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GEOMETRY OF BOUNDARY EXCHANGES

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ABSTRACT. The geometric concept of self-similarity, from fractal geometry, may be used to compress portions of long, linear boundaries separating contrasting landuse types, like lake and shore or forest and meadow, into small, compact regions. This process is applied to maps of an actual location on the northern shore of Lake St. Clair and to a hypothetical map of a national forest. Using fractal geometry to determine boundaries can offer visitors to such regions the opportunity to enjoy exchange across them and can simultaneously minimize the potential for overall damage to broad expanses of shore or forest.

VISITORS to forests and to shorelines share a desire to experience the variety that the landscape has to offer, to do so in relative privacy, and to have direct access to the sites they come to visit. Of concern are finding a way to offer a full measure of public access and minimizing the potential for environmental damage. The fractal concept of self-similarity is such an approach to this issue of maximum exposure in minimum space. An analogous problem arises in telecommunications with the introduction of facsimile transmissions and other data-intensive media. The problem is to increase data transmission without exceeding capacity. Fractal methods are employed to achieve data compression, so that additional information may be transmitted within a fixed capacity (Barnsley 1988). We seek ways to increase exchange with the environment without exceeding its capacity for exposure to people.

In this article we apply the fractal compression of boundaries used in electronic systems to the terrestrial environment. Previous research has shown such geometric boundary compression to be an effective means of reducing the entire geometry of central-place theory to single fractal transformations; in such cases, the compression comes about by finding a fractallike form to fit a landscape form (Arlinghaus 1985; Arlinghaus and Arlinghaus 1989). We begin our study with a brief explanation of self-similarity. We then illustrate how self-similarity might be applied in seashore and forest settings to construct interdigitated patterns of land and water or of forest and meadow, by compressing the boundaries separating them within a small parcel of land. This approach, we believe, can help resolve the dilemma of maximizing contact between humans and natural landscapes in a minimum of terrestrial space and with a minimum of environmental hazard.

SELF-SIMILARITY

Many fractal sets can be formed as a sequence of self-similarity transformations (Mandelbrot 1983). Self-similarity is present in all studies that con-

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sider a shift in scale that preserves shape. Inset maps showing enlargements at various scales exhibit self-similarity; urban studies that envision a small city as a reduction of a large one also draw on the concept.

More abstractly, self-similarity transformations may be used to fill a bounded, two-dimensional space using one-dimensional forms. As an example, one might replace each of the three line segments constituting the letter N with a scaled-down, self-similar copy of N. Each successive iteration of the process, within a fixed frame around the N, will fill more space than the previous one (Mandelbrot 1983). Constructions such as this suggest how a one-dimensional winding path might be used to fill selected portions of two-dimensional space (Arlinghaus and Nystuen 1987).

A wider notion posits fractal sets of fractional dimension: space-filling curves of dimension greater than one and less than or equal to two. This distinction reflects the fact that some curves, with iteration carried out infinitely, may not cover an entire region of two-dimensional space. The idea extends in an obvious way to higher dimensions. Computer graphics can show highly complicated curves of fractional dimension between one and two, as well as the familiar, beautiful landscapes of higher fractional dimensions. A fractal is a point-set whose fractional dimension strictly exceeds its topological dimension (Mandelbrot 1983).

Geographical Routing through a Marina

Many individuals who enjoy visiting the shore wish to buy a residence nearby, a reality reflected in the presence of new condominium developments along seashores and around the margins of the Great Lakes. This type of construction, popular with real-estate developers and unpopular with many of the surrounding indigenous residents, raises the dilemma posed at the outset of this article: how can individuals be offered the opportunity to secure desirable shore sites without doing extensive damage to the overall waterscape? A fractal approach, compressing the land-water boundary within a relatively small, compact portion of the shore, may resolve the dilemma.

To select a fractal whose geometric characteristics will fit the situation, we must first consider what traits might be viewed as desirable for a marina. Generally the goal is to fill space with both boat slips and land, with the two separated so that homeowners have private access to individual slips, community access to channels leading to the large body of water on which the development is located, and convenient access to private parking and community roads. Each condominium might have a garage backing onto a widening trunk road that never crosses water, a view of the water uninterrupted by the road network, and direct access to the water: water in front, parking for vehicles in back (Nystuen 1963).

