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We consider a setting where the firm sells a main service (e.g., air travel) and an ancillary service (e.g., baggage delivery) to two types of consumers (e.g., business travelers and leisure travelers). We study how the firm's ability to charge discriminatory main service prices affects its decision of whether to unbundle the ancillary service from the main service and charge separate prices. Unlike a firm using uniform pricing of main service that unbundles the ancillary service if the consumers that value the main service higher have a high likelihood of purchasing the ancillary service, a firm using discriminatory pricing of main service unbundles the ancillary service if the consumers that value the main service higher have a low likelihood of purchasing the ancillary service. Moreover, discriminatory pricing of main service makes unbundling more (less) likely to be the optimal ancillary service strategy when consumers' main service valuations and ancillary service valuations are negatively (positively) correlated. Finally, we characterize how firms' use of main service price discrimination and consumers' valuation structure (i.e., whether the correlation between consumers' main service valuations and ancillary service valuations is positive or negative) jointly determine the ancillary service strategies in an industry.

1. Introduction

Many firms provide an ancillary service in addition to a main service to enhance the experience of consumers. When taking a flight, consumers may need to transport bags or have a meal during the flight. When staying at a hotel, consumers may need to have breakfast or use internet connection. In many service industries such as travel, consumers book the main service in advance, at which time they may be uncertain about their valuations for the ancillary service. As the travel time approaches, valuation uncertainty is resolved. Consumer valuations for the ancillary service are heterogeneous, indeed some consumers may not need the ancillary service at all.

Even in the same industry, firms adopt different strategies regarding whether to unbundle the same type of ancillary service. For example, while many airlines have unbundled the checked baggage service, Southwest Airlines offers the first two checked bags for free, meaning that the baggage fee is built into the ticket price. The tradeoff that a firm faces regarding unbundling the ancillary service is as follows. On the one hand, by unbundling the ancillary service, the firm gains additional flexibility from being able to charge a separate price for the ancillary service and extracts more consumer surplus. On the other hand, it incurs inconvenience costs by unbundling the ancillary service, which may include the additional labor cost to process the ancillary service payments, the cost of congestion (e.g., if an airline has to process the payments for carry-on bags at the gate, its flights could be easily delayed, resulting in an undesirable on-time performance record), and loss of consumer goodwill from having to pay for the ancillary service.

In this paper, we consider a firm that sells a main service and an ancillary service to two types of consumers (e.g., business travelers and leisure travelers). We first study the optimal ancillary service strategy for a firm that does not price-discriminate when selling the main service, i.e., the firm charges a uniform main service price to both consumer types. Note that under the commodity bundling setting, for each product type, the firm also charges the same price to all consumers. The difference between ancillary service unbundling and commodity bundling is that a consumer cannot purchase or use the ancillary service if she has not purchased the main service, whereas in commodity bundling products can be purchased separately. Despite the difference, if the firm charges a uniform service price to both consumer types, we find consistent results with the commodity bundling literature regarding the conditions for unbundling to be optimal. These results would indicate that airlines with lower percentages of business travelers such as Southwest should be more likely to unbundle checked baggage service than airlines with higher percentages of business travelers such as Delta (Section 4 provides the detailed analysis). This is clearly inconsistent with the airline industry practice. Although uniform pricing of main service is common in some other industries (e.g., many hotels, especially economy hotels, do not charge different room rates to different consumers), it is common airline practice to charge business travelers, who usually book tickets closer to the travel date and have a higher willingness to pay, a higher price than leisure travelers, who usually book tickets well in advance and have a lower willingness to pay. Therefore, we study the optimal ancillary service strategy for a firm that can price-discriminate when selling the main service, that is, the firm charges different main service prices to different consumer types (Section 5). We develop insights about how the firm's ability to price-discriminate when selling the main service affects its optimal ancillary service strategy (Section 6). Since the extent to which main service price discrimination is used differs across industries, studying both the firms using uniform pricing of main service and the firms using discriminatory pricing of main service generate insights for multiple industries.

We find that whether the firm uses discriminatory pricing of main service or not significantly changes the optimal strategy for ancillary services. For a uniform-pricing firm, it is optimal to unbundle the ancillary service if the consumers that value the main service higher have a high enough likelihood of purchasing the ancillary service. This indicates that for unbundling to be the optimal strategy for the ancillary service, the correlation between consumers' valuations for the main service and their valuations for the ancillary service needs to be positive enough, which is consistent with the findings of the commodity bundling literature (Adams and Yellen 1976, McAfee et al. 1989, and Schmalensee 1984). However, for a discriminatory-pricing firm, it is optimal to unbundle the ancillary service if the consumers that value the main service higher have a low enough likelihood of purchasing the ancillary service. Thus, the optimality condition for unbundling does not depend on the consumer valuation correlation, and the findings of the commodity bundling literature no longer hold. Moreover, by comparing the uniform-pricing case and the discriminatory-pricing case directly, we find that compared to a uniform-pricing firm, unbundling is more (less) likely to be the optimal strategy for a price-discriminating firm if consumers' valuations for the main service and the ancillary service are negatively (positively) correlated. This result indicates that as a firm starts to adopt price discrimination for the main service, the optimal ancillary service strategy may be changed, and the key determinant is consumers' valuation structure. Finally, if firms use discriminatory pricing of main service, the way that the consumer valuation structure affects the differentiation of optimal ancillary service strategies across firms is reversed from the case of uniform pricing. In the case of uniform pricing, if consumers' valuations for the main service and the ancillary service are positively (negatively) correlated, unbundling is more (less) likely to be the optimal ancillary service strategy for firms with higher proportions of high-type consumers. The result is exactly the opposite in the case of discriminatory pricing, i.e., if consumers' valuations for the main service and the ancillary service are negatively (positively) correlated, unbundling is more (less) likely to be the optimal ancillary service strategy for firms with higher proportions of high-type consumers.

Therefore, the structure of firms' optimal ancillary service strategies in an industry is jointly determined by firms' use of main service price discrimination and consumers' valuation structure. This provides insights for ancillary service strategies across different industries. For example, breakfast service is among the most common ancillary services offered by hotels. Business travelers would be more likely to eat at the hotel and pay for it, while leisure travelers may have breakfast at a nearby restaurant at a much lower price. Similarly, airlines offer in-flight beverage service. Business travelers would be more likely to purchase a beverage during the flight, while leisure travelers may consider bringing their beverage with them on board rather than purchasing if the airline charges for it. For such ancillary services, our results indicate that unbundling is more likely to be optimal for hotels with higher proportions of business travelers (e.g., luxury and upscale hotels) than hotels with lower proportions of business travelers (e.g., economy and budget hotels), and it is more likely to be optimal for airlines with lower proportions of business travelers (e.g., low-cost airlines) than airlines with higher proportions of business travelers (e.g., legacy airlines). The current industry practice is that luxury hotels usually charge for breakfast and economy hotels usually offer breakfast for free. According to the 2012 Lodging Survey by American Hotel & Lodging Association, the percentages of hotels in different tiers charging for breakfast are: luxury hotels (67%), upscale hotels (33%), midprice hotels (14%), economy hotels (8%), budget hotels (15%). However, in the airline industry, it is the low-cost airlines (e.g., Spirit and Frontier) that charge for in-flight beverages.

For airlines, the checked baggage service should involve a negative correlation between consumers' main service valuations and ancillary service valuations, because business travelers are less likely to check bags than leisure travelers (Schaal 2014). Thus, our results indicate that unbundling is more likely to be optimal for airlines with higher percentages of business travelers. In practice, legacy airlines charge for checked bags. Primarily serving leisure travelers, Southwest Airlines does not charge for the first two bags. (Some low-cost airlines have switched to a different pricing scheme for the ancillary service with late-payment penalty and started to charge for bags, e.g., Spirit and Frontier. We discuss this case in Section 6.) Moreover, as other airlines started to charge for checked bags, consumers with higher baggage needs may switch to Southwest, which results in an overall increase in Southwest's baggage demand. This would consolidate bundling as Southwest's optimal strategy. Additionally, as the only major U.S. airline that does not use online travel agencies to sell ticket, Southwest does not have the incentive to reduce the commissions paid to the intermediaries by separately charging for the ancillary service. The above three explanations shed light on the interesting phenomenon in the airline industry that while most airlines charge for checked bags, Southwest Airlines provides this ancillary service for free.

Therefore, the contribution of this paper is two-fold. First, we study the ancillary service unbundling problem for firms that use discriminatory pricing of main service. Second, by contrasting the results to firms that use uniform pricing of main service, we highlight the fact that the ability to price-discriminate when selling the main service has significant impact on a firm's optimal ancillary service strategy and an industry's ancillary service strategy structure.

2. Literature Review

Although there are not many papers that study ancillary pricing (also called add-on pricing in some papers), researchers have used both competition models and monopolistic models to address related issues. Ellison (2005), Gabaix and Laibson (2006), Shulman and Geng (2013), Lin (2015), and Geng and Shulman (2015) study the competition between firms that sell both a main service and an ancillary service. Papers that study ancillary pricing under monopolistic settings include Allon et al. (2011) and Fruchter et al. (2011). Allon et al. (2011) study airlines' baggage pricing problem and find that the firm should set the fee for the baggage service at the same level the social planner would. Their result also suggests that the way in which airlines have implemented baggage fees is more consistent with attempts to control consumer behavior (i.e., induce consumers to reduce their baggage needs) than segmenting consumers based on their need to check a bag. Fruchter et al. (2011) consider a firm that charges the same price to different consumer segments and find that a free add-on (i.e., bundling the ancillary service) is more profitable than offering it for a fee (i.e., unbundling the ancillary service) if one consumer segment has a high valuation for the add-on but a relatively low valuation for the primary service, and another segment has a higher valuation for the primary service but places no value on the add-on. Our paper is one of the first to study the question of whether the firm should unbundle the ancillary service in the first place, and our paper is the first to study this question for both a firm that charges a uniform main service price and a firm that charges discriminatory main service prices.

A related stream of literature studies commodity bundling. By analyzing a bundling setting with two commodities, Adams and Yellen (1976), McAfee et al. (1989), and Schmalensee (1984) provide the insight that a higher degree of negative correlation between consumers' valuations for the two commodities makes bundling more profitable relative to unbundled sales. We find consistent results for a firm that charges a uniform main service price. However, the results obtained from analyzing a model with a uniform-pricing firm do not explain the phenomenon we see in the airline industry. Allowing for main service price discrimination fundamentally changes the previous findings from the bundling literature. We find that whereas it is optimal for a uniform-pricing firm to unbundle the ancillary service if the consumers that value the main service higher have a high enough likelihood of purchasing the ancillary service (which indicates a positive correlation between consumers' valuations for the main service and their valuations for the ancillary service), it is optimal for a discriminatory-pricing firm to unbundle the ancillary service if the consumers that value the main service higher have a low enough likelihood of purchasing the ancillary service. Main service price discrimination makes unbundling more (less) likely to be the optimal ancillary service strategy if consumers' valuations for the main service and the ancillary service are negatively (positively) correlated. Thus, the correlation effect found by previous commodity bundling literature becomes very different with firm's ability to charge discriminatory prices. Recently, researchers have explored other research questions related to bundling, such as bundling with channel interaction (e.g., Bhargava 2012, Chakravarty et al. 2013, Girju et al. 2013, Cao et al. 2015), bundling information goods (e.g., Bakos and Brynjolfsson 1999, Geng et al. 2005), bundling vertically differentiated products (e.g., Banciu et al. 2010, Honhon and Pan 2014), cardinality bundling (e.g., Wu et al. 2014), and the effect of bundling on firm's ordering decision (e.g., Cao et al. 2014). Although our focus in studying ancillary pricing appears at first sight to have similarities to issues studied in the commodity bundling literature, there are significant differences between the two settings. In the setting studied by the commodity bundling literature, each commodity can be sold independently (e.g., a retailer that sells toothbrush-toothpaste bundles can sell toothbrushes and toothpastes as two independent products). In the ancillary pricing setting, the ancillary service cannot be sold by itself. Consumers can purchase the ancillary service only if they have already purchased the main service, and the purchase of the ancillary service often occurs later than the purchase of the main service.

There is also a related stream of literature on two-part pricing. Two-part pricing corresponds to the situation where the price of a service is composed of two parts – a lump-sum fee for the fixed part of the service (e.g., cover charge of a bar), and a per-unit charge for the variable part of the service (e.g., per-drink fee). Pioneered by Oi (1971) and Schmalensee (1981), the most important issue that the two-part pricing literature has focused on is when the optimal per-unit price should be above or below the marginal cost of providing the service. Rosen and Rosenfield (1997) find that whether the optimal per-unit price is above or below its marginal cost depends on whether the average consumer has higher or lower demand for the variable part of the service than the marginal consumer. If the average consumer has higher demand for the variable part of the service than the marginal consumer, the firm should set the per-unit price above the marginal cost; and vice versa. A more recent paper, Png and Wang (2010), finds that the result also depends on the correlation between marginal and total benefits from the service. The per-unit price should be set above the marginal cost if marginal and total benefits from the service are positively correlated; and vice versa. In the ancillary pricing setting, we find that for a firm that charges a uniform main service price, the result depends on the underlying consumer valuation structure (i.e., the correlation between consumers' valuations for the main service and the ancillary service) in a way that is consistent with the two-part pricing results. Moreover, we also find that the result becomes very different for a firm that charges discriminatory main service prices. In this case, if consumers are forward-looking (i.e., they take future utilities from the ancillary service into consideration when purchasing the service bundle or main service in advance), the optimal ancillary service price is equal to the marginal cost. However, if there exists a significant proportion of myopic consumers (who do not take future utilities into consideration), the optimal ancillary service price is higher than the marginal cost.

Therefore, as our literature review indicates, a key differentiator of our paper is that we study a discriminatory-pricing firm's optimal unbundling and pricing decisions for the ancillary service and how the results are changed compared to a uniform-pricing firm. We find that some key findings from the previous commodity bundling and two-part pricing literature become very different when one considers a discriminatory-pricing firm instead of a uniform-pricing firm.

3. Model

The firm sells a main service and an ancillary service to two types of consumers that have different valuations for the service. There are λ_H consumers that value the main service at v_H and λ_L consumers that value the main service at v_L, where $v_H > v_L$. In travel industries, the λ_H consumers can be considered as business travelers and the λ_L consumers can be considered as leisure travelers. Throughout the paper, we refer to consumers with main service valuation v_H as high-type consumers, and consumers with main service valuation v_L as low-type consumers. Consumers have uncertain valuations for the ancillary service. Let u_H and u_L denote the (uncertain) valuations for the ancillary service of high-type and low-type consumers, respectively. The ancillary service valuations u_H and u_L have support $[\underline{u}, \bar{u}]$, where $\bar{u} > 0$ and $\underline{u} < 0$. We assume $\bar{u} \leq v_L$ (i.e., consumers' valuations for the ancillary service cannot exceed their valuations for the main service) and $v_L + \bar{u} \leq v_H$ (i.e., any low-type consumer's valuation for the whole service does not exceed any high-type consumer's valuation for the whole service). Note that we allow consumers' valuations for the ancillary service to be negative. A consumer with a negative valuation for the ancillary service will not use the ancillary service even if it is offered for free. For example, some consumers do not have bags to check for the flight. Even if the firm does not charge for checked bags, these consumers still will not use this service. The cumulative distribution functions of u_H and u_L are denoted by $F_H(\cdot)$ and $F_L(\cdot)$, and the probability density functions are denoted by $f_H(\cdot)$ and $f_L(\cdot)$. We assume that u_H and u_L are both uniformly distributed over $[0, \bar{u}]$ but have different probability densities (we do not assume a specific functional form for the density over $[\underline{u}, 0)$). For $i = H, L$, the probability density function of u_i is given by $f_i(x) = \beta_i / \bar{u}$ for $0 \leq x \leq \bar{u}$. Furthermore, define $\beta_i = \bar{F}_i(0)$ for $i = H, L$. β_i measures type-i consumers' likelihood of purchasing the ancillary service. If $\beta_H \geq \beta_L$, high-type consumers are more likely to purchase the ancillary service than low-type consumers for any price that the firm charges for the ancillary service, and consumers' valuations for the main service and their valuations for the ancillary service exhibit a positive correlation. If $\beta_H < \beta_L$, low-type consumers are more likely to purchase the ancillary service than high-type consumers for any price that the firm charges for the ancillary service. Thus, in this case, consumers' valuations for the main service and their valuations for the ancillary service exhibit a negative correlation. Therefore, the relationship between β_H and β_L defines the consumer valuation structure. As we will see, this relationship is an important factor in determining firms' optimal ancillary service strategies.

