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Pneumatic Networks as Geographs

Sandra L. Arlinghaus[1], William C. Arlinghaus[2]

[1]School of Natural Resources and Environment, The University of Michigan, Ann Arbor, MI, [2]Department of Mathematics and Computer Science, Lawrence Technological University, Southfield, MI

The Jordan–Sylvester graph center theorem provides an established and well-known method for calculating the center of a graph. We visualize this theorem in the context of a historical network, the Berlin Rohrpost of 1901, and interpret that visualization using contemporary visualization tools (Google Earth). Technology serves to bridge a visualization gap over time and, through this example, suggests direction for updating other historical studies. It also serves to guide further the direction of methodological research in regard to space filling as Fractalyse software offers planners ideas of how densely packed, with network nodes, an urban environment might become over time.

Introduction

During the 1990s, it was our pleasure to work with Frank Harary in developing applications of graph theory in geography. Some of the ideas drew from earlier geographical materials, whereas other ideas related to mathematical ideas involving the symmetry in graphs. We joined our approaches with Harary's enduring interest in applications of graph theory in a wide range of disciplines: from anthropology to zoology, his applications and interest ran through the alphabet (Harary 1969; Harary and Robinson 1975; Hage and Harary 1983, 1991, 1996). In 2002, our *Graph Theory and Geography* was published; it was John Wiley and Sons' first e-book (Arlinghaus, Arlinghaus, and Harary 2002). This work represents more than a decade of collaboration among the three of us.

Harary passed away in 2004. We have to imagine that his excitement over applications of current technology to graph theory, network science, and other related topics would have been unbounded in subsequent years. His collaboration with John Hayes throughout the last part of his life, coupled with his computer science interests at New Mexico State University, testifies to a deep interest in the changes taking place (Harary, Hayes, and Wu 1988; Harary and Hayes 1989, 1993, 1996).

Correspondence: Sandra L. Arlinghaus, Mathematical Geography, School of Natural Resources and Environment, The University of Michigan, Ann Arbor, MI 48109
e-mail: sarhaus@umich.edu

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In this work, we begin by reviewing how visualization of animated graphs was used in 2002 to capture the principles of the Jordan–Sylvester graph center theorem (Arlinghaus, Arlinghaus, and Harary 2002). We disassemble the animation and illustrate how it is used to convey meaning. From there, we move forward in time to 2009 to a view of graphs in the "real" three-dimensional (3-D) world of Google Earth (Google Inc., Amphitheatre Parkway Mountain View, CA) (Arlinghaus 2009). Using 3-D views permits the simultaneous visualization of both the graph and the surrounding space in which it is embedded. These "geographs" come alive to pinpoint where the abstract graphical structure fits into the real world in relation to established benchmarks. Finally, we move back to the abstract world and offer one way to bring historical maps into the contemporary scene by using the ranking of spatial materials portrayed in those maps to suggest the extent of space filling, in the real world, by those elements. We hope that this presentation offers convincing evidence regarding the integration of historical networks with contemporary virtual representations of real landscapes and that the associated illustrations are helpful in adding context to the representation of the network in geographic space.

Pneumatic networks: a century of progress

In the world of 2011, we experience pneumatic networks in various ways, such as making bank transactions at a drive-up window. A few may see them in other contexts, for conveying money or small packets (e.g., small parcels) in department stores, libraries, and so forth. What we see in today's world is, however, a mere remnant of times past. In the late 19th and early- to mid-20th centuries, pneumatic technology was in its heyday. There were, in addition to the sorts of things we still see, pneumatic clocks, pneumatic subways, and pneumatic postal networks, to name a few.

Pneumatic technology must have been as avant-garde during that time as the Internet is today. There was the mystery of inserting something into the network and then imagining the harnessing of the wind inside unseen tubing as a means to transmit materials from here to there: truly a stunning capability! Our virtual networks of today have many of the same hallmarks: the user sees the endpoints but does not see much of the necessarily associated network. Graph-theoretic structure fits both technologies. A graph, formed from a set of nodes and edges, relinquishing all else, focuses only on the network. It is ideally suited to capture this sort of abstract structure, as a geograph; therefore, theorems from graph theory are important in the analysis of geographs.

