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HSRI Digital Computer Programs for Semi-Empirical Tire Models

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16. Abstract

This document describes a set of five tire model computer programs which compute longitudinal and lateral traction forces and aligning moment developed by a pneumatic tire on a specific road surface. The preparation of input data for the tire models is discussed and a program for computing frictional performance data, from on-the-road traction measurements, is presented. Example computations are included.

The theory behind the tire models programmed in this document is described in Interim Document 6, "A Comparative Evaluation of Five Tire Traction Models," UM-HSRI-PF-74-2 (NTIS order No. PB-229-707). These tire model programs supersede the programs described in Appendix IV of Interim Document 6.

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HSRI DIGITAL COMPUTER PROGRAMS FOR SEMI-EMPIRICAL TIRE MODELS

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Project 329180

Tire Traction Characteristics
Affecting Vehicle Performance

Interim Document 8

November 1974

Sponsored by

The Motor Vehicle Manufacturers Association

HSRI Digital Computer Programs For Semi-Empirical Tire Models

PREFACE

The tire model programs described in this document supersede the programs described in Appendix IV of Interim Document 6, "A Comparative Evaluation of Five Tire Traction Models." These programs have been revised to permit greater flexibility in their use. A single main program now calls the five tire model subroutines individually or sequentially for a specific data set. Provision has also been made for calling a plot subroutine to produce plots of selected tire model output.

The preparation of input data is discussed and a data preparation program is described which computes values of the friction performance parameters required by each of the tire model subroutines.

This document contains all of the computer programs and information necessary to fit traction curves calculated from any of the five tire models discussed in Document 6 to tire traction data measured in the laboratory or on the road.

CONTENTS

1.0 TIRE MODEL PROGRAMS

The five tire models whose descriptive equations are summarized in Appendix II of Document 6* are programmed as FORTRAN subroutines. The programming has employed the normalization scheme described on page 125 of Document 6, and the subroutines return dimensionless forces (FX, FY) and moment (XMZ) as well as the dimensional values (X, Y, and Z). The dimensional values are printed by the main program (if switch ISW2=1), the dimensionless values are used for plotting (ISW2=2).

1.1 MAIN PROGRAM (TMMAIN)

The main program is structured to obtain comprehensive traction response data from any or all of the tire model subroutines. The selection of models to be exercised, for a specific set of input data, is determined by reading model calling integers into the array MODEL by FORMAT(5I1). For example, to obtain the responses of models 1, 4, and 3, in that order, for a specific data set, simply punch 143 into the first three columns of the data card which holds input for the array MODEL. There must be at least one and at most five model calling numbers specified; no particular order is necessary.

The value of the input datum ISW1 selects the slip variable (s_x or α) to be swept at discrete values of the path variable (α or s_x). When ISW1=1, α-paths are taken; s_x is swept over N points separated by intervals of size DSX, beginning at SX11. If N=0, each s_x-sweep is made over six default values (s_x=0., .2, .4, .6, .8, 1.0). The number of α-paths taken is determined by the

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*<u>A Comparative Evaluation of Five Tire Traction Models</u>, J. T. Tielking and N. K. Mital, UM-HSRI-PF-74-2, Sponsored by the Motor Vehicle Manufacturers Association, January 1974, NTIS Order No. PB-229-707.

input integer M. If M=0, s_x is swept only for the path $\alpha=0$. If M > 0, s_x-sweeps are made for M α-paths at intervals DALFA, beginning at ALF11. When ISW1=2, s_x-paths are taken; α is swept over M points separated by intervals of size DALFA, beginning at ALF11. If M=0, each α-sweep is made over six default values (α=0., 4., 8., 12., 16., 20.). The number of s_x-paths taken is determined by the input integer N. If N=0, α is swept only for the path $s_x=0$. If N > 0, α-sweeps are made for N s_x-paths at intervals DSX, beginning at SX11.

The input datum ISW2 selects the mode of output; printing (ISW2=1), or plotting (ISW2=2). When ISW2=2, a third switch, ISW3, is active. ISW3 selects one of the eight possible data plots which can be produced by subroutine TMPLOT, described in Section 1.3.

