A CARTOGRAPHIC PERSPECTIVE ON THE SECURITY OF AN URBAN WATER SUPPLY NETWORK

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Water, water, every where;
Nor any drop to drink—
COLERIDGE, *The Rime of the Ancient Mariner*

Introduction

Mapping the positions of pockets of illness is a geographical procedure that may be employed by scientists interested in deriving insight into the spatial pattern of disease distribution; when such maps are accompanied by reports and other written documentation, factors that might be thought to have direct bearing on communicating a disease can be isolated. Perhaps the best-known classical instance of such a "medical map" (as Robinson calls it [1, p. 170]) involves the dot map used by the mid-nineteenth-century physician John Snow to chart the topography of a cholera outbreak surrounding the Broad Street public water pump in London [2]. Snow charted each death from cholera (by address) as a dot on a map and used this cartographic evidence concerning the distribution pattern of dots in relation to the Broad Street water pump, to produce visual and empirical support for the idea that water systems in general, and public wells in particular, were one source significant in communicating cholera [1–3]. Once municipal authorities understood the possible implications of Snow's observations and acted to remove the handle of the suspect public pump, a substantial reduction in numbers

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of new cases of cholera resulted in the regions surrounding the well [1–3]. The dot map had served as an aid useful in understanding the nature of the communication of cholera.

Contemporary interest in the geography of disease is expressed in the works of Pyle, Haggett, and others. Pyle [4] ties cholera diffusion to central place structures and the degree of urbanization in the United States, while Cliff et al. [5] cast the same sort of study into a multiple-regression framework in order to account for variations in distance effects from point of origin.

Water quality was a focus of concern to the Detroit Police Department. At the request of Coleman A. Young, mayor of Detroit, a team of experts from the University of Michigan (and associated outside consultants) was commissioned to determine how to secure an entire urban water system from either deliberate or accidental abuse that might cause health problems in the population served by that network. Mayor Young's interest in what might ordinarily be expected to be a politically unpopular topic (many citizens might not express their appreciation for such a low-visibility project in the voting booth) was piqued when a disgruntled city employee went on a shooting spree in a (at the time) minimally guarded water plant. The minor damage sustained by the chlorine tanks was viewed as a threat to public welfare, severe enough to move the mayor to action.

The Michigan team, comprising public health engineers and scientists, urban planners, defense experts, political scientists specializing in the history of terrorism, and mathematical/urban geographers (the authors of this paper), created a comprehensive program to secure the Detroit Water Supply Network from various biological and chemical threats as well as from physical attacks on components of the distribution network [6, 7]. What follows is a description of a geographic procedure

1. for determining the positions of zones of high risk; and
2. for assessing the size of the affected population, by land-use type, within the primary distribution network of the Detroit Water Supply Network.

In some regards the geographic strategy parallels that employed by Snow. Like Snow's work, geographic dot maps were superimposed on those parts of the water supply network deemed critical to successful water distribution to the public. Unlike Snow's earlier study, this one rested on

1. the use of mathematical technique drawn from algebraic graph theory [8] to select a set of water pipe junctions critical, in some fashion, to the distribution of water throughout the entire metropolitan region; and

2. the use of a planning process which leaps over the inductive process to forecast regions that *might* become infected, or affected, rather than analyzing data garnered after disaster has already struck.

Thus, dot maps were used to relate a topic of continuing biological significance, that of the effective distribution of a safe water supply to a living (urban) system, to the larger community as well as to municipal authorities.

Graph Theoretical Material Tailored to the Task at Hand

Any system may be represented abstractly as a network formed from

1. vertices, or points of concentration of a material or activity distinguished from the environment surrounding the system; and
2. edges, or channels of focused materials or activities linking the vertices.

Such an abstract representation is often called a graph, and any subset of vertices and edges that maintains the linkage pattern present in the original graph is called, quite naturally, a subgraph [9]. What is of fundamental importance in any application of mathematical structure to a real-world setting is to tailor the choice of tools to fit carefully the problem at hand [10].