The Jordan–Sylvester graph center theorem

Finding the center of a geographic distribution is an enduring problem. The "center" of a graph can be interpreted in a variety of ways. One standard to find the center of a given graph is to employ the Jordan–Sylvester center theorem. The work of Camille Jordan traditionally finds application in breaking curves on maps and nodes where the curve crosses itself so that the inside of the curve as the software sees it fits the intuitive notion of the inside of the curve as geometry sees it (Jordan curve theorem [Jordan 1869]). Again, with graphs, Jordan's work is critical.

Theorem. *Jordan–Sylvester Center Theorem. Every graph-theoretical tree has a center composed of either one node or two adjacent nodes.*

Often, the proof is expressed formally in a manner such as the following.

Proof. The result is obvious for the graph-theoretical trees K_1 and K_2 (complete graphs on one and two nodes). Any other tree T has the same central nodes as the tree T' obtained by removing all end nodes (at the tips of branches) of T. The maximum of the distances from a given node u of T to any other node v of T will occur only when v is an end node. Thus, the eccentricity of each node in T' is exactly one less than the eccentricity of the same node in T. Hence, the nodes of T that possess minimum eccentricity in T are the same nodes having minimum eccentricity in T'; that is, T and T' have the same center. If the process of removing end nodes is repeated, successive trees having the same center as T are obtained. Because T is finite, a tree that is either K_1 or K_2 is eventually reached. In either case, all nodes of this ultimate tree constitute the center of T that thus consists of just a single node or two adjacent nodes: QED.

One problem with proof expressed in this manner is that even though the underlying concepts are easy for nonexperts to follow, their expression is encumbered with structure that makes those concepts appear difficult to follow. We believe, when possible, revealing mathematical structure is important for the nonexperts who may have a great deal to offer if only they could get a grasp of the underlying conceptual framework. In our e-book (Arlinghaus, Arlinghaus, and Harary 2002), we construct an animated map, as a "proof without words" (Nelsen 1993, among others), to illustrate the process of deconstruction stated more formally in the preceding proof. We apply the process to one particular real-world historical graph, which is that of the pneumatic mail network, the Berlin Rohrpost of 1901, to visualize the proof (Fig. 1). The map in that figure is adapted from an original earlier source ("Rohrpostnetz von Berlin" 1901). Fig. 2a–d show several of the seven stages of peeling off edges to reduce this graph to its center. Fig. 2e offers readers with smartphones that can read quick response (QR) code an opportunity to see the animation directly from the printed text. In either visualization, the center of the graph, as a structural form independent of function, is not the same as the functional (highest order) center of the pneumatic network (represented as the geometrically largest node).

Fig. 2 furnishes the selected stages in the reduction of the Rohrpost graph to its center: in Fig. 2a, remove end nodes, a, and adjacent edges to produce Fig. 2b;