The main program reads the following data in input modules of either Type 1 or Type 2 (described below).

ISW1	selects sweep variable, s_x(=1) or α(=2)
ISW2	selects output mode, print (=1) or plot (=2)
ISW3	selects plot number (1-8) when ISW2=2
ISW4	selects input module type
SX11	initial value of s_x for s_x-sweep
DSX	step interval in s_x-sweep
N	number of evaluations in s_x-sweep
ALF11	initial value of α for α-sweep

DALFA	step interval in α-sweep	
M	number of evaluations in α-sweep	
MODEL	selects model to be exercised (1-5, one at least, five at most)	
MUO	adhesive friction limit, μ_o	
AS	speed sensitivity parameter, A_s	AS*V < 1
V	traveling velocity	
MUX	longitudinal sliding friction coefficient, μ_x	
MUY	lateral sliding friction coefficient, μ_y	
CS	longitudinal traction stiffness, C_s	
CALFA	lateral traction stiffness, C_α	
KX	longitudinal carcass stiffness, K_x	
KY	lateral carcass stiffness, K_y	
BX	longitudinal patch relocation factor, β_x(=1.)	
BY	lateral patch relocation factor, β_y(=1.)	
L	contact patch length	
FZ	tire load, F_z	

If the speed sensitivity factor, A_s(AS), causes the product A_sV to be greater than unity, the speed sensitive friction coefficient, $\mu = \mu_o(1-A_sV_s)$, may become negative. If this occurs, the traction forces and moment calculated by tire models HSRI-I, -II, -III are invalid and the program execution may be stopped by a FORTRAN exponentiation error in subroutine TMHS3.

The input data must appear in the following eight card image formats (A-H).

Card A (4I1)

ISW1, ISW2, ISW3, ISW4

Card B (2F4.0, I3)

SX11, DSX, N

Card C (2F4.0, I3)

ALF11, DALFA, M

Card D (2F8.3)

CS, CALFA

Card E (4F8.3)

KX, KY, BX, BY

Card F (2F8.3)

L, FZ

Card G (5I1)

MODEL(1), MODEL(2), MODEL(3), MODEL(4), MODEL(5)

Card H (5F8.3)

MUO, AS, V, MUX, MUY

The data card images are arranged in one or more input modules which are read by the main program. There are two types of input modules, the type being identified by the input datum ISW4. The card order for the two input module types is shown in Table 1. There are no restrictions on the number or order of the input modules.

TABLE 1

INPUT DATA MODULES

	Type 1	Type 2
	ISW4=1	ISW4=2
Cards/ Module	Constant; 8	Variable; 7+2n, where n is an integer*
Card Order	A B C D E F G H	A B C D E F G H G H . . . G H [blank]**

*n = number of times that cards G and H are repeated.

**The last card of a Type 2 input module must be blank.

The Type 2 input module is useful for computer runs where output is desired for fixed ranges of s_x and α in a tire model study with

i) Constant: KX, KY, BX, BY, L, FZ

ii) Varying: (Velocity) V

iii) Various tire model numbers with MUO, AS, MUX, MUY being computed for each tire model by the friction performance data program (described in Section 2.0).

To facilitate conversion of this program for execution on other computing equipment, I/O device numbers are given by variables, IRD (reader) and IPR (printer), which are set before the first READ statement.

1.2 TIRE MODEL SUBROUTINES

The tire model subroutines are identified by a five-character name and a model calling integer as listed in Table 2. These subroutines are called from the main program (TMMAIN). In addition to explicit input/output arguments in the subroutine calling statements, the subroutines receive input data from the main program via the common block labeled BLK1.

TABLE 2

TIRE MODEL CALLING INTEGERS AND SUBROUTINE NAMES

Calling Integer	Subroutine Name	Tire Model	Document 6 Page
1	TMHS1	HSRI-I	21
2	TMHS2	HSRI-II	27
3	TMHS3	HSRI-III	69
4	TMSKI	Sakai	55
5	TMGDR	Goodyear	44

The tire model subroutines employ the following argument variables.

