# Web-based supporting materials for "Modeling Short- and Long-term Characteristics of Follicle Stimulating Hormone as Predictors of Severe Hot Flashes in Penn Ovarian Aging Study" by

Bei Jiang, Michael R. Elliott, Mary D. Sammel and Naisyin Wang

November 28, 2014

## Details of posterior computations when assuming t model for longitudinal observations

**note:** in the case of assuming normal model for longitudinal observations, we are not updating $m_{ij}$ but let $m_{ij}$ be constant 1; and the missing longitudinal observation $y_{ij}$ is drawn from $\mathrm{N}\left(\mu(t_{ij}), \sigma_i^2\right)$.

**(1) update for longitudinal submodel**

- **update** the mean profile class memberships $D_i$, $i = 1, ..., n$: the full conditional posterior distribution $[D_i|\cdot] \sim \text{Multinomial}(\tilde{\pi}_{i1}^D, ..., \tilde{\pi}_{iK_D}^D)$, where

$$\tilde{\pi}_{id}^D = \Pr(D_i = d|.) = \frac{\pi_d^D |\Sigma_d|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}(\boldsymbol{b}_i - \boldsymbol{\beta}_d)' \Sigma_d^{-1} (\boldsymbol{b}_i - \boldsymbol{\beta}_d)\right]}{\sum_{d=1}^{K_D} \pi_d^D |\Sigma_d|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}(\boldsymbol{b}_i - \boldsymbol{\beta}_d)' \Sigma_d^{-1} (\boldsymbol{b}_i - \boldsymbol{\beta}_d)\right]}$$

- **update** the mean profile class parameters:
  - **update** $\boldsymbol{\beta}_d = (\beta_{d1}, ..., \beta_{dL})^T$:
    assuming the prior for $\boldsymbol{\beta}_{d1} \sim \mathrm{N}(0, v)$ and first order random walk prior $\beta_{dl} \sim \mathrm{N}(\beta_{d,l-1}, \tau_{\beta d}^2)$, $l = 2, ..., L$, then the prior for $\boldsymbol{\beta}_d$ can be written as: $\pi(\boldsymbol{\beta}_d) = \left(\frac{1}{\sqrt{2\pi\tau_{\beta d}}}\right)^{L-1} \exp\{-\frac{1}{2}\boldsymbol{\beta}_d^T V_d \boldsymbol{\beta}_d\}$, where $V_d = \begin{pmatrix} v^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} + P^T P/\tau_{\beta d}^2$ and $P = \begin{pmatrix} 1 & -1 & & & \\ & 1 & -1 & & \\ & & \ddots & \ddots & \\ & & & 1 & -1 \end{pmatrix}$ is the $(L-1) \times L$ penalty matrix. Then the full conditional posterior density for $\boldsymbol{\beta}_d$ for $d = 1, ..., K_D$ is $[\boldsymbol{\beta}_d|.] \sim \mathrm{MVN}\left(\tilde{\boldsymbol{\nu}}_d, \tilde{V}_d\right)$

$$\tilde{\boldsymbol{\nu}}_d = \left[V_d + \Sigma_d^{-1} \sum_{i=1}^n \mathrm{I}(D_i = d)\right]^{-1} \left[\Sigma_d^{-1} \sum_{i=1}^n \mathrm{I}(D_i = d) \boldsymbol{b}_i\right]$$

$$\tilde{V}_d = \left[V_d + \Sigma_d^{-1} \sum_{i=1}^n \mathrm{I}(D_i = d)\right]^{-1}$$

  - **update** $\Sigma_d$: assuming the prior for $\Sigma_d \overset{ind}{\sim} \text{Inverse-Wishart}(m, \Lambda)$, where $m$ and $\Lambda$ are the degrees of freedom and scale matrix, respectively, then the full conditional posterior density is, $[\Sigma|\cdot] \sim \text{Inverse-Wishart}\left(\tilde{m}_d, \tilde{\Lambda}_d\right)$ where

$$\tilde{m}_d = m + n$$

$$\tilde{\Lambda}_d = \left[\Lambda + \sum_{i=1}^n (\boldsymbol{b}_i - \boldsymbol{\beta}_d)(\boldsymbol{b}_i - \boldsymbol{\beta}_d)'\right]$$