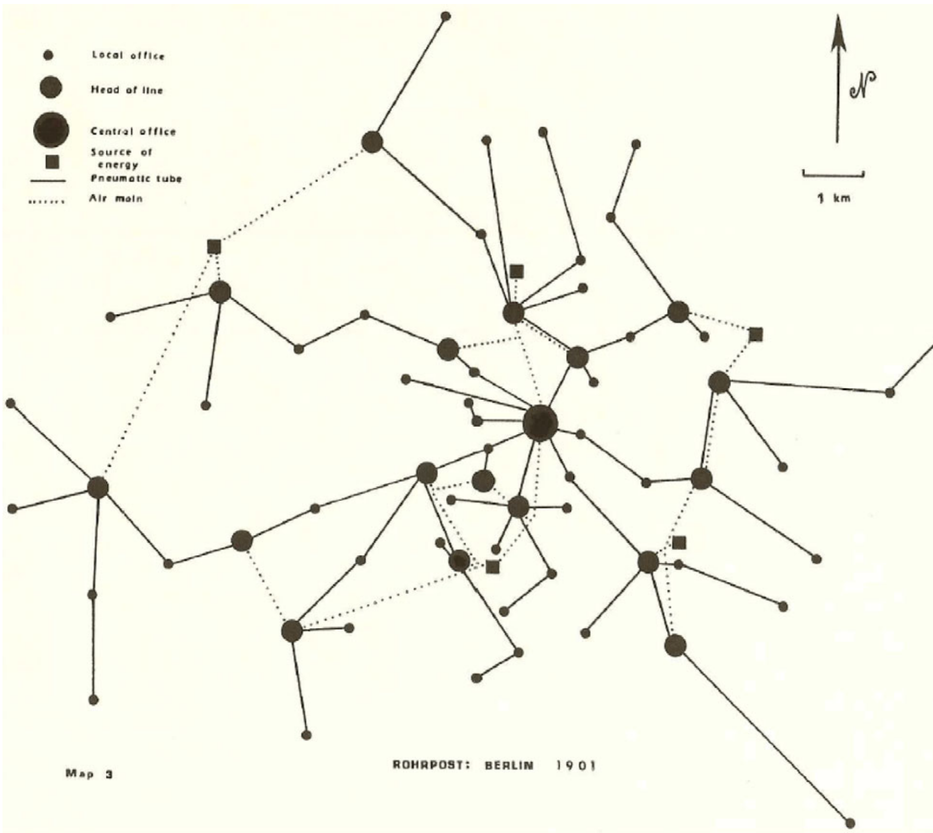
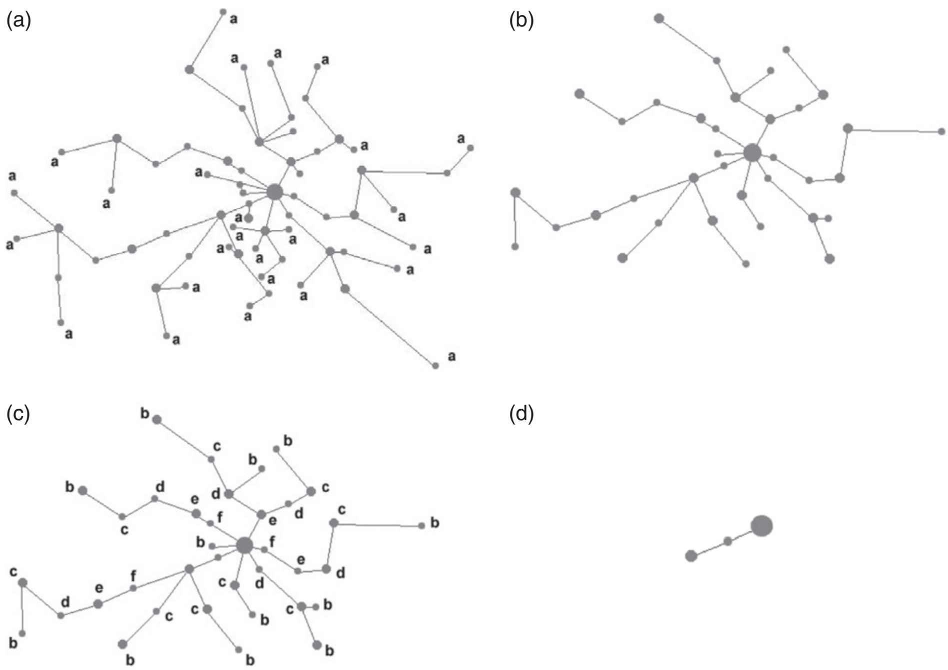


Figure 1. Berlin pneumatic postal network: schematic representation based on "Rohrpostnetz von Berlin" (1901).

