first thin layer of ice becomes continuous, further growth is in the nature of thickening, either by freezing of water directly below the ice or by agglomeration of ice crystals on the under surface (Barnes, 1929). Accumulations of snow on the top of the ice also accomplish an increase in thickness.

The rate of growth of lake ice:-

The rate of thickening of a sheet of ice is controlled by the air temperature, the effectiveness of radiation and conduction in removing heat, the presence of a snow cover on the ice, evaporation from the ice surface, and other factors. Barnes (1928) states that radiation ceases when the ice forms. However, the thermal conductivity of ice is high (0.0057) and it is thought that radiation not from the underlying water surface but from the surface of the ice is an effective cooling process. However, it is apparent that the cooling of the water takes place largely by means of the conduction of heat through the ice, regardless of the manner in which the heat is disposed of from the ice surface.

Various empirical relationships have been established, from which the rate of thickening can be determined. The following notation will be used in presenting such formulas:

L = heat of fusion = 80 cal/gm

ρ = density of ice = 0.9166 gm/cm^3

K = thermal conductivity of ice = 0.0057 cal/sec·cm·°C

θ = temperature of the ice surface in °C

E = ice thickness in cm

t = time in seconds

C = an experimentally determined constant.

Vedel (1895) derived the following formula:

$$E = \sqrt{-Ct\theta}$$

where C is an experimentally determined constant and is of the order of 10 for cgs units.

Tamura (1905) established a formula which takes into account more definitely the physical properties of ice:

$$E = \sqrt{-\frac{2Kt\theta}{L\rho}}$$

Barnes (1928) derived a useful formula for calculation, on the basis of analysis of observed data:

$$E = -\frac{1}{4} - \sqrt{\frac{1}{16} - \frac{tK\theta}{L\rho}}$$

Barnes' and Tamura's formulas can be compared on the basis of the time necessary for the formation of a layer of ice E cm thick:

$$t_{\text{Barnes}} - t_{\text{Tamura}} = \frac{L\rho}{K}\frac{E}{\theta} = 0.000155\,\frac{E}{\theta}$$

The difference in results is largest for large thickness and for temperature near 0°C. The rate of thickening as derived from Tamura's formula is:

$$\frac{dE}{dt} = \frac{K}{L\rho}\frac{\theta}{E}$$

Therefore, the rate of thickening is greatest for low temperature and for thin ice, as might be expected.

Devik (1944) has shown that when a sheet of ice is thin, the temperature gradient through it is steep. The heat loss is great and the thickness increases rapidly. When the ice has attained considerable thickness, the temperature gradient is much smaller and the rate of thickening is decreased. If the ice is covered by a snow layer, the temperature gradient in the ice is a minimum, since the snow acts as an insulating layer.

Barnes (1928) also gives a more refined development of formulas for the rate of thickening which attempts to take into account the effect of convection currents in the water, evaporation from the ice, and the effect of radiation.

Table I was taken from Dorsey (1940) and was computed by means of Barnes' formula.

TABLE I

The Rate of Thickening of an Ice-Sheet

(From Dorsey)

The body of the table comprises values of <u>t</u>, the time necessary to form a layer of ice E cm thick at a surface temperature of θ.

θ E	-5	-10	-20	-30	-40
1	64 min	32 min	16 min	11 min	8.0 min
2	2.9 hr	1.4 hr	43 min	29 min	21 min
10	1.79 da	21.4 hr	10.7 hr	7.1 hr	5.4 hr
15	3.80 da	1.90 da	22.8 hr	15.2 hr	11.4 hr
20	6.55 da	3.28 da	1.64 da	26.2 hr	19.7 hr
30	14.29 da	7.15 da	3.57 da	2.38 da	1.79 da
60	55.4 da	27.69 da	13.85 da	9.23 da	6.92 da
90	123.3 da	61.6 da	30.8 da	20.6 da	15.4 da

Metamorphism of an ice layer:-

The term "metamorphism" is used loosely in this connection. The development of rotten ice during the spring is well known, but apparently there are earlier changes in the physical structure of ice. Dobrowolski (1923) includes a sub-section on the metamorphism of an ice layer in chapter VIII. It was not possible to have this translated so correlation of his statements with those made here must remain a subject for future investigation.

Barnes (1929) has stated that surface ice becomes coarser with age, the larger crystals growing at the expense of the smaller ones. Bentley (1907) discusses the structure of old ice more thoroughly. He states, "When closely examined, old ice, as a result of slight internal melting, or of changes of structure due to its being repeatedly subjected to cold and changes of temperature, often reveals traces of its former open crystalline 'pre-solid' character. Such old ice presents faint evidence of a cellular or honey-comb-like structure, the cell walls being mainly normal to the surface of the ice." Bentley attributes the formation of long slender air tubes to the aging process also. He reports that the air tubes "are arranged perpendicularly to its (the ice's) surface, but ofttimes parallel to and at the lines of intersection of two or more of the faintly outlined cell walls. ...the main cause, in most cases, must be attributed to the fact that such ice sheets undergo lateral expansion and contraction during and subsequent to solidifying. Such internal stresses tend to squeeze the air into the ice along the lines of fracture and of least resistance, i.e., into the so-called cell walls, or into their points of intersection."

This is the only detailed statement concerning changes in layer ice subsequent to formation which was found in the literature (with the exception of Dobrowolski).

STRENGTH OF ICE

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Description and Tabulation of Experimental Results

Crushing strength (strength in compression):-

Determinations of the strength of ice in linear compression are summarized in Table II. It is seen that complete descriptions of the conditions under which the determinations were made are usually lacking. The average of all the determinations listed in this table is 625 psi for those with the load applied parallel to the crystallographic axis but without regard for other sources of variation, and 607 psi for those made with the load applied perpendicular. This 1.5 percent variation is probably smaller than the error introduced by ignoring the possibility of other controlling factors than the direction of application of the load.

Brown (1926) states that the rate of application of the load and the temperature both affect the magnitude of the crushing strength. His measurements were made as nearly as possible at a constant rate of loading (1000 pounds per 2 seconds). His results show a significant variation with temperature. Similarly the results of Vitman and Shandrikov (1938) indicate increasing strength with decreasing temperature. The rate of loading is not stated and their results differ materially from those of Brown. (See figure 1.)

Barnes (1914) reports that "the only effect of varying the position of the axis of the ice with respect to the direction of the pressure appeared to be the way the block burst." There is, however, some consistency in the relationship between the average values, with those measured parallel to the crystallographic axis being slightly higher (Barnes, 1914 and Bell, 1911). This variation is small and deviations from the normal are large for individual specimens.

With regard to the effect of temperature, the results of Brown (1926) and Vitman and Shandrikov (1938) have been cited previously. Tarr and Von Engeln (1915) state in their conclusions that "variations of temperature within limits between 10° and 25°F do not seem to exert a notable effect on the crushing strength of the ice." It is possible that the slow and variable rate of loading employed in their measurements obscured the effect of temperature. Barnes (1928) stresses the relationship between temperature and crushing strength although he points out that the increase in hardness observed by many may be only a reflection of the increase in viscosity with decrease in temperature. He comes to no conclusion and states only that further experimentation is desirable. The article by Weinberg (1929), which was not examined, would undoubtedly be pertinent in this connection.

As indicated by Vitman and Shandrikov (1938), the structure and temperature history of the ice exert a dominant influence on crushing strength, with clear young ice being stronger than old porous ice.

Weinberg (1936) gives a summary of determinations, mostly by Russian investigators. These values have been corrected to a temperature of -3°C according to Weinberg's method (Weinberg, 1929). The data are reproduced in Table III. The sources of these data, which for the most part have not been examined, are listed in the original reference.

Sharp (1947), in discussing the suitability of ice for aircraft landings, uses an average value for crushing strength in theoretical calculations of the thickness of ice necessary to support a given load. Although there may be some question as to the applicability of crushing strength in this sense, his results for river, lake and sea ice should stand in correct relation to each other. He finds that the required thickness of young sea ice for a given load is about three times that of river ice, with lake ice being intermediate.

TABLE II

Summary of Measurements of Compressive Strength of Ice

Crushing Strength psi	Temperature °F	Direction of Force with respect to optic axis	Source of Data	Type of Ice
356	32	normal	Barnes (1914)[1]	River ice
370	32	parallel	"	" "
590	25 to 40	normal	Bell (1911)[2]	Artificial ice
659	25 to 40	parallel	"	" "
363	32	?	Barnes (and McKay) (1928)	River ice
300	28	normal	Brown (1926)	River ice
693	14	normal	"	" "
811	2	normal	"	" "
400	?	?	Barnes, Hayward, and McLeod (1914)	Assumed to be an average value.
568	18(?)	?	Romanowicz and Honigman (1932)[3]	Artificial ice
611	"	"	"	" "
626	"	"	"	" "
198	32	?	Tarr and Rich (1912)[4]	Pond ice
1000	18 to 20	parallel	Tarr and Von Engeln (1915) and Von Engeln (1915)	Pond ice
350	" "	normal	" "	" "
312	?	?	Koechlin (1944)	?
308	above freezing	?	Moseley (1870)	?

TABLE II

(continued)

Crushing Strength psi	Temperature °F	Direction of force with respect to optic axis	Source of Data	Type of ice
355	?	?	Hess (1904)	?
1800	11	parallel	Finlayson (1927)	River ice
1050	11	normal	"	" "
700	?	parallel	Vitman and Shandrikov (1938)	River ice (?)[5]
640	?	normal	"	River ice (?)[6]
244	?	parallel	"	River ice (?)[7]
314	?	parallel	"	River ice (?)[8]
227	?	parallel	"	River ice (?)[9]
473	?	parallel	"	River ice (?)[10]
318	?	parallel	"	River ice (?)[11]
650	?	parallel	"	River ice (?)[12]
298	?	parallel	"	River ice (?)[13]
559	?	parallel	"	River ice (?)[14]

Remarks:-

[1] Blocks were heard to crack at a pressure approximately one half the ultimate crushing force.

[2] One of Bell's values was high enough (1128 psi) so that he considered it to be unrepresentative and ignored it in computing the average.

[3] 7-cm cubes were loaded at the rate of 43 psi/sec.

[4] These data are considered unreliable by the authors due to the conditions under which the measurements were made.

[5] Average of 14 values.

[6] Average of 9 values. Young clear ice with numerous air bubbles.