In most systems, some vertices are more critical to the effective functioning of the entire system than are other vertices. For example the four panels, labeled *a–d* in figure 1, represent the graph of the expressway system within the city of Detroit (the panel margin is topologically equivalent to the Detroit city boundary). Circles represent interchanges along the expressways; circles labeled $(v_1, \ldots, v_8)$ represent the set of all intersection points of limited access roadways (with each other), within the city of Detroit. If flow across the network is stopped as a result of rendering useless the entire set of eight vertices $(v_1, \ldots, v_8)$, 10 distinct network segments (labeled [1, . . . , 10] in fig. 1*a*) form downstream from the highlighted subgraph (fig. 1*a*). If flow is shut down at all of $(v_1, \ldots, v_8)$ except at v_6, the result is the same; 10 expressway segments are choked off downstream from this subgraph of the network. Generally, we refer to vertices of a graph that are critical to the functioning of a graph in the fashion of $(v_1, v_2, v_3, v_4, v_5, v_7, v_8)$ as "choke points"; the vertex v_6 is *not* a choke point because no segment of the expressway is detached from the highlighted subgraph as a result of stopping flow at v_6 (vertices such as v_6, that are totally "interior" to the highlighted subgraph, do not function as choke points). (Thanks to our colleague, John F. Kolars, for suggesting the picturesque terminology of "choke point";

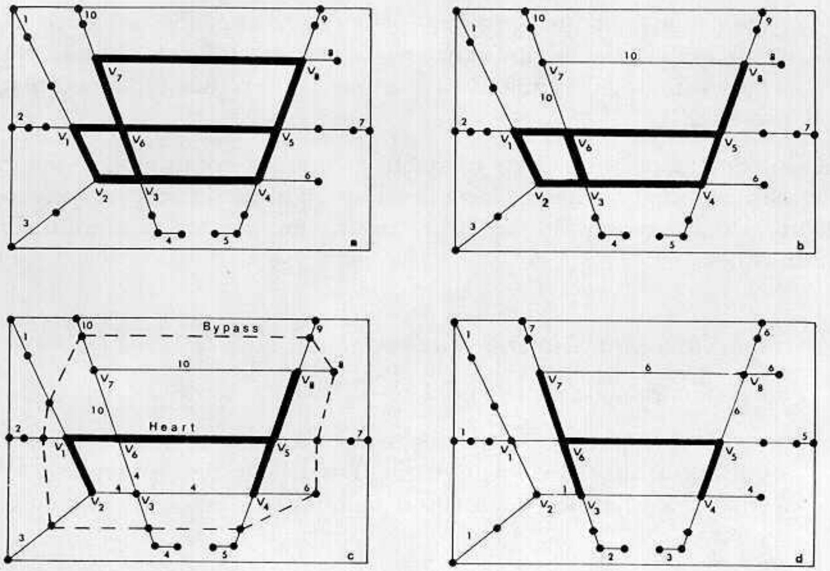

FIG. 1.—The graph of the expressway system within the city of Detroit.

mathematical terminology, used by W. T. Tutte, is "vertex of attachment" [a notion more general than that of "cutpoint"] [9], an apparently heretofore untapped theoretical source for application.)

Finding choke points in a simple system, such as the one described above, is straightforward. Often, however, complicated systems are arranged in some sort of hierarchical structure in which some of the edges, as well as some of the vertices, represent elements of substantial importance to the effective functioning of the entire system. When subsets of these "important" edges and the vertices with which they are incident are singled out as subgraphs, it is possible to select, from the finite number of choices, the subgraph of "important" edges that contains the fewest choke points. We call such a subgraph the "heart" of the graph, and we refer to network segments that are cut off from the heart by removing heart choke points as fragments.