Fig. 2b results from removing end nodes a and adjacent edges; in Fig. 2c, the process continues in succession, removing end nodes b, c, d, e, and f, resulting in Fig. 2d; in Fig. 2d the final two end nodes and edges are peeled off to isolate the center. Note that the center is, in this case, not the node offering the largest function to the network (the geometrically largest node offers the largest amount of service to the network). Fig. 2e portrays the sequence of graphs in animation when viewed on a smartphone capable of reading QR codes, a merging of conventional and contemporary publication methods.

Really all one needs to know, then, to visualize how to find the center of a graph, using the reduction scheme offered in the Jordan–Sylvester theorem, is the meaning of the term end node. This concept clarifies why one or two nodes might be in the center and why more than two can never exist (peel off an end node!). Simplicity is often the hallmark of elegance and elegance the hallmark of clarity. Current visualization capability can offer added clarity of the logic behind useful mathematical tools.



(e)

Figure 2. Selected networks. Use a smartphone to view the graphs in animation.

Pneumatic networks in Google Earth

Creating a graph to analyze a real-world structure is generally a straightforward process. Graph nodes represent small, tightly bounded geographical entities; a bus stop, a subway station, a sewer lid, and so forth are natural choices as graph nodes. Less obvious choices might involve a human mouth or an idea. Graph edges might be anything in the world that can be channeled: the movement of a train (from one node to another) or the movement of communication (from mouth to ear). This process of creating a geograph is clear-cut at least partially because the less well-defined (a real-world phenomenon) is being mapped to the clearly defined (a graph). That mapping, however, is not symmetric because the ends of the mapping are different.

Visualizing a graph in the real world often is not straightforward and even more so when the elapsing of a number of years causes change in the real world but not in the associated graph. Thus, the Rohrpost of 1901 was embedded in a Berlin that is very different from post-World War II Berlin, which is very different from today's Berlin. The path of the Rohrpost, though, has been unchanged. To get an idea of the surroundings through time, 3-D software such as Google Earth offers good visualization. Fig. 3 shows a map of Berlin from around 1901 overlaid on the Google globe. Placemarks in Google Earth represent nodes on the graph. Researchers interested in physical remnants of this early communications system might supple-

