geographical analysis

Geographical Analysis ISSN 0016-7363

The City Is a Tree; The Real World Is Not a Tree!

Frank Harary

The guest coeditors of this issue set a context in which to view a previously unpublished article of Frank Harary. The Harary article is followed here by contemporary material of Joseph Kerski that addresses similar real-world issues. The two together, along with the context, serve as a springboard to launch the reader into the rest of the materials in this special issue devoted to graph theory and network science.

Interpretation of archived materials in context

We remember Frank sitting in our living room, stating vehemently, "the city is a tree," then a moment of thought and then a wry laugh: "and, it's not a semi-lattice, either!" To the one of us more acquainted with literature outside the realm of pure mathematics, these apparently strange comments made perfect sense. Frank did not agree with architect Christopher Alexander's 1965 article entitled "A City Is Not a Tree" (Alexander 1965), where he argues that cities built along the lines of a graph-theoretic tree are sterile, devoid of interest, and destined to fail. The reason for such failure is attributed to the lack of shared components among the laterally equivalent nodes represented in a top-down-oriented tree. Alexander argues that the semilattice better fits natural cities that evolve over time because in contrast to the tree, the semilattice shares various elements in hierarchical layers that one might associate with a tree-like structure.

Generally, Alexander appears to see the whole city as created by the joining of parts. Perhaps the eye of this great architect fell naturally to the buildings and other components that go together to make a city. Frank, on the other hand, apparently viewed the city as a whole that is more than the sum of its parts. He worked first with the entire system and then with its components. Perhaps he saw the city as a

Correspondence: Sandra L. Arlinghaus, Mathematical Geography, School of Natural Resources and Environment, The University of Michigan, Ann Arbor, MI 48109
e-mail: sarhaus@umich.edu

As interpreted from posthumous materials and oral conversations with the guest coeditors of this special issue of *Geographical Analysis:* Sandra L. Arlinghaus and William C. Arlinghaus. Materials in the Appendix are from Joseph Kerski.

Submitted: November 5, 2010. Revised version accepted: July 8, 2011.

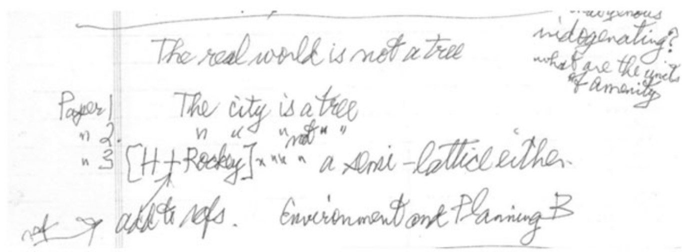

Figure 1. Note written by Frank Harary describing a set of articles to write, c. 1976.

dynamic entity, based on streams of water, streams of trash, streams of roads, and so forth that allowed people to benefit from economies of scale brought about by the clustering of resources and amenities. Clearly, these two great scholars looked at the concept of *city* from diametrically opposed viewpoints. Both approaches have merit, and how one looks at the concept determines what one sees. Indeed, their divergent viewpoints played out later in the world of computer science as Alexander influenced object-oriented programming and Harary influenced parallel processing (Salingaros c. 1997; Shirky 2004).

Viewpoint can make a world of difference! When Frank considered the entire world, he seemed to do so by assembling a view of components: the real world as the sum of its parts. Frank saw then, as Alexander had with the city, that the composite structure was *not* a tree. Indeed, in the note in Fig. 1, in Frank's own hand, he seems to acknowledge this sort of idea in constructing a hierarchy of articles to write beginning with "The city is not a semi-lattice either" (e.g., some urban systems should not share with each other drinking water supply lines and sewer lines). Harary and Rockey published an article by this name in *Environment and Planning A* (Harary and Rockey 1976). Next, Frank inserted, "The city is not a tree," then, "The city is a tree," apparently seeing both sides of the coin. Finally, he notes, "The real world is not a tree." Even in this sequence, he begins with the global and works to the local. One might debate, however, whether the emphasis in "real world" is on "real" or on "world." Perhaps it is on both; in any event, its place in this hierarchy is identical.