  - **update** $\tau_{\beta d}^2$: assuming $\tau_{\beta d}^2 \sim \text{Inverse-Gamma}(v, e)$, where $v$ and $e$ are the shape and rate parameters, then the full conditional posterior distribution is

$$\left[\tau_{\beta d}^2|\cdot\right] \sim \text{Inverse-Gamma}\left(v + \frac{L-1}{2}, e + \frac{1}{2}\boldsymbol{\beta}_d^T P \boldsymbol{\beta}_d\right)$$

where $L$ is the number of B spline basis functions.

- **update** the mixing proportion $\{\pi_d^D\}_d$: assuming $\left[\{\pi_d^D\}_d\right] \sim \text{Dirichlet}(e_1^D, ...., e_{K_D}^D)$ then the full conditional posterior distribution is

$$[\{\pi_d^D\}_d|\cdot] \sim \text{Dirichlet}(\{e_d^D + \sum_{i=1}^n \text{I}(D_i = d)\}_d)$$

- **update** the variance class memberships $C_i$, $i = 1, ..., n$: the full conditional posterior distribution $[C_i|\cdot] \sim \text{Multinomial}(\tilde{\pi}_{i1}^C, ..., \tilde{\pi}_{iK_C}^C)$ where

$$\tilde{\pi}_{ic}^C = \Pr(C_i = c|.) = \frac{\pi_c^C \exp\left[-\frac{1}{2}\left(\log\sigma_i^2 - \mu_c\right)^2/\tau^2\right]}{\sum_{c=1}^{K_C} \pi_c^C \exp\left[-\frac{1}{2}\left(\log\sigma_i^2 - \mu_c\right)^2/\tau^2\right]}$$

- **update** the variance class parameters:
  - **update** $\mu_c$: assuming the prior for $\mu_c \overset{ind}{\sim} \text{N}(a, b)$, then the full conditional posterior distribution is, $[\mu_c|\cdot] \sim \text{N}\left(\tilde{a}, \tilde{b}\right)$ where

$$\tilde{a} = \frac{\sum_{i=1}^n \text{I}(C_i = c)\log\sigma_i^2/\tau^2 + a/b}{1/b + \sum_{i=1}^n \text{I}(C_i = c)/\tau^2}$$

$$\tilde{b} = \left(1/b + \sum_{i=1}^n \text{I}(C_i = c)/\tau^2\right)^{-1}$$

  - **update** $\tau^2$: assuming $\tau^2 \sim \text{Inverse-Gamma}(v, e)$, then the full conditional posterior distribution is $\left[\tau^2|\cdot\right] \sim \text{Inverse-Gamma}\left(v + \frac{n}{2}, e + \sum_{i=1}^n \sum_{c=1}^{K_C} \frac{1}{2}\text{I}(C_i = c)\left(\log\sigma_i^2 - \mu_c\right)^2\right)$.

- **update** the mixing proportions $\{\pi_c^C\}_c$: assuming $\left[\{\pi_c^C\}_c\right] \sim \text{Dirichlet}(e_1^C, ...., e_{K_C}^C)$ then the full conditional posterior distribution is

$$[\{\pi_c^C\}_c|\cdot] \sim \text{Dirichlet}\left(\{e_c^C + \sum_{i=1}^n \text{I}(C_i = c)\}_c\right)$$

- **update** the random effects $\boldsymbol{b}_i$, $i = 1, ..., n$ the full conditional posterior distribution is $\boldsymbol{b}_i$ $[\boldsymbol{b}_i|.] \sim \text{MVN}\left(\tilde{\boldsymbol{\beta}}_i, \tilde{\Sigma}_i\right)$, where

$$\tilde{\Sigma}_i = \left[\Sigma^{-1} + \frac{1}{\sigma_i^2}\sum_{j=1}^{n_i} m_{ij}\boldsymbol{\phi}_{ij}\boldsymbol{\phi}_{ij}' + \boldsymbol{M}_{bi}\boldsymbol{M}_{bi}^T\right]^{-1}$$