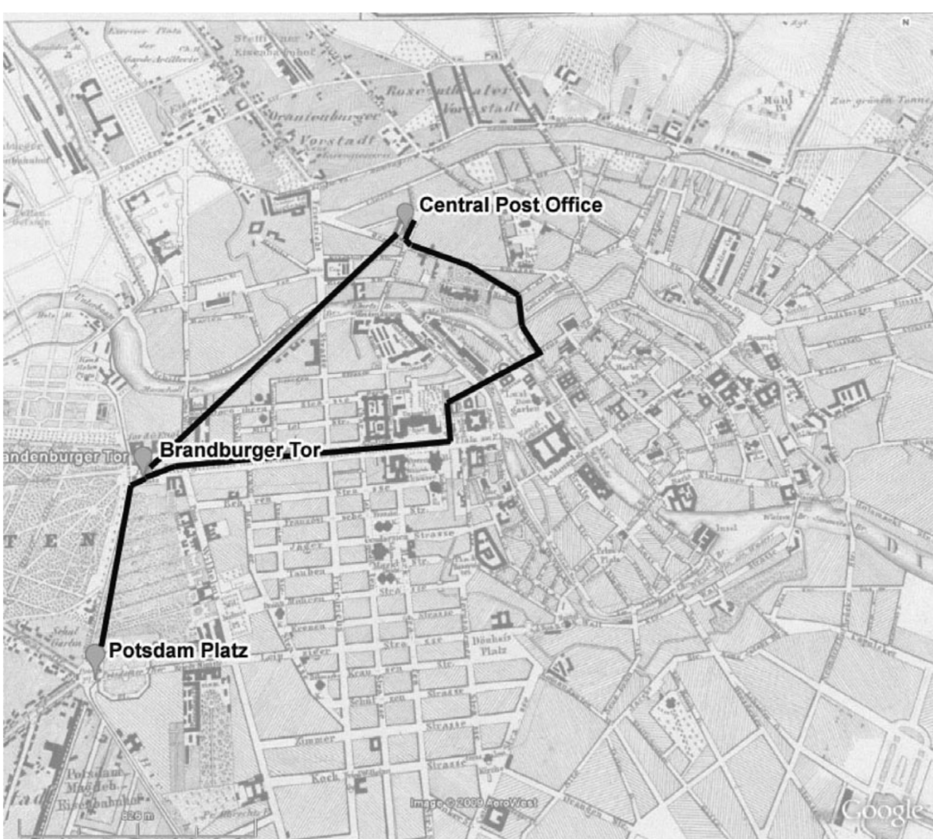


Figure 3. Early 20th-century Berlin map overlaid on Google Earth. A few placemarks and connections along the Rohrpost are aligned with the map.

ment textual evidence from the time with contemporary visual evidence (which presumably will improve steadily into the future).

The Brandenburg Gate is a good benchmark for fixing the pneumatic network through time. Its location in 1901 and today is the same. With the layer for 3-D buildings turned on in Google Earth, selecting it is easy, although one must know what to select. A persistent problem with visualization of the real world through a computerized replication of it is whether the *virtual* reality shown portrays an *accurate* reality. Conventional photographic evidence and field testing offer ways to verify reality. Fig. 4a shows a virtual reality view of the Brandenburg Gate. Fig. 4b shows a Google Earth "street view," captured in the upper right bubble of Fig. 4a as conventional photographic evidence of the same scene. The fit appears to be good, although the photographic evidence is not independent of the virtual model; it is embedded in the model by the same creator. Frequently, extra photographs are available for various sites around the world. Nevertheless, to test the fit accurately, one might go to Berlin with a camera and global positioning system and check both position and appearance of the model. For our purposes, however, the fit seems sufficient. Benchmarking in the virtual world offers new challenges!

Fig. 5 shows the model in relation to elements of the Rohrpost. The view of the network link running along Unter den Linden (still there) is surrounded mostly by structures that obviously were not present in 1901 (Fig. 6). Virtual reality models, as well as conventional maps, may guide exploration, discovery, and consequent explanation. Newer approaches to graph theory, which combine current technology with historical knowledge and documentation, offer ways to mesh time with space and to overcome the digital divide that often has made historical documents the stepchildren of the Internet.

Space filling and historical maps

As urban space changes over time, typically so does the amount of space that is filled. Downtown New York City appears packed with buildings; downtown Detroit is sparse now but becoming more fully packed. Effective planning should make efficient use of the land and also provide the goods and services people need and want in an appropriate manner. What seems "efficient" or "appropriate" is likely to vary with time and place. New analytic tools become available and offer a different set of vantage points from which to view data.

Returning again to the Rohrpost example, the ranking of locations in the historical map of Fig. 1 reflects the importance of the service provided by individual stations to that network. One also might think that the ranking reflects the level of provision of desired and needed service to the population of Berlin. To measure the extent of service provision, one might measure the amount of space filled by fanning the network out, in stages, to capture successively less and less important pneumatic stations.

Fractalyse (Gilles Vuidel, CNRS-Université de Franche-Comté et de Bourgogne, France) (available at http://www.fractalyse.org/) is a free stand-alone software that

Figure 4. The Brandenbrug Gate. (a) Virtual reality model in Google Earth. Note the "street view" bubbles, suggesting available conventional photographic images. (b) Conventional photographic evidence as seen from and within the right bubble closest to the gate in (a).

measures the amount of space filled by geometric objects. It offers various ways to calculate the amount of black existing within a bounded white area. Thus, it differs from the conventional generation of fractals where the outcome is not influenced by the physical representation of a dot or a line. We use Fractalyse to capture this space filling by Rohrpost service nodes. We chose to calculate the process as a "radius mass" process, selecting the "circular" option and using barycentric coor-