TABLE II (continued)

[7] Average of 13 values. Samples taken from lower, extremely porous layer.

[8] Average of 11 values.. Samples from upper, less porous layer. The ice was dark-colored but almost transparent.

[9] Average of 12 values. Samples from lower, porous layer. Light-colored opaque ice.

[10] Average of 11 values. Samples from upper, less porous layer.

[11] Average of 11 values. Lower, porous layer.

[12] Average of 10 values. Upper layer, absolutely devoid of cavities.

[13] Average of 10 values. Lower, very porous layer.

[14] Average of 10 values. Middle layer, entirely non-porous, coarsely crystalline.

TABLE III

Compressive Strength of Ice (after Weinberg)

	Upper layers						Lower layers					
θ	parallel			normal			parallel			normal		
Authors	n	median	mean	n	median	mean	n	median	mean	n	median	mean
Bezsonov (1923)	2	-	735	1	-	219	11	907	907	7	282	286
Pineghin (1923)	(11)	-	371	(11)	-	307	(22)	-	459	(22)	-	381
Pineghin (1924)	16	296	482	11	333	340	25	472	508	22	310	411
Sergheev (1928)	2	-	523	2	-	351	4	476	481	4	228	234
Arnold-Alabieff (1929)	-	-	-	11	539	550	-	-	-	32	430	442
Ice Station (1930-36)	11	478	430	11	407	376	28	566	587	22	275	272
Sergheev (1928) dir. of axis unknown	-	4	232	259	-	-	-	8	185	448	-	-

θ is the orientation of the force with respect to the crystallographic axis.
Values of crushing strength given in pounds per square inch.
n is the number of observations (values in parentheses are assumed).

TABLE III

(continued)

Authors	n	median	mean	n	median	mean
Vasenko (1899) (natural ice)	3	367	316	1	-	465
Makarov (1911) (artificial ice)	8	533	566	-	-	-
Bell (1911)	6	670	674	2	-	626
Barnes (and McKay) (1928)	7	280	311	9	306	300
Krayger (1922)	7	867	2091	8	597	1070
Finlayson (1927)	7	-	1920	1	-	112

	n	median	mean
Moseley (1870)	1	-	309
Vasenko (1899) (artificial ice)	13	375	326
Gzovsky (Barnes 1928)	(10)	-	209
Mees (Barnes, 1928)	(1)	-	401
Frühling (Vasenko, 1899)	(10)	-	148
Bell (1911)	3	-	882
Von Engeln (1915)	3	-	933
Katanskij (1917)	2	-	1004
Brown (1926)	(15)	-	483
Ludlow (Barnes, 1928)	(20)	-	398

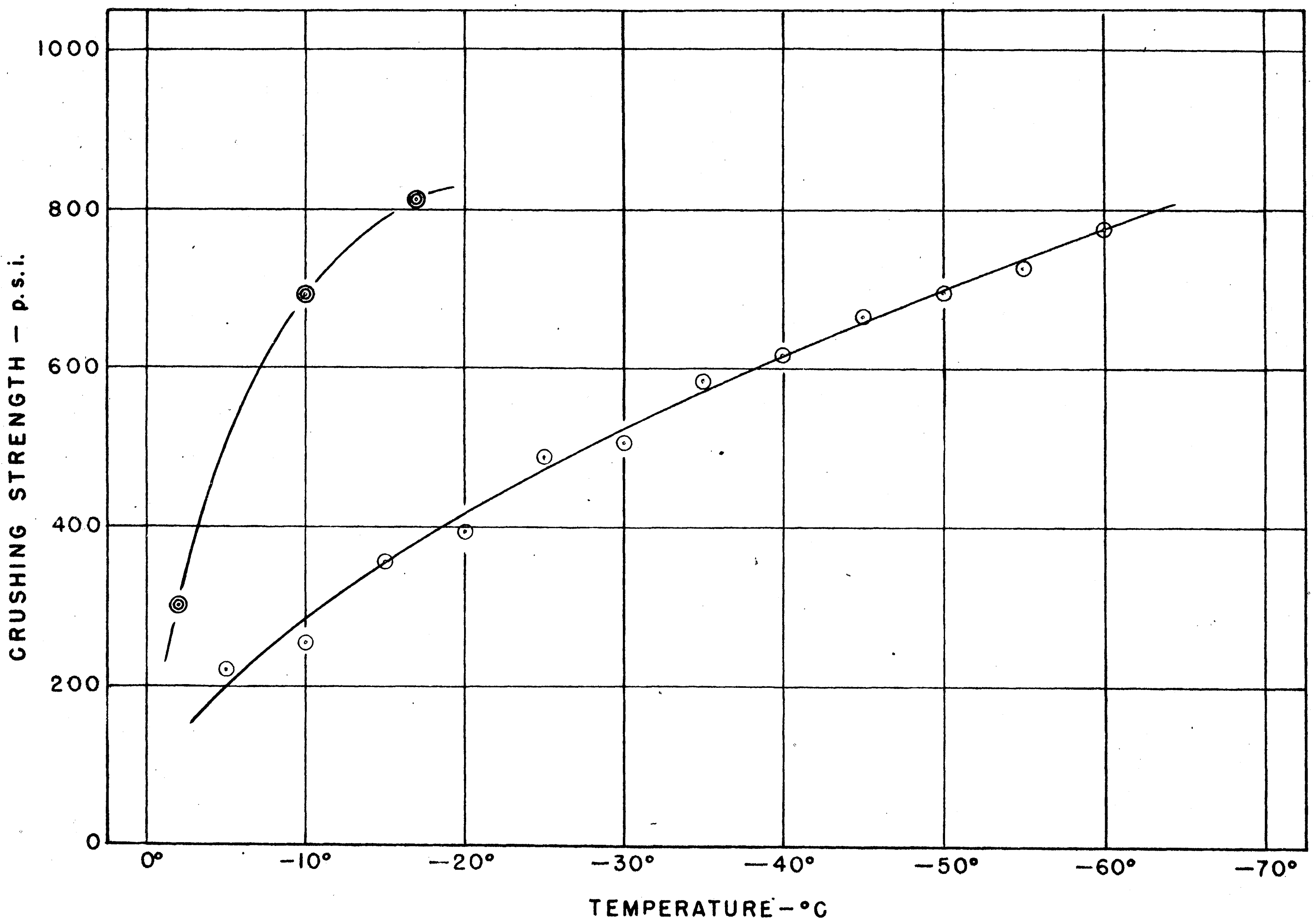

Figure I - Relation between crushing strength and temperature. ◎ — Brown (1926).
⊙ — Vitman and Shandrikov (1938)

Tensile strength:-

Measurements of the tensile strength of ice are rare and recorded experimentation has been somewhat haphazard. Dorsey (1940) makes the following observations concerning tensile strength: "The tensile strength of ice may be expected to depend upon the structure of the ice and the direction of the line of stress with reference to the optic axis of the crystal, or crystals, of which the specimen is composed."

Reusch (1864) gives 967 psi as the maximum tension at the moment of breaking of a bar supported at the ends and loaded at the middle. The temperature at the time was a few degrees above freezing. Moseley (1870) made a series of measurements at temperatures of 70° to 75°F. The average of eight determinations is 99.3 psi. Hess (1904) and Koechlin (1944), who is probably quoting from Hess, give 100 to 114 psi as the tensile strength. The conditions of the determination are not given. Barnes, Hayward, and McLeod (1914) state that the tensile strength of ice is probably less than 200 psi. Finlayson (1927) tested samples of artificial ice frozen in cement testing briquette molds and obtained the values 103 psi for water ice and 136 psi for ice formed from a mixture of snow and water. The observed variation from the average value was small. Barnes (1928) quotes the American Civil Engineers Handbook as giving 142 to 223 psi for tensile strength. Romanowicz and Honigman (1932), working at -8°C, obtained the three mean values 229, 260 and 251 psi. The rate of loading was 1.42 psi/sec. (See Dorsey, 1940, p.448.) These results are summarized in Table IV. Weinberg (1936) summarizes Russian work on tensile strength and also cites values by Moseley, Finlayson, and McConnel and Kidd. These data are reproduced in Table V. However, McConnel and Kidd were investigating plasticity and apparently did not break specimens for the purpose of determining tensile strength. Since Weinberg does not attempt to justify the inclusion of a value based on McConnel and Kidd's work, this value has been omitted. All values in Table V have been corrected to -3°C by Weinberg's method. (See bibliography on crushing strength: Weinberg, 1929).

TABLE IV

Summary of Tensile Strength Measurements

Tensile Strength psi	Author	Remarks
967	Reusch (1864)	Temp. a few degrees above freezin
99.3	Moseley (1870)	Temp. 70° to 75°F
100-114	Hess (1904) and Koechlin (1944)	
less than 200	Barnes, Hayward, and McLeod (1914)	
103	Finlayson (1927)	Water ice
136	"	Ice formed from mixture of snow and water
144-223	Barnes (1928) quoting Am. Civ. Eng. Handbook	
229 (mean)	Romanowicz and Honigman (1932)	Temp. -8°C. Rate of loading: 1.42 psi/sec.
260 (mean)	" "	
251 (mean)	" "	
353 (max)	" "	
210 (min)	" "	

TABLE V

Russian Determinations of Tensile Strength
(After Weinberg)

Tensile Strength psi	Author	Remarks
182	Pineghin (1924)	Upper layer, force parallel to axis.
102	"	Upper layer, force normal to axis.
172	"	Lower layer, force parallel to axis.
125	"	Lower layer, force normal to axis.
100-113	Schumacher, Moritz Apparently quoted from Mousson (1858)	Values quoted by Hess (1904) and Koechlin (1944) may also be from Mousson.
100	Moseley (1870)	Average of 8 specimens.
34	Fabian (1877)	
179	Frühling; quoted from Vasenko (1899)	Average of 9 specimens.
90 ± 46	Vasenko (1899)	Average of two specimens.
193 ± 20	Finlayson	Average of 10 specimens.

All values are corrected to -3°C according to Weinberg's method.

Shear strength:-

The shear strength of ice usually has been determined by the bending of ice bars or by punching or cutting ice bars in such a way that they rupture in shear. It can be shown in the case of an ice beam h cm high lying on two supports l cm apart and loaded at two points s cm apart ($s < l$) that if $2l - s/h$ is less than K where K is the ratio of tensile strength to shear strength (2 to 4), the beam will break in shear; if $2l - s/h$ is greater than K, the beam will break in tension.