$$\tilde{\boldsymbol{\beta}}_i = \tilde{\Sigma}_i\left[\Sigma^{-1}\boldsymbol{\beta}_{D_i} + \frac{1}{\sigma_i^2}\sum_{j=1}^{n_i} y_{ij}m_{ij}\boldsymbol{\phi}_{ij} + (W_i - \alpha_0 - \boldsymbol{x}_i^T\lambda_0)\boldsymbol{M}_{bi}\right]$$

where, $\boldsymbol{\phi}_{ij} = (\phi_1(t_{ij}), \cdots, \phi_L(t_{ij}))^T$ a vector of B spline basis functions evaluated at time $t_{ij}$ such that $\mu_i(t_{ij}) = \boldsymbol{b}_i^T \boldsymbol{\phi}_{ij}$ where $y_{ij} \sim \text{N}\left(\mu_i(t_{ij}), \sigma_i^2/m_{ij}\right)$ and $\boldsymbol{M}_{bi}$ is defined such that $\int_T \mu_i(t)\theta_0(t)dt = \int_T \boldsymbol{b}_i^T \boldsymbol{\phi}(t)\boldsymbol{\psi}^0(t)^T \tilde{\boldsymbol{\theta}}_0 dt = \boldsymbol{b}_i^T G_T^0 \tilde{\boldsymbol{\theta}}_0 = \boldsymbol{b}_i^T \boldsymbol{M}_{bi}$.

- **update** the variances $\sigma_i^2$, $i = 1, ..., n$

$$\pi(\sigma_i^2|.) \propto (\sigma_i^2)^{-\frac{n_i}{2}-1} \exp\left[-\sum_{c=1}^{K_C} \text{I}(C_i = c)\frac{\left(\log\sigma_i^2 - \mu_c\right)^2}{2\tau^2} - \frac{1}{2\sigma_i^2}\sum_{j=1}^{n_i} m_{ij}\left(y_{ij} - \boldsymbol{b}_i^T\boldsymbol{\phi}_{ij}\right)^2\right]$$
$$\times \exp\left[-\frac{1}{2}\left(W_i - \alpha_0 - \boldsymbol{x}_i^T\boldsymbol{\lambda}_0 - \int_T \mu_i(t)\theta_0(t)dt\right)^2\right]$$

  where, $\boldsymbol{x}_i$ is a vector of baseline covariate including subject specific residual variance $\frac{v_i}{v_i-2}\sigma_i^2$. Since there is no closed form of the full conditional posterior density, the draws for $\sigma_i^2$, $i = 1, ..., n$ at each iteration of the Gibbs sampling are obtained using the inverse cumulative distribution sampling method.

- **update** $m_{ij}$, $j = 1, ..., n_i$, given that $m_{ij} \sim \text{gamma}(v/2, v/2)$ where $v/2$ and $v/2$ are the shape and rate parameter in gamma distribution, then the full conditional posterior distribution for $m_{ij}$ is $m_{ij} \sim \text{gamma}\left(\frac{v+1}{2}, \frac{1}{2}\left(\frac{(y_{ij}-\mu(t_{ij}))^2}{\sigma_i^2} + v\right)\right)$.

- **update** missing longitudinal data: missing $y_{ij}$ at time $t_{ij}$ is drawn from $\text{t}\left(\mu(t_{ij}), \sigma_i^2, v\right)$.

**(2) update for outcome probit submodel:**

- **update** $W_i$, $i = 1, ..., m$

$$[W_i|o_i = 0, \cdot] \sim \text{N}(\eta_i^W, 1)I_{(-\infty,0)}(\cdot)$$
$$[W_i|o_i = 1, \cdot] \sim \text{N}(\eta_i^W, 1)I_{(0,\gamma_2)}(\cdot)$$
$$[W_i|o_i = 2, \cdot] \sim \text{N}(\eta_i^W, 1)I_{(\gamma_2,\infty)}(\cdot)$$

  where, $\eta_i^W = \alpha_0 - \boldsymbol{x}_i^T\boldsymbol{\lambda}_0 - \int_T \mu_i(t)\theta_0(t)dt$ and $\gamma_2$ is the cutoff.