In the example above, based on the Detroit expressway graph, the highlighted subgraph of figure 1*a* is not the "heart" of the network; because this subgraph has redundancy of connection in it, one suspects that a subgraph with fewer edges might also break the network into 10 network segments. When the subgraph associated with the smaller set of vertices (v_1, v_2, v_3, v_4, v_5, v_6, v_8) is highlighted (fig. 1*b*), the entire system still falls into 10 network segments; in this case, each distinguished vertex serves as a choke point because each has a network segment detached from it. Again, this subgraph still has a circuit in it, and again there is linkage redundancy. Indeed, using even fewer choke points (v_1, v_2, v_4, v_5, v_6, v_8) forces the system to break into 10 network segments deprived of access to flow from the network subgraph highlighted in figure 1*c*. The highlighted subgraph in figure 1*d* is based on even fewer choke points; however, it forces the system to fall into only seven, rather than ten, network segments. Thus, the highlighted subgraph in figure 1*c* is the "heart" of the Detroit expressway graph, and the 10 network segments that may be detached from it are the corresponding "fragments."

The mathematical conception of the notion of "heart" is in keeping with "max-min" ideas for optimizing flows across a network [11]: the heart is a subgraph that forces maximal fragmentation of the entire graph using a minimum number of choke points to do so. It is not too difficult to find hearts, using an iterative algorithm based on coloring (rank-ordering) water pipes according to their topological remoteness from water plant pumping stations [6]. In this fashion, hearts may be seen to exist, although it is not the object of this essay to provide the detail required to do so [12]. Issues involving the mathematical determination of when a heart is unique and of what sorts of optimal algorithms for finding choke points exist are apparently open questions [11, 13, 14]. The uniqueness issue is more critical to the development of theory than is the computational complexity issue, although the latter is, perhaps, of greater concern in practical application.

The Case of the Detroit Water Supply Network: Overview

The Detroit Water Supply Network comprises water pipes of varying diameter, five water plants and associated tunnels and pumping mechanisms, a variety of other buildings and mechanical equipment, and a wide range of personnel to support the functioning of the network. Viewed broadly, the supply pipes link households and industries to the pumping mechanisms at the water plants, which in turn draw water from water intakes at various locations in Lake Huron and in the Detroit River. The service area includes businesses and residences scattered across a five-county area of southeastern Michigan [15], embracing a population of over 4,000,000.

The primary network of supply pipes under consideration is formed from:

1. trunk line piping of 4.5 or more feet in diameter that leads directly from the water pumping stations; and
2. supply piping of "small" diameter that branches out from the trunk lines to serve individual streets in residential neighborhoods.

(We do not include, as part of the primary water supply network under consideration, the leads from individual homes or businesses to water supply pipes in the streets. Nor do we include other pipes of relatively narrow diameter that function as "twigs" of piping on the primary water supply "tree.")

The primary Water Supply Network of the city of Detroit includes many miles of water pipes hooked together at 727 distinct pipe junctions below the streets of metropolitan Detroit [15]. When these junctions and the piping linking them are viewed as a graph [8, 9], it becomes possible to identify the "heart," "choke points," and "fragments" detached downstream from the heart of the primary water supply network.

Water pipes of width sufficient to carry a volume of water to flush the system of dangerous levels of contaminants constitute a trunk line of piping of particular "importance" to the primary Water Supply Network. Population located close to this trunk line has a water supply that is more secure in time of disaster than does that of the population topologically remote from this trunk. When redundant edges were removed from this subset of "important" edges, and the remaining dendritic subgraphs viewed as candidates for the heart of the Detroit water graph, it was not difficult to determine which of the candidates had the fewest choke points and to use that one as the heart of the Detroit Water Supply Network. Indeed, the set of "important" edges, in the Detroit case, was incident with about 100 of the 727 vertices representing pipe junctions. The network heart was based on around 30 choke points, and only about a dozen distinct fragments, downstream from the heart, resulted from rendering useless the entire set of 30 choke points.