Consumers make the purchasing decision in two stages. First, consumers decide whether to purchase the service bundle (if the firm bundles), or whether to purchase the main service (if the firm unbundles) before their valuations for the ancillary service are realized. Then, after their valuations for the ancillary service are realized, consumers decide whether to use the ancillary service (if the firm bundles), or whether to purchase the ancillary service (if the firm unbundles). We assume that consumers are forward-looking, that is, when making the purchasing decision for the service bundle or main service in advance, they take future utilities from the ancillary service into consideration. In Section 5.1, we incorporate myopic consumers (who do not consider future utility from the ancillary service when making the purchasing decision for the service bundle or main service in advance) for a discriminatory-pricing firm and study the effect of myopic consumers on firm's optimal ancillary service strategy.

The firm's key decision is whether to sell the whole service as a bundle, or to unbundle the ancillary service from the main service and sell the two services separately. We will first present results for a firm that charges a uniform price for the main service, and show that the results are consistent with existing results from the commodity bundling literature. Then, we will study a firm that charges discriminatory prices for the main service. The firm's pricing decisions and notations are as follows:

Uniform pricing of main service In the bundling case, the firm charges price p_b to both consumer types for the service bundle. In the unbundling case, the firm charge price p_m for the main service and p_a for the ancillary service.

Discriminatory pricing of main service In the bundling case, the firm charges price p_{bH} to high-type consumers and p_{bL} to low-type consumers for the service bundle. In the unbundling case, the firm charge prices p_{mH} and p_{mL} to two types of consumers for the main service and p_a for the ancillary service.

Consistent with industry practice, we assume that the firm charges the same ancillary service price to both types of consumers when the ancillary service is unbundled. For example, if a consumer buys a coach ticket, price of in-flight meal does not depend on how much the consumer has paid for the ticket.

The firm incurs several variable costs serving its consumers. The marginal cost of providing one unit of main service is c_m $(0 < c_m < v_L)$. The marginal cost of providing one unit of ancillary service is c_a $(0 < c_a < \bar{u})$. Moreover, when the firm unbundles the ancillary service, it incurs an inconvenience cost $c(\cdot)$ due to consumers' separate purchases of the ancillary service. The marginal cost is incurred whenever a consumer uses the ancillary service, no matter whether the ancillary service is bundled or unbundled. For example, the marginal cost of airline baggage service would include the fuel cost and labor cost (e.g., loading and unloading the bag). On the other hand, the inconvenience cost is incurred when the ancillary service is purchased separately. If the ancillary service is unbundled, the inconvenience costs may include the additional labor cost to process the ancillary service payments and the cost of congestion. For example, passengers paying for carry-on bags at the gate can delay the boarding process and affect airlines' on-time performances. Moreover, the inconvenience cost may include firm's potential profit loss because of consumers' loss of goodwill that is caused by unbundling. By studying consumer perception at a travel resort, Naylor and Frank (2001) find that not receiving an all-inclusive package lessens perceptions of value for first-time guests. Recently some airlines have been considering charging for in-flight lavatory use (Pawlowski 2010). One could easily imagine the consumer dissatisfaction that is brought about if she was asked to pay a fee for lavatory use during the flight. The cost of unbundling the ancillary/add-on service has also been modeled by other papers, e.g., Allon et al. (2011) model consumers' costly effort to reduce the likelihood of using the ancillary service (effort cost is transferred to the firm), Geng and Shulman (2015) model the cost of unbundling as a potential loss of market share. We define the inconvenience cost $c(\cdot)$ as a function of the number of consumers who purchase the ancillary service in the unbundling case. We assume $c(0) = c'(0) = 0$, $c'(\cdot) \geq 0$ and $c''(\cdot) \geq 0$. In practice, it would be difficult to significantly reduce the marginal cost, but it may be possible to significantly reduce the inconvenience cost (e.g., by using mechanisms that induce consumers to pay for the ancillary service in advance). The firm's goal is to choose the optimal strategy (i.e., unbundle or not) and price the main service and the ancillary service optimally so that the total profit from selling the whole service is maximized.

4. Uniform Pricing of Main Service

In this section, we study the optimal ancillary service strategy for a firm that charges a uniform price for the main service to both types of consumers. Note that under discriminatory pricing, both types of consumers are served. Under uniform pricing, it may be optimal to serve only high-type consumers. However, to make a fair comparison, we consider a uniform-pricing firm that serves both types of consumers, that is, the firm charges the uniform price at low-type consumers' willingness to pay.

We analyze the bundling case and the unbundling case separately, and then compare these two cases to obtain the optimal ancillary service strategy. First, consider the bundling case. For each consumer type $i = H, L$, given that a consumer purchases the service bundle, she uses the ancillary service if $u_i \geq 0$ after u_i is realized. The firm sells the bundle at price $p_b^* = v_L + E(u_L)^+$ which is low-type consumers' willingness to pay. Note that $\bar{u} \leq v_H - v_L$ ensures that when $p_b = v_L + E(u_L)^+$, high-type consumers purchase the service bundle as well, that is, charging at low-type consumers' willingness to pay induces high-type consumers to purchase as well even if high-type consumers may value the ancillary service lower. This also holds in the unbundling case. Moreover, the firm incurs marginal costs for the ancillary service used by consumers who have non-negative valuations for the ancillary service. Thus, the optimal profit in the bundling case is

$$\Pi_{b,n}^* = [v_L + E(u_L)^+ - c_m](\lambda_H + \lambda_L) - c_a[\lambda_H \bar{F}_H(0) + \lambda_L \bar{F}_L(0)].$$

The second subscript of "n" means that the firm does not use price discrimination when selling the main service.

Second, consider the unbundling case. For each consumer type $i = H, L$, given that a consumer purchases the main service, she purchases the ancillary service if $u_i \geq p_a$ after u_i is realized. The firm's main service price p_m should satisfy $p_m = v_L + E(u_L - p_a)^+$ which makes low-type consumers' individual rationality constraint binding. Moreover, the firm incurs marginal and inconvenience costs from those consumers who purchase the ancillary service (i.e., those who have $u_i \geq p_a$). The firm's profit maximization problem in the unbundling case can be reduced to a single-variable optimization problem of the ancillary service price p_a with the following profit function:

$$\begin{aligned}\Pi_{u,n}(p_a) = {} & [v_L + E(u_L - p_a)^+ - c_m](\lambda_H + \lambda_L) \\ & + (p_a - c_a)[\lambda_H \bar{F}_H(p_a) + \lambda_L \bar{F}_L(p_a)] - c(\lambda_H \bar{F}_H(p_a) + \lambda_L \bar{F}_L(p_a)).\end{aligned}$$

In this section, we assume $c'''(\cdot) \geq 0$ which holds at least if the inconvenience cost function is polynomial with non-negative coefficients or exponential. This assumption is only needed to ensure the quasi-concavity of $\Pi_{u,n}(p_a)$ (hence to guarantee that the optimal solution $p^*_{a,n}$ is unique), and is not needed for the rest of the analysis in this section or the analysis in the remaining sections of the paper.

THEOREM 1. *Under uniform pricing of main service, in the unbundling case, if high-type consumers are more likely to purchase the ancillary service than low-type consumers (i.e., $\beta_H \geq \beta_L$), the optimal ancillary service price is greater than the total marginal cost of ancillary service (i.e., $p^*_{a,n} \geq c_a + c'(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n}))$); otherwise, the result reverses (i.e., if $\beta_H < \beta_L$, $p^*_{a,n} < c_a + c'(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n}))$).*

Theorem 1 states that under uniform pricing of main service, if the firm unbundles, the optimal ancillary service price should be above the total marginal cost of ancillary service when high-type consumers are more likely to purchase the ancillary service than low-type consumers, or equivalently, consumers' valuations for the main service and the ancillary service are positively correlated. The optimal ancillary service price should be below the total marginal cost when low-type consumers are more likely to purchase the ancillary service than high-type consumers, or equivalently, consumers' valuations for the main service and the ancillary service are negatively correlated. This result is consistent with the previous findings in two-part pricing literature (e.g., Rosen and Rosenfield 1997, Png and Wang 2010). Under uniform pricing, the firm only extracts full surplus from low-type consumers and leaves some surplus from high-type consumers un-captured. A uniform-pricing firm needs to adjust the main service price and the ancillary service price to extract more surplus from high-type consumers, while keeping low-type consumers willing to purchase. If high-type consumers are more likely to purchase the ancillary service than low-type consumers, to capture more of high-type consumers' surplus from the ancillary service, the firm should increase the ancillary service price and decrease the main service price accordingly. On the other hand, if low-type consumers are more likely to purchase the ancillary service than high-type consumers, the firm should decrease the ancillary service price and increase the main service price accordingly.

THEOREM 2. *Under uniform pricing of main service, there exists an increasing threshold function $\bar{\beta}_{H,n}(\beta_L)$ such that unbundling is more profitable than bundling if and only if $\beta_H \geq \bar{\beta}_{H,n}(\beta_L)$.*

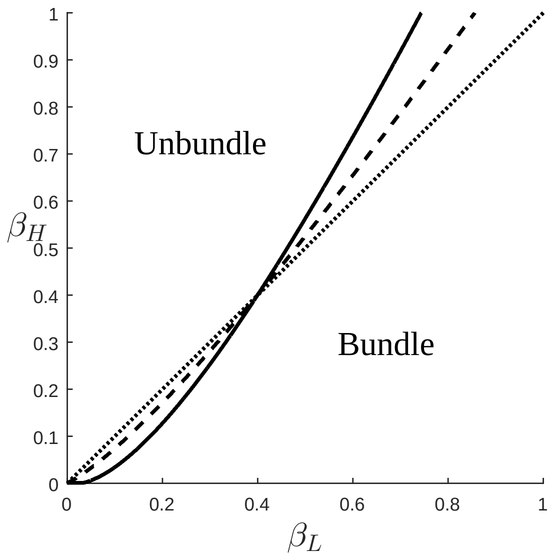

Figure 1 **Optimal ancillary service strategy and threshold $\bar{\beta}_{H,n}(\beta_L)$ under uniform pricing of main service ($v_H = 300$, $v_L = 200$, $\bar{u} = 50$, $\underline{u} = -20$, $c_m = 150$, $c_a = 20$, $c(x) = 0.5x^2$; solid curve: $\lambda_H = 20$, $\lambda_L = 80$; dashed curve: $\lambda_H = 50$, $\lambda_L = 50$; dotted curve: $45°$ line)**

Theorem 2 characterizes the firm's optimal ancillary service strategy under uniform pricing of main service. Figure 1 illustrates the optimal ancillary service strategy by showing the threshold function $\bar{\beta}_{H,n}(\beta_L)$. Theorem 2 essentially states that it is optimal to unbundle the ancillary service if high-type consumers' likelihood of purchasing the ancillary service is high enough and low-type consumers' likelihood of purchasing the ancillary service is low enough, which is equivalent to requiring that the correlation between consumers' main service valuations and ancillary service valuations is positive enough. Thus, although the ancillary service unbundling setting is in nature different from the commodity bundling setting that has been studied by previous literature (i.e., the ancillary service cannot be sold independently), our result in Theorem 2 for a uniform-pricing firm is consistent with the commodity bundling literature.

While unbundling the ancillary service gives the firm more flexibility and allows the firm to extract more consumer surplus, it results in higher inconvenience costs. Under uniform pricing, if high-type consumers are very likely to purchase the ancillary service while low-type consumers are very unlikely to purchase the ancillary service, bundling the ancillary service would mean that the firm is leaving too much surplus to high-type consumers. In this case, the firm should unbundle and charge a high price for the ancillary service to extract more surplus from high-type consumers. On the other hand, if high-type consumers are very unlikely to purchase the ancillary service while low-type consumers are very likely to purchase the ancillary service, the firm benefits from bundling the ancillary service. If the firm unbundles in this case, it will charge a low price for the ancillary service (Theorem 1), which would not generate much revenue but result in a high inconvenience cost. Overall, unbundling the ancillary service assists the firm in extracting more surplus from high-type consumers at the expense of distorting the prices charged to low-type consumers. A positive enough correlation between consumers' valuations for the main service and the ancillary service indicates that high-type consumers have significantly more surplus from the ancillary service compared to low-type consumers, and hence the firm should capture it by unbundling the ancillary service.

THEOREM 3. *Consider two scenarios for a firm using uniform pricing of main service. In the first scenario, the demand sizes are λ_{H1} and λ_{L1}. In the second scenario, the demand sizes are λ_{H2} and λ_{L2}. Suppose $\lambda_{H1}+\lambda_{L1}=\lambda_{H2}+\lambda_{L2}=\lambda$ and $\lambda_{H1}<\lambda_{H2}$ (hence $\lambda_{L1}>\lambda_{L2}$). Then, $\bar{\beta}_{H,n1}(\beta_L)\geq\bar{\beta}_{H,n2}(\beta_L)$ in the region of $\beta_H\geq\beta_L$ and $\bar{\beta}_{H,n1}(\beta_L)<\bar{\beta}_{H,n2}(\beta_L)$ in the region of $\beta_H<\beta_L$; $\bar{\beta}_{H,n1}(\beta_L)$ and $\bar{\beta}_{H,n2}(\beta_L)$ intersect on the 45° line $\beta_H=\beta_L$.*

Theorem 3 characterizes how the optimal ancillary service strategy is affected by the firm's demand portfolio in the uniform-pricing case. Figure 1 illustrates how the threshold function $\bar{\beta}_{H,n}(\beta_L)$ moves as a result of a change in the firm's demand portfolio. If high-type consumers are more likely to purchase the ancillary service than low-type consumers (i.e., $\beta_H\geq\beta_L$), then increasing the proportion of high-type consumers expands the region in which unbundling is optimal. If low-type consumers are more likely to purchase the ancillary service than high-type consumers (i.e., $\beta_H<\beta_L$), then increasing the proportion of high-type consumers shrinks the region in which unbundling is optimal. If consumers' valuations for the main service and the ancillary service are positively correlated, compared to a firm with fewer high-type consumers, a firm with more high-type consumers has more incentive to capture the ancillary service surplus from high-type consumers, hence unbundling is more likely to be the optimal ancillary service strategy. On the other hand, if consumers' valuations for the main service and the ancillary service are negatively correlated, unbundling is less likely to be the optimal ancillary service strategy for a firm with more high-type consumers.