Input Arguments

SX	longitudinal slip parameter, s_x
ALFA	slip angle, α (degrees)

Output Arguments

FX	longitudinal force	dimensionless
FY	lateral force	
XMZ	aligning moment	
X	longitudinal force	dimensional
Y	lateral force	
Z	aligning moment	
XIA	adhesion limit fraction, ξ_a/L	
XIS	transition limit fraction, ξ_s/L	

It should be noted that XIS is relevant only for the HSRI-II and HSRI-III models as these are the only models which include a transition region between adhesive and sliding contact.

1.3 PLOT SUBROUTINE (TMPLOT)

The plot subroutine is called from the main program when ISW2=2. The subroutine argument, ISW, should be zero on the first call (to check validity of switches ISW1 and ISW3—certain combinations are illegal) and nonzero on succeeding calls. The plot data is carried in the common block labeled BLK2. The following plots are produced, according to the integer value of ISW3.

ISW3	Plot
1	F_x vs. s_x for various α
2	F_y vs. s_x for various $\alpha \neq 0$
3	M_z vs. s_x for various $\alpha \neq 0$
4	F_y vs. F_x for various α (s_x sweep)
5	M_z vs. F_x for various α (s_x sweep)
6	F_y vs. α for various s_x
7	M_z vs. α for various s_x
8	F_y vs. α and F_z (carpet plot)*

*The carpet plot facility has not yet been implemented by coding in the main program.

1.4 PROGRAM LISTINGS

The following pages show listings of the main program (TMMAIN), the tire model subroutines (TMHS1, TMHS2, TMHS2, TMSKI, TMGDR), and the plot subroutine (TMPLOT), which have been compiled and executed on the PDP 11/45 computer at the Highway Safety Research Institute.