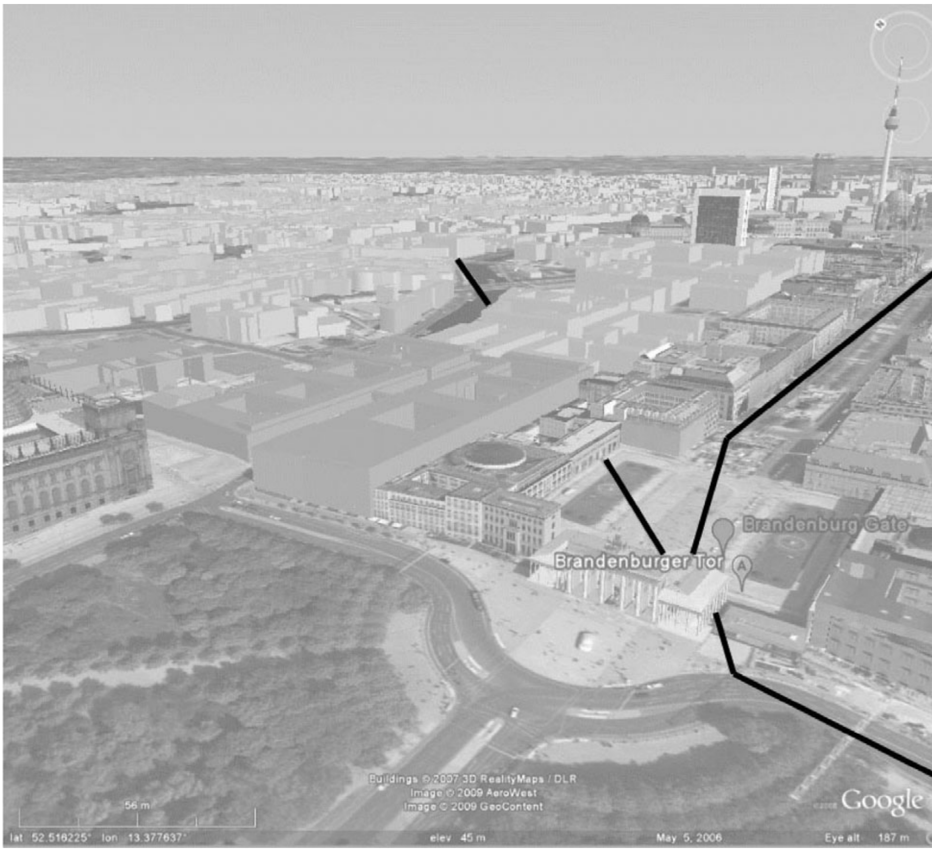


Figure 5. With the three-dimensional layer switched on, the Brandenburg Gate is easy to find in relation to edges and nodes in the Rohrpost graph. The thick black lines denote the approximate path of the Rohrpost (subterranean).

dinates (marked as a small crosshair) to calculate the center of the dot distribution. The result is a measure, expressed as a "dimension," of the amount of white space filled by the black dot scatter. Fig. 7 illustrates the outcome of one application of this process. The influence of dot size on the generated values is an advantage in this case. Deciding at the outset whether to let the physical size of the dots influence the space-filling measure is critical when using this software; Fractalyse is a good choice if one wishes to do so. It focuses on the relative, rather than the absolute, filling of space. Naturally, the more dots that are included (such as in a wider hierarchical range), the more space they fill.

Because space filling is influenced by physical representation of the entities, the historical ranking assigned in 1901 is captured in the measure generated in 2010. Fig. 8a–c show the succession of Fractalyse values: the scatter in Fig. 8a fills the least space, $D = 0.00009766$; the scatter in 8(b) the next most, $D = 0.7357$; and the scatter in Fig. 8c the most, $D = 0.8063$. The highest value is still less than

Figure 6. Looking from the top of the virtual Brandenburg Gate along Unter den Linden. Note the mix of old and new virtual reality buildings. The thick black line denotes the approximate path of Rohrpost (subterranean).

