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STREET GEOMETRY AND FLOWS*

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ABSTRACT. Deadly collisions between vehicles and people or between vehicles are a fact of urban life. A geometric approach to reducing the potential for collisions rests on using the symmetry of the underlying street pattern to sort antagonistic flows into separate channels and on disrupting that symmetry to refine the separation to the level of bicyclist and pedestrian. This disruption is part of a non-Euclidean, hyperbolic street geometry.

URBAN streets are often a confused hodgepodge in which throngs of pedestrians and automotive vehicles share congested streets and in which spillover traffic to nearby residential streets threatens anyone moving on foot or by bicycle. Antagonistic flows may result in deadly collisions and create tension and hostility in the urban environment. One societal solution is legal—laws confining trucks spatially or temporally. This type of institutional solution is costly. Increased police patrols add to the taxpayers' burden and impinge on motorists' perceptions of their freedom of movement. Limitations on vehicular size and performance constrain what is technically feasible and lead to tension among the drivers, who then resort to evasion of laws and regulations. An alternative to these solutions is a passive geometric approach that reduces the potential for deadly collisions by spatially separating vehicular and pedestrian flows on the basis of characteristics inherent in the design of the underlying street geometry.

In the United States a common urban street pattern is a rectangular grid, as exists, for example, on Manhattan Island north of Fourteenth Street. In this type of geographical arrangement, vehicular flows can choose which way they go around the block; there are many shortest paths or geodesics between two noncollinear points in this grid geometry (Krause 1975). In a sample three-block-by-four-block area, there are thirty-five geodesics from the upper left corner (**A**) to the lower right corner (**B**) (Mohanty 1979; Hilton and Pedersen 1989), two of which are shown on Figure 1. Some of these thirty-five geodesic routes require more turns by a driver than do others. Thus number of turns will sort vehicular-traffic flows onto distinct geodesics. Total separation is not possible in this street geometry, because there are only two routes into and out of each of **A** and **B**. Around the outside of the grid only one turn is necessary. To penetrate the interior along another geodesic requires at least one additional turn and may require as many as five, for a total of one less than the sum of the number of units on the axes

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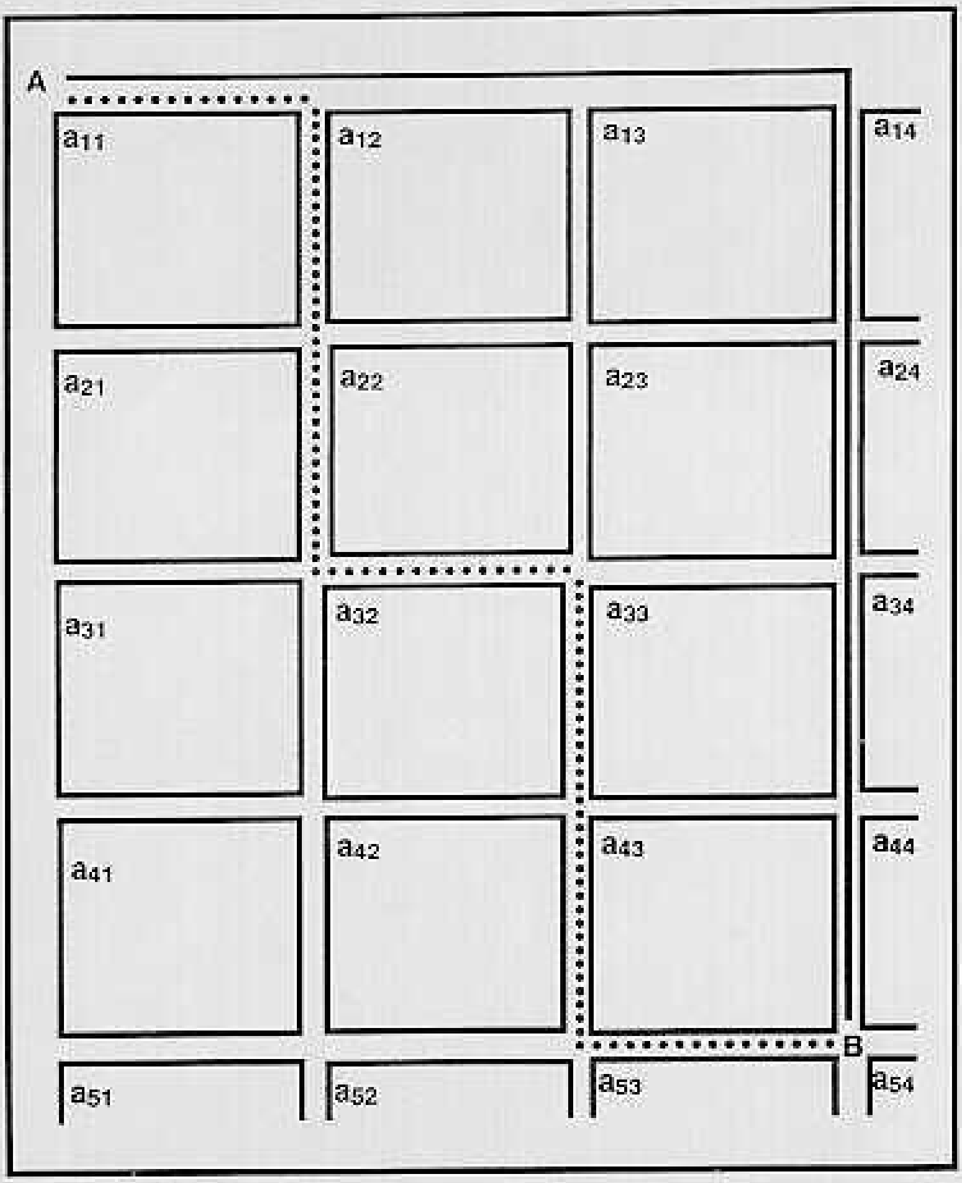