Frank's archived articles offer insight not only into the articles he may have wished to write but into his pattern of thinking about systems; they also reflect his intense enthusiasm for graph theory and for academic endeavor of all sorts (Harary Publication Archive 2011[1]; Wikipedia). His interests perhaps have appeared far-flung, but they had method. Thus, there are extensive collections of notes about physical streams as trees with the numerical ordering of them from one tributary to the next (Shreve 1966; Kirkby 1976; Stevens 1976; Moon 1980). Other streams he appeared to think about involve polluting materials; he categorized pollution types as a form of ordering of these streams (ENACT 1970; Thermal Pollution 1970) and people and crowding, all as part of viewing the urban dynamic in terms of feedback expressed as a signed graph. Again, he looked at the whole system as an abstract

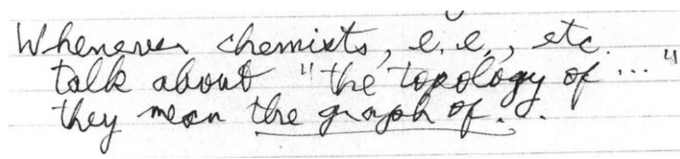

Figure 2. Quotation from Frank Harary.

entity and fit various concepts into that viewpoint—not to "fill" the space and answer all questions but to place items in places where they did fit. Consequently, he considered important urban concepts, such as "centrality," in the context of his view of the city as a tree. He claims there are only four types of graphical centrality: center (Jordan 1869), centroid (Jordan 1869), median, and cutting center (Harary 1969; Harary Publication Archive 2011). Beyond these somewhat natural concepts associated with cities, we find, juxtaposed with a graph representing urban dynamics, one representing streams of attitudes toward mental illness and their characterization as planar or nonplanar, corresponding (presumably) to states of rationality. In all cases, however, the view of the whole dominates: the pieces fit into the whole view.

By reading widely, Frank supported his various big-picture concepts; often, he appeared to feel the frustration that a person heavily trained in formal logic and pure mathematics (and to a lesser extent, in theoretical physics) encounters when trying to study how other disciplines use mathematics. Fig. 2 illustrates the frustration with one particular issue that both guest editors have felt as well.

We leave it to the reader to imagine whether Frank thought there was anything "new" about "network science." Whatever conclusion is drawn, at least selected graph theorists clearly have been considering whole systems for many years. Their work is not always easy to read and therefore, often not accessible to a wide variety of readers in various disciplines. This possible nonaccessibility, however, does not mean no work exists. Indeed, Frank, as a young logician/mathematician, appears to have been inspired by the work of Herbert Robbins (1939), who looked at urban street patterns, from the standpoint of the whole, to understand how one might assign flows to balance traffic. Robbins' work apparently continued to inspire Frank, not only in terms of particular content but also in its general approach of working from the global to the local rather than local to global.

Thus, we include here a previously unpublished article (with permission of the Harary Publication Archive at New Mexico State University and of Frank's son, Joel Harary, and with a light copyediting for style), undated but clearly written after 1980, in which Frank explains some of the material in an article written by two of his many coauthors that expands Robbins' earlier work. This latter work reflects Harary's "global first" approach and supports his view that the city (or at least one view of it) is a tree. In his usual style, with appreciation tempered by precision, Frank not only adds to the exposition of this article on one-way streets as a means to balance traffic flows but also suggests, following a critique of the work, directions

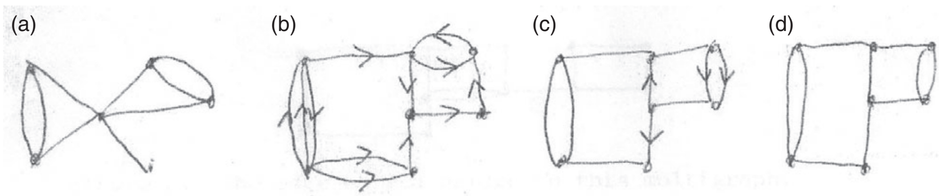

Figure 3. Illustrations: (a) and (d) a multigraph; (b) a multidigraph; (c) a mixed multigraph; and (d) the underlying multigraph of (c).