One curve that fits is generated by the idea of self-similarity applied to a straight line segment with a V attached (Fig. 1). In the first stage of the transformation, an upside-down V is attached to the center of a line segment,

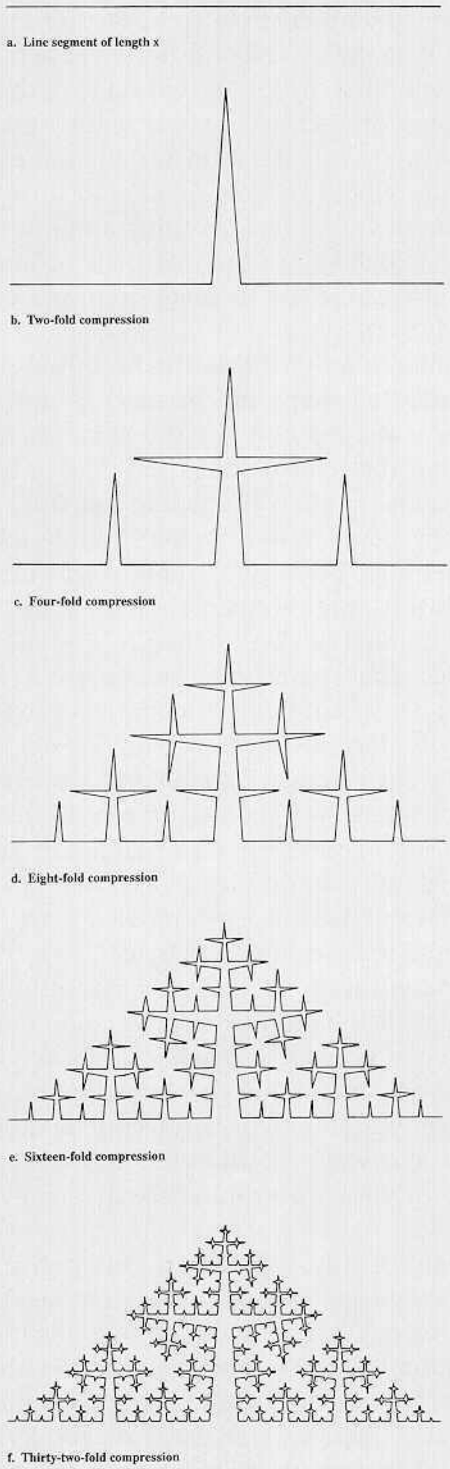

FIG. 1—Self-similarity transformation. Successive attachment of wedges produces an increasingly compressed curve. Wedges are cut at a 95° angle.

fracturing the line into a jointed curve composed of four segments of equal length (Fig. 1a). The V shape corresponds to the idea of making a cut in the land for a boat slip or a canal. In the second stage of the transformation, the process is repeated once on each of the four segments of the previous stage (Fig. 1b). In theory, after infinite iteration a triangular plane space enveloped by the road trunk-lines would become filled with lines of the road network (Mandelbrot 1983). However, in this planning application, we stop after five iterations, a point at which the individual pods of land in the land-water boundary are of a size sufficient to meet zoning and other institutional requirements (Piku 1988) (Figs. 1c–f).

Iteration of the self-similarity transformation produces a bilaterally symmetrical overall shape; this shape can be used as the geometric base for a marina (Fig. 2). The V-shaped cuts for the slips fill the space under this transformation, so that the road network emerges as a consequence of the slip geometry, with no extra effort in calculation. The land edge close to the water is penetrated by many water avenues, while the land edge close to the highway is penetrated by only a single road entrance to the marina. Access to the water is thus more convenient than access to the highway, and substantially more locations can be reached in a small number of turns by boat than by car, a desirable characteristic in a marina (Fig. 3). The 466 small squarish shapes are pods of land on which to erect condominiums or other living quarters for boundary dwellers (Nystuen 1967).