Measurement of Transmission Reliability across the Network

A network fragment that is attached to the network heart at a number of choke points is more secure from detachment to the water supply than is a fragment hooked to the heart at only one choke point. A simple index of reliability can be used to characterize the redundancy of connection from each fragment to the heart. The index is defined as the percent connection of the fragment to the heart when one choke point is removed. The higher the value of the "Index of Reliability," the more secure is the fragment from isolation from the heart. Thus, if N is the number of choke points a given fragment has with the heart, then R, the index of reliability, has a value of $R = [(N - 1)/N] \times 100$.

Using this index, we can see clearly that a fragment attached to the heart at but one choke point has reliability of transmitting water from the heart to the fragment of $R = [(1 - 1)/1] \times 100 = 0$ percent (remove the choke point and nothing passes into the fragment from the heart). A fragment attached to the heart at three choke points, however, has reliability of transmitting water from the heart to the fragment of $R = [(3 - 1)/3] \times 100 = 66.67$ percent (remove the choke point and two linkages of the three from the heart to the fragment remain).

Improvement in Transmission Reliability: Heart Bypasses

In the Detroit system more than one of the water supply graph fragments had a 0 percent transmission reliability with the heart; clearly, choke points associated with such fragments were particularly vulnerable. A passive means to improve transmission reliability across the entire network, is to construct a bypass beyond the heart that would link fragments with 0 percent transmission reliability, at the periphery. Bypass edges pass through a fragment, across a location where there is a pipe junction (vertex) already present, into another fragment. (In fig. 1*c*, the bypass links each of the 10 numbered expressway fragments in a circuit near the city limits.) This procedure serves to increase the redundancy in connection by providing extra choke points that need to be disrupted in order to isolate a fragment (fig. 1*c*).

The percentage increase in the improvement of transmission reliability that comes from introducing bypass edges may be evaluated using the same sort of index as was employed in calculating, directly, the initial reliability of the transmission. Thus, if N' is the number of added bypass links, and if N is the number of choke points a given fragment has with the heart, then I, the index of improvement in reliability, has a value of $I = [(N + N' - 1)/N] \times 100$. Because I depends on N, as well as on N', the percentage improvement coming from added bypass links is a function of the number of bypass links to be installed as well as of the number of choke points already present in the initial network configuration.

Using this index, we see that a fragment attached to the heart at but one choke point has an improvement in reliability of transmitting water from the heart to the fragment of $I = [(1 + 2 - 1)/1] \times 100 = 200$ percent when two bypass links are added. A fragment attached to the heart at three choke points, however, has an improvement in reliability of transmitting water from the heart to the fragment of only $I = [(3 + 2 - 1)/3] \times 100 = 133.33$ percent when two bypass links are added. Again, this index fits with common sense: bypass links are more significant to fragments with fewer vertices hooking into the heart. In the case of the actual Detroit network, when the figures for percentage improvement in reliability (by fragment) were ranked, they served to investigators Feldt and Rycus as a specific variable fundamental to the development of an open-ended "Richter"-type logarithmic scale to measure the extent of disaster that might befall a water network [6].

Population Served by the Water Supply Network

When land-use criteria such as the size of the residential or employed population served by a fragment of the water supply network are also included, decisions regarding investments that add redundancy to the system (for security purposes) may be ordered according to water needs of the underlying population. Population dot maps (figs. 2 and 3), constructed from *1980 Census* data and mapped by South East Michigan Council of Government (*SEMCOG*) analysis zones, show the geographic positions of the residential and of the industrial populations (accurate to within one *SEMCOG* analysis zone), served by the Detroit Water Supply Network [6, 7]. The theoretical rationale for using dot maps rests on the fact that geographic regions within a set of discrete fragments detached from water networks are unlikely to match corresponding zones of a continuous population distribution in equal proportion; dot maps allocate proportions, removing problems of incommensurable unit areas.