Moseley (1870) broke 13 specimens in shear and determined the average shear strength to be 111 psi. The temperature varied from 70° to 75°F. Using the same experimental procedure, Moseley (1869 and 1871) had previously found 74.5 psi to be the average shear strength of two specimens of ice formed by tamping small pieces of ice, a few at a time, into the test apparatus. The later (1870) value seems more applicable although it also was in part based on specimens formed by tamping.

Finlayson (1927) measured the shear strength of 3 by 3 inch bars of river ice in a 30,000 pound Riehlé testing machine. Loading was done at a slow rate but was otherwise unspecified. Specimens were tested both parallel and perpendicular to the crystallographic axis and at various temperatures. He found 98 psi and 114 psi as average shear strengths parallel and perpendicular to the crystallographic axis respectively. Finlayson also found that the shear strength of artificial ice was about 80 percent that of river ice.

Weinberg (1936) lists values of shear strength determined by Pineghin (also by Moseley and Finlayson). The average of Pineghin's values, corrected to -3°C, for all orientations and types of ice and on the basis of the assumed (by Weinberg) number of observations is 106 psi.

Wilson and Horeth (1948) tested the shear strength of artificial ice frozen to simulate lake ice and of Lake Michigan ice parallel to the crystallographic axis. The average of their results for 30 specimens of artificial ice was 99.7 psi. Thirteen specimens of lake ice had an average shear strength of 75 psi. These determinations were made at various temperatures (see Table VI).

Table VI is a compilation of the results cited above. Barnes (1928) and Dorsey (1940) also give brief summaries of shear strength determinations.

The dependence of shear strength on temperature is not clearly established. The data of Wilson and Horeth (1948) and of Finlayson (1927) show a random relationship. On the other hand, Weinberg has apparently assumed, erroneously or not, that such a relationship exists, as indicated by his correction of Pineghin's data to -3°C. Matsuyama (1920) has noted a significant dependence of the rigidity modulus on temperature, which should be reflected in shear strength. He has expressed this relationship as follows:

$$n = (0.18 - 0.095t - 0.0020t^2) \times 10^6$$

where n is the modulus of rigidity, not in cgs units as Matsuyama indicates, but in gms/cm^2, and t is temperature in °C. It is to be expected that careful attention to the details of temperature, orientation, structure, absence of stresses other than shear, and rate of application of the load may eventually reveal a relationship between temperature and shear strength.

TABLE VI

Shear Strength of Ice

Shear Strength psi	Temp. °F	Author	Remarks
75	70-75	Moseley (1869)	Specimens formed by tamping and regelation.
111	70-75	Moseley (1870)	Some specimens of solid ice, others formed by tamping and regelation.
88	corr. to -3°C	Pineghin (Weinberg, 1938)	Upper layer. Shear parallel to axis. Crystal axis.
102	27°F	"	Upper layer. Shear normal to axis.
107	"	"	Lower layer. Shear parallel to axis.
117	"	"	Lower layer. Shear normal to axis.
101	-3	Finlayson (1927)	River ice. Shear parallel to axis.
102	4	"	
98	5	"	
99	8	"	
90	26	"	
94	28	"	
101	29	"	
115	-10	Wilson and Horeth (1948)	Averages of 10 specimens each of artificial ice. Shear parallel to axis.
90	2	" "	
94	32	" "	
75	12-20	" "	Average of 13 samples of Lake Michigan ice. Shear parallel to crystal axis.

TABLE VI

(continued)

Shear Strength psi	Temp. °F	Author	Remarks
131	-10	Finlayson	River ice. Shear normal to crystal axis.
111	- 9	"	
108	- 7	"	
109	- 3	"	
110	2	"	
103	4	"	
112	8	"	
120	11	"	
94	23	"	
132	26	"	
114	28	"	
110	30	"	

Bending (determination of tensile strength):-

The determination of tensile strength by the bending of ice bars has been undertaken by few investigators. As indicated in the section on shear strength, if a bar of ice h cm high lies on supports l cm apart and is loaded in two places s cm apart ($s < l$), then if $2l - s/h$ is greater than K where K is the ratio of the tensile strength to the shear strength (2 to 4), the bar will break in tension. Compressive stress is present but the crushing strength is so high, comparatively, that the specimen breaks in tension only.

The dependence of tensile strength as determined by bending on temperature seems clearly defined. The results of bending tests by Brown (1926) and Wilson and Horeth (1948) are consistent and are represented approximately by the relation:

$$\text{tensile strength (psi)} = 240 - 1.7t - 0.01t^2$$

where t is the temperature in °F. (See Wilson and Horeth, 1948).

The results of Brown (1926) and Wilson and Horeth (1948) are given in Table VII. Weinberg (1936) has included a summary table marked "Mean values of the limit of plasticity of ice by bending" which can probably be interpreted as tensile strength determinations. All data are apparently corrected to -3°C by Weinberg. (See Weinberg, 1929; bibliography on crushing strength.) In rechecking Weinberg's summary, the writer was unable to find certain values cited by Weinberg in the original references. This was true of values credited to Hess, Koch, Pfaff, Reusch, and Brown. The value credited to Tarr and Rich (1912) was found to agree with data in the original reference. New computations were made using the data given by Hess (1902). The average of these values is 502 psi for samples composed of single ice crystals. Only the values credited by Weinberg to Russian authors are given in Table VIII. The references cited were not examined and the data, therefore, are not verified.

TABLE VII

Tensile Strength of Ice as Determined by Bending

Tensile Strength psi	Temp. °F	Author	Remarks
155	28-30	Brown (1926)	9 specimens of St. Lawrence River ice. Crystals horizontal.
184	28-30	"	12 specimens of St. Lawrence River ice. Crystals vertical.
239	14-16	"	12 specimens of St. Lawrence River ice. Crystals horizontal.
214	14-16	"	12 specimens of St. Lawrence River ice. Crystals vertical.
180	32	Wilson and Horeth (1948)	11 specimens of artificial ice. Crystals vertical.
214	12-20	" "	6 specimens of Lake Michigan ice. Crystals vertical.
256	-9	" "	9 specimens of artificial ice. Crystals vertical.
158	?	Tarr and Rich (1912)	3 specimens of pond ice. Fourth specimen was apparently overloaded and breaking stress was too high.

TABLE VIII

Summary of Determinations of Tensile Strength by Bending
by Russian Investigators
(After Weinberg)

Layer	Orientation	No. of Obs.	Mean Values (psi)		
Upper	axis ∥ force	93	208	199	229
	axis ⊥ force	25	230		
	unknown	31	151		
Lower	axis ∥ force	260	216	228	
	axis ⊥ force	62	185		
	axis ∥ length	30	458		
	unknown	79	179		
Not Indicated	axis ∥ force	5	242	271	
	axis ⊥ force	2	492		
	axis ∥ length	4	848		
	unknown	110	247		

ICE FRICTION

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The previous two references are taken from Arnold-Alabieff (1938) and apparently deal with the experimental procedures on which the data in the article cited are based.

Belokonj, P. N. (1938), On the coefficient of friction in the ice cover: Meteorologia i Hydrologia, vol. 4, pp. 116-131. Probably in Russian.

Description and Tabulation of Experimental Data

The power required to drive an ice-breaking ship onto a layer of ice is partly a function of the coefficient of friction between the plates of the ice-breaker and the ice. The question of ice friction has been investigated largely with reference to the process of ice skating. Pressure melting due to concentration of load on a small area of contact is thought to be effective in forming a film of water which serves as a lubricant between the ice and the blade of the skate. The friction between a flat, painted, rusted or otherwise roughened plate of metal (such as the plates of a ship) has been studied only recently and by few investigators. Of the papers cited, the three which were examined treated aspects of the latter problem.

Morphy's (1913) treatment is the least applicable to the problem of ice-breaking. In order to investigate the static friction of ice, he placed a small aluminum sled on an inclined ice surface and, under varying loads, determined the angle at which sliding occurred. He found that the tangent of the angle at which sliding occurred was 0.36±0.01 if the load were less than 14.3 grams weight. If the load were greater than 15 grams weight, the tangent of the corresponding angle was 0.17±0.01. In each range the angle was found to be independent of the load.

Bowden and Hughes (1939) investigated the friction of miniature ski of various materials moving in contact with ice. Their results demonstrate the independence of the coefficient of kinetic friction from load, apparent area of contact, and speed of sliding. Temperature of the ice, however, affected the friction markedly, the friction rising as the temperature of the ice fell. Variation in friction of the ski of different materials indicated the dependence of friction on thermal conductivity of the ski material. This suggests frictional melting as an important factor in the formation of the water film during sliding. This theory, together with the non-dependence of friction on load and area of contact, minimizes greatly the effect of pressure melting in sliding under conditions comparable to those of the experiments. The authors agree qualitatively with Morphy by stating, "The F/W curve was at first linear showing that μ_s was independent of the load, but at heavier loads the curve became concave to the load axis, showing that μ_s becomes less at heavier loads."

Tables IX to XI, all from Bowden and Hughes, show the relationship between friction and various quantities: Table IX shows the influence of area of contact on friction in the case of ice sliding on ice; Table X shows the effect of temperature on static friction for ice on ice; Table XI shows the effect of increasing the thermal conductivity and heat capacity of a hollow ebonite ski whose bottom was covered with a thin sheet of copper. This is done by filling the hollow ski above the copper sheet with mercury; Table XI shows further the effect of variable thermal conductivity; Table XII shows the influence of wax on the ski bottom; Table XIII shows the effect of temperature on the coefficient of kinetic friction between a waxed hickory ski and ice. The figures are all from Bowden and Hughes.

Table IX

Relation between Friction and Area of Contact

Exp. no.	Apparent area of contact cm^2.	Mean temperature °C	μ_k
1	0.6	-1.4	0.019
	2.3	-2.0	0.019
2	0.6	-3.0	0.017
	2.5	-3.0	0.019
3	0.02	-1 to -10	0.016
	3.1	-3.0	0.021

Table X

Relation between Friction and Temperature

Temperature, °C	0	-12	-71	-82	-110
μ_s	0.05-0.15	0.3	0.5	0.5	0.5

Table XI

Relation between Friction and Thermal Conductivity of Ski

A.

Metal	μ_k (no mercury)	μ_k (mercury on ski)	$\mu_k Hg/\mu_k$
Copper	0.022	0.031	1.4
	0.027	0.034	1.3
	0.032	0.043	1.3
	0.032	0.054	1.7
Constantan	0.021	0.020	1.0

B.