FIG. 1—Street grid of three blocks by four blocks. Two different geodesics are shown from A to B.

bounding the area. It would be easiest, in turning, for all vehicles to occupy the two, one-turn outside routes. As these exterior channels congest, alternative routes become attractive. The extra effort of added turns is less for automobiles than for trucks, which have a larger turning radius. Thus a separation of vehicles, by length, occurs as congestion increases.

The enumeration of routes with more than one turn quickly becomes complicated; in a grid of infinite extent, a binary tree would represent the

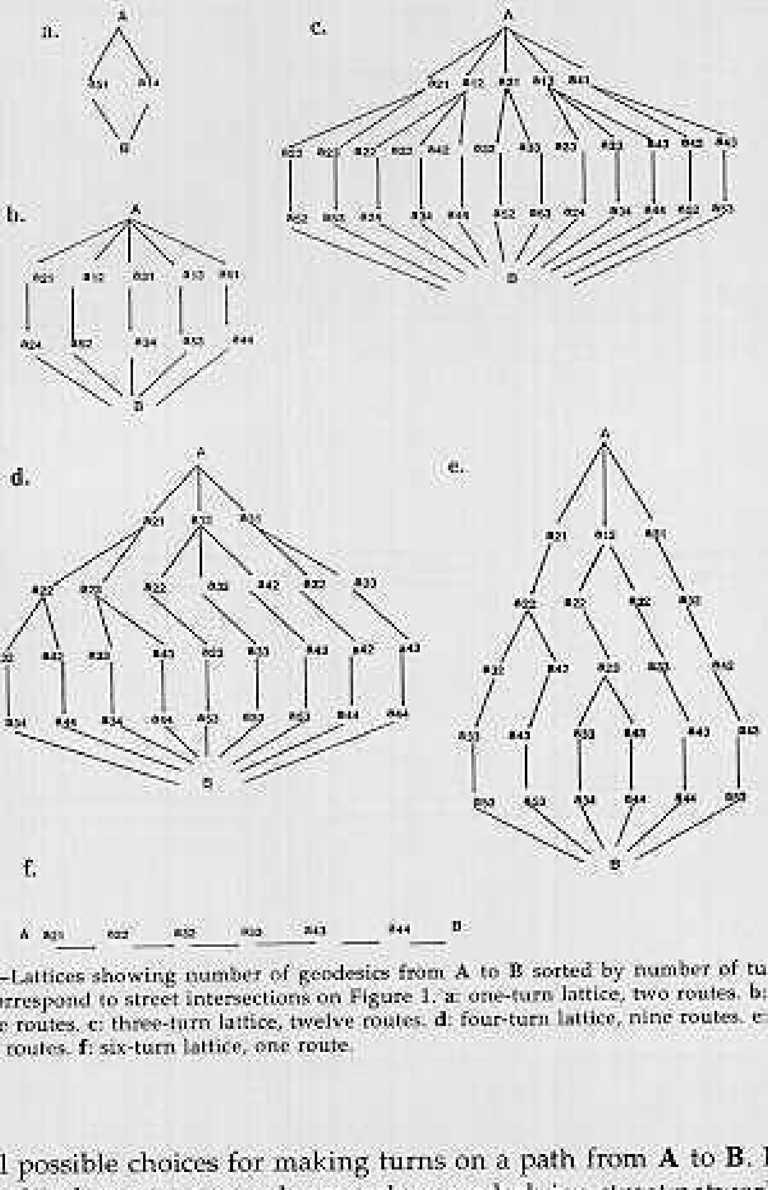

FIG. 2—Lattices showing number of geodesics from A to B sorted by number of turns. All a vertices correspond to street intersections on Figure 1. **a**: one-turn lattice, two routes. **b**: two-turn lattice, five routes. **c**: three-turn lattice, twelve routes. **d**: four-turn lattice, nine routes. **e**: five-turn lattice, six routes. **f**: six-turn lattice, one route.

set of all possible choices for making turns on a path from **A** to **B**. Because a geographical area can never have such an underlying street network, these trees are truncated to correspond to municipal or other boundaries. The trees or lattices were used to sort all the logical possibilities for numbers of shortest routes from **A** to **B** with a prescribed total of turns (Fig. 2). Each level represents a turn along a geodesic on the grid in Figure 1. The complexity of the lattice construction corresponds directly to the size of the street grid under consideration.

Imagine that the grid area described above is a residential neighborhood. As the outer one-turn routes from **A** to **B** become congested with through traffic, some automobiles or small trucks may seek other shortest paths. When vehicular through traffic mixes with residential pedestrian traffic, which often includes a large percentage of children, the collision-hazard potential increases, in part because the pedestrian is not confined to the grid. One method to reduce the risk of automobile-people collisions is to remove the option of next-best two-turn routes for through traffic. When "street parks" are placed at strategically chosen intersections, the removal of two-turn routes can force vehicular shortcuts through the residential neighborhood along unattractive route choices with large numbers of turns. In this configuration, one park impedes vehicular flow and two other half-parks permit flow through one-half of each street intersection (Fig. 3). The number of shortest routes through the neighborhood decreases from thirty-three to three, each with three turns.