Subroutine PLOTST (start plot) and PLOTND (end plot), called by the main program, and GRID (draw grid), GRDNUM (number grid), PLABEL (label axes), PLOTPT (plot point), SYMBOL (label plot), called by the plot subroutine, are not included in the following listings as they are highly dependent on the particular computing and plotting equipment utilized.

```
C     TMMAIN ... MAIN PROGRAM FOR SEMI-EMPIRICAL TIRE MODELS
C
C     --------------------------------------------------------------------
C
      DIMENSION MODEL(5),ALF2(6),SX2(6)
      DIMENSION FFZ(9)
      REAL MUO,MUY,KX,KY,L,MUX,KXU,KYU
      REAL KKX,KKY
C.....VARIABLES USED IN MODEL ROUTINES
      COMMON/BLK1/MUO,AS,V,MUX,MUY,CS,CALFA,
     +                 KX,KY,BX,BY,L,FZ
C.....VARIABLES USED BY PLOT- X,Y,Z DIMENSIONAL COUNTERPART OF FX,FY,XMZ
      COMMON/BLK2/ALF1(101),SX1(101),FX(101),FY(101),XMZ(101),
     +                 X(101),Y(101),Z(101),
     +                 SCALX,SCALY,XMIN,YMIN,IPR,ISW1,ISW3,NUMBER,N,M,ITYPE
      REAL*8 NAME(5)
      DATA NAME/'MODEL 1','MODEL 2','MODEL 3','SAKAI  ','GOODYEAR'/
C.....DEFAULT VALUES IF NONE GIVEN
      DATA ALF2/0.,4.,8.,12.,16.,20./,SX2/0.,.2,.4,.6,.8,1./
C
C.....I/O DEVICE NUMBERS
      IRD=8
      IPR=5
C     CALL PLOTSTART TO BE READY FOR PLOTTING
C     CALL PLOTST---------TEMPORARILY UNUSABLE
C     --------------------------------------------------------------------
C.....READ IN SETS OF VARIABLES UNTIL END OF FILE
   10 READ(IRD,1400,END=9999)ISW1,ISW2,ISW3,ISW4
C.....IF ONLY PARTIAL SET OF VARIABLES, PRINT ERROR, EXIT
      READ(IRD,2000,END=1900)SX11,DSX,N,ALF11,DALFA,M
      READ(IRD,3000,END=1900)CSU,CALFAU,KXU,KYU,BX,BY,L,FZ
   14 READ(IRD,1400,END=1900)MODEL
C.....IF MODEL(1) EQUALS ZERO, REINITIALIZE MODE
C.....OF OPERATION SWITCH ISW4 AND READ IN NEW DATA
      IF(MODEL(1).EQ.0)GO TO 10
      READ(IRD,3500,END=1900)MUO,AS,V,MUX,MUY
C
      WRITE(IPR,7000)MUO,AS,V,MUX,MUY,CSU,CALFAU,
     +                    KXU,KYU,BX,BY,L,FZ
C
C     --------------------------------------------------------------------
C
      GOTO(15,20)ISW2
C.....PRINT OUT HEADING IF ISW2=1
   15 WRITE(IPR,5000)
      WRITE(IPR,6000)
C
C.....NORMALIZE VARIABLES
   20 CS=CSU/FZ
      CALFA=CALFAU/FZ
      KX=KXU/FZ
      KY=KYU/FZ
C.....TAKE PATH DESIGNATED BY ISW1
C.....  1 = ALPHA PATH
C.....  2 = SX PATH
```

```
      GO TO (30,40),ISW1
C     ----------------------------------------------------------------------
C.....ALPHA PATH,SWEEP SX
C
C.....N=0 ; SX NOT SPECIFIED; DEFAULT SX VALUES FOR N=6 TAKEN
   30 IF(N.GT.0) GO TO 50
      N=6
      DO 11 I=1,N
   11 SX1(I)=SX2(I)
      GO TO 150
C.....N>0 ; SX VALUES AS SPECIFIED ARE USED IN SWEEP
   50 SX1(1)=SX11
      DO 100 I=2,N
  100 SX1(I)=SX1(I-1)+DSX
C.....M=0 ; SX VALUES SWEPT FOR ALPHA=0 ONLY
C.....M>0 ; SX VALUES SWEPT FOR EACH ALPHA VALUE GIVEN
  150 ALF1(1)=0.
      IF(M.EQ.0)GO TO 250
      ALF1(1)=ALF11
      DO 200 I=2,M
  200 ALF1(I)=ALF1(I-1)+DALFA
C
C.....FOR EACH ALPHA VALUE, CALL MODEL NUMBERS DESIRED WITH ALL SX;
C.....5 POSSIBLE MODELS ARE CALLED UNTIL MODEL NUMBER 0 IS FOUND