for further research. It was our privilege and pleasure to work with Frank in developing a number of structural models in geography (Arlinghaus, Arlinghaus, and Harary 1993, 1994, 2002; Arlinghaus et al. 1994). We thank Daniel A. Griffith, the current editor of *Geographical Analysis*, for his forward-looking idea of resurrecting some of Frank's materials so that others might benefit as well. We present this previously unpublished (to our knowledge) article of Frank Harary (from the Harary Publication Archive) complete with his original hand-drawn graphs.

Introduction

The article "Robbins' Theorem for Mixed Multigraphs" by Frank Boesch and Ralph Tindell (1980) is an extension of an earlier article by H. E. Robbins (1939). It attempts to answer the question, "When is it possible to find an assignment of one-way directions for all the streets in a town while preserving the property that it is possible to reach any point in town from any other point?" Robbins answered this question for a town that has no one-way streets. Boesch and Tindell solved it for the case when some, but not all, of the streets are already one-way. They show that all the streets can be made one-way so that reachability is preserved if and only if it was possible to do so before any streets were made one-way.

In order to illustrate their traffic problem, I begin by defining a few terms and describing the graph theoretic model. In a *multigraph*, more than one line can join two points, and in a *multidigraph*, more than one arc in the same direction can join two points. A *mixed multigraph* can have both arcs and lines between its points. A multidigraph and a multigraph are special cases of a mixed multigraph. The *underlying multigraph* of a mixed multigraph is the multigraph obtained by undirecting all the directed edges (arcs). Fig. 3 illustrates these definitions.

A multigraph is connected if there exists a path between every distinct pair of points, and a multidigraph is strongly connected if there exists a dipath between every ordered pair of points. A walk from u to v in a mixed multigraph is an alternating sequence of points and edges or arcs, u (u, u_1), u_1,., u_i, $[u_i, u_{i+1}]$, u_{i+1},., v, in which all arcs are traversed in their proper orientation: denote an edge between u and v by (u, v), and an arc from u to v by $[u, v]$. The point v is accessible from u in a mixed multigraph if a walk exists from u to v. A mixed multigraph is connected if a walk exists between every ordered pair of points. In Fig. 3, the multigraphs in

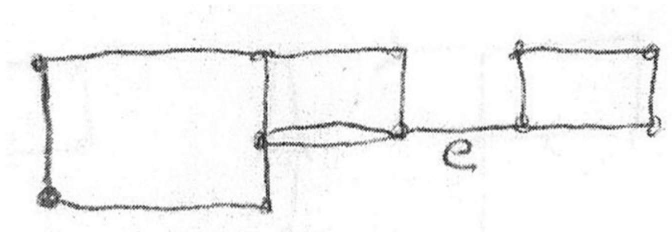

Figure 4. The edge *e* is a bridge in this multigraph.

(a) and (d) are connected as is the mixed multigraph in (c). The multidigraph in (b) is strongly connected. If a direction can be chosen for all edges in a multigraph such that there exists a dipath between every ordered pair of points, then the multigraph *G* is called strongly orientable. A bridge in a graph is an edge whose removal disconnects the graph (Fig. 4), illustrating this concept.

Robbins' theorem states that a multigraph is strongly orientable if and only if it is connected and bridgeless. Boesch and Tindell's extension, which they call "The General Robbins Theorem," may be stated as follows.