Like the pods, the road network stems directly from the self-similarity transformation, which forces each thoroughfare to be the correct width; trunk lines are wider than local collectors. Water channels are similarly tailored: individual slips are narrow; community access channels are wider than slips and broaden further as the need to accommodate more boat traffic increases. No bridges are required to reach any pod by car, so sailboats with tall masts can approach all condominium sites. Each condominium has its own slip, the number of slips equals the number of pods, and unique, nonoverlapping assignment of slip to pod is ensured by choosing a single orientation, clockwise or counterclockwise, across the geometric figure. Thus rationalization of both the road and the water networks, selection of desirable condominium sites relative to land and water boundaries, and establishment of a traffic pattern requiring no bridges are simultaneously achieved when a curve of this sort is used.

One other attribute of this approach merits notice: the marina, as designed, can accommodate persons with varying needs. Because five self-similarity iterations were used to produce this abstract marina plan, the number of turns required to reach any specific pod either by car or by boat does not exceed five. Number of turns is crucial in measuring the ease of navigating boats in the water as well as of maneuvering cars and trailer-hauled boats on land. For example, to reach an interior pod, P, requires five turns by car and five turns by boat; to reach one coastal site, Q, takes one

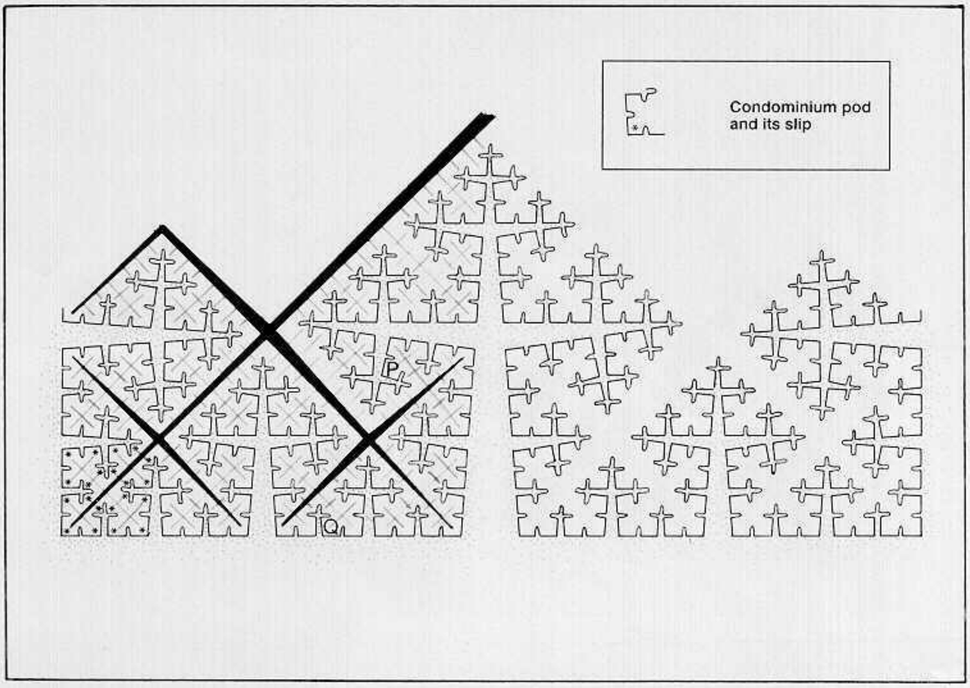

FIG. 2—A marina-routing design with a geometric base. Appropriately selected V-shaped cuts transform a shoreline length of x into one of 32x, a thirty-two-fold compression, without adding any new area. The chosen self-similarity transformation for generating slips and canals simultaneously produces appropriately scaled road networks from trunk lines to driveways, as well as 466 pods for condominiums.

turn by boat and five turns by car (Fig. 2). An elderly or infirm person who enjoys the marina lifestyle but whose needs for emergency services are greater than the norm might choose a pod with few road turns and more water turns, while an avid sailor might select a pod with the reverse configuration.

APPLICATION AT EXISTENT MARINA SITES

The real world does not conform exactly to the characteristics of geometric self-similarity. However, it is not especially difficult to modify purely fractal concepts to fit various situations in the terrestrial environment. Such modification requires a delicate hand, sensitive to issues of indigenous tastes and preferences and to local laws and landuse concerns. Indeed, it is here that the art of planning needs to be exercised.