Additional street parks would further decrease through traffic in this neighborhood, but at the expense of the convenience of local traffic. The configuration provides great reduction in the number of shortest routes through the neighborhood and does so without adding too many extra turns or without stranding residents. Generally, added turns afford residents extra privacy and security from theft, but they reduce convenience and access to main thoroughfares and emergency services.

Another method of separating people and vehicles is to disturb the underlying symmetry of the grid by inserting a strategically placed diagonal between street parks (Fig. 3). This method reduces the length of the pedestrian geodesic and offers a more attractive option than traversing the edges of a block, all else being equal. The number of distinct pedestrian geodesics drops: there are two in the 1×1 grid subspace anchored at **A** and three in the 2×1 subspace anchored at **B**. Thus the two sets of possibilities, linked by the diagonal, present the pedestrian with $2 \times 3 = 6$ distinct choices for geodesics from **A** to **B**. Four of these geodesics contain four turns, and two of them contain five turns. Additional turns do not cause significant navigational difficulty for pedestrians and may be attractive. The presence of the single diagonal decreased the number of geodesic choices from thirty-five to six; the diagonal introduces a slight imperfection in the grid symmetry, causing a planned disorder that improves the system by concentrating pedestrian flows between **A** and **B** (Kirkpatrick, Gelatt, and Vecchi 1983).

A change in the number of turns, together with installation of barriers to restrict flow, that is, street parks, and of a diagonal to shorten the length of the pedestrian geodesic, reduces total geodesics from **A** to **B**. The combined effect is to channel through traffic into one set of routes, local vehicular traffic into another set, and pedestrian traffic into yet another set, patterns achieved without institutional stricture. No signs or police direct vehicles and pedestrians to one route or another; the underlying geometry is the guide.