	Load 200 g.	Load 1000 g.
Brass ski	0.010	0.005
Ebonite ski	0.025	0.010

Table XII

Influence of Waxed Ski on Friction

Load 200 g.

Nature of ski surface	Temp. -3°C	Temp. -7°C
Unwaxed hickory	0.08	----
Waxed hickory	0.05	0.04
Unwaxed brass	0.030	0.05
Waxed brass	0.025	0.045

Table XIII

Influence of Temperature on Kinetic Friction of Waxed Ski

Temperature, °C ...	0	-3	ca.-10	-40
μ_k	0.04	0.09	0.18	0.4

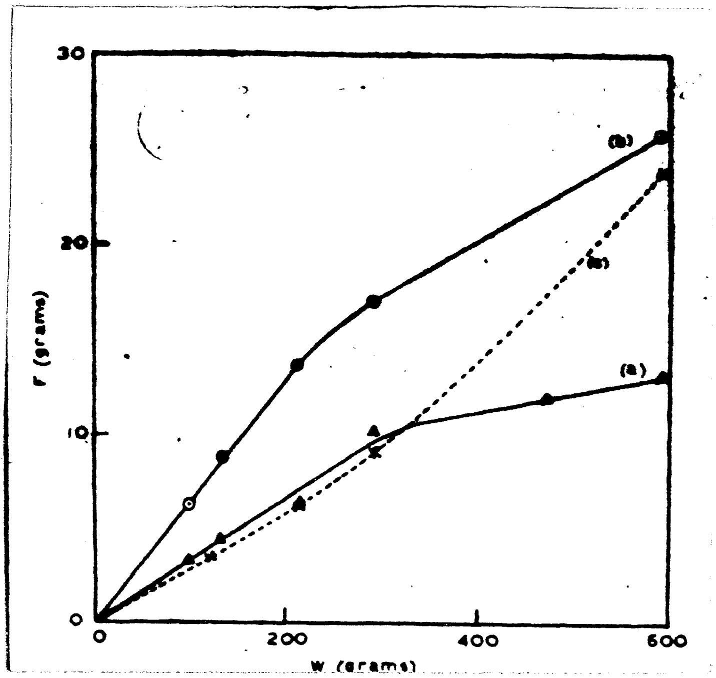

Figure 2. The relation between the frictional force (F) and the load (W) for ice sliding on ice. Curve (a) mean surface temperature -3.3°C; curve (b) mean surface temperature -27.5°; curve (c) mean surface temperature 0°.

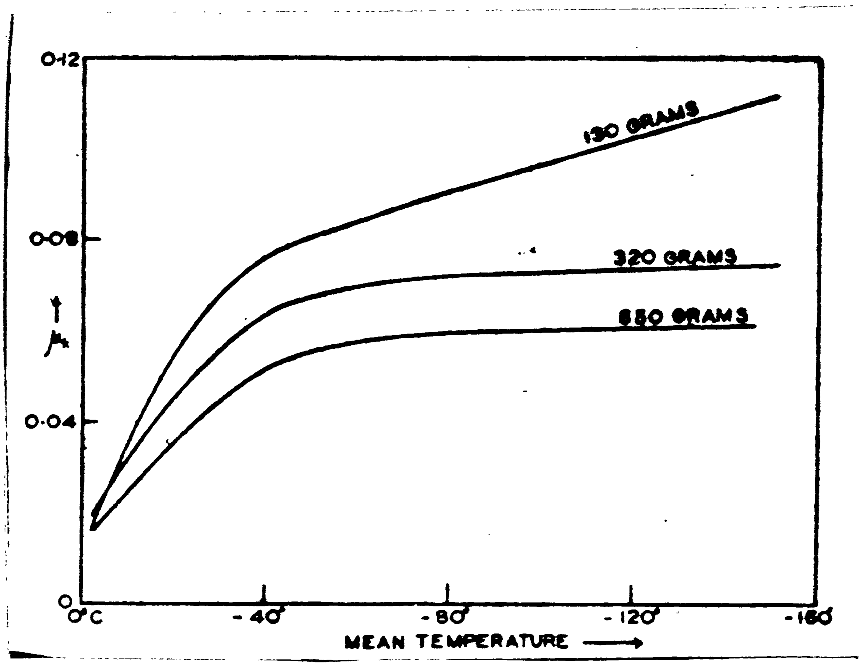

Figure 3. The influence of temperature on the kinetic friction between ice surfaces.

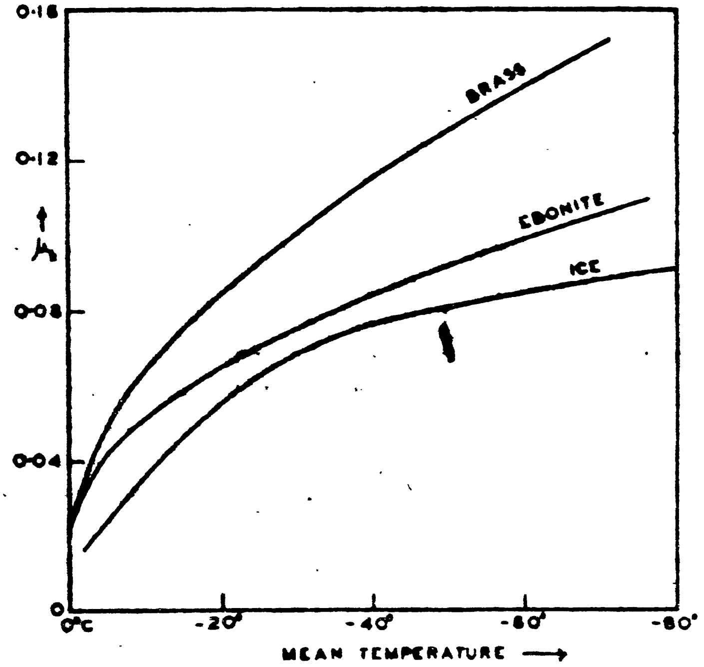

Figure 4. The influence of temperature on the friction of brass, ebonite and ice sliding on ice.

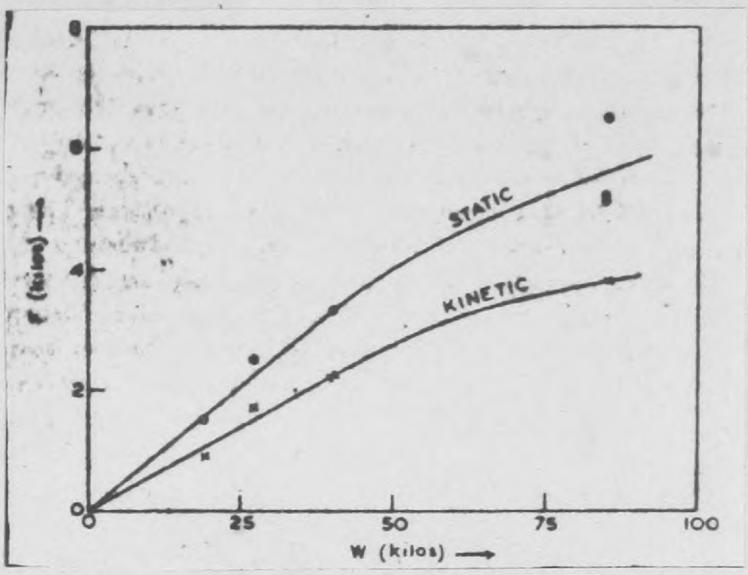

Figure 5. The F/W relation for real ski on snow.

Arnold-Alabieff (1938) studied ice friction according to the following plan:

- Kind of friction:
  - Friction of rest (static friction)
  - Friction of motion (kinetic friction)
- Character of friction:
  - Dry friction
  - Fluid friction
  - Friction with self lubrication
  - Type of ice: fresh water, transparent, turbid, sea, polar, etc.
  - Material: ship steel, smooth, new, old, rusty, painted; concrete, cement covering, etc.

The following statements are of interest in a qualitative sense: "From the practice of navigation in ice it is known e.g. that ice mixed with snow and covered with it produces greater friction than that without snow."

"Also crystalline ice should give different coefficients depending on the direction of the surface of friction relative to the axis of the crystals. This suggestion follows from the statements of McConnel (1891) and Deeley (1908) according to whom ice has a different viscosity dependent on direction to the axis of the crystals. For the same reason granular ice of crystalline structure ought to have a different coefficient of friction than that of monocrystalline ice with identical directions of the axes of separate crystals."

The results of experiments by Arnold-Alabieff are shown in Tables XIV and XV.

Table XIV

Coefficients of Friction

Kind of Friction	Coefficient of Friction
Friction at rest	0.30 - 0.50
Friction in motion	0.03 - 0.05

Table XV

Coefficients of Friction for Various Surfaces and Types of Ice

Kind of ice:	Neva ice Fresh water ice		Baltic Sea ice De-salted ice		Kara Sea ice Polar salt-water ice	
Kind of Friction	Smooth metal	Painted metal	Smooth metal	Painted metal	Smooth metal	Painted metal
Friction of rest	0.15-0.25	0.35-0.40	0.15-0.20	---	0.15-0.25	0.30-0.35
Friction of motion	0.10-0.15	---	0.10-0.15	---	0.10-0.20	0.20

HARDNESS OF ICE

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Description and Tabulation of Experimental Data

Hardness may be expressed as resistance to abrasion or as resistance to indentation. The two properties are not necessarily related in a given substance.

Resistance to Abrasion: Mohs' scale is commonly used to express hardness in terms of resistance to abrasion. This scale is applied almost exclusively to the determination of the relative hardness of minerals.

Textbooks on mineralogy (Rogers, 1937 and Palache and others, 1944) give 1-1/2 as the relative hardness of ice on Mohs' scale (between talc and gypsum). Later work by Teichert (1939) and Blackwelder (1940) in connection with the corrasive power of wind-blown snow shows an increase in relative hardness with a decrease in temperature. Table XVI gives a summary of the values of the hardnesses of ice for different temperatures.