Theorem 1. *Let e be an undirected edge of a connected mixed multigraph G. Then e may be directed to produce another connected mixed multigraph if and only if e is not a bridge of the underlying multigraph of G.*

The authors prove Theorem 1 and then claim that an obvious inductive proof yields Theorem 2, which is their version of Robbins' theorem. From Theorem 1, if no bridges exist in the underlying graph of a mixed multigraph *G*, then the undirected edges can be directed, one at a time, to produce a strongly connected multidigraph. In other words, all the two-way streets of a city can be made one-way and preserve reachability if the city with all two-way streets has no "bridges"; Fig. 5 illustrates this concept. Consequently,

Theorem 2. *A mixed multigraph G has a strongly connected orientation if and only if G is connected and the underlying multigraph of G is bridgeless.*

Explanation of proof

The "only if" part of the proof (Theorem 1) is obvious. If *e* is a bridge, then the mixed multigraph would not be connected when *e* is directed; Fig. 6 shows this feature.

The authors next show that if neither orientation of *e* (let *e* be any edge between any two points *u* and *w*) produces a connected mixed multigraph, then *e* is a bridge. They show that no walks from *u* to *w*, or from *w* to *u*, can exist because if either one existed, then *e* could be oriented so that *G* would still be connected. Fig. 7 illustrates this feature.

Next, Boesch and Tindell divide the points of *G* into the sets *U* and *W*, where *U* is the set of points accessible from *U* in $G-e$, and *W* is $V-U$, *V* being the set of

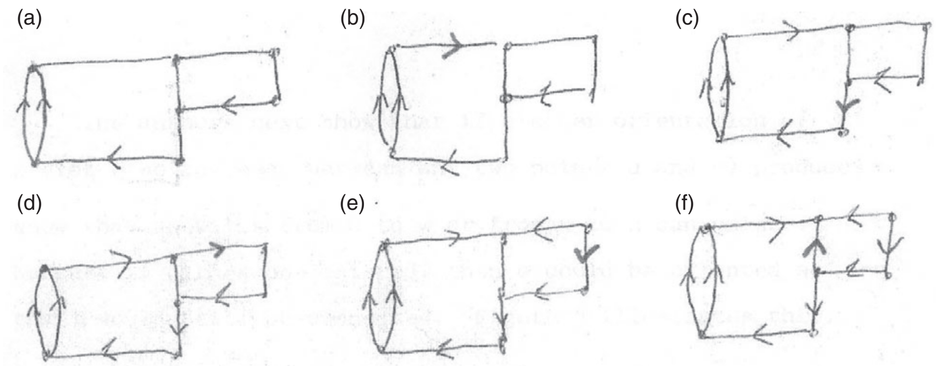

Figure 5. The mixed multigraph in (a) is connected and has a bridgeless underlying multigraph. Sequentially in (b) through (f), one additional undirected edge is directed, always yielding a connected mixed multigraph, until *G* becomes a strongly connected multidigraph.

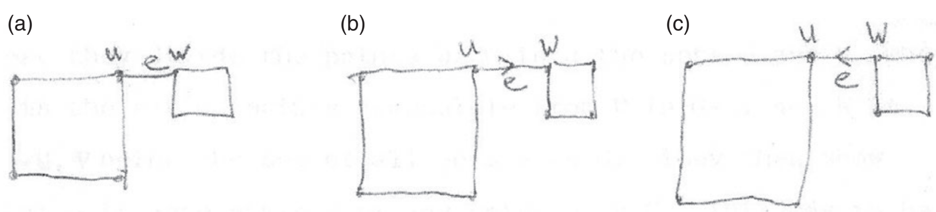

Figure 6. The mixed multigraph in (a) has a bridge *e* between points *u* and *w*. If it is directed from *u* to *w* (case [b]), *u* is not accessible from *w*, and vice versa, when *e* is oriented from *w* to *u* (case [c]).