For airline checked baggage service, the consumer valuation correlation is negative, because business travelers who have higher willingness to pay for the tickets actually are less likely to have bags to check. In this case, applying Theorem 3 would indicate that airlines with lower business traveler percentages are more likely to charge for checked bags compared to airlines with higher business traveler percentages. This is clearly inconsistent with checked baggage strategies used by airlines. As of 2015, all legacy airlines charge for checked bags. On the other hand, Southwest Airlines has a much lower percentage of business travelers compared to legacy airlines, but it does not charge for the first two checked bags. One motivation of this paper is to study airlines' ancillary service strategies. In order to explain the airline industry practice, we need to analyze the case where firms use price discrimination when selling the main service. Next, we are going to explore how the results and insights are changed by firm's ability to price-discriminate when selling the main service.

5. Discriminatory Pricing of Main Service

Now, we consider a firm that can charge discriminatory prices for the service bundle and main service. For example, in the airline industry, leisure travelers usually plan their trips in advance and business travelers usually make reservations closer to the travel date. Because of this demand characteristic, airlines have implemented price discrimination by changing prices over time (i.e., inter-temporal price discrimination).

We analyze the bundling case and the unbundling case separately, and then compare these two cases to obtain the optimal ancillary service strategy. In the bundling case, the firm sells the bundle at prices $p_{bH}^* = v_H + E(u_H)^+$ and $p_{bL}^* = v_L + E(u_L)^+$ to different consumer types. The optimal profit in the bundling case is

$$\Pi_b^* = [v_H + E(u_H)^+ - c_m]\lambda_H + [v_L + E(u_L)^+ - c_m]\lambda_L - c_a[\lambda_H \bar{F}_H(0) + \lambda_L \bar{F}_L(0)].$$

Note that if $v_i + E(u_i)^+ - c_m - c_a\bar{F}_i(0) < 0$ for type-i consumers, the firm should not sell to this consumer type. We assume $v_L + E(u_L)^+ - c_m - c_a\bar{F}_L(0) \geq 0$, that is, the firm earns profits by selling to low-type consumers. Since this condition implies $v_H + E(u_H)^+ - c_m - c_a\bar{F}_H(0) > 0$, the firm also earns profits by selling to high-type consumers. Allowing the possibility that the firm may want to only sell to some consumer type does not result in different insights regarding the firm's optimal ancillary service strategy. In the unbundling case, the firm's main service prices charged to different consumer types, p_{mH} and p_{mL}, should satisfy $v_H - p_{mH} + E(u_H - p_a)^+ = 0$ and $v_L - p_{mL} + E(u_L - p_a)^+ = 0$, respectively. The profit function in the unbundling case as a function of the ancillary service price p_a is

$$\begin{aligned}\Pi_u(p_a) &= (p_{mH} - c_m)\lambda_H + (p_{mL} - c_m)\lambda_L \\ &\quad + (p_a - c_a)[\lambda_H \bar{F}_H(p_a) + \lambda_L \bar{F}_L(p_a)] - c(\lambda_H \bar{F}_H(p_a) + \lambda_L \bar{F}_L(p_a)) \\ &= [v_H + E(u_H - p_a)^+ - c_m]\lambda_H + [v_L + E(u_L - p_a)^+ - c_m]\lambda_L \\ &\quad + (p_a - c_a)[\lambda_H \bar{F}_H(p_a) + \lambda_L \bar{F}_L(p_a)] - c(\lambda_H \bar{F}_H(p_a) + \lambda_L \bar{F}_L(p_a)).\end{aligned}$$

THEOREM 4. *(i) Under discriminatory pricing of main service, in the unbundling case, the optimal ancillary service price is equal to the total marginal cost of ancillary service (i.e., p_a^* is the solution to $p_a^* = c_a + c'(\lambda_H \bar{F}_H(p_a^*) + \lambda_L \bar{F}_L(p_a^*))$).*

(ii) For each consumer type $i = H, L$, the optimal prices satisfy $p_{mi}^* < p_{bi}^* < p_{mi}^* + p_a^*$.

(iii) The price reduction from the optimal bundle price to the optimal main service price when the firm unbundles is greater for the consumer type with a higher likelihood of purchasing the ancillary service (i.e., if $\beta_H \geq \beta_L$, $p_{bH}^* - p_{mH}^* \geq p_{bL}^* - p_{mL}^*$; if $\beta_H < \beta_L$, $p_{bH}^* - p_{mH}^* < p_{bL}^* - p_{mL}^*$).

Theorem 4(i) characterizes the optimal ancillary service price for a discriminatory-pricing firm in the unbundling case, which is given by the condition that marginal benefit is equal to total marginal cost. Combining Theorem 4(i) and Theorem 1 which characterizes the optimal ancillary service price for a uniform-pricing firm, we obtain that compared to the uniform-pricing case, the firm should charge a lower ancillary service price under discriminatory pricing when high-type consumers are more likely to purchase the ancillary service than low-type consumers, or equivalently, consumers' valuations for the main service and the ancillary service are positively correlated. The firm should charge a higher ancillary service price under discriminatory pricing when low-type consumers are more likely to purchase the ancillary service than high-type consumers, or equivalently, consumers' valuations for the main service and the ancillary service are negatively correlated. Under discriminatory pricing, the firm is able to extract full surplus ex ante from both types of consumers, and does not need to distort the ancillary service price in order to reduce the un-captured surplus from high-type consumers. Thus, the relationship between the optimal ancillary service price and the total marginal cost of ancillary service no longer depends on the correlation of consumers' valuations for the main service and the ancillary service as it did in the uniform pricing case. Recall that consumers make forward-looking decisions when purchasing the main service, i.e., they take future utilities from the ancillary service into consideration. Thus, the firm's optimal ancillary and main service prices are interrelated. In Section 5.1, we consider a model that also includes myopic consumers who do not make forward-looking purchasing decisions as an extension. As we will show, when myopic consumers exist, a discriminatory-pricing firm's optimal ancillary service price can also be higher than the total marginal cost.

Theorem 4(ii) states that compared to the optimal bundle price, in the unbundling case, the firm should charge a lower main service price but a higher total price including the ancillary service to both types of consumers. Moreover, Theorem 4(iii) states that the consumer type with a higher likelihood of purchasing the ancillary service should see a more significant price reduction of the main service when the firm unbundles the ancillary service. For airlines, since business travelers usually check fewer bags than leisure travelers, our result indicates that the fare reduction resulting from unbundling the baggage service should be more significant for leisure travelers. Our results in Theorem 4(ii) and (iii) are consistent with the empirical findings of Brueckner et al. (2014) that after airlines started charging for baggage fees, leisure fares (as measured by the 25th percentile fare) fell by one-half to one-third of the baggage fee. Correspondingly, the full trip price for a passenger paying the baggage fee rose by one-half to two-thirds of the baggage fee. Their empirical analysis also reveals that the fare impact of imposing a baggage fee is larger at the lower percentiles (i.e., leisure travelers) and smaller at the higher percentiles (i.e., business travelers), which is exactly what we find in Theorem 4(iii). Thus, our model explains the empirical findings of Brueckner et al. (2014).

THEOREM 5. *There exists a decreasing threshold function $\bar{\beta}_H(\beta_L)$ such that unbundling is more profitable than bundling if and only if $\beta_H \leq \bar{\beta}_H(\beta_L)$.*

Next, we derive the optimal ancillary service strategy for a firm using discriminatory pricing of main service. Theorem 5 states that it is optimal to unbundle the ancillary service when both types of consumers' likelihoods of purchasing the ancillary service are low enough. Figure 2 illustrates when unbundling or bundling the ancillary service is optimal for a discriminatory-pricing firm through the same example used in Figure 1. A lower likelihood of consumers purchasing the ancillary service keeps the inconvenience cost less

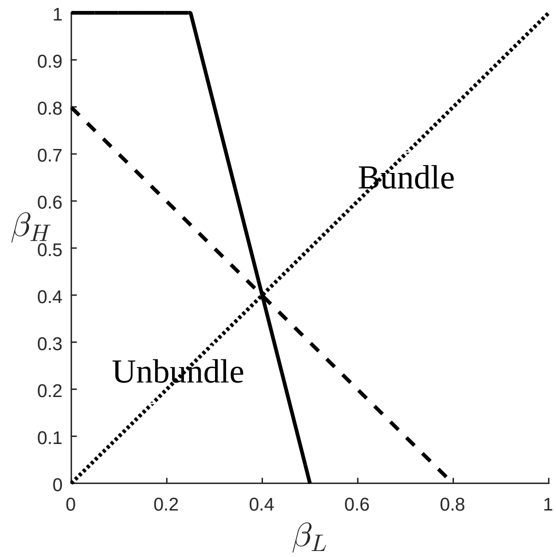

Figure 2 **Optimal ancillary service strategy and threshold $\bar{\beta}_H(\beta_L)$ under discriminatory pricing of main service ($v_H = 300$, $v_L = 200$, $\bar{u} = 50$, $\underline{u} = -20$, $c_m = 150$, $c_a = 20$, $c(x) = 0.5x^2$; solid curve: $\lambda_H = 20$, $\lambda_L = 80$; dashed curve: $\lambda_H = 50$, $\lambda_L = 50$; dotted curve: 45° line)**

significant. For example, airlines usually charge for the ancillary services that are needed by very few consumers, such as fees for carrying pets on board. On the other hand, with a high enough likelihood of consumers purchasing the ancillary service, it is optimal for the firm to bundle the ancillary service into the main service. For example, since everyone needs to eat during long international flights, airlines usually offer "free" meals (i.e., meal price is included in ticket price) for international flights that are long enough (while they usually do not offer inclusive meals for domestic flights).

Recall that we characterized the regions where unbundling and bundling are optimal in Theorem 2 and Figure 1 for a firm using uniform pricing of main service. First, under uniform pricing, unbundling is more profitable than bundling if high-type consumers' likelihood of purchasing the ancillary service is high enough; whereas under discriminatory pricing, unbundling is more profitable if high-type consumers' likelihood of purchasing the ancillary service is low enough. Second, the threshold function that separates the unbundling region and the bundling region is an increasing function under uniform pricing and a decreasing function under discriminatory pricing. Under discriminatory pricing, by charging a different price to high-type consumers for the service bundle or main service, the firm can capture the surplus from high-type consumers directly. Thus, whether unbundling is profitable or not no longer depends on the consumer valuation correlation as it did in the uniform-pricing case. If the firm is able to price-discriminate when selling the main service, for unbundling to be the optimal strategy, both consumer types' likelihoods of purchasing the ancillary service should be low enough.

THEOREM 6. *Consider two scenarios for a firm using discriminatory pricing of main service. In the first scenario, the demand sizes are λ_{H1} and λ_{L1}. In the second scenario, the demand sizes are λ_{H2} and λ_{L2}. Suppose $\lambda_{H1}+\lambda_{L1}=\lambda_{H2}+\lambda_{L2}=\lambda$ and $\lambda_{H1}<\lambda_{H2}$ (hence $\lambda_{L1}>\lambda_{L2}$). Then, $\bar{\beta}_{H1}(\beta_L)\geq\bar{\beta}_{H2}(\beta_L)$ in the region of $\beta_H\geq\beta_L$ and $\bar{\beta}_{H1}(\beta_L)<\bar{\beta}_{H2}(\beta_L)$ in the region of $\beta_H<\beta_L$; $\bar{\beta}_{H1}(\beta_L)$ and $\bar{\beta}_{H2}(\beta_L)$ intersect on the 45° line $\beta_H=\beta_L$.*

Theorem 6 describes how the optimal ancillary service strategy is affected by the firm's demand portfolio in the discriminatory-pricing case. Figure 2 illustrates how the threshold function $\bar{\beta}_H(\beta_L)$ moves as a result of a change in the firm's demand portfolio. The threshold function $\bar{\beta}_H(\beta_L)$ is less steep for a firm with a higher proportion of high-type consumers ($\bar{\beta}_H(\beta_L)$ spins counterclockwise as the proportion of high-type consumers increases). If high-type consumers are more likely to purchase the ancillary service than low-type consumers (i.e., $\beta_H\geq\beta_L$), then increasing the proportion of high-type consumers shrinks the region in which unbundling is optimal. If low-type consumers are more likely to purchase the ancillary service than high-type consumers (i.e., $\beta_H<\beta_L$), then increasing the proportion of high-type consumers expands the region in which unbundling is optimal. Therefore, if consumers' valuations for the main service and the ancillary service are positively correlated, bundling is more likely to be the optimal strategy for a firm with more high-type consumers than a firm with fewer high-type consumers; if consumers' valuations for the main service and the ancillary service are negatively correlated, unbundling is more likely to be the optimal strategy for a firm with more high-type consumers than a firm with fewer high-type consumers.

5.1. Myopic Consumers

In this section, we investigate the effect of myopic consumers on the firm's optimal ancillary service strategy. In travel industries, consumers usually purchase the service bundle (when the firm bundles the ancillary service) or the main service (when the firm unbundles the ancillary service) in advance. Different from forward-looking consumers who take future utilities from the ancillary service into consideration when purchasing the service bundle or main service in advance, myopic consumers do not consider future utilities. For some ancillary services that do not cost significant amounts of money, consumers are likely to be myopic. For example, it would be very unusual that a consumer takes the possible purchase of a can of coke during the flight (and the price of a can of coke) into consideration when booking the ticket several months in advance.

To capture the effect of myopic consumers, we now introduce a model with a more general demand composition comprised of both forward-looking and myopic consumers. We assume α_H proportion of high-type consumers and α_L proportion of low-type consumers are forward-looking, the other consumers are myopic. In the bundling case, type-i $(i = H, L)$ myopic consumers are willing to pay v_i for the service bundle when making purchasing decisions in advance, which is lower than forward-looking consumers' willingness to pay, $v_i + E(u_i)^+$. For each consumer type $i = H, L$, the firm can choose to price the service bundle at $p_{bi} = v_i + E(u_i)^+$ to induce only forward-looking consumers to purchase, or at $p_{bi} = v_i$ to induce both forward-looking and myopic consumers to purchase. Thus, the firm has four price combinations to choose from: "HH", "HL", "LH", "LL", where the former notation refers to the price for high-type consumers and the latter refers to the price for low-type consumers, "H" means pricing high and "L" means pricing low. The resulting profits are as follows, where we add a subscript "m" to represent the case with myopic consumers, and use the superscript to represent the price choice of the firm:

$$\begin{aligned}
\Pi_{b,m}^{HH*} &= [v_H + E(u_H)^+ - c_m]\alpha_H\lambda_H + [v_L + E(u_L)^+ - c_m]\alpha_L\lambda_L - c_a[\alpha_H\lambda_H\bar{F}_H(0) + \alpha_L\lambda_L\bar{F}_L(0)], \\
\Pi_{b,m}^{HL*} &= [v_H + E(u_H)^+ - c_m]\alpha_H\lambda_H + (v_L - c_m)\lambda_L - c_a[\alpha_H\lambda_H\bar{F}_H(0) + \lambda_L\bar{F}_L(0)], \\
\Pi_{b,m}^{LH*} &= (v_H - c_m)\lambda_H + [v_L + E(u_L)^+ - c_m]\alpha_L\lambda_L - c_a[\lambda_H\bar{F}_H(0) + \alpha_L\lambda_L\bar{F}_L(0)], \\
\Pi_{b,m}^{LL*} &= (v_H - c_m)\lambda_H + (v_L - c_m)\lambda_L - c_a[\lambda_H\bar{F}_H(0) + \lambda_L\bar{F}_L(0)].
\end{aligned}$$

The optimal profit in the bundling case is $\Pi_{b,m}^* = \max(\Pi_{b,m}^{HH*}, \Pi_{b,m}^{HL*}, \Pi_{b,m}^{LH*}, \Pi_{b,m}^{LL*})$.