Temperature °C	Relative Hardness Mohs' Scale	Equivalent Mineral	Source of Data
?	1-1/2	Talc to gypsum	(Rogers (1937) (Palache and others (1944)
-15	2-3	Gypsum to calcite	Teichert (1939)
-30	3-4	Calcite to fluorite	Teichert (1939)
-40	approx. 4	Fluorite	Teichert (1939)
-44	exactly 4	Fluorite	Teichert (1939)
-78.5	approx. 6	Orthoclase	Blackwelder (1940)

Table XVI. Variation in relative hardness of ice (Mohs' scale) with temperature.

Resistance to Indentation: Hardness is expressed in terms of resistance to indentation as Brinell hardness, Rockwell hardness or Shore scleroscope hardness. Various other tests have been suggested recently, some of which are merely refinements of these. In the Brinell hardness testing machine an impression is left in the sample being tested by a steel ball under a given load applied for a given length of time. The Brinell hardness number is the ratio of the load in kilograms to the area of the indentation in square millimeters. The Rockwell tester uses either a steel ball or a 120°

diamond cone under minor and major loads applied successively. The increase in the depth of the impression formed by the minor load to that formed by the major load determines the hardness. The value of hardness is read directly from the machine. The Shore scleroscope, which in a strict sense does not measure resistance to indentation, employs a diamond-tipped hammer which is allowed to fall on a polished surface of the test sample. The height of rebound measured on an arbitrary linear scale is the hardness.

Dorsey (1940) states that no hardness data determined by the above methods have been found but that no difficulty should be anticipated in determining the Shore scleroscope hardness of ice. As for the Brinell test, he states: "The Brinell hardness number for ice would have no significance except at temperatures so low that the rate at which ice yields progressively under the action of a constant load applied to a small area of its surface is negligible; say, at temperatures below -30°C."

A more recent method of testing hardness has been developed by Knoop and others (1939). This method employs a diamond-shaped indenter, the ratio of whose diagonals is one to seven. It is stated that this method of determining hardness is applicable to brittle materials and to those with low elastic limits since recovery of the impression takes place only along the minor diagonal. This enables the unrecovered value of the major diagonal and hence the unrecovered area of the indentation to be determined. Instruments manufactured for applying this test employ impact loading. Thus, in testing ice, the plastic deformation which occurs upon slow application of a load would be minimized, lending greater significance to the results than is the case for Brinell and Rockwell hardness values. No measurements on ice were reported in the paper cited. The hardness of the minerals of Mohs' scale as determined by the Knoop method is shown in Table XVII for purposes of comparison with Table XVI.

Mineral	Knoop Hardness	Mohs' Hardness
Gypsum	32	2
Calcite	135	3
Fluorite	163	4
Apatite	360-430	5
Orthoclase	563	6
Quartz	710-790	7
Topaz	1250	8
Diamond	8000-8500	10

Table XVII. Relation between Knoop hardness and Mohs' scale.

Further discussion of the applicability of this method of testing is given by Winchell (1945).

Measurements of the penetrability of ice by Andrews (1886) were interpreted by him as representing the relative hardness of ice. Barnes (1928) refers to these determinations in his discussion of the plasticity of ice. Dorsey (1940) points out the evident connection between penetrability and progressive deformation under a constant load.

Andrews' measurements were made by allowing a 0.292 by 16 inch blunt steel rod to penetrate ice under a constant load of 181-1/2 pounds. Observations of the rate of penetration were made at temperatures from -35°F to 32°F. His observations show that ice is very resistant to penetration from -40°C to about 6°C. From 6°C to the melting point, the resistance rapidly decreases.

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Description and Tabulation of Experimental Data

Data for the computation of Young's modulus and the modulus of rigidity have been derived from the bending of bars of ice, from bars of ice mounted as torsion pendulums, from the longitudinal extension of ice samples under a load, by determining the pitch of the tone given off by a bar of ice vibrating at its resonant frequency, by compression, and by determining the velocity of transmission of longitudinal and transverse waves through ice. The dynamic methods which make use of longitudinal and transverse vibrations have thus far given much more consistent results than the static methods involving bending, compression, and tension.

Weinberg (1936) states that "it will be safer to assume that (on the basis of) our present knowledge on the moduli of elasticity of ice we can only say that Young's modulus is of the order 700 to 800 kg/mm^2 and the modulus of rigidity of the order 250 to 300 kg/mm^2." However, his compilation of values does not include the important dynamic determinations of Boyle and Sproule (1931) or of Ewing, Crary, and Thorne (1934) which Dorsey (1940) states are the most accurate available. The values determined by Boyle and Sproule for Young's modulus vary from 918 to 1070 kg/mm^2. This variation, according to them, is much smaller than that observed in the results of static tests but is too large to be assigned to instrumental error. They list the possible sources of error in applications of their ultrasonic method as follows: "(1) Error in measurement of the length of the ice rods; (2) Error due to possibly mistaking the identity of one mode or type of vibration for a totally different mode or type, e.g., assuming that the fifth mode of transverse vibration is the first mode of longitudinal vibration; (3) Error in measurement of frequency." (1) and (3) together were not over one per cent. An error in (2) would result in a value of velocity which was in error by some multiple of the true velocity; in practice this type of error is easily avoided.

Weinberg (1936) summarizes the possible causes of variation in experimental results. Most tables of values are grouped so as to take into account the first three, but, according to Weinberg, most authors ignore the last five possible causes of variation in reporting their results. The causes are:

1. Type of ice.
2. Relation between the direction of application of the force and the direction of the crystallographic axis.
3. The temperature at which the experiments are done.
4. The conditions of formation of the ice.
5. The temperature history of the ice from the time of formation to the time of the test.
6. The rate of application of the load.
7. The size of the sample and the precision of its preparation.
8. The character of the pressing surface. Presumably this refers to the method of application of the load.

To this list might be added such factors as salinity in the case of brackish and sea ice, and the elastic history of the sample.

Most of these factors have been taken into account by one or the other of the investigators on ice but in none of the sets of experimental results examined by the writer were even a majority of the factors considered by a single author. For instance, Brown (1926) has investigated the effect of the rate of application of the load in static determinations of Young's modulus (figures 1 to 5) and has carefully noted the sizes of the samples used in his tests. At the same time he fails to consider the fact that in most cases his apparatus has exceeded the elastic limit of ice and hence is not measuring a true value of Young's modulus. On the other hand, most of the investigators who have employed dynamic methods involving vibrations have not accounted for the possible effect of variation in frequency of vibration, i.e., the rate of application of the load, on the values of the elastic constants.

By applying Maxwell and Schwedoff's (see Viscosity and Plasticity of Ice, Maxwell (1868) and Schwedoff (1890)) theory of relaxation to ice, Weinberg computes the elastic limit of sheet ice (Neva River) to be 0.57 kg/cm^2 and of glacier ice to be 0.09 kg/cm^2. On the basis of this determination he proposes a valid objection to most determinations of the moduli of elasticity. He says, "almost all authors have been investigating ice beyond its limits of elasticity and.....this one circumstance does not permit us to consider their results on the moduli with full confidence." Most authors have not measured Young's modulus of an ice sample under different loads. However the results of Brown (1926) show decreasing values of Young's modulus with increasing load. It appears that the elastic limit has been exceeded in this case. The above objection may apply equally well to dynamic determinations of elasticity.

The tables of values of the various elastic constants contain the more important data on this subject. Tables XVIII and XIX (after Dorsey) include the results of static and dynamic determinations of Young's modulus. Table XX lists static and dynamic determinations of the modulus of rigidity. The single value which was determined by dynamic methods (Ewing, Crary, and Thorne, 1934) is as yet (1940) unsupported but, according to Dorsey, is to be preferred over the static values. Table XXI lists values of Poisson's ratio. Weinberg (1936) has erroneously quoted Koch (1914) as having given 0.314 ±0.007 and 0.248 ±0.007 for Poisson's ratio. The values given by Koch refer instead to the ratio of the modulus of rigidity to Young's modulus in a given sample of ice. Figures 6, 7 and 8 (from Brown, 1926) show the relationship between Young's modulus and the rate of application of the load in compression tests. Figures 9 and 10 show a similar relationship for bending tests. These tests were carried on under various temperature conditions as indicated in the figures.

TABLE XVIII

Young's Modulus - Dynamic Determation
(after Dorsey)

I. Longitudinal vibrations. Units of E = kg wt/mm^2: θ = angle between optic axis and length of sample. t = temperature °C.

E	θ	t	Source	Type of ice
947	0	-26	Boyle and Sproule(1931)	River ice
967	0	-26	" "	" "
1040	0	-26	" "	" "
1110	0	-26	" "	" "
970	0	-26	" "	" "
900	45	-26	" "	" "
990	90	-26	" "	" "
945	90	-26	" "	" "
990	90	-26	" "	" "
945	90	-26	" "	" "
877	90	-5	Trowbridge and McRae(1934)	Fresh pond ice
960	90	-4	Reich and Stierstadt(1931)	Artificial ice

II. Transverse vibrations

622	?	-13(?)	Trowbridge and McRae(1934)	Artificial ice
884	90	-7	Koch (1885)	Lake ice
236	90	0(?)	Reusch (1880)	Artificial ice
710	90	?	Kohler (1929)	Lake ice

TABLE XIX

Young's Modulus of Ice - Static Method
Bending of Beams
(after Dorsey)

Unit of E = kg/mm^2, temperature = t°C.

E, optic axis parallel to			t	Author	Type of Ice
length	width	depth			
182		383	0 to -1	Hess (1902)	Single crystals
59			-2 to -5	"	" "
	254	418	-1 to -5	"	" "
67	194	336	(?)	"	" "
185	60	92	-3.5	Matsuyama (1920)	?

E, orientation of optic axis with respect to length:		t	Author	Type of Ice
parallel	normal			
	642	-5.4	Koch (1885)	Lake ice
	860	-5 to -7	Trowbridge and McRae (1885)	Pond ice
609	622	-6.5 to -7.8	Koch (1913)	Clear lake ice
	656	-6.5 to -7.8	Koch (1913)	" " "
696	696	-9	Koch (1885)	Lake ice
1120	958	(?)	Koch (1914)	Lake ice
	950	(?)	Moseley (1870)	(?)
	500	(?)	Bevan (1826)	(?)

TABLE XX

Rigidity of Ice
(after Dorsey)

N = modulus of rigidity in kmegadynes/cm^2·radian = 1019 kg/cm^2·radian.
t = temperature in °C.

A. Dynamic Method.

N = 91.7 = 0.5; t = -5 to -15; Ewing, Crary, and Thorne (1934); artificial ice.