When the fragments downstream from the heart of the Detroit Water Supply network are mapped to scale and superimposed on the population dot maps, a quick estimate (by land-use type) of the potential disaster to the populations of each fragment, arising from water deprivation or contamination, is possible. We show, as a parallel, how this strategy might be applied to the Detroit area expressway system. When the graph of the expressway system heart (fig. 1*c*) is mapped to scale (figs. 2, 3—highlighted lines) and when the territory served by expressways is broken into areal fragments corresponding to the linear network fragments (linear fragment number 2 in fig. 1*c* corresponds to areal fragment number 2 in figs. 2 and 3), then the number of individuals at home affected by stopping the flow at choke point v_1 (for example) can be quickly estimated as $1{,}150 \times 436 = 501{,}400$ (fig. 2) and the number coping with this difficulty at places of employment would be approximately $1{,}150 \times 188 = 216{,}200$ (fig. 3).

Wall-sized versions of these dot maps (figs. 2 and 3) are in use by the Detroit Water and Sewerage Department, along with a transparent overlay displaying the actual network heart (not shown here) of the Detroit Water Supply Network and corresponding fragments detached from the heart. These dot maps and the graph of the heart and fragments provide a quick gauge to approximate affected populations in times of disaster; they are easy to use, despite the reliance of the design of the network heart on sophisticated mathematical tools designed specifically for the occasion. The value of the approximation is not easily perturbed by changes in data, so that the process has an element of dynamism; only radical, sudden population shifts would diminish the value of the dot map (normally, dots can be added and erased easily without needing to reconstruct the map).

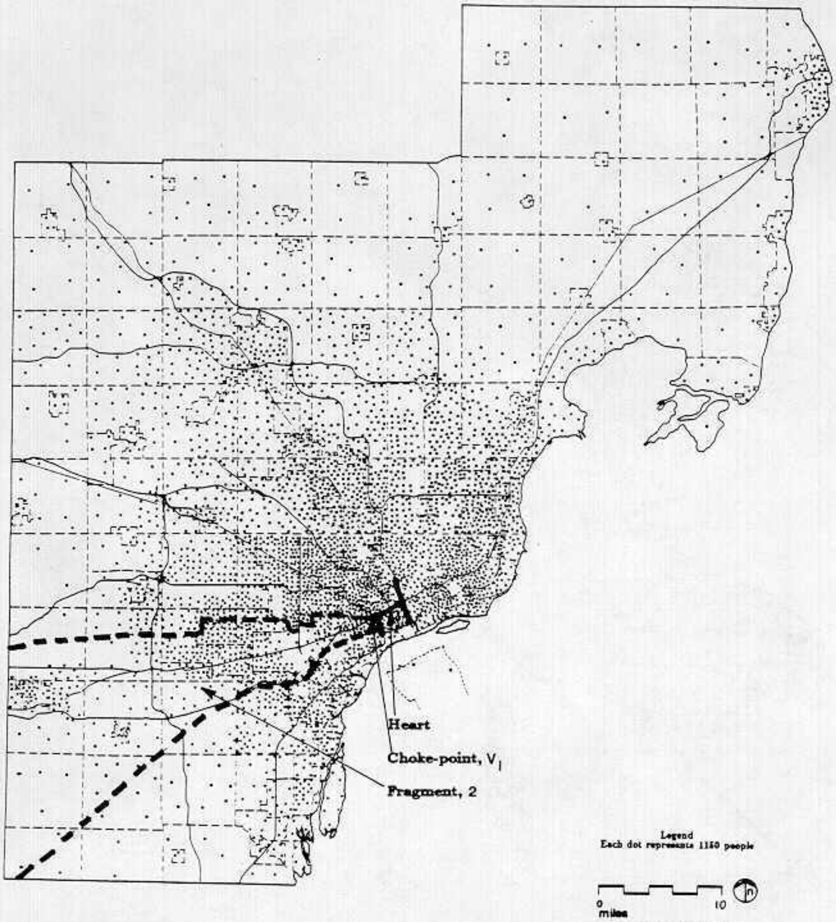

FIG. 2.—Residential dot map constructed from *1980 Census* data and mapped by South East Michigan Council of Government.