In the unbundling case, type-i $(i = H, L)$ myopic consumers are willing to pay v_i for the main service when making purchasing decisions in advance, and forward-looking consumers have a higher willingness to pay, $v_i + E(u_i - p_a)^+$. For each consumer type $i = H, L$, the firm can choose to price the main service at $p_{mi} = v_i + E(u_i - p_a)^+$ to induce only forward-looking consumers to purchase, or at $p_{mi} = v_i$ to induce both forward-looking and myopic consumers to purchase, hence also leading to four price combinations. The resulting profits are as follows, as functions of the ancillary service price:

$$\Pi_{u,m}^{HH}(p_a) = [v_H + E(u_H - p_a)^+ - c_m]\alpha_H\lambda_H + [v_L + E(u_L - p_a)^+ - c_m]\alpha_L\lambda_L$$

$$
\begin{aligned}
&+(p_a-c_a)[\alpha_H\lambda_H\bar{F}_H(p_a)+\alpha_L\lambda_L\bar{F}_L(p_a)]-c(\alpha_H\lambda_H\bar{F}_H(p_a)+\alpha_L\lambda_L\bar{F}_L(p_a)),\\
\Pi_{u,m}^{HL}(p_a) =\ &[v_H+E(u_H-p_a)^+-c_m]\alpha_H\lambda_H+(v_L-c_m)\lambda_L\\
&+(p_a-c_a)[\alpha_H\lambda_H\bar{F}_H(p_a)+\lambda_L\bar{F}_L(p_a)]-c(\alpha_H\lambda_H\bar{F}_H(p_a)+\lambda_L\bar{F}_L(p_a)),\\
\Pi_{u,m}^{LH}(p_a) =\ &(v_H-c_m)\lambda_H+[v_L+E(u_L-p_a)^+-c_m]\alpha_L\lambda_L\\
&+(p_a-c_a)[\lambda_H\bar{F}_H(p_a)+\alpha_L\lambda_L\bar{F}_L(p_a)]-c(\lambda_H\bar{F}_H(p_a)+\alpha_L\lambda_L\bar{F}_L(p_a)),\\
\Pi_{u,m}^{LL}(p_a) =\ &(v_H-c_m)\lambda_H+(v_L-c_m)\lambda_L\\
&+(p_a-c_a)[\lambda_H\bar{F}_H(p_a)+\lambda_L\bar{F}_L(p_a)]-c(\lambda_H\bar{F}_H(p_a)+\lambda_L\bar{F}_L(p_a)).
\end{aligned}
$$

The optimal profit in the unbundling case is $\Pi_{u,m}^{*}=\max(\Pi_{u,m}^{HH}(p_a^*),\Pi_{u,m}^{HL}(p_a^*),\Pi_{u,m}^{LH}(p_a^*),$ $\Pi_{u,m}^{LL}(p_a^*))$.

Theorem 7. *In the unbundling case, the optimal ancillary service price is strictly higher than the total marginal cost if the firm sells to both forward-looking and myopic consumers; the optimal ancillary service price is equal to the total marginal cost if the firm only sells to forward-looking consumers.*

In the unbundling case, Theorem 7 states that as long as the firm sells to myopic consumers (either high-type or low-type), it should price the ancillary service above the marginal cost. In fact, we can characterize the condition for the firm to price the ancillary service above or equal to the marginal cost. It can be shown that there exist two threshold functions, $\tilde{\alpha}_H(\alpha_L)$ and $\tilde{\alpha}_L(\alpha_H)$, such that $\Pi_{u,m}^{*}=\Pi_{u,m}^{HH*}$ (hence the optimal ancillary service price is equal to the marginal cost) if $\alpha_H>\tilde{\alpha}_H(\alpha_L)$ and $\alpha_L>\tilde{\alpha}_L(\alpha_H)$, and $\Pi_{u,m}^{*}\neq\Pi_{u,m}^{HH*}$ (hence the optimal ancillary service price is above the marginal cost) otherwise. Unlike forward-looking consumers, myopic consumers' decisions on purchasing the main service and purchasing the ancillary service are made independently. Thus, when selling to myopic consumers, the firm no longer wants to decrease the ancillary service price to the marginal cost so that it could extract more consumer surplus overall by increasing the main service price accordingly. In reality, "small-item" ancillary services are usually priced well-above their marginal costs, e.g., a can of coke is priced more than 10 times the cost of it if ordered during the flight. Since consumers are myopic, the firm extracts high margins from selling the ancillary service.

Theorem 8. *(i) If unbundling is more profitable when all consumers are forward-looking (i.e., $\alpha_H=\alpha_L=1$), then it is more profitable for all α_H and α_L.*

(ii) If bundling is more profitable when all consumers are forward-looking (i.e., $\alpha_H = \alpha_L = 1$), then there exist two thresholds $\hat{\alpha}_H$, $\hat{\alpha}_L$ and a decreasing threshold function $\bar{\alpha}_H(\alpha_L)$ such that when $\alpha_H \le \hat{\alpha}_H$, $\alpha_L \le \hat{\alpha}_L$, and $\alpha_H \le \bar{\alpha}_H(\alpha_L)$, unbundling is more profitable.

Now we compare the unbundling profit to the bundling profit when some of the firm's consumers are myopic. Theorem 8 states that as the firm's proportion of myopic consumers increases, it may become optimal for the firm to switch from bundling to unbundling but not the other way around. If it is optimal to unbundle the ancillary service when all consumers are forward-looking, then it is also optimal to unbundle the ancillary service with any proportion of myopic consumers. If it is optimal to bundle the ancillary service when all consumers are forward-looking, we find a sufficient condition such that the firm should actually unbundle the ancillary service when the proportion of myopic consumers is significant enough (i.e., α_H and α_L are small enough). When selling to myopic consumers, the firm does not capture any consumer surplus from the ancillary service in the bundling case, because myopic consumers' utilities from the ancillary service do not affect their willingness to pay for the service bundle. However, by unbundling the ancillary service, the firm is able to capture myopic consumers' surplus from the ancillary service, because the firm induces myopic consumers to actually pay for the ancillary service. Thus, the existence of myopic consumers may switch the firm's optimal ancillary service strategy from bundling to unbundling but not the other way around.

Theorem 9. *The optimal profits from bundling and unbundling are both increasing in the proportion of forward-looking consumers, α_H and α_L.*

Theorem 9 states that regardless of whether the firm bundles or unbundles the ancillary service, its profit becomes higher when more consumers are forward-looking. For a firm that sells an ancillary service in addition to a main service, having more forward-looking consumers is beneficial because by accounting for future utilities from the ancillary service, forward-looking consumers are willing to pay more for the service bundle and main service than myopic consumers when purchasing in advance. Thus, the firm benefits from providing guidance to consumers for their ancillary service needs and making the information of the ancillary services easily accessible to consumers. Notice that forward-looking (strategic) consumers play a different role in the ancillary service pricing setting than in the markdown pricing setting which has been extensively studied by previous literature.

Although forward-looking consumers have been perceived as harmful to firms that salvage product leftovers at the end of the selling season, they actually benefit firms that manage the sales of a main service and an ancillary service simultaneously.

6. Comparison and Industry Insights

So far, we have analyzed the ancillary service unbundling problem for a firm using uniform pricing of main service and a firm using discriminatory pricing of main service separately. In this section, we compare the results for the two types of firms and discuss insights for industry practice. We first compare the cases where unbundling/bundling is the optimal ancillary service strategy under uniform pricing and discriminatory pricing. Then, we compare the way that the optimal ancillary service strategy is affected by the firm's demand portfolio under uniform pricing and discriminatory pricing. Finally, we use the results to explain the ancillary service strategies used in the airline industry and hotel industry. We discuss why airlines and hotels exhibit different patterns of ancillary service strategies, and why Southwest Airlines chooses not to charge for checked bags while most other airlines do.

THEOREM 10. *(i) If high-type consumers are more likely to purchase the ancillary service than low-type consumers (i.e., $\beta_H \geq \beta_L$), when unbundling is more profitable under discriminatory pricing, it is also more profitable under uniform pricing (i.e., when $\Pi_u^* \geq \Pi_b^*$, we also have $\Pi_{u,n}^* \geq \Pi_{b,n}^*$).*

(ii) If low-type consumers are more likely to purchase the ancillary service than high-type consumers (i.e., $\beta_H < \beta_L$), when unbundling is more profitable under uniform pricing, it is also more profitable under discriminatory pricing (i.e., when $\Pi_{u,n}^* \geq \Pi_{b,n}^*$, we also have $\Pi_u^* \geq \Pi_b^*$).

Theorem 10 compares the optimal ancillary service strategy for a uniform-pricing firm (Theorem 2) and a price-discriminating firm (Theorem 5). If high-type consumers are more likely to purchase the ancillary service than low-type consumers, or equivalently, if consumers' valuations for the main service and the ancillary service are positively correlated, unbundling is less likely to be the optimal ancillary service strategy for a discriminatory-pricing firm than a uniform-pricing firm. If low-type consumers are more likely to purchase the ancillary service than high-type consumers, or equivalently, if consumers' valuations for the main service and the ancillary service are negatively correlated, unbundling is more

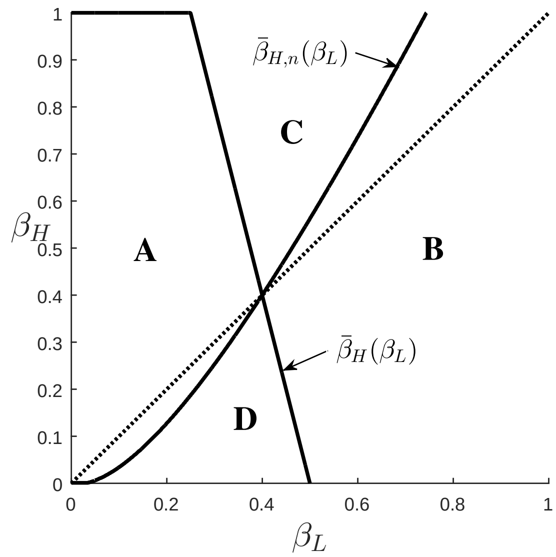

Figure 3 **Comparison of optimal ancillary service strategies under uniform pricing and discriminatory pricing of main service ($v_H = 300$, $v_L = 200$, $\bar{u} = 50$, $\underline{u} = -20$, $c_m = 150$, $c_a = 20$, $c(x) = 0.5x^2$, $\lambda_H = 20$, $\lambda_L = 80$; Region A: unbundle in both cases; Region B: bundle in both cases; Region C: unbundle under uniform pricing, bundle under discriminatory pricing; Region D: bundle under uniform pricing, unbundle under discriminatory pricing)**

likely to be the optimal ancillary service strategy for a discriminatory-pricing firm than a uniform-pricing firm. Figure 3 illustrates the result in Theorem 10 by plotting together the threshold functions under uniform pricing and discriminatory pricing, using the same example in Figures 1 and 2. As Figure 3 shows, when consumers' valuations are positively correlated (i.e., in the region above the dotted line), the unbundling region is smaller for a discriminatory-pricing firm; when consumers' valuations are negatively correlated (i.e., in the region below the dotted line), the unbundling region is larger for a discriminatory-pricing firm.

Therefore, when a firm switches from uniform pricing to discriminatory pricing for the main service, it should re-evaluate its policy for the ancillary service. For a firm managing an ancillary service that involves a positive consumer valuation correlation, a shift from unbundling to bundling may be needed; for a firm managing an ancillary service that involves a negative consumer valuation correlation, a shift from bundling to unbundling may be needed. Firms in several industries, such as sporting event organizers and hotels, are currently trying to enforce inter-temporal price discrimination. Along with the adoption of main service price discrimination, it is important for these firms to identify which of their consumer segments values the ancillary service more and adjust the strategy for the ancillary service accordingly.

In previous sections, we have shown how the optimal ancillary service strategy is affected by the firm's demand portfolio under uniform pricing (Theorem 3) and discriminatory pricing (Theorem 6). If we compare Theorem 3 to Theorem 6, we see that the result is exactly reversed. Again, the fundamental reason is that the ancillary service price plays a different role under uniform pricing than it does under discriminatory pricing. A uniform-pricing firm uses the ancillary service price as a lever to capture more of the high-type consumers' surplus that is not captured by the uniform main service price, while a discriminatory-pricing firm does not do so. Table 1 summarizes the findings from this paper about the effect of firm's demand portfolio on its optimal ancillary service strategy. It characterizes how the optimal ancillary service strategies in an industry is jointly determined by firms' use of main service price discrimination as well as consumers' valuation structure.

	Uniform pricing	Discriminatory pricing
Positive consumer valuation correlation	Higher $\lambda_H\% \Rightarrow$ unbundle Lower $\lambda_H\% \Rightarrow$ bundle	Higher $\lambda_H\% \Rightarrow$ bundle Lower $\lambda_H\% \Rightarrow$ unbundle
Negative consumer valuation correlation	Higher $\lambda_H\% \Rightarrow$ bundle Lower $\lambda_H\% \Rightarrow$ unbundle	Higher $\lambda_H\% \Rightarrow$ unbundle Lower $\lambda_H\% \Rightarrow$ bundle

Table 1 Comparison of the effects of firm's demand portfolio on the optimal ancillary service strategy in the uniform-pricing case and in the discriminatory-pricing case

Different from the airline industry where it is very common that consumers in different segments pay different prices for the same type of seats, discriminatory pricing of room rates is much less used in the hotel industry. Moreover, the most common ancillary services offered by hotels (e.g., breakfast, in-room internet connection) would usually involve a positive correlation between consumers' main service valuations and ancillary service valuations. Higher-valuation consumers are more likely to purchase these ancillary services from the hotel, whereas lower-valuation consumers may seek cheaper outside options (e.g., having breakfast in a nearby fast-food store at a lower price). Thus, Theorem 3 and Table 1 indicate that unbundling is more likely to be optimal for hotels with higher proportions of higher-valuation consumers (e.g., luxury hotels) than hotels with lower proportions of higher-valuation consumers (e.g., economy hotels). The current industry practice is that luxury hotels usually charge for such ancillary services and economy hotels usually offer them for free. Our result here provides an explanation for this phenomenon.

Next, consider airline baggage policies. Since business travelers are less likely to check bags than leisure travelers (i.e., $\beta_H < \beta_L$), Theorem 6 and Table 1 indicate that unbundling is more likely to be optimal for airlines with higher proportions of business travelers than airlines with lower proportions of business travelers. As of 2015, legacy airlines charge for checked bags; the airline that stands firm on not charging for checked bags is Southwest (Southwest does not charge for the first or second checked bag) which primarily serves leisure travelers (some low-cost airlines, including Spirit and Frontier, unbundle the baggage service under a different pricing structure; we discuss this case later in this section). Additionally, after some firms unbundle the ancillary service, consumers with higher needs for the ancillary service may switch to the firms that are still bundling the ancillary service for their lower total prices, and consumers without ancillary service needs may switch to the unbundling firms for their lower main service prices. This would result in an increase in consumers' likelihood of using the ancillary service for the bundling firms, and a decrease in consumers' likelihood of purchasing the ancillary service for the unbundling firms. Thus, following from Theorem 5, firms' differentiated ancillary service strategies will be consolidated. This type of consumer self-selection regarding airlines' checked bag fees (which is empirically supported by Nicolae et al. 2013) provides another reason for Southwest to bundle the checked bags. Moreover, the bundling firms can increase the bundle price due to the increased consumer valuations for the ancillary service. For example, Henrickson and Scott (2012) consider the top 150 domestic routes from 2007 to 2009, and find that a one dollar increase in baggage fees reduced airline ticket prices on the baggage-fee-charging airlines by \$0.24 and increased Southwest Airlines' ticket prices on routes in which they compete with baggage-fee-charging airlines by \$0.73.