Interdigitated marinas abound in Florida and in California and are beginning to emerge along the rim of the Great Lakes. In no case, however, has a fractal connection for marina planning and development been explored. Instead, the evidence from topographic maps suggests that when interdigitated patterns of land and water appear, they come from building a canal network formed from two comblike structures with interlocking teeth. This

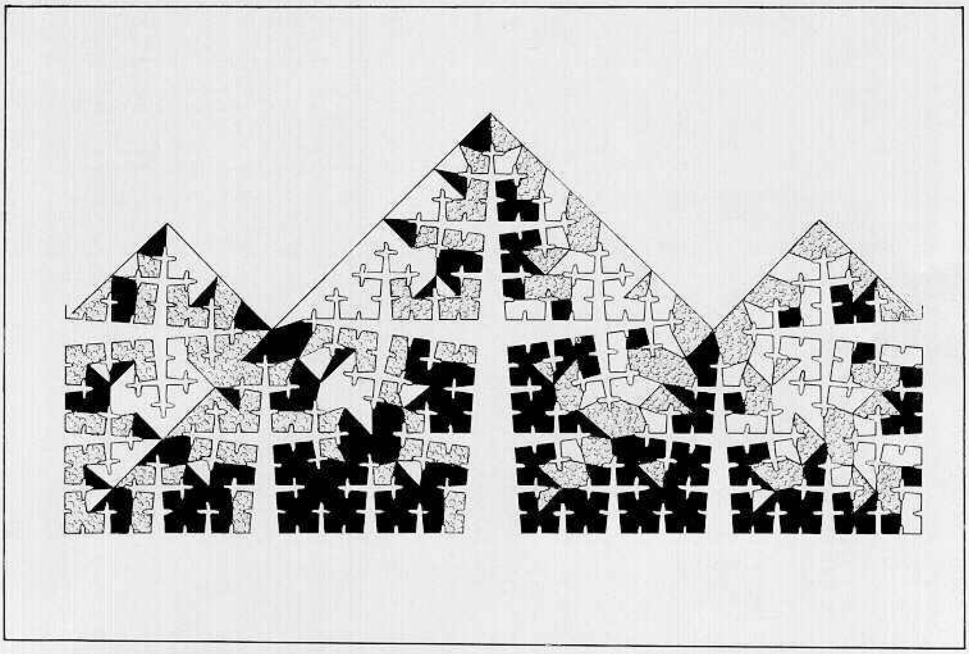

FIG. 3—Access to each pod characterized according to number of turns. Fewer water than land turns are required to reach 196 black pods; an equal number to reach 146 gray pods; and more water than land turns to reach 124 white pods. The orientation of pod to slip is clockwise across the entire configuration, which produces an asymmetrical pattern of shading.

sort of configuration seems to force the use of bridges to ensure that all parcels of land have simultaneous access to road and water networks along a route with a small number of turns. Older marinas in the St. Clair Shores area, a Detroit suburb on Lake St. Clair, have numerous bridges across the canals to overcome otherwise highly circuitous routing. In newer marinas in New Buffalo, Michigan, condominiums were built on either side of a network of slips that lay in a channel parallel to the coastline. The main road into the complex had to bridge the channel parallel to the coastline; to accommodate fairly tall ships, this bridge was built with an unusually steep vault, probably an inconvenience to year-round residents.

Bridges are, in fact, a source of considerable difficulty in a marina setting. Not only do they limit lot selection for owners of tall-masted boats, but they also concentrate automobile traffic, restrict options for extending services like water supply, electrical lines, and telephone cables in what might be territory remote from conduit and cable trunk lines, expose infrastructure pipes, tubes, and wiring to weather damage, and are themselves costly to maintain. A desirable real-world marina plan would provide land and water access with relatively direct routes without the use of bridges. That, coupled with the more general imperative to achieve the maximum exposure to land and water networks in minimum space, suggests the appropriateness of applying the abstract fractal marina plan already detailed as a geometric base for an actual development.