B. Static Method.

N, orientation of optic axis with respect to length:		t	Author	Type of ice
parallel	normal			
27.2	29.4	?	Koch (1914)	Lake ice
10		0	Weinberg (1907B)	?
17		-5	"	?
	1.6	-7.5	Matsuyama(1920)	Artificial ice
1.8		-6	"	" "
orientation unknown				
28.2		?	Hargis (1922)	Artificial ice
27		?	Kohler (1929)	Lake ice
glacier ice				
8		0	Weinberg (1907B)	?
34		-5	"	?

TABLE XXI

Poisson's Ratio for Ice

Poisson's Ratio	Author	Remarks
0.365 ± 0.007	Ewing, Crary, and Thorne (1934)	Computed from the velocity of longitudinal waves in the range -5°C to -15°C.
0.39	Kohler (1929)	Computed from horizontal velocity of waves in an "isotropic" ice sheet.
0.38 ± 0.49	Weinberg (1907A)	Computed from static observations. Direction of extension not indicated.
0.326	Pineghin (1927)	Computed from two different series of observations. Static method-either bending or longitudinal compression.
0.358 ± 0.047	Pineghin (1927)	
0.361	Brockamp and Mothes (1930)	Computed from velocities of longitudinal and transverse waves in glacier ice. Weinberg thinks it applies to river ice also.

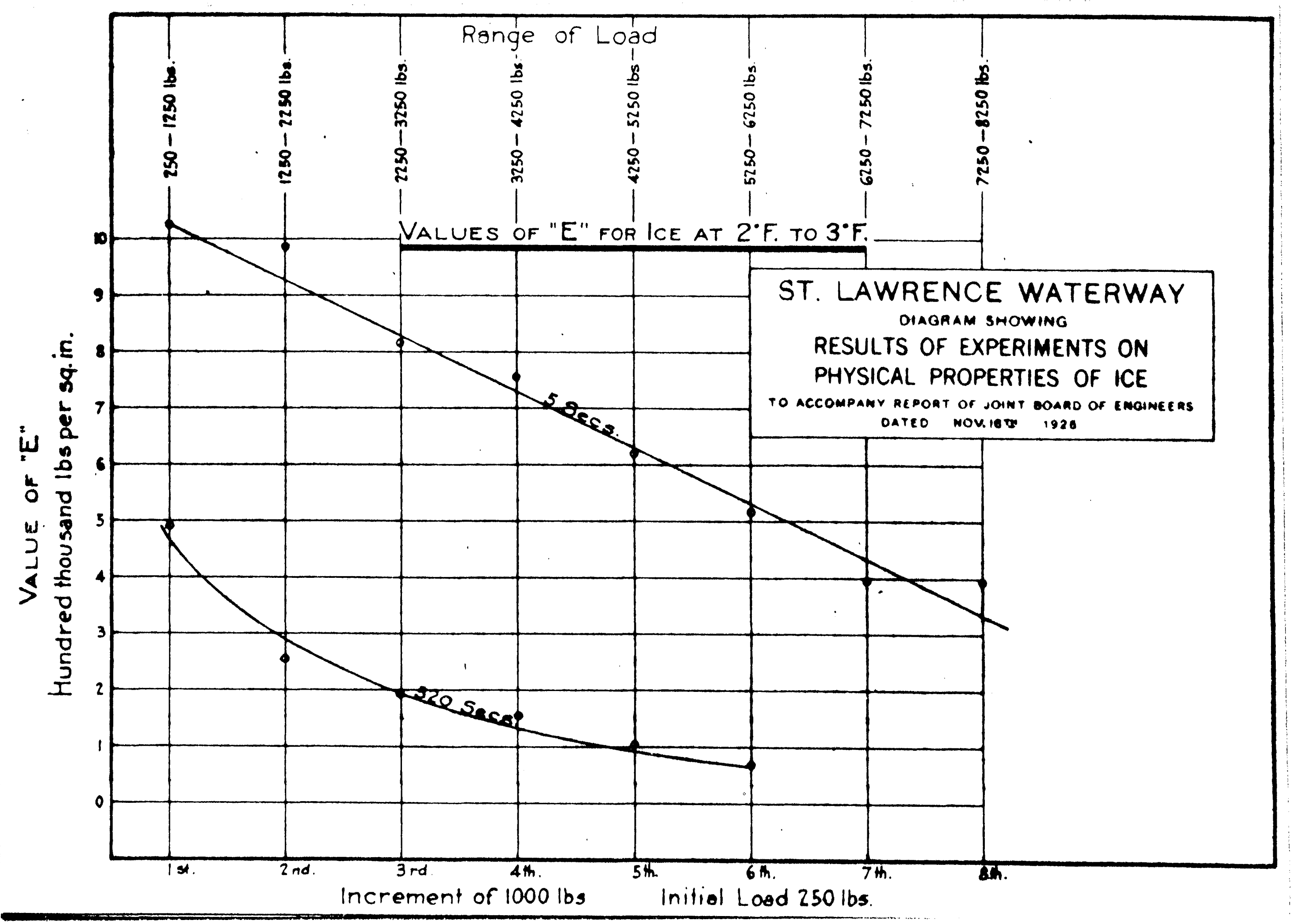

Fig. 6. Relationship between Young's modulus (E) and rate of application of load. Temperature 2°F to 3°F. Compression tests. From Brown (1926).

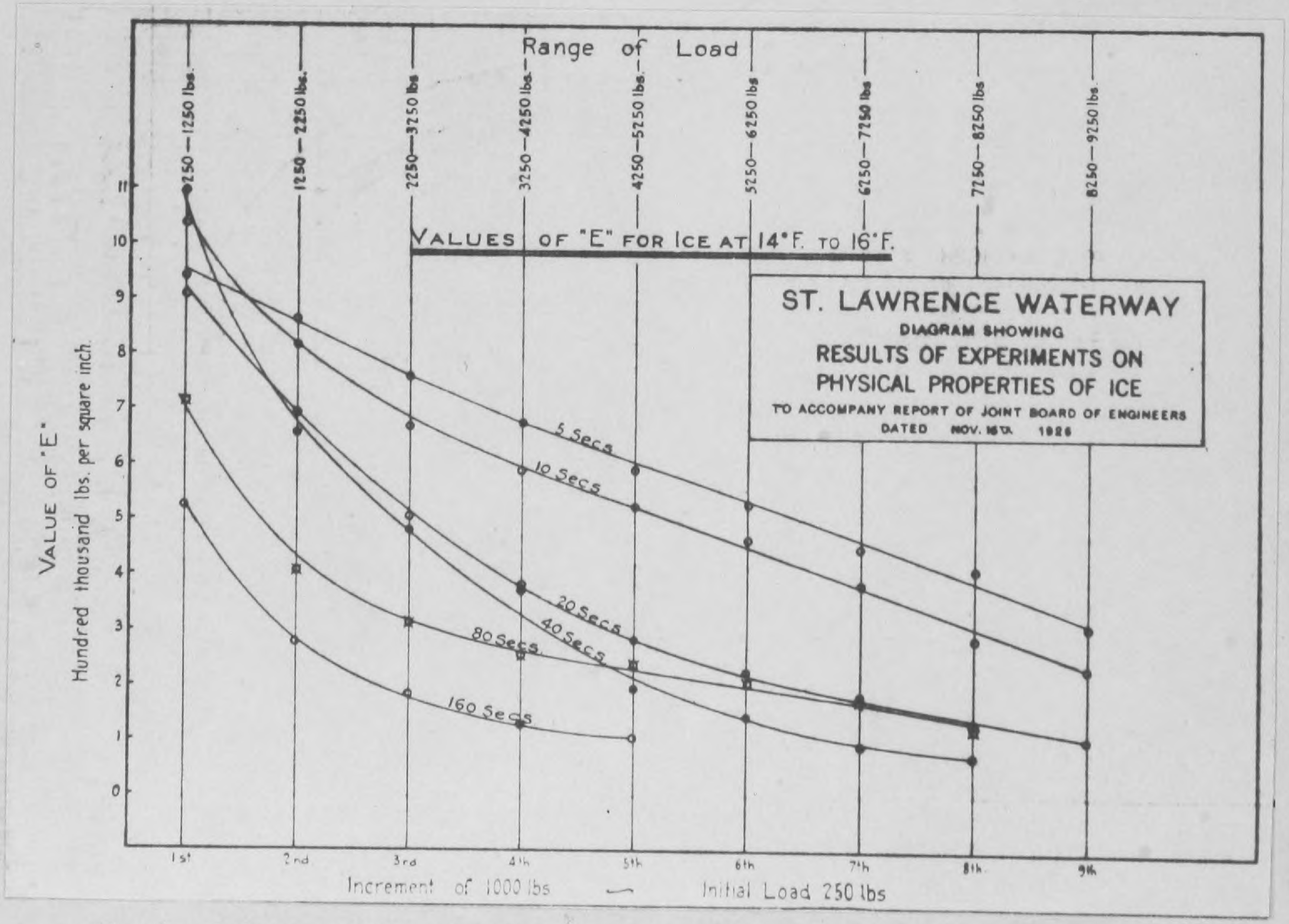

Fig. 7. Relationship between Young's modulus (E) and rate of application of load. Temperature 14°F to 16°F. Compression tests. From Brown (1926).