Another reason for Southwest to bundle the checked baggage service is its non-dependency on intermediary sales channels such as online travel agencies (OTAs). When the firm bundles the ancillary service into the main service, it has to pay commissions to the OTA for the whole service price. By unbundling the ancillary service, the firm only pays commissions to the OTA for the main service price and still collects the full price of the ancillary service. Thus, unbundling the ancillary service helps firms earn more revenues back from OTAs, which is what a lot of travel firms are trying to achieve nowadays. For a firm that is facing a higher OTA commission or is more dependent on OTAs (i.e., OTAs account for a larger proportion of the firm's sales), unbundling the ancillary service is more valuable. In fact, one can easily analyze a model extension with intermediary and show that as the OTA's commission increases, or as consumers shift from purchasing directly from the firm to purchasing from the OTA, unbundling the ancillary service becomes more profitable relative to bundling. Formally, consider a model extension where γ_H proportion of high-type consumers and γ_L proportion of low-type consumers purchase directly from the firm, the other consumers purchase from the intermediaries. The firm pays a commission of τ (which is defined as a percentage of the revenue collected by the OTA) to the OTA for each unit of sale. It can be proved that the difference between the optimal unbundling profit and the optimal bundling profit is increasing in the intermediary commission τ, and decreasing in the proportion of direct sales, γ_H and γ_L.

In order to benefit from unbundling, the firm needs to reduce the inconvenience cost. One way to reduce the inconvenience cost is to induce consumers to pay for the ancillary service in advance. Spirit and Frontier Airlines have recently started to unbundle the baggage service while resorting to a new pricing structure for the ancillary service with late-payment penalty. Spirit and Frontier are now charging baggage fees contingent on when consumers pay for their bags. The later a consumer pays for the bag, the higher the fee is. For example, Spirit charges \$100 for any bag (checked and carry-on) that is paid for at the gate, which is three to four times higher than the baggage fees other airlines normally charge and Spirit's advance baggage fee itself. The following explanation has been given by Spirit's spokesperson: "The fee is intentionally set high to encourage customers to reserve their bags in advance, and it is meant to deter customers from waiting until they get to the boarding gate. When customers wait until the boarding gate, this delays the boarding process for everyone." (Brown 2012) Because the new pricing structure significantly reduces the inconvenience cost, Spirit and Frontier also charge for carry-on bags. Being recognized as the airline with the lowest fares, Spirit may not lose too many consumers even if its consumers are dissatisfied with the high late-payment penalty, because price-sensitive consumers are not very likely to get even lower ticket prices elsewhere if they refuse to accept the new baggage policy and pay in advance. So far, Spirit's implementation of the new baggage policy appears to be a success. However, resorting to a pricing structure with the late-payment penalty may be riskier for other airlines.

Different from checked baggage service, other ancillary services offered by airlines such as in-flight services (e.g., beverages, snacks) would usually involve a positive correlation between consumers' valuations for the main service and the ancillary service. Same as the ancillary services offered by hotels, higher-valuation consumers are more likely to purchase these ancillary services while lower-valuation consumers can seek outside options (e.g., bringing their own snacks or simply not having snacks during the flight). Thus, Theorem 6 and Table 1 indicate that airlines with higher percentages of business travelers are less likely to charge for these ancillary services compared to airlines with lower percentages of business travelers. As of 2015, all legacy airlines offer this service as complementary. In general, low-cost airlines have embraced the concept of a la carte pricing for in-flight services to a greater extent than legacy airlines.

7. Conclusion

In this paper, we study whether and when a firm should unbundle the ancillary service from the main service and separately charge for the ancillary service for two types of firms: firms that charge a uniform main service price and firms that charge discriminatory main service prices. We find that the ability to price-discriminate when selling the main service plays an important role in the decision of unbundling the ancillary service or not. While the results for a uniform-pricing firm are consistent with the commodity bundling literature, some classic findings from the previous commodity bundling literature and two-part pricing literature actually do not carry through to the discriminatory-pricing case. Thus, our paper offers unique contributions to the existing literature. We find that whereas it is optimal for a uniform-pricing firm to unbundle the ancillary service if the consumers that value the main service higher have a high enough likelihood of purchasing the ancillary service, it is optimal for a discriminatory-pricing firm to unbundle the ancillary service if the consumers that value the main service higher have a low enough likelihood of purchasing the ancillary service. Firm's ability to price-discriminate when selling the main service makes unbundling more (less) likely to be the optimal ancillary service strategy when consumers' valuations for the main service and the ancillary service are negatively (positively) correlated. This result highlights the need to re-evaluate the ancillary service strategy to firms that are adopting main service price discrimination, and provides the insight that the direction of change in the ancillary service strategy depends on the underlying consumer valuation structure.

This paper also provides the insight that firms' use of main service price discrimination and consumers' valuation structure jointly determine the structure of optimal ancillary service policies in an industry. For similar ancillary services that involve the same type of consumer valuation structure and are offered by different industries (e.g., hotels' breakfast service and airlines' in-flight beverage service), which firms in the industry are more likely to unbundle the ancillary service could differ significantly depending on the industry's use of main service price discrimination.

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Unbundling of Ancillary Service: How Does Price Discrimination of Main Service Matter?

Appendix: Proofs of Theorems

Proof of Theorem 1

Proof. Taking derivatives of the profit function yields

$$
\begin{aligned}
\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a} &= \lambda_H[\bar{F}_H(p_a) - \bar{F}_L(p_a)] + [c'(\lambda_H\bar{F}_H(p_a) + \lambda_L\bar{F}_L(p_a)) - (p_a - c_a)][\lambda_H f_H(p_a) + \lambda_L f_L(p_a)],\\
\frac{\mathrm{d}^2\Pi_{u,n}}{\mathrm{d}p_a^2} &= -2\lambda_H f_H(p_a) + \lambda_H f_L(p_a) - \lambda_L f_L(p_a) - c''(\lambda_H\bar{F}_H(p_a) + \lambda_L\bar{F}_L(p_a))[\lambda_H f_H(p_a) + \lambda_L f_L(p_a)]^2,\\
\frac{\mathrm{d}^3\Pi_{u,n}}{\mathrm{d}p_a^3} &= c'''(\lambda_H\bar{F}_H(p_a) + \lambda_L\bar{F}_L(p_a))[\lambda_H f_H(p_a) + \lambda_L f_L(p_a)]^3.
\end{aligned}
$$

If $\beta_H \geq \beta_L$, it is easy to see that $\frac{\mathrm{d}^2\Pi_{u,n}}{\mathrm{d}p_a^2} < 0$, so $\Pi_{u,n}$ is concave. If $\beta_H < \beta_L$, since $\frac{\mathrm{d}^3\Pi_{u,n}}{\mathrm{d}p_a^3} \geq 0$, $\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a}$ is convex. Moreover,

$$\left.\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a}\right|_{p_a=\bar{u}} = -(\bar{u} - c_a)[\lambda_H f_H(p_a) + \lambda_L f_L(p_a)] < 0.$$

Thus, $\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a}$ can cross the zero line at most once, from positive to negative, which means $\Pi_{u,n}$ is quasi-concave.

Therefore, $p^*_{a,n} = \inf\{0 < p_a < \bar{u} : \lambda_H[\bar{F}_H(p_a) - \bar{F}_L(p_a)] + [c'(\lambda_H\bar{F}_H(p_a) + \lambda_L\bar{F}_L(p_a)) - (p_a - c_a)][\lambda_H f_H(p_a)+\lambda_L f_L(p_a)] \leq 0\}$. Then, if $\beta_H \geq \beta_L$, we have $p^*_{a,n} \geq c_a + c'(\lambda_H\bar{F}_H(p^*_{a,n}) + \lambda_L\bar{F}_L(p^*_{a,n}))$; if $\beta_H < \beta_L$, we have $p^*_{a,n} < c_a + c'(\lambda_H\bar{F}_H(p^*_{a,n}) + \lambda_L\bar{F}_L(p^*_{a,n}))$.

□

Proof of Theorem 2

Proof. Consider $\Pi^*_{u,n} - \Pi^*_{b,n}$ as a function of β_H. First consider the case of $p^*_{a,n} = 0$. $p^*_{a,n} = 0$ occurs when $\left.\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a}\right|_{p_a=0} = \lambda_H(\beta_H - \beta_L) + [c'(\lambda_H\beta_H + \lambda_L\beta_L) + c_a] \cdot \frac{\lambda_H\beta_H + \lambda_L\beta_L}{\bar{u}} \leq 0$, which requires β_H is small enough (if $p^*_{a,n} = 0$ ever occurs). When $p^*_{a,n} = 0$, we have $\Pi^*_{u,n} - \Pi^*_{b,n} = -c(\lambda_H\beta_H + \lambda_L\beta_L)$ which is negative and decreasing in β_H.

Second, consider the case of $p^*_{a,n} > 0$ which occurs when β_H is large enough. Taking derivatives of the optimal profit functions with respective to β_H yields:

$$
\begin{aligned}
\frac{\partial\Pi^*_{u,n}}{\partial\beta_H} &= [p^*_{a,n} - c_a - c'(\lambda_H\bar{F}_H(p^*_{a,n}) + \lambda_L\bar{F}_L(p^*_{a,n}))]\lambda_H \cdot \frac{\bar{u} - p^*_{a,n}}{\bar{u}},\\
\frac{\partial\Pi^*_{b,n}}{\partial\beta_H} &= -c_a\lambda_H,
\end{aligned}
$$

where the derivative of $\Pi^*_{u,n}$ follows from the Envelope Theorem. Thus,

$$\frac{\partial(\Pi^*_{u,n}-\Pi^*_{b,n})}{\partial\beta_H}=\lambda_H\left\{[p^*_{a,n}-c_a-c'(\lambda_H\bar{F}_H(p^*_{a,n})+\lambda_L\bar{F}_L(p^*_{a,n}))]\cdot\frac{\bar{u}-p^*_{a,n}}{\bar{u}}+c_a\right\}. \tag{1}$$

Recall from the proof of Theorem 1 that the first-order condition in the uniform pricing case is $p^*_{a,n}-c_a-c'(\lambda_H\bar{F}_H(p^*_{a,n})+\lambda_L\bar{F}_L(p^*_{a,n}))=\frac{\lambda_H[\bar{F}_H(p^*_{a,n})-\bar{F}_L(p^*_{a,n})]}{\lambda_H f_H(p^*_{a,n})+\lambda_L f_L(p^*_{a,n})}$. Thus, if $\beta_H\geq\beta_L$, $p^*_{a,n}-c_a-c'(\lambda_H\bar{F}_H(p^*_{a,n})+\lambda_L\bar{F}_L(p^*_{a,n}))\geq 0$, and hence $\frac{\partial(\Pi^*_{u,n}-\Pi^*_{b,n})}{\partial\beta_H}>0$. If $\beta_H<\beta_L$, by using the first-order condition, we can equivalently write $\frac{\partial(\Pi^*_{u,n}-\Pi^*_{b,n})}{\partial\beta_H}$ as

$$\frac{\partial(\Pi^*_{u,n}-\Pi^*_{b,n})}{\partial\beta_H}=\frac{\lambda_H}{\lambda_H\beta_H+\lambda_L\beta_L}\left\{\lambda_H\cdot\frac{\beta_H-\beta_L}{\bar{u}}\cdot(\bar{u}-p^*_{a,n})^2+c_a(\lambda_H\beta_H+\lambda_L\beta_L)\right\}.$$

If $p^*_{a,n}$ is increasing in β_H, then $\lambda_H\cdot\frac{\beta_H-\beta_L}{\bar{u}}\cdot(\bar{u}-p^*_{a,n})^2+c_a(\lambda_H\beta_H+\lambda_L\beta_L)$ is increasing in β_H, and hence $\frac{\partial(\Pi^*_{u,n}-\Pi^*_{b,n})}{\partial\beta_H}$ is first decreasing then increasing in β_H. We now show that $p^*_{a,n}$ is increasing in β_H. By applying the Implicit Function Theorem to the first-order condition $\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a}=0$, we obtain

$$\frac{\mathrm{d}p^*_{a,n}}{\mathrm{d}\beta_H}=-\frac{\frac{\partial}{\partial\beta_H}\left(\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a}\right)\big|_{p_a=p^*_{a,n}}}{\frac{\partial}{\partial p_a}\left(\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a}\right)\big|_{p_a=p^*_{a,n}}}=-\frac{\frac{\partial}{\partial\beta_H}\left(\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a}\right)\big|_{p_a=p^*_{a,n}}}{\frac{\mathrm{d}^2\Pi_{u,n}}{\mathrm{d}p_a^2}\big|_{p_a=p^*_{a,n}}}. \tag{2}$$

The numerator of (2) is

$$\begin{aligned}\frac{\partial}{\partial\beta_H}\left(\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a}\right)\Big|_{p_a=p^*_{a,n}} &= \lambda_H\cdot\frac{\bar{u}-p^*_{a,n}}{\bar{u}}+c''(\lambda_H\bar{F}_H(p^*_{a,n})+\lambda_L\bar{F}_L(p^*_{a,n}))\cdot\frac{\lambda_H(\bar{u}-p^*_{a,n})(\lambda_H\beta_H+\lambda_L\beta_L)}{\bar{u}^2}\\ &\quad -[p^*_{a,n}-c_a-c'(\lambda_H\bar{F}_H(p^*_{a,n})+\lambda_L\bar{F}_L(p^*_{a,n}))]\cdot\frac{\lambda_H}{\bar{u}}.\end{aligned}$$

Since $\beta_H<\beta_L$, the first-order condition implies that $p^*_{a,n}-c_a-c'(\lambda_H\bar{F}_H(p^*_{a,n})+\lambda_L\bar{F}_L(p^*_{a,n}))<0$. Then, since $c(\cdot)$ is convex, we know that $\frac{\partial}{\partial\beta_H}\left(\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a}\right)\big|_{p_a=p^*_{a,n}}>0$. Moreover, in the proof of Theorem 1, we already know that $\frac{\mathrm{d}\Pi_{u,n}}{\mathrm{d}p_a}$ can only cross the zero line from positive to negative. Thus, $\frac{\mathrm{d}^2\Pi_{u,n}}{\mathrm{d}p_a^2}\big|_{p_a=p^*_{a,n}}<0$, and hence $\frac{\mathrm{d}p^*_{a,n}}{\mathrm{d}\beta_H}>0$.