Clearly, the relative positions of pedestrians and street patterns are crucial in determining the potential for deadly collisions. A typical, Euclidean within-block solution to reducing conflict is for pedestrians to have an easy-to-navigate, paved sidewalk parallel to, and therefore separate from, the street. This type of sidewalk, however, may not offer the shortest within-block path

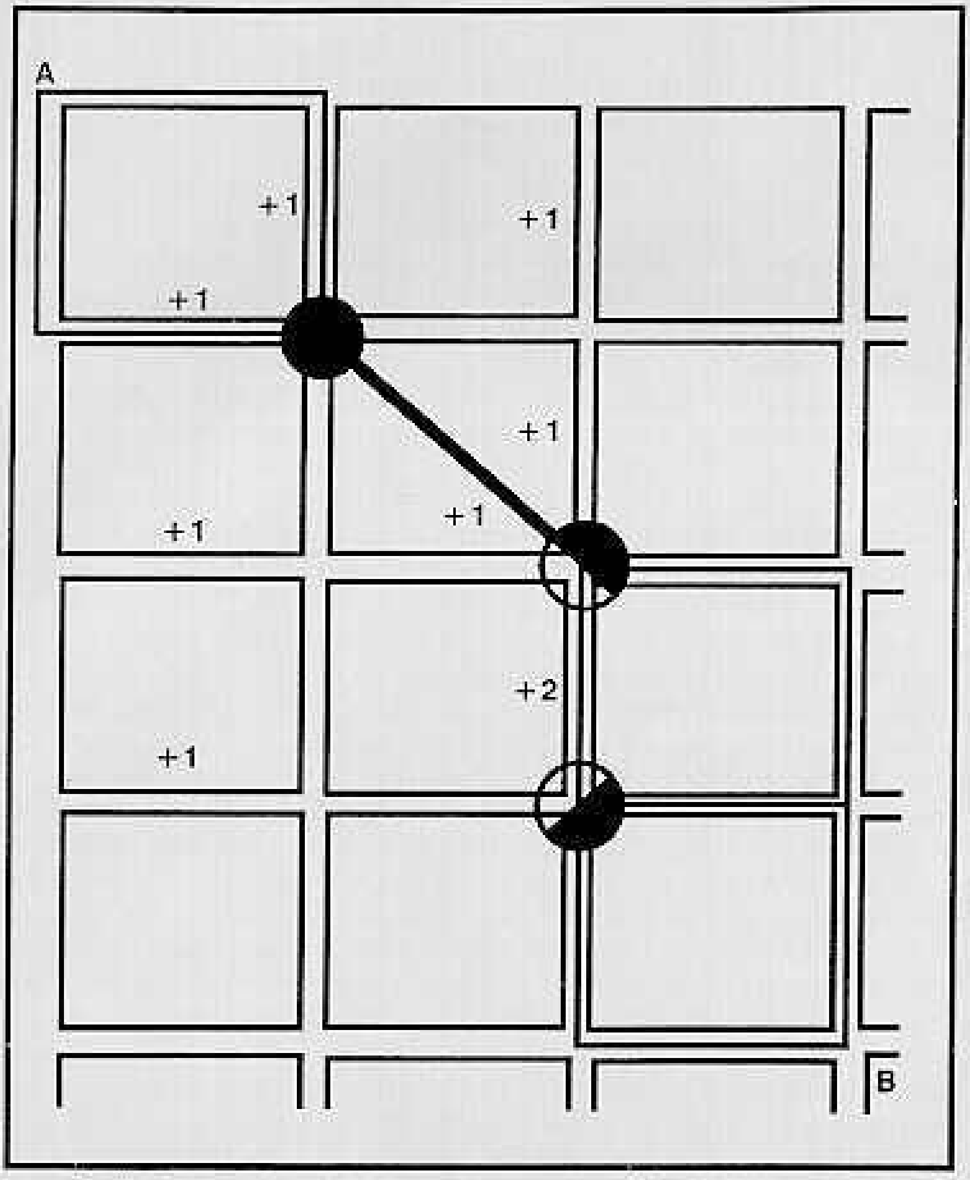

FIG. 3—Separation of pedestrian and vehicular traffic by full barrier (circular) and partial-barrier (semicircular) street parks. The diagonal is not accessible to vehicular traffic. The edge weights indicate the increased number of turns, as a result of the street parks, needed to reach B from labeled street segments. Solid lines in the streets, together with the diagonal, trace the pedestrian geodesics from A to B.

from one corner to the next. "People trails," well-worn shortcuts usually through grass or lawns, indicate users' view of the shortest routes in many varied settings.