- **update** cutoff $\gamma_2$: assuming flat prior on $\gamma_2$, then the full conditional posterior density for $\gamma_2$ is $\text{Unif}(\text{Max}_{o_i=1}W_i, \text{Min}_{o_i=2}W_i)$.

- **update** $(\alpha, \boldsymbol{\lambda})'$: Assuming independent prior for $(\alpha, \boldsymbol{\lambda})' \sim \text{MVN}(\boldsymbol{\nu}_{\alpha\lambda}, V_{\alpha\lambda})$, then the full conditional posterior density $\sim \text{MVN}\left(\tilde{\boldsymbol{\nu}}_{\alpha\lambda}, \tilde{V}_{\alpha\lambda}\right)$ where

$$\tilde{\boldsymbol{\nu}}_{\alpha\lambda} = \left[V_{\alpha\lambda}^{-1} + \sum_{i=1}^{n} \boldsymbol{z}_i\boldsymbol{z}_i'\right]^{-1}\left[V_{\alpha\lambda}^{-1}\boldsymbol{\nu}_{\alpha\lambda} + \sum_{i=1}^{n}\left(W_i - \int_T \mu_i(t)\theta_0(t)dt\right)\boldsymbol{z}_i\right]$$
$$\tilde{V}_{\alpha\lambda} = \left[V_{\alpha\lambda}^{-1} + \sum_{i=1}^{n} \boldsymbol{z}_i\boldsymbol{z}_i'\right]^{-1}$$

  where $\boldsymbol{z}_i = (1, \boldsymbol{x}_i)'$ is a vector of constant 1, baseline covariates and residual variance $\frac{v}{v-2}\sigma_i^2$

- **update** $\tilde{\boldsymbol{\theta}}_{\mathbf{0}} = (\tilde{\theta}_{01}, ..., \tilde{\theta}_{0K})'$: assuming the prior for $\tilde{\theta}_{01} \sim \mathrm{N}(0, v_{\theta_0})$ and first order random walk prior $\tilde{\theta}_{0k} \sim \mathrm{N}(\tilde{\theta}_{k-1}, \tau^2_{\theta_0})$, $k = 2, ..., K$, then the prior for $\tilde{\boldsymbol{\theta}}_0$ can be written as: $\pi(\tilde{\boldsymbol{\theta}}) = \left(\frac{1}{\sqrt{2\pi\tau_{\theta_0}}}\right)^{L-1} \exp\{-\frac{1}{2}\tilde{\boldsymbol{\theta}}_0^T V_{\theta_0} \tilde{\boldsymbol{\theta}}_0\}$, where $V_{\theta_0} = \begin{pmatrix} v_{\theta_0}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} + P_{\theta_0}^T P_{\theta_0} / \tau^2_{\theta_0}$ with $P_{\theta_0} =$

$$\begin{pmatrix} 1 & -1 & & & \\ & 1 & -1 & & \\ & & \ddots & \ddots & \\ & & & 1 & -1 \end{pmatrix}$$

  is the $(K-1) \times K$ penalty matrix. Then the full conditional posterior density for $\tilde{\boldsymbol{\theta}}_0$ is $\left[\tilde{\boldsymbol{\theta}}_0 | .\right] \sim \mathrm{MVN}\left(\tilde{\boldsymbol{\nu}}_{\boldsymbol{\theta}_0}, \tilde{V}_{\boldsymbol{\theta}_0}\right)$ where

$$\tilde{\boldsymbol{\nu}}_{\boldsymbol{\theta}} = \left[V_{\boldsymbol{\theta}_0}^{-1} + \sum_{i=1}^{n} \boldsymbol{z}_{\theta i} \boldsymbol{z}_{\theta i}'\right]^{-1} \left[V_{\boldsymbol{\theta}_0}^{-1} \boldsymbol{\nu}_{\boldsymbol{\theta}_0} + \sum_{i=1}^{n} \left(W_i - \alpha_0 - \boldsymbol{x}_i^T \boldsymbol{\lambda}_0\right) \boldsymbol{z}_{\theta i}\right]$$
$$\tilde{V}_{\boldsymbol{\theta}_0} = \left[V_{\boldsymbol{\theta}_0}^{-1} + \sum_{i=1}^{n} \boldsymbol{z}_{\theta i} \boldsymbol{z}_{\theta i}'\right]^{-1}$$