So far, we have obtained that 1) for small β_H (if $p^*_{a,n}=0$ ever occurs), $\Pi^*_{u,n}-\Pi^*_{b,n}$ is negative and decreasing in β_H; 2) for large β_H (i.e., $\beta_H\geq\beta_L$), $\Pi^*_{u,n}-\Pi^*_{b,n}$ is increasing in β_H; 3) for medium β_H (i.e., $\beta_H<\beta_L$ and $p^*_{a,n}>0$), $\Pi^*_{u,n}-\Pi^*_{b,n}$ is first decreasing then increasing in β_H. Thus, overall, $\Pi^*_{u,n}-\Pi^*_{b,n}$ is quasi-convex (i.e., first decreasing then increasing) in β_H. If $p^*_{a,n}=0$ occurs for small β_H, $\Pi^*_{u,n}-\Pi^*_{b,n}$ first decreases from a negative value and then becomes increasing in β_H, thus it is negative for small β_H and positive for large β_H. If $p^*_{a,n}=0$ never occurs, we may have two scenarios. First, if $\Pi^*_{u,n}-\Pi^*_{b,n}$ is increasing in β_H at $\beta_H=0$, then it is always increasing in β_H, and hence $\Pi^*_{u,n}-\Pi^*_{b,n}$ can only be negative for small β_H and positive for large β_H. Second, if $\Pi^*_{u,n}-\Pi^*_{b,n}$ is decreasing in β_H at $\beta_H=0$, we now show that we must have $\Pi^*_{u,n}-\Pi^*_{b,n}<0$ at $\beta_H=0$ in this case, so that $\Pi^*_{u,n}-\Pi^*_{b,n}$ is negative for small β_H and positive for large β_H. At

$\beta_H = 0$, we have

$$\Pi^*_{u,n} - \Pi^*_{b,n} = -\lambda_H \cdot \frac{\beta_L}{2\bar{u}} \cdot p^*_{a,n}(2\bar{u} - p^*_{a,n}) + \lambda_L \cdot \frac{\beta_L}{2\bar{u}} \cdot p^*_{a,n}(2c_a - p^*_{a,n}) - c(\lambda_L \bar{F}_L(p^*_{a,n})).$$

$\frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \beta_H}\Big|_{\beta_H=0} < 0$ can be simplified to $-\lambda_H \cdot \frac{\beta_L}{\bar{u}} \leq -\frac{c_a \lambda_L \beta_L}{(\bar{u} - p^*_{a,n})^2}$. Thus, we have

$$\begin{aligned}
\Pi^*_{u,n} - \Pi^*_{b,n} &\leq -\frac{c_a \lambda_L \beta_L}{2(\bar{u} - p^*_{a,n})^2} \cdot p^*_{a,n}(2\bar{u} - p^*_{a,n}) + \lambda_L \cdot \frac{\beta_L}{2\bar{u}} \cdot p^*_{a,n}(2c_a - p^*_{a,n}) - c(\lambda_L \bar{F}_L(p^*_{a,n})) \\
&= \frac{\lambda_L \beta_L}{2\bar{u}} \cdot p^*_{a,n} \cdot \left[-c_a \cdot \frac{2\bar{u} - p^*_{a,n}}{\bar{u} - p^*_{a,n}} \cdot \frac{\bar{u}}{\bar{u} - p^*_{a,n}} + (2c_a - p^*_{a,n}) \right] - c(\lambda_L \bar{F}_L(p^*_{a,n})) \\
&< -\frac{\lambda_L \beta_L}{2\bar{u}} \cdot (p^*_{a,n})^2 - c(\lambda_L \bar{F}_L(p^*_{a,n})) \\
&< 0,
\end{aligned}$$

where the first inequality follows from using $-\lambda_H \cdot \frac{\beta_L}{\bar{u}} \leq -\frac{c_a \lambda_L \beta_L}{(\bar{u} - p^*_{a,n})^2}$ and the second inequality follows from $\frac{2\bar{u} - p^*_{a,n}}{\bar{u} - p^*_{a,n}} > 2$ and $\frac{\bar{u}}{\bar{u} - p^*_{a,n}} > 1$. Therefore, combining all cases analyzed above, we obtain that there exists a threshold function $\bar{\beta}_{H,n}(\beta_L)$ such that $\Pi^*_{u,n} \geq \Pi^*_{b,n}$ if and only if $\beta_H \geq \bar{\beta}_{H,n}(\beta_L)$.

Next, we show that $\bar{\beta}_{H,n}(\beta_L)$ is an increasing function. By applying the Implicit Function Theorem to the equation $\Pi^*_{u,n} - \Pi^*_{b,n} = 0$ which defines $\bar{\beta}_{H,n}(\beta_L)$, we have

$$\frac{\mathrm{d}\bar{\beta}_{H,n}}{\mathrm{d}\beta_L} = -\frac{\frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \beta_L}\Big|_{\beta_H = \bar{\beta}_{H,n}(\beta_L)}}{\frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \beta_H}\Big|_{\beta_H = \bar{\beta}_{H,n}(\beta_L)}}.$$

We have shown that $\Pi^*_{u,n} - \Pi^*_{b,n}$ can only cross the zero line from negative to positive, thus $\frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \beta_H}\Big|_{\beta_H = \bar{\beta}_{H,n}(\beta_L)} > 0$. It remains to show that $\frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \beta_L}\Big|_{\beta_H = \bar{\beta}_{H,n}(\beta_L)} \leq 0$. Taking derivatives of the optimal profit functions with respect to β_L yields:

$$\begin{aligned}
\frac{\partial \Pi^*_{u,n}}{\partial \beta_L} &= \frac{(\bar{u} - p^*_{a,n})^2}{2\bar{u}} \cdot (\lambda_H + \lambda_L) + [p^*_{a,n} - c_a - c'(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n}))]\lambda_L \cdot \frac{\bar{u} - p^*_{a,n}}{\bar{u}}, \\
\frac{\partial \Pi^*_{b,n}}{\partial \beta_L} &= \frac{\bar{u}}{2} \cdot (\lambda_H + \lambda_L) - c_a \lambda_H,
\end{aligned}$$

where the derivative of $\Pi^*_{u,n}$ follows from the Envelope Theorem. Thus,

$$\frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \beta_L} = \frac{p^*_{a,n}(p^*_{a,n} - 2\bar{u})}{2\bar{u}} \cdot (\lambda_H + \lambda_L) + \lambda_L \left\{ [p^*_{a,n} - c_a - c'(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n}))] \cdot \frac{\bar{u} - p^*_{a,n}}{\bar{u}} + c_a \right\}.$$

At $\beta_H = \bar{\beta}_{H,n}(\beta_L)$, we have $\Pi^*_{u,n} = \Pi^*_{b,n}$ which is equivalent to the following equation after rearranging terms:

$$\begin{aligned}\frac{\beta_L}{2\bar{u}} \cdot p^*_{a,n}(p^*_{a,n} - 2\bar{u})(\lambda_H + \lambda_L) &= -(p^*_{a,n} - c_a)(\lambda_H\beta_H + \lambda_L\beta_L) \cdot \frac{\bar{u} - p^*_{a,n}}{\bar{u}} \\ &\quad + c(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n})) - c_a(\lambda_H\beta_H + \lambda_L\beta_L).\end{aligned}$$

By using $c(x) \leq c'(x)x$, we obtain from the above equation that at $\beta_H = \bar{\beta}_{H,n}(\beta_L)$,

$$\begin{aligned}\frac{\beta_L}{2\bar{u}} \cdot p^*_{a,n}(p^*_{a,n} - 2\bar{u})(\lambda_H + \lambda_L) &\leq -[p^*_{a,n} - c_a - c'(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n}))](\lambda_H\beta_H + \lambda_L\beta_L) \cdot \frac{\bar{u} - p^*_{a,n}}{\bar{u}} \\ &\quad - c_a(\lambda_H\beta_H + \lambda_L\beta_L).\end{aligned}$$

By using the above inequality, at $\beta_H = \bar{\beta}_{H,n}(\beta_L)$, we have

$$\begin{aligned}\frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \beta_L} &\leq -[p^*_{a,n} - c_a - c'(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n}))] \cdot \frac{\lambda_H\beta_H + \lambda_L\beta_L}{\beta_L} \cdot \frac{\bar{u} - p^*_{a,n}}{\bar{u}} \\ &\quad - c_a \cdot \frac{\lambda_H\beta_H + \lambda_L\beta_L}{\beta_L} \\ &\quad + \lambda_L \left\{ [p^*_{a,n} - c_a - c'(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n}))] \cdot \frac{\bar{u} - p^*_{a,n}}{\bar{u}} + c_a \right\} \\ &= -\lambda_H \cdot \frac{\beta_H}{\beta_L} \cdot \left\{ [p^*_{a,n} - c_a - c'(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n}))] \cdot \frac{\bar{u} - p^*_{a,n}}{\bar{u}} + c_a \right\}. \quad (3)\end{aligned}$$

Thus, by comparing (1) and (3), we obtain that $\frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \beta_L}\Big|_{\beta_H = \bar{\beta}_{H,n}(\beta_L)} \leq \frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \beta_H}\Big|_{\beta_H = \bar{\beta}_{H,n}(\beta_L)} \cdot \big[- \frac{\bar{\beta}_{H,n}(\beta_L)}{\beta_L} \big] \leq 0$. Therefore, $\bar{\beta}_{H,n}(\beta_L)$ is increasing in β_L.

□

Proof of Theorem 3

Proof. When we increasing λ_H and decreasing λ_L such that $\lambda_H + \lambda_L = \lambda$, applying the Implicit Function Theorem to the equation $\Pi^*_{u,n} - \Pi^*_{b,n} = 0$ yields

$$\begin{aligned}\frac{\mathrm{d}\bar{\beta}_{H,n}}{\mathrm{d}\lambda_H} &= -\frac{\frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \lambda_H} - \frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \lambda_L}}{\frac{\partial(\Pi^*_{u,n} - \Pi^*_{b,n})}{\partial \beta_H}} \\ &= -\frac{(\bar{\beta}_{H,n} - \beta_L)\left\{ [p^*_{a,n} - c_a - c'(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n}))] \cdot \frac{\bar{u} - p^*_{a,n}}{\bar{u}} + c_a \right\}}{\lambda_H \left\{ [p^*_{a,n} - c_a - c'(\lambda_H \bar{F}_H(p^*_{a,n}) + \lambda_L \bar{F}_L(p^*_{a,n}))] \cdot \frac{\bar{u} - p^*_{a,n}}{\bar{u}} + c_a \right\}} \\ &= \frac{\beta_L - \bar{\beta}_{H,n}}{\lambda_H}.\end{aligned}$$

Thus, $\bar{\beta}_{H,n}(\beta_L)$ is decreasing in λ_H when $\bar{\beta}_{H,n}(\beta_L) \geq \beta_L$ and increasing in λ_H when $\bar{\beta}_{H,n}(\beta_L) < \beta_L$. Also, note that when $\bar{\beta}_{H,n}(\beta_L) = \beta_L$, $\bar{\beta}_{H,n}(\beta_L)$ does not change with λ_H if we keep $\lambda_H + \lambda_L = \lambda$. Thus, $\bar{\beta}_{H,n}(\beta_L)$ intersects at the same point on $\beta_H = \beta_L$ when we change λ_H and keep $\lambda_H + \lambda_L = \lambda$.

□

Proof of Theorem 4

Proof. (i) The first-order derivative of $\Pi_u(p_a)$ is

$$\frac{\mathrm{d}\Pi_u}{\mathrm{d}p_a} = [c'(\lambda_H \bar{F}_H(p_a) + \lambda_L \bar{F}_L(p_a)) - (p_a - c_a)][\lambda_H f_H(p_a) + \lambda_L f_L(p_a)].$$

Since $c'(\lambda_H \bar{F}_H(p_a) + \lambda_L \bar{F}_L(p_a)) - (p_a - c_a)$ is decreasing in p_a, $\Pi_u(p_a)$ is quasi-concave in p_a. Thus, the optimal ancillary service price is the solution to the first-order condition, i.e., $p_a^* = c_a + c'(\lambda_H \bar{F}_H(p_a^*) + \lambda_L \bar{F}_L(p_a^*))$.

(ii) Since $p_a^* > 0$, for $i = H, L$, we have $p_{mi}^* = v_i + E(u_i - p_a^*)^+ < v_i + E(u_i)^+ = p_{bi}^*$. Since $p_{mi}^* + p_a^* = v_i + E[\max(u_i, p_a^*)]$, we have $p_{bi}^* < p_{mi}^* + p_a^*$.

(iii) Since $p_{bi}^* - p_{mi}^* = E(u_i)^+ - E(u_i - p_a^*)^+ = \int_0^{p_a^*} \bar{F}_i(x)\,\mathrm{d}x$ for $i = H, L$, the result follows.

□

Proof of Theorem 5

Proof. Since $E(u_i - x)^+ = \frac{\beta_i}{2\bar{u}}(\bar{u} - x)^2$ for $i = H, L$, taking derivatives of the optimal profit functions with respect to β_H and β_L yields

$$\begin{aligned}
\frac{\partial \Pi_u^*}{\partial \beta_H} &= \frac{(\bar{u} - p_a^*)^2}{2\bar{u}} \cdot \lambda_H + (p_a^* - c_a)\lambda_H \cdot \frac{\bar{u} - p_a^*}{\bar{u}} - c'(\lambda_H \bar{F}_H(p_a^*) + \lambda_L \bar{F}_L(p_a^*))\lambda_H \cdot \frac{\bar{u} - p_a^*}{\bar{u}} \\
&= \frac{(\bar{u} - p_a^*)^2}{2\bar{u}} \cdot \lambda_H, \\
\frac{\partial \Pi_b^*}{\partial \beta_H} &= \left(\frac{\bar{u}}{2} - c_a\right)\lambda_H, \\
\frac{\partial \Pi_u^*}{\partial \beta_L} &= \frac{(\bar{u} - p_a^*)^2}{2\bar{u}} \cdot \lambda_L + (p_a^* - c_a)\lambda_L \cdot \frac{\bar{u} - p_a^*}{\bar{u}} - c'(\lambda_H \bar{F}_H(p_a^*) + \lambda_L \bar{F}_L(p_a^*))\lambda_L \cdot \frac{\bar{u} - p_a^*}{\bar{u}} \\
&= \frac{(\bar{u} - p_a^*)^2}{2\bar{u}} \cdot \lambda_L, \\
\frac{\partial \Pi_b^*}{\partial \beta_L} &= \left(\frac{\bar{u}}{2} - c_a\right)\lambda_L,
\end{aligned}$$

where the derivation for derivatives of Π_u^* follows from the Envelope Theorem and the first-order condition. Thus,

$$\frac{\partial(\Pi_u^* - \Pi_b^*)}{\partial \beta_H} = \left[\frac{(\bar{u} - p_a^*)^2}{2\bar{u}} - \frac{\bar{u}}{2} + c_a\right]\lambda_H, \qquad \frac{\partial(\Pi_u^* - \Pi_b^*)}{\partial \beta_L} = \left[\frac{(\bar{u} - p_a^*)^2}{2\bar{u}} - \frac{\bar{u}}{2} + c_a\right]\lambda_L.$$

By applying the Implicit Function Theorem to the first-order condition, we obtain

$$\begin{aligned}
\frac{\partial p_a^*}{\partial \beta_H} &= \frac{c''(\lambda_H \bar{F}_H(p_a^*) + \lambda_L \bar{F}_L(p_a^*))\lambda_H \frac{\bar{u}-p_a^*}{\bar{u}}}{1 + c''(\lambda_H \bar{F}_H(p_a^*) + \lambda_L \bar{F}_L(p_a^*))[\lambda_H f_H(p_a^*) + \lambda_L f_L(p_a^*)]} > 0, \\
\frac{\partial p_a^*}{\partial \beta_L} &= \frac{c''(\lambda_H \bar{F}_H(p_a^*) + \lambda_L \bar{F}_L(p_a^*))\lambda_L \frac{\bar{u}-p_a^*}{\bar{u}}}{1 + c''(\lambda_H \bar{F}_H(p_a^*) + \lambda_L \bar{F}_L(p_a^*))[\lambda_H f_H(p_a^*) + \lambda_L f_L(p_a^*)]} > 0.
\end{aligned}$$

Thus p_a^* is increasing in β_H and β_L. Then, since $\frac{\partial(\Pi_u^*-\Pi_b^*)}{\partial \beta_H}$ is decreasing in p_a^*, it is decreasing in β_H, hence $\Pi_u^* - \Pi_b^*$ is concave in β_H. Similarly, $\Pi_u^* - \Pi_b^*$ is concave in β_L.