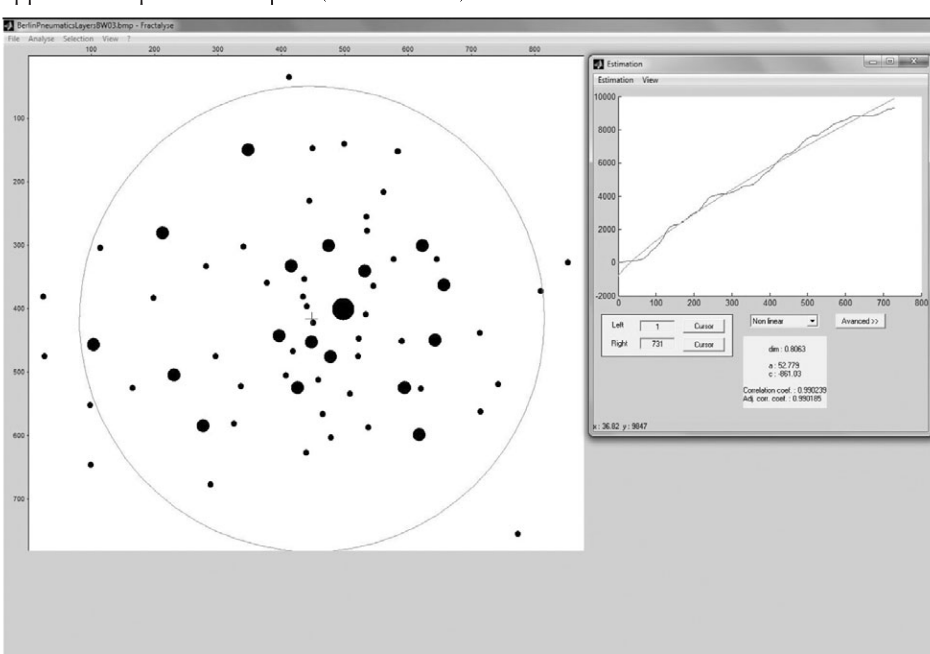


Figure 7. A screen capture of the output of Fractalyse for service nodes of the Berlin Rohrpost.

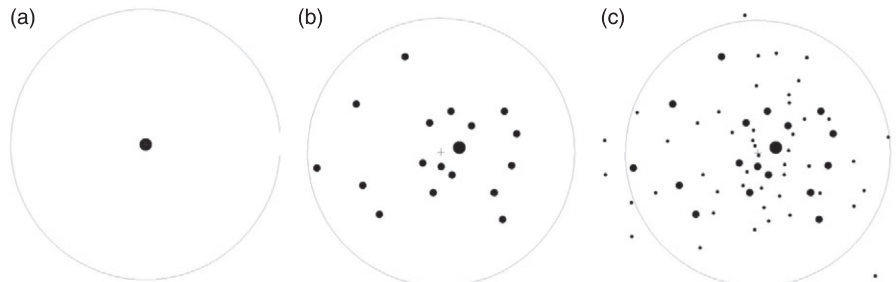


Figure 8. Fractalyse applied to the node hierarchy present in the Berlin Rohrpost to capture numerically evident visual space filling. (a) Least space filling: D = 0.00009766; correlation coefficient, 0.46. (b) Moderate space filling: D = 0.7357; correlation coefficient, 0.98. (c) Most space filling: D = 0.8063; correlation coefficient, 0.99.

D = 1, the maximum value of space filled by dot scatter that eventually becomes a line (where the measure is not influenced by the physical representation of dot sizes). That value is clearly less than D = 2, the corresponding highest value of space filled by a dot scatter when physical representation of dot size is factored in.

Conclusion

Integration of the new with the old is critical. Often, simply to forge ahead in the excitement of the new is tempting. However, without bringing existing materials into the current realm, one is destined to waste precious time and resources in reinventing the past. We suggest here one way to incorporate an array of works in a current context to take advantage, in a variety of ways, of the exciting revolutionary times in which we live. Regardless, whether we call it graph theory or network science, the important feature in geographic applications is to integrate spatial elements from diverse times and contexts with the broad, abstract, mathematical framework behind all of it. That is a principle that has endured in application, and it is one that we expect to endure in the future.

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