  where $\boldsymbol{z}_{\theta i}$ is a vector of $K$ elements defined such that $\int_T \mu_i(t)\theta_0(t)dt = \int_T \boldsymbol{b}_i^T \boldsymbol{\phi}(t) \boldsymbol{\psi}^0(t)^T \tilde{\boldsymbol{\theta}}_0 dt = \boldsymbol{b}_i^T G_T^0 \tilde{\boldsymbol{\theta}}_0 = \tilde{\boldsymbol{\theta}}_0^T \boldsymbol{z}_{\theta i}$.

- **update** $\tau^2_{\theta_0}$: assuming $\tau^2_{\theta_0} \sim \mathrm{IG}(f, g)$, where $f$ and $g$ are the shape and rate parameters, then the full conditional posterior distribution is $\left[\tau^2_{\theta_0} | \cdot\right] \sim \mathrm{IG}\left(f + \frac{K-1}{2}, g + \frac{1}{2}\tilde{\boldsymbol{\theta}}_0^T P_{\theta_0} \tilde{\boldsymbol{\theta}}_0\right)$, where $K$ is the number of B spline basis functions to express $\theta_0(t)$ in the ordinal probit submodel.

Table 1: Estimates of model parameters by our best fitting models with $\mu_i(t)$ $i = 1, ..., n$ within the time window $T = [45, 55]$ as a functional predictor in primary outcome submodel.