When $\beta_H = \beta_L = 0$, $\Pi_u^* - \Pi_b^* = 0$; also, $p_a^* = c_a$, hence $\left.\frac{\partial(\Pi_u^*-\Pi_b^*)}{\partial \beta_H}\right|_{\beta_H=\beta_L=0} = \frac{c_a^2}{2\bar{u}} \cdot \lambda_H > 0$, $\left.\frac{\partial(\Pi_u^*-\Pi_b^*)}{\partial \beta_L}\right|_{\beta_H=\beta_L=0} = \frac{c_a^2}{2\bar{u}} \cdot \lambda_L > 0$. Thus, when $\beta_L = 0$, there exists a threshold $\hat{\beta}_H$ such that $\Pi_u^* - \Pi_b^* \geq 0$ when $\beta_H \leq \hat{\beta}_H$, and $\Pi_u^* - \Pi_b^* < 0$ when $\beta_H > \hat{\beta}_H$. Similarly, when $\beta_H = 0$, there exists a threshold $\hat{\beta}_L$ such that $\Pi_u^* - \Pi_b^* \geq 0$ when $\beta_L \leq \hat{\beta}_L$, and $\Pi_u^* - \Pi_b^* < 0$ when $\beta_L > \hat{\beta}_L$.

Next, notice that $\frac{\partial(\Pi_u^*-\Pi_b^*)}{\partial \beta_H}$ and $\frac{\partial(\Pi_u^*-\Pi_b^*)}{\partial \beta_L}$ have the same sign. If $\beta_L > \hat{\beta}_L$, we have $\left.\frac{\partial(\Pi_u^*-\Pi_b^*)}{\partial \beta_L}\right|_{\beta_H=0} < 0$, hence we also have $\left.\frac{\partial(\Pi_u^*-\Pi_b^*)}{\partial \beta_H}\right|_{\beta_H=0} < 0$. Then, since $\Pi_u^* - \Pi_b^*$ is concave in β_H, we have $\frac{\partial(\Pi_u^*-\Pi_b^*)}{\partial \beta_H} < 0$ for any $\beta_L > \hat{\beta}_L$. Thus, if $\beta_L > \hat{\beta}_L$, since $\Pi_u^* - \Pi_b^* < 0$ when $\beta_H = 0$, we have $\Pi_u^* - \Pi_b^* < 0$ for all β_H. Similarly, if $\beta_H > \hat{\beta}_H$, $\Pi_u^* - \Pi_b^* < 0$ for all β_L. Thus, the solution to $\Pi_u^* - \Pi_b^* = 0$ must satisfy $\beta_H \leq \hat{\beta}_H$ and $\beta_L \leq \hat{\beta}_L$. For any β_L, because $\Pi_u^* - \Pi_b^*$ is concave in β_H and $\Pi_u^* - \Pi_b^* \geq 0$ at $\beta_H = 0$, $\Pi_u^* - \Pi_b^*$ crosses the zero line once from positive to negative when varying β_H. Let $\bar{\beta}_H(\beta_L)$ denote this threshold. We must have $\left.\frac{\partial(\Pi_u^*-\Pi_b^*)}{\partial \beta_H}\right|_{\beta_H=\bar{\beta}_H(\beta_L)} < 0$ and $\left.\frac{\partial(\Pi_u^*-\Pi_b^*)}{\partial \beta_L}\right|_{\beta_H=\bar{\beta}_H(\beta_L)} < 0$. Thus, by applying the Implicit Function Theorem to $\Pi_u^* - \Pi_b^* = 0$ which is the equation that defines $\bar{\beta}_H(\beta_L)$, we know that $\bar{\beta}_H(\beta_L)$ is decreasing in β_L. Note that $\bar{\beta}_H(\beta_L)$ intersects with the β_H-axis and β_L-axis at $\hat{\beta}_H$ and $\hat{\beta}_L$, respectively.

□

Proof of Theorem 6

Proof. When we increase λ_H and decrease λ_L such that $\lambda_H + \lambda_L = \lambda$, applying the Implicit Function Theorem to the equation $\Pi_u^* - \Pi_b^* = 0$ yields

$$\begin{aligned}
\frac{\mathrm{d}\bar{\beta}_H}{\mathrm{d}\lambda_H} &= -\frac{\frac{\partial(\Pi_u^*-\Pi_b^*)}{\partial \lambda_H} - \frac{\partial(\Pi_u^*-\Pi_b^*)}{\partial \lambda_L}}{\frac{\partial(\Pi_u^*-\Pi_b^*)}{\partial \bar{\beta}_H}} \\
&= -\frac{\bar{\beta}_H \left[\frac{(\bar{u}-p_a^*)^2}{2\bar{u}} - \frac{\bar{u}}{2} + c_a\right] - \beta_L \left[\frac{(\bar{u}-p_a^*)^2}{2\bar{u}} - \frac{\bar{u}}{2} + c_a\right]}{\lambda_H \left[\frac{(\bar{u}-p_a^*)^2}{2\bar{u}} - \frac{\bar{u}}{2} + c_a\right]} \\
&= \frac{\beta_L - \bar{\beta}_H}{\lambda_H}.
\end{aligned}$$

Thus, $\bar{\beta}_H(\beta_L)$ is decreasing in λ_H when $\bar{\beta}_H(\beta_L) \geq \beta_L$ and increasing in λ_H when $\bar{\beta}_H(\beta_L) < \beta_L$.

Also, note that when $\bar{\beta}_H(\beta_L) = \beta_L$, $\bar{\beta}_H(\beta_L)$ does not change with λ_H if we keep $\lambda_H + \lambda_L = \lambda$. Thus, $\bar{\beta}_H(\beta_L)$ intersects at the same point on $\beta_H = \beta_L$ when we change λ_H and keep $\lambda_H + \lambda_L = \lambda$. □

Proof of Theorem 7

Proof. For each of the four cases ("HH", "HL", "LH", "LL"), by using the same approach in the proof of Theorem 4, we can prove the quasi-concavity of the profit function, and hence the optimal ancillary service price is given by the first-order condition as follows:

- "HH" case: The optimal ancillary service price $p_{a,m}^{HH*}$ is the solution to $p_{a,m}^{HH*} = c_a + c'(\alpha_H\lambda_H\bar{F}_H(p_{a,m}^{HH*}) + \alpha_L\lambda_L\bar{F}_L(p_{a,m}^{HH*}))$.
- "HL" case: The optimal ancillary service price $p_{a,m}^{HL*}$ is the solution to $p_{a,m}^{HL*} = c_a + c'(\alpha_H\lambda_H\bar{F}_H(p_{a,m}^{HL*}) + \lambda_L\bar{F}_L(p_{a,m}^{HL*})) + \frac{\lambda_L\bar{F}_L(p_{a,m}^{HL*})}{\alpha_H\lambda_H f_H(p_{a,m}^{HL*}) + \lambda_L f_L(p_{a,m}^{HL*})}$.
- "LH" case: The optimal ancillary service price $p_{a,m}^{LH*}$ is the solution to $p_{a,m}^{LH*} = c_a + c'(\lambda_H\bar{F}_H(p_{a,m}^{LH*}) + \alpha_L\lambda_L\bar{F}_L(p_{a,m}^{LH*})) + \frac{\lambda_H\bar{F}_H(p_{a,m}^{LH*})}{\lambda_H f_H(p_{a,m}^{LH*}) + \alpha_L\lambda_L f_L(p_{a,m}^{LH*})}$.
- "LL" case: The optimal ancillary service price $p_{a,m}^{LL*}$ is the solution to $p_{a,m}^{LL*} = c_a + c'(\lambda_H\bar{F}_H(p_{a,m}^{LL*}) + \lambda_L\bar{F}_L(p_{a,m}^{LL*})) + \frac{\lambda_H\bar{F}_H(p_{a,m}^{LL*}) + \lambda_L\bar{F}_L(p_{a,m}^{LL*})}{\lambda_H f_H(p_{a,m}^{LL*}) + \lambda_L f_L(p_{a,m}^{LL*})}$.

The result then follows.

□

Proof of Theorem 8

Proof. (i) We will show that when $\Pi_u^* \geq \Pi_b^*$, the following four results hold: 1) $\Pi_{u,m}^{HH*} \geq \Pi_{b,m}^{HH*}$, 2) $\Pi_{u,m}^{HL*} \geq \Pi_{b,m}^{HL*}$, 3) $\Pi_{u,m}^{LH*} \geq \Pi_{b,m}^{LH*}$, 4) $\Pi_{u,m}^{LL*} \geq \Pi_{b,m}^{LL*}$.

- $\Pi_{u,m}^{HH*} \geq \Pi_{b,m}^{HH*}$: Notice that $\Pi_{u,m}^{HH}(p_a)$ is equal to $\Pi_u(p_a)$ with λ_H replaced by $\alpha_H\lambda_H$ and λ_L replaced by $\alpha_L\lambda_L$. By following the same approach in the proof of Theorem 5, we can obtain that there exists a decreasing threshold function $\bar{\lambda}_H(\lambda_L)$ such that unbundling is more profitable than bundling if and only if $\lambda_H \leq \bar{\lambda}_H(\lambda_L)$. Thus, when $\Pi_u^* \geq \Pi_b^*$, we also have $\Pi_u^* \geq \Pi_b^*$ with smaller λ_H and λ_L. This means that when $\Pi_{u,m}^{HH*} \geq \Pi_{b,m}^{HH*}$ with $\alpha_H = \alpha_L = 1$, we have $\Pi_{u,m}^{HH*} \geq \Pi_{b,m}^{HH*}$ for all α_H and α_L.
- $\Pi_{u,m}^{HL*} \geq \Pi_{b,m}^{HL*}$: Our discussion above indicates that by replacing λ_H with $\alpha_H\lambda_H$, we also have $\Pi_u^* \geq \Pi_b^*$, i.e.,

$$
\begin{aligned}
& [v_H + E(u_H - p_a^*)^+ - c_m]\alpha_H\lambda_H + [v_L + E(u_L - p_a^*)^+ - c_m]\lambda_L \\
& +(p_a^* - c_a)[\alpha_H\lambda_H\bar{F}_H(p_a^*) + \lambda_L\bar{F}_L(p_a^*)] - c(\alpha_H\lambda_H\bar{F}_H(p_a^*) + \lambda_L\bar{F}_L(p_a^*)) \\
\geq\ & [v_H + E(u_H)^+ - c_m]\alpha_H\lambda_H + [v_L + E(u_L)^+ - c_m]\lambda_L - c_a[\alpha_H\lambda_H\bar{F}_H(0) + \lambda_L\bar{F}_L(0)].
\end{aligned}
$$

Note that in the above inequality, p_a^* is the optimal ancillary service price in the basic model with λ_H replaced by $\alpha_H\lambda_H$. Next, subtracting the left-hand side of the above inequality by $E(u_L-p_a^*)^+\lambda_L$ and subtracting the right-hand side by a larger amount $E(u_L)^+\lambda_L$, we obtain

$$\begin{aligned}
& [v_H+E(u_H-p_a^*)^+-c_m]\alpha_H\lambda_H+(v_L-c_m)\lambda_L \\
& +(p_a^*-c_a)[\alpha_H\lambda_H\bar{F}_H(p_a^*)+\lambda_L\bar{F}_L(p_a^*)]-c(\alpha_H\lambda_H\bar{F}_H(p_a^*)+\lambda_L\bar{F}_L(p_a^*)) \\
> \; & [v_H+E(u_H)^+-c_m]\alpha_H\lambda_H+(v_L-c_m)\lambda_L-c_a[\alpha_H\lambda_H\bar{F}_H(0)+\lambda_L\bar{F}_L(0)],
\end{aligned}$$

which is equivalent to $\Pi_{u,m}^{HL}(p_a^*)>\Pi_{b,m}^{HL*}$. Since $\Pi_{u,m}^{HL*}\geq\Pi_{u,m}^{HL}(p_a^*)$, we have $\Pi_{u,m}^{HL*}>\Pi_{b,m}^{HL*}$.

- $\Pi_{u,m}^{LH*}\geq\Pi_{b,m}^{LH*}$: This follows from the same approach that we used above to prove $\Pi_{u,m}^{HL*}\geq\Pi_{b,m}^{HL*}$.
- $\Pi_{u,m}^{LL*}\geq\Pi_{b,m}^{LL*}$: This is true because $\Pi_{u,m}^{LL*}>\Pi_{u,m}^{LL}(\bar{u})=v_H\lambda_H+v_L\lambda_L>\Pi_{b,m}^{LL*}$. Note that $\Pi_{u,m}^{LL*}\geq\Pi_{b,m}^{LL*}$ is actually always true and is not dependent on $\Pi_u^*\geq\Pi_b^*$.

Therefore, combining these four results, we conclude that when $\Pi_u^*\geq\Pi_b^*$, $\Pi_{u,m}^*\geq\Pi_{b,m}^*$ for all α_H and α_L.

(ii) In Part (i), we have proved that $\Pi_{u,m}^{LL*}\geq\Pi_{b,m}^{LL*}$ always holds. Thus, when $\Pi_{u,m}^*=\Pi_{u,m}^{LL*}$ and $\Pi_{b,m}^*=\Pi_{b,m}^{LL*}$, we must have $\Pi_{u,m}^*\geq\Pi_{b,m}^*$. We first consider the unbundling case and characterize when $\Pi_{u,m}^*=\Pi_{u,m}^{LL*}$. $\Pi_{u,m}^*=\Pi_{u,m}^{LL*}$ requires 1) $\Pi_{u,m}^{LL*}\geq\Pi_{u,m}^{HH*}$, 2) $\Pi_{u,m}^{LL*}\geq\Pi_{u,m}^{HL*}$, 3) $\Pi_{u,m}^{LL*}\geq\Pi_{u,m}^{LH*}$.