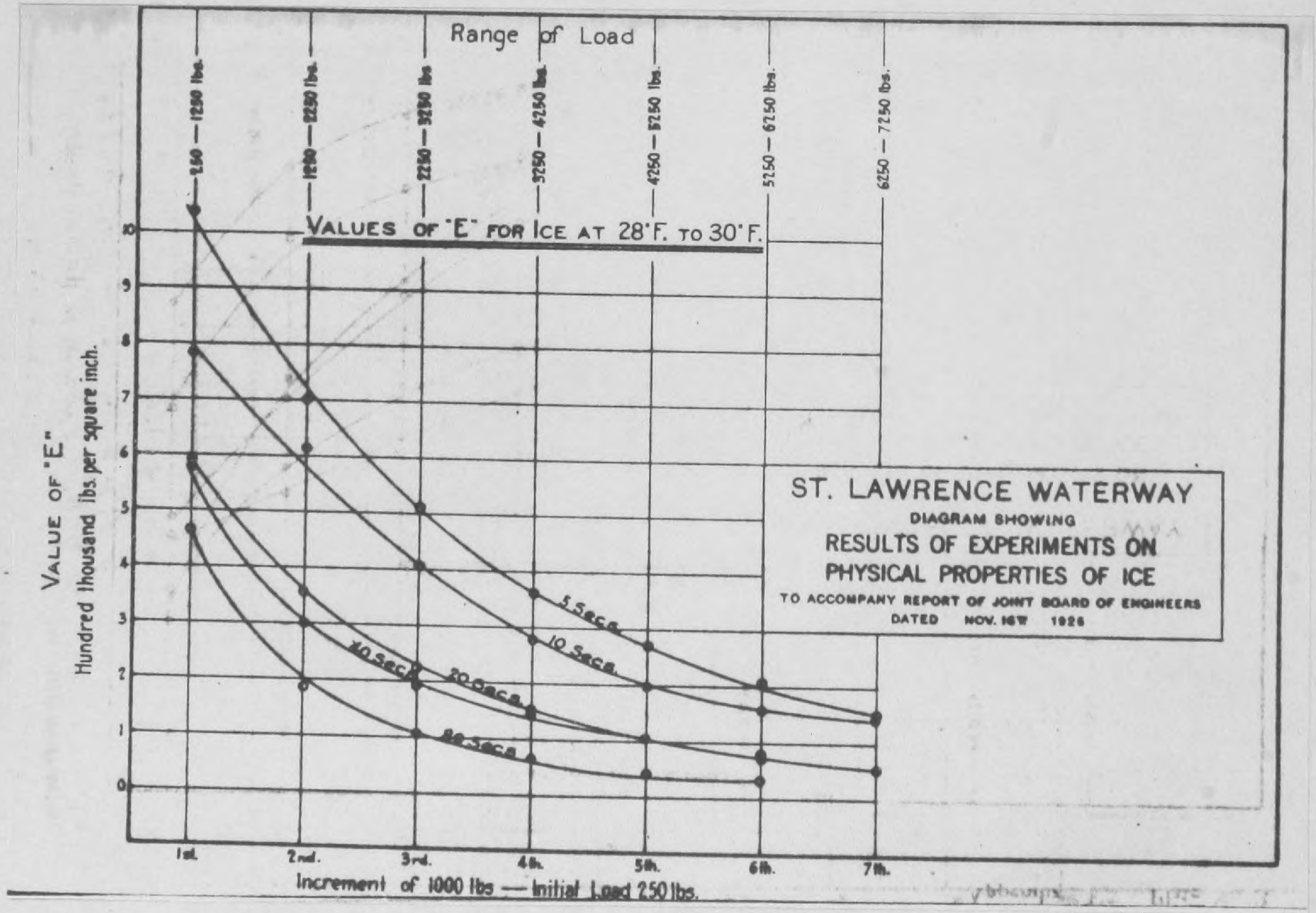

Fig. 8. Relationship between Young's modulus (E) and rate of application of load. Temperature 28°F to 30°F. Compression tests. From Brown (1926).

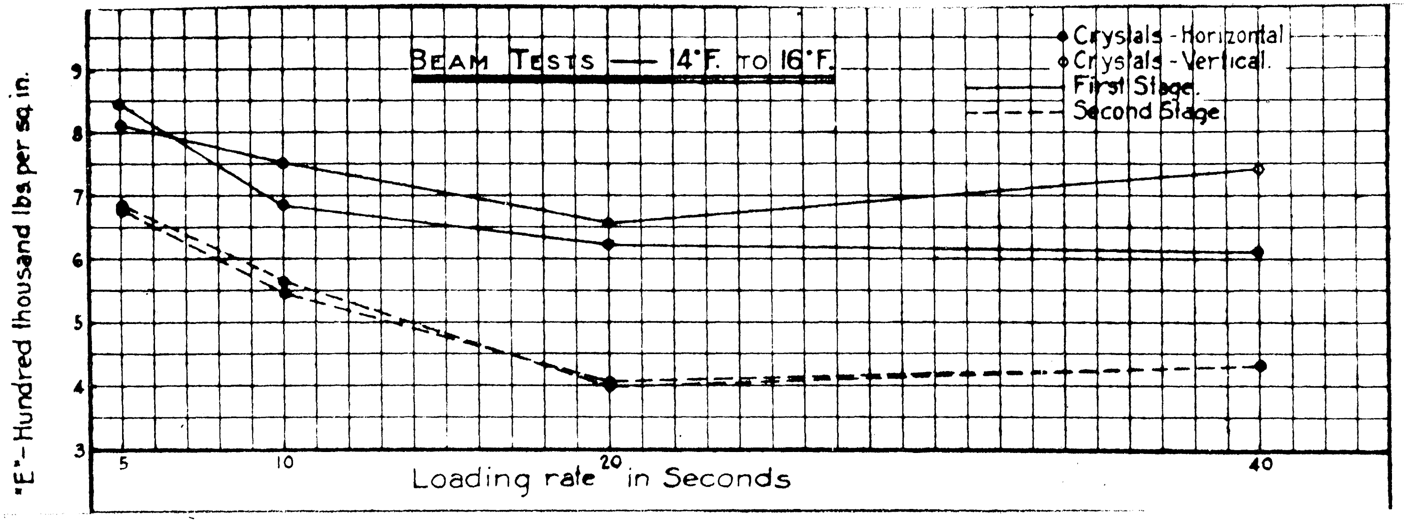

Fig. 9. Relationship between Young's modulus (E) and rate of application of load. Temperature 14°F to 16°F. Bending tests. From Brown (1926).

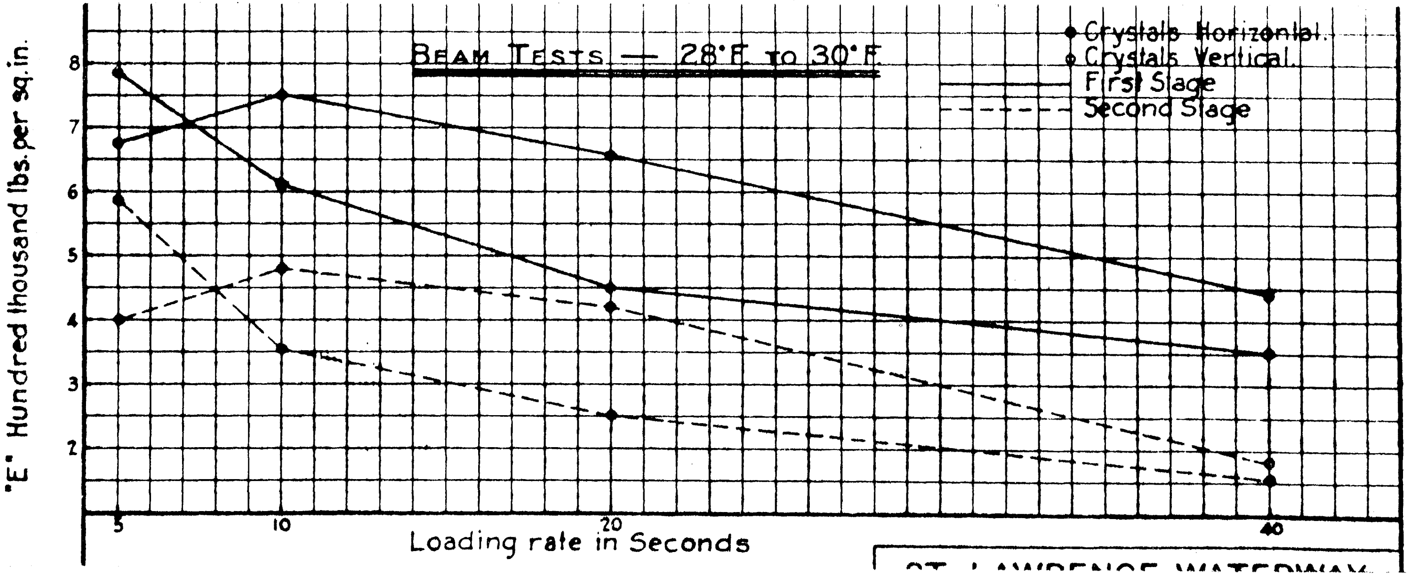

Fig. 10. Relationship between Young's modulus (E) and rate of application of load. Temperature 28°F to 30°F. Bending tests. From Brown (1926).

VISCOSITY AND PLASTICITY OF ICE

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Description and Tabulation of Experimental Data

Ice is a plastic solid in that it possesses an elastic limit for small forces but is continuously deformed by stresses exceeding the elastic limit. Dorsey (1940) follows Maxwell's (1868) treatment as extended by Schwedoff (1889-90) in his discussion of the viscosity and plasticity of ice. He considers a plastic solid, ice, bounded by two parallel plates, one of which is moving relative to the other. A velocity gradient dV/dx is established in the solid and each plate experiences a tangential drag of P units per unit area. The elastic limit is taken as p units per unit area. Equation (1) is a statement of the proportionality of the tangential drag to the velocity gradient:

$$P - p = \mu \frac{dV}{dx} \qquad (1)$$

The constant of proportionality, μ, is the viscosity and is a characteristic property of the solid, being independent of the other variables. Equation (1) may be taken as a definition of a plastic solid. Unless P exceeds p, dV/dx is zero. Dorsey further chooses to denote the usual expression Pdx/dV as apparent viscosity, μ_e. Equation (2) relates viscosity and apparent viscosity:

$$\mu_e = \mu - p \frac{dx}{dV} \qquad (2)$$

Weinberg (1905, 1907A) was the first to investigate quantitatively the inverse relationship between the apparent viscosity and the velocity gradient. He expressed his experimental data by means of an equation of the form of (3):

$$\mu_e = \mu_0 (a - \frac{b}{t})^{-t} - \frac{c}{\psi} \qquad (3)$$

where t is the temperature in °C, ψ is the rate of shear in radians per second, and μ_0 is the value of viscosity at 0°C and $\psi = \infty$. Dorsey cites several irregularities in application of this formula and has recomputed the values which appear in Table XXII.

Weinberg (1907A, 1907B) also observed that when the stress is in the nature of a torsion, the elastic limit of ice is not a function of temperature. J. Thomson (1857), Pfaff (1875), Bianconi (1876) and Andrews (1886) maintain, on the other hand, that ice is a perfectly viscous solid (elastic limit equal to zero). Mallet (1845) has also attempted to demonstrate the non-plastic nature of ice.