  250 DO 400 IMODE=1,5
      NUMBER=MODEL(IMODE)
      IF(NUMBER.EQ.0.OR.NUMBER.GT.5)GO TO 500
      XIS=0.
C ....PRINT LABEL OR PLOT GRID
      IF(ISW2.EQ.1) WRITE(IPR,4000)NAME(NUMBER),FZ
      IF(ISW2.EQ.2)CALL TMPLOT(0)
C.....IF ILLEGAL PLOT (ISW3=0 AFTER TMPLOT CALL);
C.....1) GET NEXT SET OF DATA IF ISW4=1
C.....2) EXIT PROGRAM IF ISW4=2
      IF(ISW2.EQ.2.AND.ISW3.EQ.0)GO TO(10,9999),ISW4
      DO 400 I=1,M
      IF(ISW2.EQ.1)WRITE(IPR,4000)
      DO 300 J=1,N
      K=J
      GO TO(1,2,3,4,5),NUMBER
  300 CONTINUE
C.....DO THE PLOTS NOW IF ISW2=2
C
      IF(ISW2.EQ.1) GO TO 400
      CALL TMPLOT(1)
C
  400 CONTINUE
C
  500 CONTINUE
C
C.....READ NEXT SET OF VALUES
      GO TO (10,14),ISW4
C     ----------------------------------------------------------------------
C.....SX PATH, SWEEP ALPHA
C
C.....M=0; ALPHA NOT SPECIFIED ; DEFAULT ALPHA VALUES FOR M=6 TAKEN
```

```
   40 IF(M.GT.0) GO TO 550
      M=6
      DO 12 I=1,M
   12 ALF1(I)=ALF2(I)
      GO TO 675
C.....M>0: ALPHA VALUES AS SPECIFIED ARE USED FOR SWEEP
  550 ALF1(1)=ALF11
      DO 600 I=2,M
  600 ALF1(I)=ALF1(I-1)+DALFA
C.....N=0:ALPHA VALUES SWEPT FOR SX=0 ONLY
C.....N>0:ALPHA VALUES SWEPT FOR EACH ALPHA VALUE GIVEN
  675 SX1(1)=0.
      IF(N.EQ.0) GOTO 750
      SX1(1)=SX11
      DO 700 I=2,N
  700 SX1(I)=SX1(I-1)+DSX
C
C.....FOR EACH SX VALUE, CALL MODEL NUMBERS DESIRED WITH ALL ALPHAS,
C.....5 POSSIBLE MODELS ARE CALLED UNTIL MODEL NUMBER 0 IS FOUND
  750 DO 900 IMODE=1,5
C
      NUMBER=MODEL(IMODE)
      IF(NUMBER.EQ.0.OR.NUMBER.GT.5)GO TO 1000
      XIS=0.
C ....PRINT LABEL OR PLOT GRID
      IF(ISW2.EQ.1) WRITE(IPR,4000) NAME(NUMBER),FZ
      IF(ISW2.EQ.2)CALL TMPLOT(0)
C.....IF ILLEGAL PLOT (ISW3=0 AFTER TMPLOT CALL):
C.....1) GET NEXT SET OF DATA IF ISW4=1
C.....2) EXIT PROGRAM IF ISW4=2
      IF(ISW2.EQ.2.AND.ISW3.EQ.0)GO TO(10,9999),ISW4
      DO 900 J=1,N
      IF(ISW2.EQ.1)WRITE(IPR,4000)
C
      DO 800 I=1,M
      K=I
      GO TO(1,2,3,4,5),NUMBER
  800 CONTINUE
C
C.....DO THE PLOTS NOW IF ISW2=2
C
      IF(ISW2.EQ.1)GO TO 900
      CALL TMPLOT(1)
C
  900 CONTINUE
C
C
 1000 CONTINUE
C.....READ NEXT SET OF VALUES
      GO TO (10,14),ISW4
C
    1 CALL TMHS1(SX1(J),ALF1(I),FX(K),FY(K),XMZ(K),X(K),Y(K),Z(K),XIA)
      GO TO 350
    2 CALL TMHS2(SX1(J),ALF1(I),FX(K),FY(K),XMZ(K),X(K),Y(K),Z(K),XIA,
     +XIS)
      GO TO 350
```

```
0093          3 CALL TMHS3(SX1(J),ALF1(I),FX(K),FY(K),XMZ(K),X(K),Y(K),Z(K),XIA,
              +XIS)
0094            GO TO 350
0095          4 CALL TMSKI(SX1(J),ALF1(I),FX(K),FY(K),XMZ(K),X(K),Y(K),Z(K),XIA)
0096            GO TO 350
0097          5 IF(K.EQ.1) CALL TMGDR(SX1(J),ALF1(I),FX(K),FY(K),XMZ(K),X(K),Y(K),
              +Z(K),XIA,0.,0.,0.)
0098            IF(K.GT.1)CALL TMGDR(SX1(J),ALF1(I),FX(K),FY(K),XMZ(K),X(K),Y(K),
              +Z(K),XIA,FX(K-1),FY(K-1),XMZ(K-1))
        C
        C.....PRINT OUT DIMENSIONAL VALUES IF ISW2=1; ELSE PLOT LATER
0099      350 IF(ISW2.EQ.1)
            +     WRITE(IPR,8000)ALF1(I),SX1(J),X(K),Y(K),Z(K),XIA,XIS
        C