| | **normal** | | | $t_7$ | | | $t_4$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | mean | se | 95% CI | mean | se | 95% CI | mean | se | 95% CI |
| $\beta_{11}$ | 1.666 | 0.145 | (1.374, 1.95) | 1.508 | 0.152 | (1.195, 1.794) | 1.475 | 0.152 | (1.166, 1.766) |
| $\beta_{12}$ | 1.993 | 0.094 | (1.808, 2.177) | 2.118 | 0.094 | (1.937, 2.308) | 2.142 | 0.091 | (1.963, 2.321) |
| $\beta_{13}$ | 1.286 | 0.153 | (0.985, 1.585) | 0.933 | 0.147 | (0.643, 1.22) | 0.889 | 0.144 | (0.608, 1.175) |
| $\beta_{14}$ | 5.416 | 0.225 | (4.981, 5.862) | 5.506 | 0.24 | (5.029, 5.969) | 5.586 | 0.238 | (5.115, 6.051) |
| $\beta_{15}$ | 4.253 | 0.445 | (3.358, 5.114) | 4.272 | 0.418 | (3.44, 5.09) | 4.122 | 0.396 | (3.345, 4.897) |
| $\omega^2_{11}$ | 0.94 | 0.265 | (0.535, 1.556) | 2.321 | 1.86 | (0.786, 6.4) | 2.357 | 1.597 | (0.811, 6.39) |
| $\omega^2_{12}$ | 0.606 | 0.137 | (0.379, 0.915) | 2.588 | 2.556 | (0.698, 7.717) | 2.838 | 2.357 | (0.808, 8.107) |
| $\omega^2_{13}$ | 2.022 | 0.396 | (1.343, 2.887) | 3.942 | 2.215 | (1.327, 9.512) | 3.644 | 1.911 | (1.291, 8.433) |
| $\omega^2_{14}$ | 5.68 | 0.927 | (4.08, 7.692) | 4.612 | 2.91 | (1.376, 12.195) | 4.232 | 2.51 | (1.306, 10.495) |
| $\omega^2_{15}$ | 9.43 | 3.235 | (4.643, 17.328) | 23.262 | 25.851 | (3.639, 87.769) | 17.166 | 16.618 | (3.364, 61.141) |
| $\rho_{1,12}$ | -0.763 | 0.073 | (-0.877, -0.592) | -0.633 | 0.213 | (-0.914, -0.096) | -0.653 | 0.202 | (-0.917, -0.133) |
| $\rho_{1,13}$ | 0.661 | 0.098 | (0.436, 0.816) | 0.527 | 0.223 | (0.002, 0.862) | 0.54 | 0.214 | (0.042, 0.857) |
| $\rho_{1,14}$ | -0.199 | 0.146 | (-0.468, 0.102) | -0.374 | 0.257 | (-0.786, 0.203) | -0.381 | 0.249 | (-0.783, 0.165) |
| $\rho_{1,15}$ | -0.017 | 0.167 | (-0.338, 0.31) | 0.17 | 0.308 | (-0.462, 0.707) | 0.163 | 0.299 | (-0.445, 0.688) |
| $\rho_{1,23}$ | -0.818 | 0.047 | (-0.894, -0.711) | -0.632 | 0.19 | (-0.902, -0.169) | -0.644 | 0.182 | (-0.903, -0.216) |
| $\rho_{1,24}$ | 0.459 | 0.103 | (0.239, 0.64) | 0.378 | 0.273 | (-0.229, 0.808) | 0.369 | 0.268 | (-0.219, 0.802) |
| $\rho_{1,25}$ | -0.172 | 0.15 | (-0.452, 0.133) | -0.076 | 0.347 | (-0.7, 0.589) | -0.061 | 0.333 | (-0.671, 0.581) |
| $\rho_{1,34}$ | -0.552 | 0.078 | (-0.69, -0.385) | -0.861 | 0.116 | (-0.974, -0.537) | -0.863 | 0.096 | (-0.971, -0.606) |
| $\rho_{1,35}$ | 0.262 | 0.14 | (-0.031, 0.517) | 0.563 | 0.271 | (-0.111, 0.909) | 0.56 | 0.251 | (-0.055, 0.897) |
| $\rho_{1,45}$ | -0.744 | 0.08 | (-0.865, -0.554) | -0.733 | 0.199 | (-0.953, -0.198) | -0.722 | 0.197 | (-0.948, -0.192) |
| $\beta_{21}$ | | | | 2.5 | 0.527 | (1.481, 3.574) | 2.599 | 0.524 | (1.612, 3.705) |
| $\beta_{22}$ | | | | 1.233 | 0.439 | (0.38, 2.118) | 1.146 | 0.432 | (0.275, 1.982) |
| $\beta_{23}$ | | | | 4.129 | 0.493 | (3.13, 5.103) | 4.263 | 0.479 | (3.336, 5.233) |
| $\beta_{24}$ | | | | 5.267 | 0.544 | (4.234, 6.387) | 5.211 | 0.526 | (4.196, 6.279) |
| $\beta_{25}$ | | | | 3.996 | 1.371 | (1.109, 6.594) | 4.006 | 1.316 | (1.313, 6.584) |
| $\omega^2_{21}$ | | | | 0.895 | 0.262 | (0.507, 1.521) | 0.909 | 0.265 | (0.51, 1.535) |