Public Policy Recommendations

A. C. Davanzo, chief engineer of the Detroit Water and Sewerage Department, had suggested a set of positions for proposed new water pipes based purely on engineering considerations internal to the mechanics of the water supply network itself [15]. He found that our rank-ordered list of bypass segments supported his recommendations. Thus academic and municipal recommendations came together to signal which bypass linkages should receive top priority for actual construction. This listing, based on transmission improvement through adding choke points, served as an agenda for the Detroit City Council in establishing

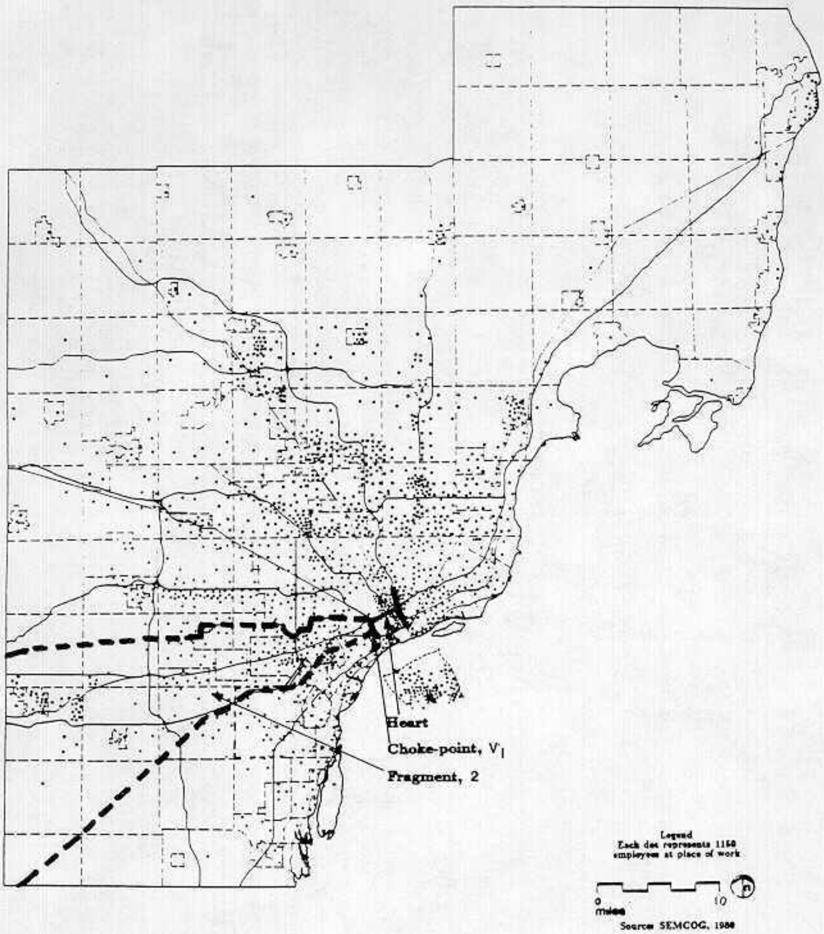

FIG. 3.—Employment dot map constructed from *1980 Census* data and mapped by South East Michigan Council of Government.

budgetary priorities for choosing which new water pipes should be built as bypass links. Thus, in the tradition of Dr. Snow, the creators of the dot maps (and of associated reports) were able to work effectively with municipal authorities to produce constructive changes in the distribution of water.

Conclusion

The theoretical strategy employed here might be applied to a variety of networks [12]. In general, this strategy consists of

1. identifying the heart of the network;
2. identifying fragments formed downstream from that heart;
3. establishing reliability of network transmission;
4. establishing topological means to improve the reliability of network transmission (bypasses);
5. measuring the affected population, by land-use type, within each fragment (using dot maps);
6. using steps 4 and 5 together, to rank-order fragments after they have been weighted by population, in order to select appropriate bypass links; and
7. supplying the results of this procedure to city authorities in order to assist them in establishing priorities within limited budgets.