We examined three contiguous parcels of land at the northern end of Lake St. Clair as a potential fractal-marina site and combined the results with specific site information from a Detroit-area developer (Piku 1988). Under the accompanying plan, a single long cut would be made perpendicular to the shoreline to minimize battering effects on dredged channels from wave action and to serve as the water entry to the marina (Fig. 4). The cut would be more than twice as wide as the widest recreational boats used on the lake. This long channel would correspond to the shoreline in the abstract plan, and further cuts to the interior of the parcel would be made at an angle to this entry channel. A second level of narrower, modified V-shaped cuts leading from the entry channel and into the interior of the parcel would be made parallel, with angle of placement determined by the outer boundaries of the entire parcel. This set of cuts, as well as the ones to follow, would be narrower as a group than the entry channel; however, within each group there would be variation, so that individuals with smaller boats might position themselves on smaller channels or might pay a premium to occupy a residence on a pod adjacent to a large channel. A third level of cuts would be made at an angle to the second level so that the sides of the canals would be parallel to the lot lines. A fourth level of cuts would be made at an angle to those of the third, in this case so that canal sides would be parallel to the highway. Iteration of the self-similarity transformation would cease at this point, because additional cuts would consume too much land.

This sequence of crisscrossing cuts subdivides the interior of the parcel into pods. Using the outer boundaries of the parcel as oblique axes to guide the shape of the pods is superior to imposing prescribed, arbitrary, perhaps rectangular axes on the entire parcel. Any other set of axes would leave irregularly shaped pods at both the northern and southern ends of the development. Our choice of axes leaves irregular pods at one end only, along the shoreline, where a yacht club, a public marina, beaches, and other amenities might easily fit into otherwise awkwardly shaped parcels. Access to public marina facilities often involves substantial queuing difficulties. The presence of individual slips, in addition to a public marina, offers a wide variety of persons efficient access to docking facilities.

As in the abstract case, decreasing the width of the water channels through successive levels of water cuts engenders a corresponding decline in road width, appropriate to distance from the highway, that produces with no extra effort the desired interdigitated pattern of road and water networks. This self-similarity transformation might be iterated indefinitely; we stopped it after four repetitions to produce an average pod size of more than 7,200 square feet, which conforms to local zoning regulations for a variety of housing types. The smallest pods are 6,400 square feet and might require zoning variances.

Our abstract, fractal-based design easily adjusted to fit an actual parcel of land but retained all of its desirable characteristics in the process. Each condominium site has simultaneous, direct access to both road and water