| $\omega^2_{22}$ | | | | 0.55 | 0.125 | (0.344, 0.835) | 0.552 | 0.122 | (0.347, 0.824) |
| $\omega^2_{23}$ | | | | 1.561 | 0.319 | (1.027, 2.275) | 1.597 | 0.323 | (1.042, 2.303) |
| $\omega^2_{24}$ | | | | 5.367 | 0.891 | (3.815, 7.292) | 5.391 | 0.893 | (3.835, 7.324) |
| $\omega^2_{25}$ | | | | 6.35 | 1.948 | (3.326, 10.87) | 6.017 | 1.744 | (3.278, 10.062) |
| $\rho_{2,12}$ | | | | -0.769 | 0.07 | (-0.881, -0.608) | -0.773 | 0.069 | (-0.88, -0.613) |
| $\rho_{2,13}$ | | | | 0.65 | 0.101 | (0.421, 0.814) | 0.657 | 0.1 | (0.431, 0.818) |
| $\rho_{2,14}$ | | | | -0.253 | 0.157 | (-0.545, 0.064) | -0.261 | 0.158 | (-0.554, 0.065) |
| $\rho_{2,15}$ | | | | 0.07 | 0.175 | (-0.269, 0.41) | 0.077 | 0.175 | (-0.263, 0.417) |
| $\rho_{2,23}$ | | | | -0.858 | 0.041 | (-0.923, -0.764) | -0.858 | 0.041 | (-0.922, -0.763) |
| $\rho_{2,24}$ | | | | 0.51 | 0.108 | (0.279, 0.699) | 0.511 | 0.106 | (0.281, 0.7) |
| $\rho_{2,25}$ | | | | -0.282 | 0.151 | (-0.557, 0.03) | -0.296 | 0.148 | (-0.57, 0.008) |
| $\rho_{2,34}$ | | | | -0.686 | 0.073 | (-0.809, -0.525) | -0.686 | 0.071 | (-0.808, -0.529) |
| $\rho_{2,35}$ | | | | 0.398 | 0.135 | (0.112, 0.633) | 0.4 | 0.132 | (0.119, 0.637) |
| $\rho_{2,45}$ | | | | -0.745 | 0.085 | (-0.871, -0.539) | -0.761 | 0.078 | (-0.878, -0.575) |
| $\pi^D_1$ | | | | 0.865 | 0.035 | (0.787, 0.926) | 0.861 | 0.033 | (0.788, 0.918) |
| $\pi^D_2$ | | | | 0.135 | 0.035 | (0.074, 0.213) | 0.139 | 0.033 | (0.082, 0.212) |
| $\mu_1$ | -2.859 | 0.18 | (-3.23, -2.521) | -3.03 | 0.223 | (-3.481, -2.637) | -3.128 | 0.203 | (-3.537, -2.712) |
| $\mu_2$ | -1.261 | 0.059 | (-1.378, -1.144) | -1.701 | 0.078 | (-1.859, -1.555) | -1.91 | 0.092 | (-2.102, -1.74) |
| $\pi^C_1$ | 0.205 | 0.044 | (0.126, 0.295) | 0.242 | 0.07 | (0.123, 0.384) | 0.293 | 0.08 | (0.148, 0.449) |
| $\pi^C_2$ | 0.795 | 0.044 | (0.705, 0.874) | 0.758 | 0.07 | (0.616, 0.877) | 0.707 | 0.08 | (0.551, 0.852) |
| $\tau^2$ | 0.208 | 0.046 | (0.133, 0.309) | 0.154 | 0.059 | (0.065, 0.29) | 0.134 | 0.072 | (0.03, 0.319) |
| $\gamma_2$ | 1.09 | 0.108 | (0.893, 1.316) | 1.08 | 0.106 | (0.879, 1.294) | 1.08 | 0.106 | (0.879, 1.294) |
| $\tilde{\theta}_{01}$ | -0.144 | 0.6 | (-1.453, 1.125) | -0.104 | 0.554 | (-1.3, 1.096) | -0.098 | 0.555 | (-1.303, 1.138) |
| $\tilde{\theta}_{02}$ | -0.158 | 0.624 | (-1.4, 1.055) | -0.113 | 0.579 | (-1.223, 0.945) | -0.11 | 0.565 | (-1.198, 0.981) |
| $\tilde{\theta}_{03}$ | -0.175 | 0.487 | (-1.141, 0.624) | -0.117 | 0.413 | (-0.888, 0.644) | -0.113 | 0.417 | (-0.899, 0.642) |
| $\tilde{\theta}_{04}$ | -0.139 | 0.144 | (-0.444, 0.069) | -0.1 | 0.117 | (-0.332, 0.086) | -0.101 | 0.118 | (-0.329, 0.088) |
| $\tilde{\theta}_{05}$ | 0.037 | 0.187 | (-0.268, 0.453) | 0.009 | 0.159 | (-0.29, 0.326) | 0.007 | 0.162 | (-0.297, 0.321) |
| $\tilde{\theta}_{06}$ | 0.085 | 0.317 | (-0.542, 0.709) | 0.072 | 0.278 | (-0.428, 0.671) | 0.078 | 0.286 | (-0.412, 0.697) |
| $\tilde{\theta}_{07}$ | 0.09 | 0.668 | (-1.185, 1.304) | 0.082 | 0.59 | (-0.905, 1.235) | 0.08 | 0.582 | (-0.942, 1.219) |