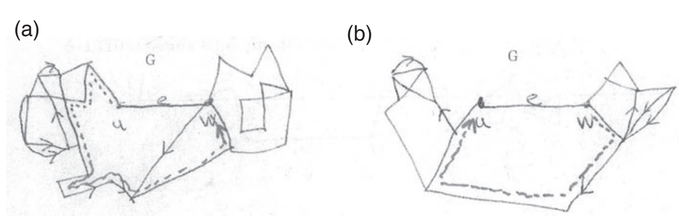

Figure 7. In (a), a walk from *u* to *w* exists, and *e* can be oriented from *w* to *u* so that the mixed multigraph *G* is connected. In (b), a walk from *w* to *u* exists so that *e* can be oriented from *u* to *w*, and again, *G* is connected.

all points in *G*. They then show that *u* is accessible from any point *v* in *U*. This result has to be true because in *G*, *u* is accessible from *v*, and if this accessibility required *e*, then *w* would be accessible from *u*, but this walk already was shown to not exist. They also show that *w* is accessible from any point *z* in *W*.

The proof is completed when Boesch and Tindell show that *e* is the only line of the underlying multigraph that has one endpoint in *U* and one in *W*. The definition

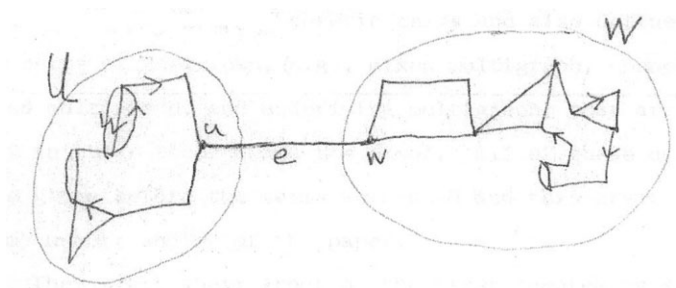

Figure 8. No arcs or edges other than *e* can exist with one endpoint in *u* and one in *w* so *e* must be a bridge.

of *U* precludes the existence of either an undirected edge between *U* and *W* or an arc from *U* to *W*. There can be no arc from *W* to *U* because there then would be a walk from *w* to *u*, which was shown to be impossible. Because *e* is the only edge with one endpoint in *U* and one in *W*, it must be a bridge, and the proof is completed. Fig. 8 illustrates the proof.

Critique

The application of graph theory to traffic control is what attracted me to Robbins' article, and that article led me to Boesch and Tindell's article. Boesch and Tindell describe a traffic problem to motivate their theorem, and this application adds color and makes the article more interesting. In the beginning of the article, the authors try to describe their theorem in terms of streets and traffic control. Their sentence "We show that if reachability has been preserved in arriving at this initial condition, then there is some assignment of directions to convert the remaining two-way streets to one-way if and only if it was originally possible to find an assignment for converting all the streets to one-way" confused me because it uses an "if-then" construction along with an "if and only if" construction in the same sentence. I had to read it several times to understand exactly what they meant, and I think that they could have worded it in a clearer way.

Before the statement of the theorem, the authors define several general graph theoretic terms and also define some new terms of their own (e.g., mixed multigraph, connected mixed multigraph, and underlying multigraph) that are necessary in their theorem and its proof. All of these definitions were given before the terms were used, which greatly aided in my understanding of the article.

They start their proof of the first theorem by saying that the "only if" portion is obvious and then proceed to the "if" portion. I agree that the "only if" part is obvious, but I think that they could have and should have included one sentence to demonstrate its obviousness. Next, they proceed to prove the "if" part (if *X*, then Y). They do it by proving "if not Y, then not X," but they do not tell the reader that the proof is "by contrapositive"; they just go ahead and do it. This forced me, as I read the proof, to look back at the theorem several times to make sure that I knew what they were trying to prove. Toward the middle of the proof, I got a little confused, specifically when they say, "points in G that are accessible in $G-e$ from u. . . ." I think that they should say, "points of V, in G-e, accessible from u. . . ." At the end of the proof, they did not bring together all of the points of the proof to conclude that the hypothesis is true, and to make matters worse, they did not include any symbol to mark the end of the proof.