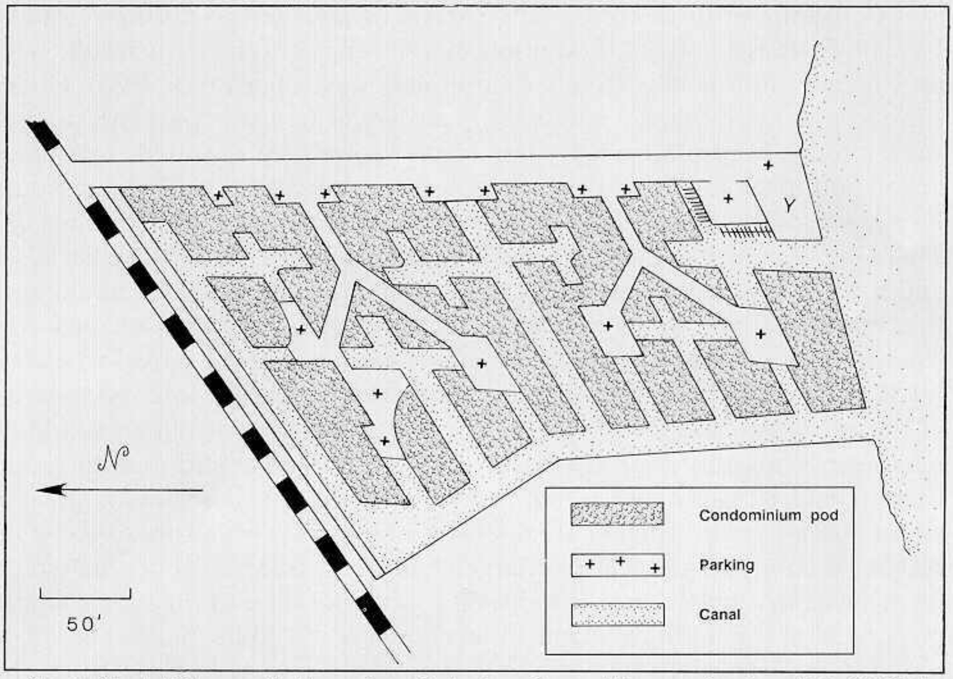

FIG. 4—Fractal ideas applied to an irregularly shaped parcel for a marina along Lake St. Clair. A highway, denoted as heavy dashed line, is to the north; a public marina, adjacent to a yacht club, denoted Y, is to the south.

networks. Each site may be reached by either network in a maximum of four turns. No bridges are required to achieve this interdigitated arrangement. The conditions for a desirable marina plan have been met in a manner that compresses the land-water boundary into a compact space with numerous features attractive to potential residents.

Geographical Routing through a National Forest

Our second example applies a self-similarity transformation to a hypothetical national forest. As sources of fire, disease, and waste, people represent a real threat to the forest; yet they need access to it as a locus of raw materials for industry and as a recreational and educational resource for the public at large. We here offer a fractal scheme addressing the latter need: locating a road network in a heavily treed region, such as a national forest, that would permit large numbers of visitors to experience the interdigitation of ecological niches and the variety that the forest contains and, at the same time, would reduce the potential for damage from visitor contact with the forest environment.

The first step in determining roadway position is to separate large regions, based on their percentage of tree cover, on a small-scale map of the forest. Forested areas, for example, might be defined to have more than 20 percent tree cover; meadow areas would be those with less than or equal to 20 percent

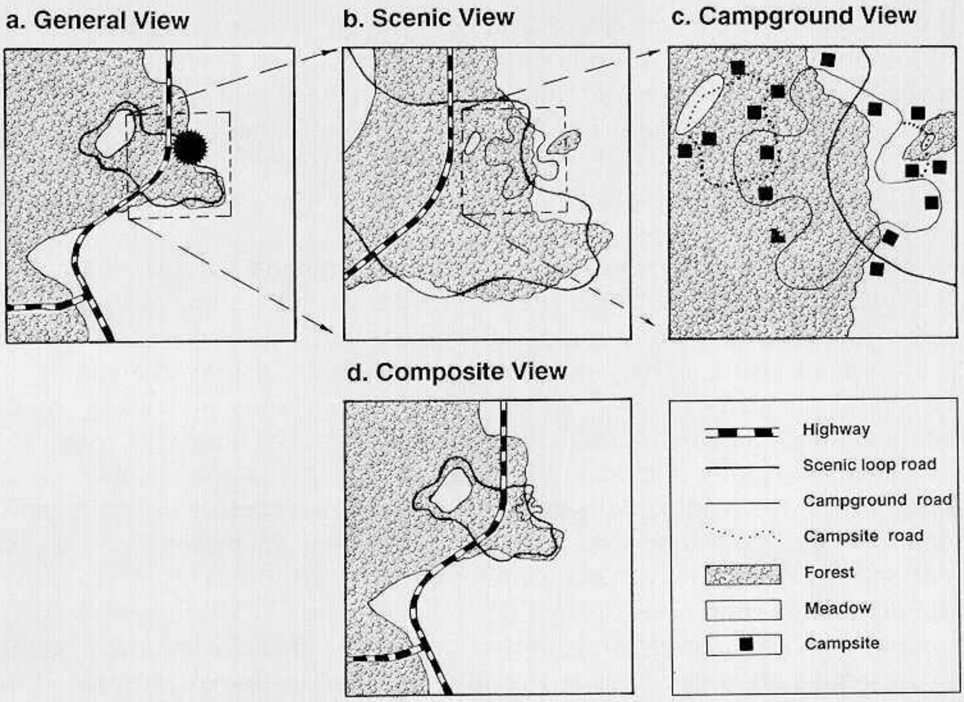

FIG. 5—Self-similarity used to place forest routes.

tree cover. As the map view takes successively larger scale, smaller areas of meadow that could not be seen at smaller scales might become visible (Fig. 5). The abstract symbol along the forest edge (Fig. 5a) represents a relatively complex forest-and-meadow grouping that comes more sharply into view through enlargement (Figs. 5b and c), much as an abstract symbol on a state map signifies an underlying, complex grid of a city revealed in a detail map. Thus an assessment of how much forest covers a specific parcel of land will vary, depending on the scale of the map that is the base for measurement.