Simple, homogeneous geographical environments are unusual. Midblock paths are often confined by fences along property lines; alleys can provide

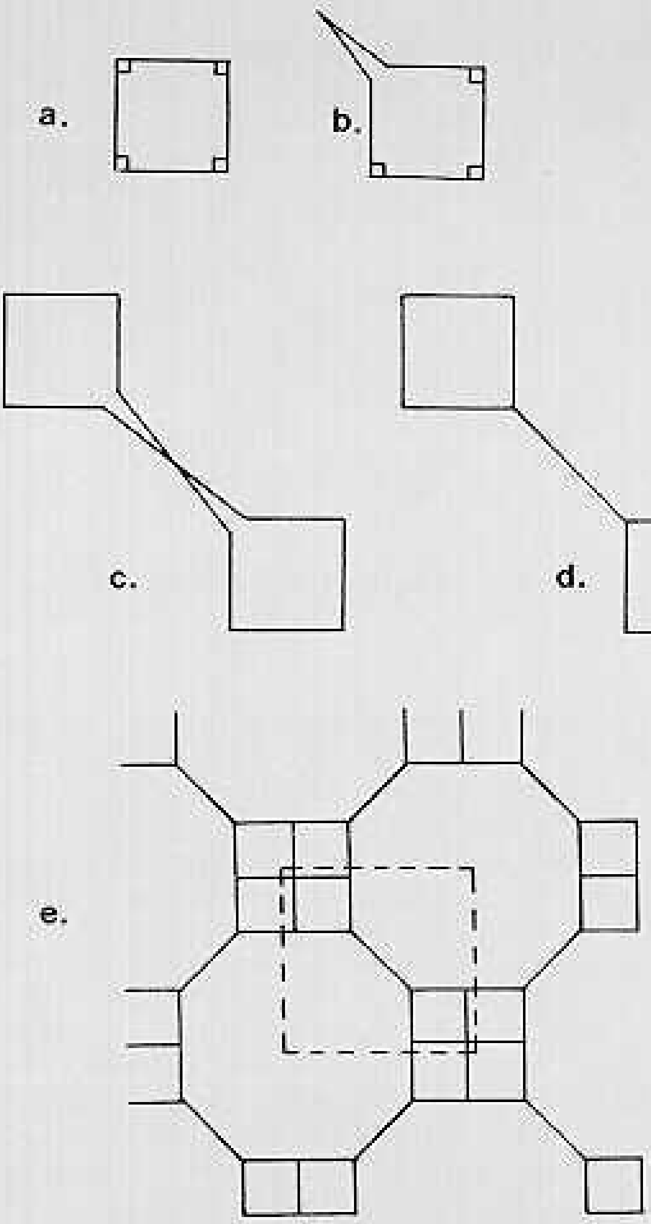

FIG. 4—a: grid quadrilateral of four right angles. b: Lambert quadrilateral. c: two Lambert quadrilaterals aligned on their acute angles. d: Lambert generator. e: Lambert design derived from rotations and translations of d; dashed box shows a single Lambert generator within this design.

shortcuts but might be perceived as dangerous; and small parks with tall grass or deep snow might present only seasonal opportunity for shortcuts. The characteristics of an urban surface limit the opportunity to choose a within-block shortest path. The advantage of the diagonal shortcut is its simplicity; to retain such geometric simplicity, but to do so within realistic

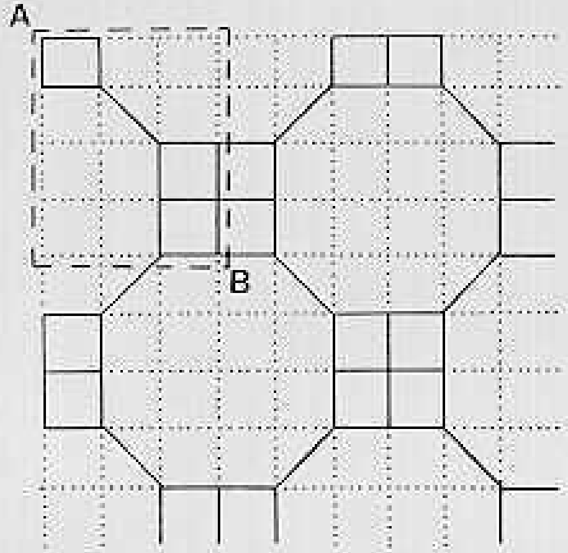

FIG. 5—Street geometry derived by superimposing Figure 3 on Figure 4e. The subset determined by corners A and B corresponds exactly to the street network and pedestrian geodesic in Figure 3 based on existence of street parks.

geographical environments, might involve a shift in the characteristics of the underlying geometry. What disturbs symmetry at one level of thought might be part of a deeper symmetry in a broader geometric context.