The evaluation of potential improvement for the network proceeded along two parallel tracks: first, along the abstract topological viewpoint (in determining a network heart and bypass links); second, along the empirical demographically weighted viewpoint (dot maps of figs. 2 and 3). Combined, these tracks permitted the ranking of alternatives for quantitative improvement in network transmission reliability. For then, passive network-defense measures might focus limited resources for network security on choke point security, especially on those choke points from which fragments serving large populations might become isolated.

Lists associated with such rank-orderings may then be appraised, in a more qualitative context, by municipal authorities operating on tight budgets. In this light, the presentation of these ideas to the Detroit Water and Sewerage Department aided municipal authorities in gaining the approval of the Detroit City Council to construct the recommended heart bypass in the Detroit Water Supply Network (personal communication from J. Snyder, 1987, from Mayor Young). Indeed, tactical ideas involved in understanding the control of real-world systems have endured not only from cholera maps to water supply maps, but throughout the history of Western civilization as well: from the Graeco-Roman phalanx to the admonition of the British geographer, Halford J. Mackinder [16], that

> Who rules East Europe commands the Heartland;
> Who rules the Heartland commands the World Island;
> Who rules the World Island commands the World.

REFERENCES

1. Robinson, A. H. *Early Thematic Mapping in the History of Cartography.* Chicago: Univ. Chicago Press, 1982.
2. Snow, J. *On the Mode of Communication of Cholera,* 2d ed. 1855. In *Snow on Cholera: Being a Reprint of Two Papers by John Snow, M.D.,* with a Biographical

Memoir by B. W. Richardson, M.D., and an Introduction by W. H. Frost, M.D. New York: The Commonwealth Fund; London: Oxford Univ. Press, 1936.

3. Shannon, G. W., and Dever, G. E. A. *Health Care Delivery: Spatial Perspectives.* New York: McGraw-Hill, 1974.
4. Pyle, G. F. Diffusion of cholera in the United States. *Geogr. Anal.* 1:59–75, 1969.
5. Cliff, A. D.; Haggett, P.; Ord, J. K.; and Versey, G. R. *Spatial Diffusion: An Historical Geography of Epidemics in an Island Community.* Cambridge: Cambridge Univ. Press, 1981.
6. Snyder, J. C.; Arlinghaus, S.; Brabec, M.; et al. *Detroit Water and Sewerage Department Pollution and Detection Abatement: Preliminary Report,* with appendices. Ann Arbor: Univ. Michigan, 1985.
7. Snyder, J. C.; Arlinghaus, S.; Brabec, M.; et al. *Security of the Water System, Final Report Tasks 1, 2, 3.* Ann Arbor: Univ. Michigan, 1986.
8. Tutte, W. T. *Connectivity in Graphs.* Toronto: Univ. Toronto Press, 1966.
9. Harary, F. *Graph Theory.* Reading, Mass.: Addison-Wesley, 1969.
10. Mac Lane, S. Proof, truth, and confusion: the 1982 Ryerson lecture. Chicago: Univ. Chicago Press, 1982.
11. Lawler, E. *Combinatorial Optimization: Networks and Matroids.* New York: Holt, Rinehart & Winston, 1976.
12. Arlinghaus, S. L. *Essays on Mathematical Geography—II.* Institute of Mathematical Geography, Monogr. Ser. no. 5. Ann Arbor: Michigan Document Services, 1987.
13. Harary, F., and Palmer, E. *Graphical Enumeration.* New York: Academic Press, 1973.
14. Saito, N., and Nishizeki, T. *Graph Theory and Algorithms: Sendai, Japan, October 1980 Proceedings.* Berlin: Springer-Verlag, 1980.
15. Davanzo, A. C. Report of water capital needs. Detroit Water and Sewerage Department Capital Committee, March 12, 1985.
16. Mackinder, H. J. *Democratic Ideals and Reality.* New York: Holt, 1942.