My last criticism is that they restate, after Theorem 2, that Robbins' theorem is a special case of their second theorem and was not used in their proof of Theorem 2, so they have a new proof of Robbins' theorem. They point this out earlier in the article and thus do not need to point it out again. Also, they refer to their proof of Theorem 2, but this proof is never shown in the article.

New directions

Determining how many different ways graph theory could be applied to problems dealing with city streets would be interesting. One could try to find the most efficient way to pick up the garbage in a city. One could draw the graph representing the streets and, for a given number of garbage trucks, find the routes that would give the least duplication.

One also could designate the points in the city that are most frequently driven to and from and try to minimize the travel time between these points. One could say that by making a street one-way, the travel time is cut in half. By considering the graph, one could assign directions to certain streets without making traveling to and from other points in the city more difficult (editors: Harary was apparently inspired by earlier works of Tucker [1973] as well as by those of Robbins. Consider the exercise to come as integrative of earlier and contemporary efforts).

Acknowledgements

We wish to thank Cassie McClure, Library Specialist II, New Mexico State University Library, Archives and Special Collections Department, for her helpful and valuable assistance in assembling materials from the Harary Special Collection.[2] Her wisdom made this project possible as did the kind granting of permission from Frank's son, Joel Harary, for us to access these materials.

Notes

1 An online archive maintained by the Computer Science Department at New Mexico State University.
2 Housed in the New Mexico State University Library.

Appendix

With input from Joseph Kerski, PhD, education manager, ESRI

Harary's suggestion to use graph theory to optimize garbage removal is an interesting one. It is one that apparently continued to intrigue Harary and that is of enduring interest. Tucker (1973) wrote about the use of perfect graphs to optimize garbage pickup in New York City. Later, Egudo (from the disparate location of Papua New Guinea) wrote an analysis of municipal garbage collection based on graph-theoretic techniques (Egudo 1992). We (the guest coeditors) think that Frank would have greatly enjoyed Joseph Kerski's commentary using ESRI's Network Analyst software to optimize the routing of trucks within and outside of New York City.

Routing using ArcGIS online

ArcGIS Online now includes a collection of tasks, including geocoding and a routing service that supports point-to-point and optimized routing for North America and Europe. It is available as a standard, no-cost service with a limit of 5,000 routes per year, and as a fee-based service for each additional block of 5,000 routes, which can be used for commercial purposes. The network analyst extension is not required.

To begin, start ArcMap 9.3.1, turn on the StreetMap toolbar, and select "Find Route using online route services." Select the desired routing service for North America or Europe. The North America routing service, based on Tele Atlas 2008 data, enables the generation of routes and driving directions for the United States and Canada. Up to 25 route barriers may be included per request.

In the Find Route box, under the Stops tab, enter the stops along a proposed route. Up to 10 stops can be added from graphics or features. For the example in Fig. 9, I set up a lesson where students are the "new owners" of a double-decker, open-top Manhattan tour bus. They have to route the bus from St. John the Divine Church to Radio City, the New York Public Library, the Empire State Building, the House of Oldies in Greenwich Village, the Woolworth Building, the American Geographical Society on Wall Street, and return.

The seven stops came from a comma-separated value (.csv) file that I geocoded using the ArcGIS Online geocoding service and saved as a shapefile. With the Options tab, add a graphic, add a callout, and save the route and the stops as shapefiles. Students compare the quickest (in white/yellow) and the shortest (in black/blue) route in terms of the map and the total distance traveled. Only the quickest route has the bus traveling through the Upper East Side of Manhattan. How does adding one stop, changing the order of stops, or adjusting the influence of local roads versus highways affect the final route?

Give the ArcGIS Online routing service a try in your classroom!
Joseph Kerski

Figure 9. Manhattan tour bus routing example. Image from ArcGIS software, courtesy of ESRI, Inc.

Frank said he was born on the site of the Empire State Building (Stop 6 on Kerski's bus tour). "Mr Graph Theory" not only saw the world as a graph/network but often interpreted his visions in the context of New York City. We imagine that drawing those interests together with his compassion for the young, their education, and their prospect, and with parallel interests in computers, would have thrilled him no end. Thus, we encourage the reader of this special issue of *Geographical Analysis* to move with us through the various fields that attracted the attention, on a regular basis, of Frank Harary.