By separating forest from meadow at different map scales, road networks may be placed in accordance with the principle of least effort, which we cited previously. At a small scale, a highway might skirt the edge of the forest or pass briefly through its leading edge; the balance of forest to meadow on one side of the road might be 80 to 20 percent, on the other side 20 to 80 percent. Broad, homogeneous forest and meadow regions are not penetrated by this route. To expand visitor contact with forest variety, a scenic loop might be added that preserves the balance, at the same ratio, between forest and meadow. Maintenance of this ratio ensures that the road passes through relatively mixed regions of the forest and thereby separates the broad, homogeneous forest and meadow regions from visitor contact.

With scale change, new forest and meadow regions come into view; new roads may be added that preserve the 80 percent forest, 20 percent meadow balance. Self-similarity of ratio as a map scale changes suggests route position and ensures separation of visitors from vast, single-species expanses of forest or meadow. Iteration of this procedure may be carried out to a highly localized level, involving, for example, the separation of open and treed sites along access roads to sets of campsites. With the iteration sequence extended as far as desired, stacking all the members of the sequence on the appropriate "windows" yields a composite view that resolves the abstract symbol used on the initial global view.

Because this route passes through a mix of forest and meadow, it and the boundary-dwelling vegetation and open spaces surrounding it buffer the larger, more remote stands from hazards that might be introduced by contact with human visitors. This compressed route exposes visitors to a variety of ecological niches in heterogeneous regions of the forest as it fills a small, compact area. Zones of interdigitated ecological types are filled with roadway, while large regions of possibly homogeneous plant communities remain distant from the road network. When this composite route is linked to park entrances and exits, a network of high scenic value emerges. Those regions not adjacent to the network are protected from visitors. The most clearly defined remote homogeneous regions might be classed as wilderness areas with the highest degree of protection. Again, the fractal dimension might provide a single index to assign relative levels of protected as opposed to scenic value to various regions of the forest.

Concern for positioning networks to take advantage of the buffering effect of heterogeneous mixtures of vegetation types need not be confined to existent national forest patterns. City dwellers have witnessed the devastation of handsome elm-shaded streets by Dutch elm disease; to them the wisdom of mixing tree types and of placing them relative to the entire network is evident. On a global scale, much current discussion of vast reforestation as an approach to soaking up the extra carbon dioxide produced by various human activities centers on increasing the total proportion of tree cover on the earth. We suggest that there may be merit to considering, at the outset, the underlying geometric properties of the proposed undertaking. As in the case of the national forests or of a city, the placement of a global network relative to stands of vegetation might contribute significantly to or detract seriously from the success of a vast reforestation effort as a method to confront a possible greenhouse effect.

Closing Comments

Geometric theory has persisted as the backbone of numerous and diverse human achievements since ancient times. When geometric theory rests only on nonmetric properties such as reflection, translation, or some other transformation from one geometric space to another, it is called synthetic; when the theory rests on some metric that permits the measurement of distance, it is analytic. Synthetic geometry thus can be seen as fundamental and, in fact, premetric. In the system of boundary exchanges proposed here, only the synthetic notion of separation in geographic space is crucial. To this synthetic concept, we have applied another synthetic idea, self-similarity, and its associated, superimposed analytical structure to separate, first, boat and automobile flows through a marina and, second, vulnerable, homogeneous stands of vegetation from visitor flows in a national forest. The work demonstrates, we contend, that when synthetic geometric concepts are carefully crafted to fit basic spatial elements of a geographical problem, the result can be a general theoretical tool with wide-ranging, even unexpected, power.

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