From experimental data on the bending of rectangular bars, Hess (1902, 1904) deduced that for moderate loads, μ_e increases with time, the rate being essentially constant after five minutes. However, under loads approaching the breaking strength, μ_e showed a decrease with time. Hess' values of apparent viscosity were on the order of 1/100 as great as those of Weinberg (Table XXIV).

McConnel (1891) made similar observations and Deeley (1908) computed from his data the viscosity and found it to be of the same order as Hess' results. (Dorsey was unable to reproduce these computations.) On the basis of these data, Dorsey has estimated that the apparent viscosity measured parallel to the optic axis is perhaps a hundred times that measured normal to the optic axis.

Deeley and Parr (1913) have computed viscosities from data by McConnel and Kidd (1888) which illustrate dependence on temperature, the structure of the ice and the direction of the shear (Table XXIII).

Hargis (1922) has computed the viscosity from data obtained by mounting a cylinder of ice as a torsion pendulum. The values varied from 3.8×10^9 to 6.33×10^9 poises.

Dorsey summarizes the available data on viscosity of ice as follows:

1. None of the available data for the plasticity or for the viscosity of ice are entirely satisfactory.

2. Values of μ_e derived from the bending of bars are of the order of 10^{10} poises, those for axial torsion and those from the longitudinal stretching of bars are of the order of 10^{12} poises.

3. Although McConnel's and McConnel and Kidd's data indicate that μ_e for shear parallel to the optic axis is about 100 times as great as for shear perpendicular to that axis, Hess' data indicate that the difference is slight.

4. The value of μ_e increases as the rate of shear decreases (Table I).

5. When the stress is kept constant, μ_e increases with the time the stress has been applied. Whether this involves other phenomena than those pertaining to the variation with the rate of shear cannot be determined from the data now available.

6. The value of μ increases very rapidly as the temperature decreases, a decrease of 10°C being accompanied by a five-fold increase in μ for river ice, and a 26-fold increase for glacier ice. This increase in μ causes a marked, but in general a smaller, increase in μ_e.

7. An attempt to fit the data of Hess, of McConnel, and of McConnel and Kidd to Weinberg's equation has been unsuccessful.

8. Owing to the absence of important data, to significant variations in the procedures followed, and to variations in the structure and the purity of the ice used, it is impossible to correlate satisfactorily the data obtained by different observers.

Table XXII
Viscosity of River and Glacier Ice
(From Dorsey, 1940, p. 454)

$\mu_e = \mu_o(a - b/t)^{-t} - c/\psi$; temperature = t°C; rate of shear = radians/sec., corresponding to a difference of v meters/year in the velocities of two planes of slipping that are 100 meters apart. In his paper of 1905, Weinberg gives $\mu_t = (12.44 - 4.02t - 0.277t^2) \times 10^{12}$ poises when the mean value of ψ is about 10^{-8} radians/sec.; this formula is probably not so good as the other. Computations by the compiler. Unit of $\mu = 10^{12}$ poises, of ψ and of v as already indicated; temp. = t°C.

I. River ice. Planes of slipping are perpendicular to optic axis. $\mu_o = 9.5$, $a = 1.12$, $b = 0.54$°C, $c = 5 \times 10^5$ poise-radian/sec.

ψ	10^{-8}(?)	$5\text{x}10^{-9}$	10^{-8}	10^{-7}	$5\text{x}10^{-7}$	∞
v	31.6(?)	15.8	31.6	316	631	∞
t	μ_t		μ_e			
0	12.4	110	60	14.5	10.5	9.5
-0.1	12.8	112	62	16.5	12.5	11.5
-0.5	14.5	114	64	19.1	15.1	14.1
-1.0	16.7	116	66	20.8	16.8	15.8
-2.0	21.6	118	68	23.3	19.3	18.3
-3.0	27.0	121	71	25.9	21.9	20.9
-4.0	33.0	124	74	28.6	24.6	23.6
-5.0	39.5	126	76	31.5	27.5	26.5
-7.5	58.2	135	85	40.3	36.3	35.3
-10.0	80.3	147	97	52.3	48.3	47.3
-12.5	93.0	163	113	67.3	63.9	62.9
-15.0	135.0	184	134	88.6	84.6	83.6

II. Glacier ice. $\mu_o = 3.8$, $a = 1.32$, $b = 0.65$°C, $c = 8 \times 10^4$ poise-radian/sec.

ψ	10^{-9}	$5\text{x}10^{-9}$	10^{-8}	10^{-7}	$5\text{x}10^{-7}$	∞
v	3.16	15.8	31.6	316	631	∞
t	μ_t		μ_e			
0	83.8	19.8	11.8	4.6	4.0	3.8
-0.1	84.7	20.7	12.7	5.5	4.8	4.7
-0.5	86.1	22.1	14.1	6.9	6.3	6.1
-1.0	87.5	23.5	15.5	8.3	7.6	7.5
-2.0	90.3	26.3	18.3	11.1	10.5	10.3
-3.0	93.8	29.8	21.9	14.6	14.0	13.8
-4.0	98.4	34.4	26.4	19.2	18.5	18.4
-5.0	104.4	40.4	32.4	25.2	24.5	24.4
-7.5	129.0	65.0	57.0	49.8	49.2	49.0
-10.0	178.0	114.4	106.4	99.2	98.6	98.4
-12.5	278.0	214.0	206.0	198.0	198.0	198.0
-15.0	475.0	411.0	403.0	396.0	395.0	395.0

Table XXIII
Viscosity of Glacier Ice
Adapted from R. M. Deeley and P. H. Parr
(From Dorsey, 1940, p. 456)

Unit of $\mu_e = 10^{12}$ poises

Observer		Computer		μ_e
Dr. Main	1888	R. M. Deeley	1912	6.0
McConnel and Kidd	1888	R. M. Deeley	1912	84.5
B. Weinberg	1907	B. Weinberg	1907	8.0
Tyndall and others	1907	R. M. Deeley	1908	78.9
Blümcke and Hess	1907	B. Weinberg	1906	17.4
Blümcke and Hess	1910	B. Weinberg	1910	17.5
Blümcke and Hess	1910	Deeley and Parr	1913	147.7[a]
Blümcke and Hess	1910	Deeley and Parr	1913	125.0[a]

a. From motion of glaciers in winter.

Table XXIV
Viscosity of River Ice
(From Dorsey, 1940, p. 456)
Computed from Hess' (1902, 1904) data

Values were derived from the bending of horizontal rectangular bars supported at the ends and loaded at the middle. P = load; M = bending moment per unit of cross sectional area = Pl/4ab; μ_τ = value of the apparent viscosity as computed from the rate of shear τ sec. after the load was applied; l = length between supports; a = vertical thickness; b = horizontal breadth; vertical and horizontal refer to position of the bar when loaded for test. All three bars were cut from the same sheet of ice.

Unit of P = 1 gram force, of M = 1 gram force/cm., of μ = 10^{12} poises.

Axis	Parallel to l			Parallel to a		
P	2000	5000	6000	1000	1500	2000
M	1350	3400	4000	2350	2350	3100
τ	μ_τ			μ_τ		
15	0.065	0.105	0.0055	0.075	0.100	0.080
60	0.175	0.115	0.0360	0.075	0.110	0.070
120	0.100	0.130	0.0365	0.075	0.090	0.110
300	0.110	0.160	0.0350	0.080	0.120	0.120
1200		0.120				

	Optic axis parallel to b			
P	1000	1500	2000	3000
M	1500	2250	3000	4450
τ	μ_τ			
15	0.037	0.037	0.024	0.110
60	0.080	0.110	0.060	0.090
120	0.120	0.100	0.100	
300	0.210	0.190	0.170	

ICE BREAKERS AND ICE BREAKING

The articles examined were mostly non-technical. Of the Russian articles, which were not available for examination, Davydov's is the only one of a technical nature, judging from the titles.

Barnes, H. T. (1928), Ice engineering, Renouf Publishing Company, Montreal, chapter 8, pp. 229-299.

Following are the paragraph headings and a brief statement of the contents of each:

Work at the Northumberland Straits: a description of the ice encountered, meteorological conditions, and methods of breaking ice.

Ice breaking on the St. Lawrence: similar discussion.

Note on the formation of ice over Lake St. Peter: similar discussion.

Measurement of water temperatures by microthermometer: a description of the electrical resistance thermometer used.

Effect of sun on general ice conditions: absorption of the sun's rays in the water.

Influence of ice on the temperature of the water.

Effect of convection: A limit to the thickness to which ice can grow is postulated due to the fact that the water under the ice is usually slightly warmer than the freezing point.

Atmospheric humidity: Even when the relative humidity is 100 per cent, evaporation may take place both from ice and from water since they are generally warmer than the air. The temperature of the layer of air next to the ice or water is raised by conduction, thus lowering the relative humidity and permitting evaporation.

Effect of radiation: This effect is difficult to evaluate but is probably small.

Ice-breakers and their services: a review of Gulston's article.

Ice-breaking tugs: a discussion of existing ice-breaking tugs and of the effect of the lines of the ship and of the bow propellor on efficiency as ice-breakers.

Bartlett, R. A. (1928), Ice navigation, Am. Geog. Soc. Sp. Pub. No. 7 (Problems of Polar research), pp. 427-444.

This article includes non-technical discussions of the following subjects with special reference to polar sea-ice:

Polar ships and their construction
The training grounds for ice navigation
Ice navigating with Peary
Varying ice conditions from year to year
Wooden ships versus steel ships
Ice movements in Baffin Bay and Labrador waters

Bregman, G. R. (?), The Atlantic influence on the processes of ice breaking and freezing of rivers (To be printed in the Transactions of the Hydrological Institute). Probably in Russian. Not examined.

The title of this article may be a mis-translation of the Russian and may actually refer to the <u>melting</u> and freezing of ice in rivers.

Davydov, V. V. (1938), Theoretical investigations of the impact of a ship on ice, Problemy Arktiki (Problems of the Arctic): No. 5/6, pp. 103-124. In Russian. Not examined.

Gulston, A. (1904), Ice-breakers and their services: Jour. Soc. Arts, vol. 52, p. 215.

This article is a history of the development of ice-breakers to the date of the paper and includes descriptions of most of the ice breakers which had been in use previous to that time.

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Data available on ice conditions
Trends of foreign designs
Coast Guard ice-breaker design developments
Detail requirements of ice-breaker designs
Selection of machinery
Propellors and shafting
Operating difficulties

Subdivision
Heeling and trimming arrangements
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