References

Alexander, C. (1965). "A City Is Not A Tree." *Architectural Forum* 122(1), 58–61; (2), 58–62.

Arlinghaus, S., W. C. Arlinghaus, and F. Harary. (1993). "Sum Graphs and Geographic Information." *Solstice* 4(1). Available at http://www.imagenet.org/ and at the persistent archive of The University of Michigan, Deep Blue, http://deepblue.lib.umich.edu/handle/2027.42/58219 (accessed January 17, 2011).

Arlinghaus, S., W. C. Arlinghaus, and F. Harary. (1994). "The Paris Metro: Is Its Graph Planar?" *Solstice* 5(2). Available at http://www.imagenet.org/ and at the persistent archive of The University of Michigan, Deep Blue, http://deepblue.lib.umich.edu/handle/2027.42/58219 (accessed January 17, 2011).

Arlinghaus, S., W. C. Arlinghaus, and F. Harary. (2002). *Graph Theory and Geography: An Interactive E-Book*. New York: Wiley.

Arlinghaus, S., W. C. Arlinghaus, F. Harary, and J. Nystuen. (1994). "Los Angeles, 1994—A Spatial Scientific Study." *Solstice* 5(1). Available at http://www.imagenet.org/ and at the persistent archive of The University of Michigan, Deep Blue, http://deepblue.lib.umich.edu/handle/2027.42/58219 (accessed January 17, 2011).

Boesch, F., and R. Tindell. (1980). "Robbins Theorem for Mixed Multigraphs." *American Mathematical Monthly* 87, 716–19.

Egudo, R. (1992). "An Analysis of Garbage Collection Operation." *International Journal of Mathematical Education in Science and Technology* 23(1), 89–95.

ENACT. (1970). "*Environmental Action for Survival*." Leaflet, Environmental Teach-In, March 11–14.

Harary, F. (1969). *Graph Theory*. Reading, MA: Addison-Wesley.

Harary, F., and J. Rockey. (1976). "A City Is Not a Semilattice Either." *Environment and Planning A* 8(4), 375–84.

Harary Publication Archive. (2011). "*New Mexico State University*." Available at http://www.cs.nmsu.edu/fnh/publ.html (accessed January 17, 2011).

Jordan, C. (1869). "Sur les Assemblage de Lignes." *Journal für die reine und angewandte Mathematik* 70, 185–90.

Kirkby, M. (1976). "Tests of the Random Network Model and Its Application to Basin Hydrology." *Earth Surface Processes* 1, 197–212.

Moon, J. (1980). "On Horton's Law for Random Channel Networks." *Annals of Discrete Mathematics* 8, 117–21.

Robbins, H. (1939). "A Theorem on Graphs with an Application to a Problem of Traffic Control." *American Mathematical Monthly* 46, 281–83.

Salingaros, N. (c. 1997). "*Some Notes on Christopher Alexander*." Available at http://zeta.math.utsa.edu/~yxk833/Chris.text.html (accessed January 17, 2011).

Shirky, C. (2004). "*Many 2 Many: A Group Weblog about Social Software*." Available at http://many.corante.com/archives/2004/04/26/a_city_is_not_a_tree.php (accessed January 17, 2011).

Shreve, R. (1966). "Statistical Law of Stream Numbers." *Journal of Geology* 74, 17–37.

Stevens, P. (1976). *Patterns in Nature*. New York: Penguin.

Thermal Pollution. (1970). University Record, University of Michigan, March 12, 1970.

Tucker, A. (1973). "Perfect Graphs and an Application to Optimizing Municipal Services." *SIAM Review* 15(3), 585–90.

Wikipedia. (2011). "Frank Harary." Available at http://en.wikipedia.org/wiki/Frank_Harary (accessed January 17, 2011).