- Condition for $\Pi_{u,m}^{LL*}\geq\Pi_{u,m}^{HH*}$: When $\alpha_H=\alpha_L=0$, $\Pi_{u,m}^{LL*}>\Pi_{u,m}^{HH*}$ trivially because $\Pi_{u,m}^{HH*}=0$. When $\alpha_H=\alpha_L=1$,

$$\begin{aligned}
\Pi_{u,m}^{LL*} &= (v_H-c_m)\lambda_H+(v_L-c_m)\lambda_L \\
&\quad +(p_{a,m}^{LL*}-c_a)[\lambda_H\bar{F}_H(p_{a,m}^{LL*})+\lambda_L\bar{F}_L(p_{a,m}^{LL*})]-c(\lambda_H\bar{F}_H(p_{a,m}^{LL*})+\lambda_L\bar{F}_L(p_{a,m}^{LL*})) \\
&< [v_H+E(u_H-p_{a,m}^{LL*})-c_m]\lambda_H+[v_L+E(u_L-p_{a,m}^{LL*})-c_m]\lambda_L \\
&\quad +(p_{a,m}^{LL*}-c_a)[\lambda_H\bar{F}_H(p_{a,m}^{LL*})+\lambda_L\bar{F}_L(p_{a,m}^{LL*})]-c(\lambda_H\bar{F}_H(p_{a,m}^{LL*})+\lambda_L\bar{F}_L(p_{a,m}^{LL*})) \\
&= \Pi_{u,m}^{HH}(p_{a,m}^{LL*}) \\
&\leq \Pi_{u,m}^{HH*}.
\end{aligned}$$

Moreover, we know from the proof of Theorem 9 that $\frac{\mathrm{d}(\Pi_{u,m}^{LL*}-\Pi_{u,m}^{HH*})}{\mathrm{d}\alpha_H}<0$ and $\frac{\mathrm{d}(\Pi_{u,m}^{LL*}-\Pi_{u,m}^{HH*})}{\mathrm{d}\alpha_L}<0$. Thus, there exists a threshold function $\bar{\alpha}_{H,u}(\alpha_L)$ such that $\Pi_{u,m}^{LL*}\geq\Pi_{u,m}^{HH*}$ when $\alpha_H\leq\bar{\alpha}_{H,u}(\alpha_L)$ Moreover, by applying the Implicit Function Theorem to the equation $\Pi_{u,m}^{LL*}-\Pi_{u,m}^{HH*}=0$ which defines $\bar{\alpha}_{H,u}(\alpha_L)$, we obtain that $\bar{\alpha}_{H,u}(\alpha_L)$ is a decreasing function.

- Condition for $\Pi_{u,m}^{LL*}\geq\Pi_{u,m}^{HL*}$: When $\alpha_H=0$, $\Pi_{u,m}^{HL*}=(v_L-c_m)\lambda_L+(p_{a,m}^{HL*}-c_a)\lambda_L\bar{F}_L(p_{a,m}^{HL*})-c(\lambda_L\bar{F}_L(p_{a,m}^{HL*}))$ which is independent of λ_H. Consider $\Pi_{u,m}^{LL*}$ as a function of λ_H. At $\lambda_H=0$, we have $\Pi_{u,m}^{LL*}=\Pi_{u,m}^{HL*}$. Moreover, by using the Envelope Theorem and the first-order condition of $\Pi_{u,m}^{LL}(p_a)$, we have

$$\begin{aligned}
\frac{\mathrm{d}\Pi_{u,m}^{LL*}}{\mathrm{d}\lambda_H} &= v_H - c_m + (p_{a,m}^{LL*} - c_a)\bar{F}_H(p_{a,m}^{LL*}) - c'(\lambda_H \bar{F}_H(p_{a,m}^{LL*}) + \lambda_L \bar{F}_L(p_{a,m}^{HL*}))\bar{F}_H(p_{a,m}^{LL*}) \\
&= v_H - c_m + \bar{F}_H(p_{a,m}^{LL*}) \cdot \frac{\lambda_H \bar{F}_H(p_{a,m}^{LL*}) + \lambda_L \bar{F}_L(p_{a,m}^{LL*})}{\lambda_H f_H(p_{a,m}^{LL*}) + \lambda_L f_L(p_{a,m}^{LL*})} \\
&> 0.
\end{aligned}$$

Thus, when $\alpha_H = 0$, $\Pi_{u,m}^{LL*} > \Pi_{u,m}^{HH*}$ for any positive λ_H. When $\alpha_H = 1$,

$$\begin{aligned}
\Pi_{u,m}^{LL*} &= (v_H - c_m)\lambda_H + (v_L - c_m)\lambda_L \\
&\quad +(p_{a,m}^{LL*} - c_a)[\lambda_H \bar{F}_H(p_{a,m}^{LL*}) + \lambda_L \bar{F}_L(p_{a,m}^{LL*})] - c(\lambda_H \bar{F}_H(p_{a,m}^{LL*}) + \lambda_L \bar{F}_L(p_{a,m}^{LL*})) \\
&< [v_H + E(u_H - p_{a,m}^{LL*}) - c_m]\lambda_H + (v_L - c_m)\lambda_L \\
&\quad +(p_{a,m}^{LL*} - c_a)[\lambda_H \bar{F}_H(p_{a,m}^{LL*}) + \lambda_L \bar{F}_L(p_{a,m}^{LL*})] - c(\lambda_H \bar{F}_H(p_{a,m}^{LL*}) + \lambda_L \bar{F}_L(p_{a,m}^{LL*})) \\
&= \Pi_{u,m}^{HL}(p_{a,m}^{LL*}) \\
&\leq \Pi_{u,m}^{HL*}.
\end{aligned}$$

Moreover, we know from the proof of Theorem 9 that $\frac{\mathrm{d}(\Pi_{u,m}^{LL*} - \Pi_{u,m}^{HL*})}{\mathrm{d}\alpha_H} < 0$. Thus, there exists a threshold $\hat{\alpha}_{H,u}$ such that $\Pi_{u,m}^{LL*} \geq \Pi_{u,m}^{HL*}$ when $\alpha_H \leq \hat{\alpha}_{H,u}$.

- Condition for $\Pi_{u,m}^{LL*} \geq \Pi_{u,m}^{LH*}$: By using the same approach of deriving the condition for $\Pi_{u,m}^{LL*} \geq \Pi_{u,m}^{HL*}$, we can obtain that there exists a threshold $\hat{\alpha}_{L,u}$ such that $\Pi_{u,m}^{LL*} \geq \Pi_{u,m}^{LH*}$ when $\alpha_L \leq \hat{\alpha}_{L,u}$.

Therefore, we have obtained that $\Pi_{u,m}^{*} = \Pi_{u,m}^{LL*}$ when $\alpha_H \leq \hat{\alpha}_{H,u}$, $\alpha_L \leq \hat{\alpha}_{L,u}$, and $\alpha_H \leq \bar{\alpha}_{H,u}(\alpha_L)$.

Next, consider the bundling case. $\Pi_{b,m}^{*} = \Pi_{b,m}^{LL*}$ requires 1) $\Pi_{b,m}^{LL*} \geq \Pi_{b,m}^{HH*}$, 2) $\Pi_{b,m}^{LL*} \geq \Pi_{b,m}^{HL*}$, 3) $\Pi_{b,m}^{LL*} \geq \Pi_{b,m}^{LH*}$. We have the following:

- $\Pi_{b,m}^{LL*} \geq \Pi_{b,m}^{HH*}$ is equivalent to

$$\begin{aligned}
\alpha_H &\geq \frac{(v_H - c_m)\lambda_H + (v_L - c_m)\lambda_L - c_a[\lambda_H \bar{F}_H(0) + \lambda_L \bar{F}_L(0)]}{[v_H + E(u_H)^+ - c_m - c_a \bar{F}_H(0)]\lambda_H} \\
&\quad - \frac{[v_L + E(u_L)^+ - c_m - c_a \bar{F}_L(0)]\lambda_L \alpha_L}{[v_H + E(u_H)^+ - c_m - c_a \bar{F}_H(0)]\lambda_H} \\
&\overset{\text{def}}{=\!=} \bar{\alpha}_{H,b}(\alpha_L).
\end{aligned}$$

- $\Pi_{b,m}^{LL*} \geq \Pi_{b,m}^{HL*}$ is equivalent to

$$\alpha_H \geq \frac{v_H - c_m - c_a \bar{F}_H(0)}{v_H + E(u_H)^+ - c_m - c_a \bar{F}_H(0)} \overset{\text{def}}{=\!=} \hat{\alpha}_{H,b}.$$

- $\Pi_{b,m}^{LL*} \geq \Pi_{b,m}^{LH*}$ is equivalent to

$$\alpha_L \geq \frac{v_L - c_m - c_a \bar{F}_L(0)}{v_L + E(u_L)^+ - c_m - c_a \bar{F}_L(0)} \stackrel{\text{def}}{=\!=} \hat{\alpha}_{L,b}.$$

Therefore, $\Pi_{b,m}^* = \Pi_{b,m}^{LL*}$ when $\alpha_H \leq \hat{\alpha}_{H,b}$, $\alpha_L \leq \hat{\alpha}_{L,b}$, and $\alpha_H \leq \bar{\alpha}_{H,b}(\alpha_L)$.

Finally, take $\bar{\alpha}_H(\alpha_L) = \min(\bar{\alpha}_{H,u}(\alpha_L), \bar{\alpha}_{H,b}(\alpha_L))$, $\hat{\alpha}_H = \min(\hat{\alpha}_{H,u}, \hat{\alpha}_{H,b})$, $\hat{\alpha}_L = \min(\hat{\alpha}_{L,u}, \hat{\alpha}_{L,b})$. Thus, when $\alpha_H \leq \hat{\alpha}_H$, $\alpha_L \leq \hat{\alpha}_L$, and $\alpha_H \leq \bar{\alpha}_H(\alpha_L)$, we have $\Pi_{u,m}^* = \Pi_{u,m}^{LL^*}$ and $\Pi_{b,m}^* = \Pi_{b,m}^{LL*}$, and hence $\Pi_{u,m}^* \geq \Pi_{b,m}^*$.

□

Proof of Theorem 9

Proof. First, consider the monotonicity of $\Pi_{b,m}^*$. We need to show that each $\Pi_{b,m}^{ij*}$ $(i, j = H, L)$ has a non-negative derivative with respect to α_H and α_L. This is true because

$$\begin{aligned}
\frac{\partial \Pi_{b,m}^{HH*}}{\partial \alpha_H} &= \frac{\partial \Pi_{b,m}^{HL*}}{\partial \alpha_H} = [v_H + E(u_H)^+ - c_m]\lambda_H - c_a \lambda_H \bar{F}_H(0) > 0, \qquad \frac{\partial \Pi_{b,m}^{LH*}}{\partial \alpha_H} = \frac{\partial \Pi_{b,m}^{LL*}}{\partial \alpha_H} = 0; \\
\frac{\partial \Pi_{b,m}^{HH*}}{\partial \alpha_L} &= \frac{\partial \Pi_{b,m}^{LH*}}{\partial \alpha_L} = [v_L + E(u_L)^+ - c_m]\lambda_L - c_a \lambda_L \bar{F}_L(0) \geq 0, \qquad \frac{\partial \Pi_{b,m}^{HL*}}{\partial \alpha_L} = \frac{\partial \Pi_{b,m}^{LL*}}{\partial \alpha_L} = 0.
\end{aligned}$$

Second, consider the monotonicity of $\Pi_{u,m}^*$. We have

$$\begin{aligned}
\frac{\partial \Pi_{u,m}^{HH*}}{\partial \alpha_H} &= [v_H + E(u_H - p_{a,m}^{HH*})^+ - c_m]\lambda_H \\
&\quad + (p_{a,m}^{HH*} - c_a)\lambda_H \bar{F}_H(p_{a,m}^{HH*}) - c'(\alpha_H \lambda_H \bar{F}_H(p_{a,m}^{HH*}) + \alpha_L \lambda_L \bar{F}_L(p_{a,m}^{HH*}))\lambda_H \bar{F}_H(p_{a,m}^{HH*}) \\
&= [v_H + E(u_H - p_{a,m}^{HH*})^+ - c_m]\lambda_H \\
&> 0,
\end{aligned}$$

where the first equality follows by using the Envelope Theorem and the second equality follows by using the first-order condition of $\Pi_{u,m}^{HH}(p_a)$. Similarly, $\frac{\partial \Pi_{u,m}^{HH*}}{\partial \alpha_L} > 0$. By applying the Envelope Theorem and the first-order condition of $\Pi_{u,m}^{HL}(p_a)$, we have

$$\begin{aligned}
\frac{\partial \Pi_{u,m}^{HL*}}{\partial \alpha_H} &= [v_H + E(u_H - p_{a,m}^{HL*})^+ - c_m]\lambda_H \\
&\quad + (p_{a,m}^{HL*} - c_a)\lambda_H \bar{F}_H(p_{a,m}^{HL*}) - c'(\alpha_H \lambda_H \bar{F}_H(p_{a,m}^{HL*}) + \lambda_L \bar{F}_L(p_{a,m}^{HL*}))\lambda_H \bar{F}_H(p_{a,m}^{HL*}) \\
&= [v_H + E(u_H - p_{a,m}^{HL*})^+ - c_m]\lambda_H + \lambda_H \bar{F}_H(p_{a,m}^{HL*}) \cdot \frac{\lambda_L \bar{F}_L(p_{a,m}^{HL*})}{\alpha_H \lambda_H f_H(p_{a,m}^{HL*}) + \lambda_L f_L(p_{a,m}^{HL*})} \\
&> 0.
\end{aligned}$$

Similarly, $\frac{\partial \Pi_{u,m}^{LH*}}{\partial \alpha_L} > 0$. Additionally, we have $\frac{\partial \Pi_{u,m}^{LH*}}{\partial \alpha_H} = \frac{\partial \Pi_{u,m}^{LL*}}{\partial \alpha_H} = \frac{\partial \Pi_{u,m}^{HL*}}{\partial \alpha_L} = \frac{\partial \Pi_{u,m}^{LL*}}{\partial \alpha_L} = 0$. Therefore, $\Pi_{u,m}^*$ is also increasing in α_H and α_L.

□

Proof of Theorem 10

Proof. (i) If $\beta_H \geq \beta_L$, we have

$$\begin{aligned}
\Pi^*_{u,n} - \Pi^*_u &\geq \Pi_{u,n}(p^*_a) - \Pi^*_u \\
&= -(v_H - v_L)\lambda_H - [E(u_H - p^*_a)^+ - E(u_L - p^*_a)^+]\lambda_H \\
&= -(v_H - v_L)\lambda_H - \lambda_H \int_{p^*_a}^{\bar{u}} [\bar{F}_H(x) - \bar{F}_L(x)]\,\mathrm{d}x \\
&\geq -(v_H - v_L)\lambda_H - \lambda_H \int_{0}^{\bar{u}} [\bar{F}_H(x) - \bar{F}_L(x)]\,\mathrm{d}x \\
&= -(v_H - v_L)\lambda_H - [E(u_H)^+ - E(u_L)^+]\lambda_H \\
&= \Pi^*_{b,n} - \Pi^*_b.
\end{aligned}$$

Rearranging terms in the inequality obtained above yields $\Pi^*_{u,n} - \Pi^*_{b,n} \geq \Pi^*_u - \Pi^*_b$. Thus, when $\Pi^*_u \geq \Pi^*_b$, we also have $\Pi^*_{u,n} \geq \Pi^*_{b,n}$.

(ii) If $\beta_H < \beta_L$, we have

$$\begin{aligned}
\Pi^*_{u,n} - \Pi^*_u &< \Pi^*_{u,n} - \Pi_u(p^*_{a,n}) \\
&= -(v_H - v_L)\lambda_H + [E(u_L - p^*_{a,n})^+ - E(u_H - p^*_{a,n})^+]\lambda_H \\
&\leq -(v_H - v_L)\lambda_H + [E(u_L)^+ - E(u_H)^+]\lambda_H \\
&= \Pi^*_{b,n} - \Pi^*_b,
\end{aligned}$$

where the second inequality follows from the same approach used in Part (i). Thus, we have $\Pi^*_{u,n} - \Pi^*_{b,n} < \Pi^*_u - \Pi^*_b$; when $\Pi^*_{u,n} \geq \Pi^*_{b,n}$, we also have $\Pi^*_u \geq \Pi^*_b$.

□