Because there may be more than one shortest within-block path that is parallel to the street (does not intersect it), the grid approach suggested above might be entirely recast in non-Euclidean hyperbolic geometry in which there is more than one parallel to a given line through a point not on that line, and in which the sum of the angles of a triangle is less than 180 degrees. The hyperbolic counterpart to the grid quadrilateral (Fig. 4a) is the Lambert quadrilateral (Fig. 4b), a four-sided figure with three right angles and one acute angle that combine to less than 360 degrees (Greenberg 1980). When two Lambert quadrilaterals are glued together at the tips of their acute angles (Fig. 4c), a diagonal path linking city blocks is suggested as the union of the stretched corners (Fig. 4d). When this stretched union is taken as the fundamental unit (sidewalk corresponds to the diagonal and streets to the horizontal and vertical elements), it can serve as a generator for a "natural" tiling composed of Lambert quadrilaterals to correspond to the Manhattan street tiling composed of Euclidean quadrilaterals (Fig. 4e).

In this generator, the acute angles of the Lambert quadrilaterals have been shrunk to a line, on the assumption that sidewalks function as a line. Because shapes of this sort, which contain segments of area zero, have been excluded as tiles (Grünbaum and Shephard 1987), we refer to the patterns produced by this generator as designs, not tilings. When this generator is reflected, rotated, and translated, several designs of the Euclidean plane

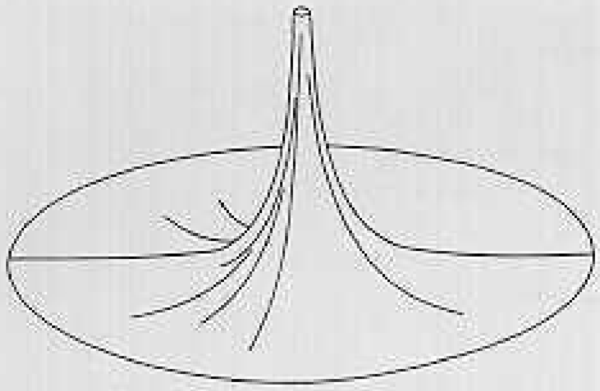

FIG. 6—A truncated tractroid, a surface of constant negative curvature that serves as a Euclidean model of the hyperbolic plane. The shape serves as a street-intersection pylon to reduce the potential for deadly collisions.

appear, which we label Lambert designs. One Lambert design is composed of quadrilaterals and octagons (Fig. 4e). Its advantage is the inclusion, as part of the basic layout, of the diagonals found useful in separating pedestrian and automobile flows. Specifically, when Figure 4e is superimposed on an urban grid of appropriate cell size (Fig. 5), the overlay of symmetric layers contains precisely the network elements of Figure 3.

In this geometry, automotive vehicles could select paths from routes across a quilt of contiguous squares, while pedestrians and bicyclists could choose paths including sidewalk diagonals. When the grid tiling and Lambert design are superimposed, intersection points of the two layers are the positions with the greatest potential for collisions between automobiles and pedestrians. The points are street corners.

The choice of such corners as sites for street parks is also a consequence of superimposing geometries. The solid lines in the layered street geometry are derived from the Lambert generator; within the dashed box of this geometry, the solid lines follow precisely the pattern of all possible routes through the underlying urban space with its street parks.

The association between the urban grid and the hyperbolic street geometry might yield additional insights into solving collision problems. Consider the curvature of the two geometries: the grid has zero curvature, and the hyperbolic one has negative curvature (Coxeter 1965). The entire hyperbolic plane cannot be represented as a single Euclidean model; however, the tractroid, a surface of revolution of constant negative curvature (Fig. 6), is often used as a Euclidean model for much, but not all, of the hyperbolic plane. Thus, at geometric interfaces in Figure 5, one might construct geometrically derived driving surfaces as a truncated tractroid or a pylon to indicate interfaces of the two geometries and to guide vehicles along the rim of the pylon through half-circle street parks. The pylon shape could affect driver behavior insofar as reverse banking occurs after the midpoint of a turn, which forces the driver to slow down in this space where automobiles should yield to pedestrians.

The layering of patterns to study interfaces is a method prevalent in many research endeavors ranging from crystallography to biology to architecture. An advantage of using the Lambert design rather than some other tiling or design that would separate flows by introducing diagonal elements is the theoretical consideration of aligning basic structures derived from different geometries, to preserve the possibility of using theorems from one to shed light on the other.

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