        C.....RETURN TO LOOP
        C
0100            GO TO(300,800),ISW1
        C.....INPUT ERROR
0101     1900 WRITE(IPR,9000)
0102     9000 FORMAT('0END OF FILE ENCOUNTERED IN READING PARAMS, ERROR EXIT')
        C       CLOSE PLOT FACILITY
        C9999 CALL PLOTND---------TEMPORARILY UNUSABLE
0103     9999 CALL EXIT
        C
        C
0104     2000 FORMAT(2F4.0,I3)
0105     1400 FORMAT(5I1)
0106     3000 FORMAT(2F8.3/4F8.3/10F8.3)
0107     3500 FORMAT(5F8.3)
0108     4000 FORMAT('0',A8,'  FZ=',F6.1/)
0109     5000 FORMAT(1H1)
0110     6000 FORMAT(10X,'SLIP ANGLE     LONG, SLIP     FORCE-LONG     FORCE-LAT
            +         MOMENT',8X,'ADHESION',6X,'TRANSITION'/11X,'(ALPHA)',10X,
            +'(SX)',11X,'(FX)',11X,'(FY)',11X,'(MZ)',10X,'(XIA)',11X,'(XIS)'/
            +11X,'DEGREES',8X,'PERCENT',11X,'LB.',12X,'LB.',11X,'LB-IN',
            +/10X,'----------     ----------     ----------     ----------
            +----------     ----------     ----------'/)
0111     7000 FORMAT('1 MUO=',F8.3/' AS=',F8.3/' V=',F8.3/' MUX=',F8.3/' MUY=',
            +F8.3/' CS=',F9.3/' CALFA=',F9.3/' KX=',F8.3/' KY=',F8.3/' BX=',
            +F8.3/' BY=',F8.3/' L=',F8.3/' FZ=',9F10.3)
        C
0112     8000 FORMAT(F17.1,F16.2,3F15.1,2F15.3)
0113          END

        ROUTINES CALLED:
        TMPLOT, TMHS1 , TMHS2 , TMHS3 , TMSKI , TMGDR , EXIT


        OPTIONS =/ON,/OP:3

        BLOCK      LENGTH
        MAIN.    1817    (007062)*
        BLK1     26      (000064)
        BLK2     1631    (006276)
```

```
        C      TMHS1            HSRI-I    (DOC.6, REF.3)
        C      -----------------------------------------------------------
0001           SUBROUTINE TMHS1(SX,ALFA,FX,FY,MZ,X,Y,Z,XIA)
        C
        C      UNIFORM PRESSURE DISTRIBUTION
        C      COMPLETE SLIDING ONLY AT WHEEL LOCK
        C      WHEEL LOCK FORCES ARE COMPUTED.
        C      MOMENT IS NOT COMPUTED.
        C      NO TRANSITION FROM ADHESION TO SLIDING.
        C
        C      -----------------------------------------------------------
        C
0002           REAL MU,MUX,MUY,MZ,KX,KY,L,MUO
0003           COMMON/BLK1/MUO,AS,V,MUX,MUY,CS,CALFA,
             +        KX,KY,BX,BY,L,FZ
        C
        C      -----------------------------------------------------------
        C
0004           ALF=ALFA*.0174533
0005           SY=SIN(ALF)/COS(ALF)
        C....SPECIAL CASE FOR SX=SY=0.
0006           IF(ABS(SX).GE..001)GOTO 21
0007           IF(ABS(SY).GE..001)GOTO 21
0008        22 FX=0.
0009           FY=0.
0010           XIA=1.
0011           GO TO 30
0012        21 CONTINUE
0013           IF(SX-1.)20,20,80
0014        80 SX=1.
0015        20 VS=V*COS(ALF)*SQRT(ABS(SX)**2.+ABS(SY)**2.)
0016           MU=MUO*(1.-AS*VS)
0017           TEMP=SQRT(ABS(SX*CS)**2.+ABS(SY*CALFA)**2.)
        C
        C.....DETERMINE ADHESION RANGE.
        C
0018           IF(SX-1.)40,10,22
0019        40 XIA=.5*MU*(1.-SX)/TEMP
0020           IF(1.-XIA)50,50,60
0021        50 XIAP=1.
0022           GO TO 70
0023        60 XIAP=XIA
        C
        C.....ADHESION AND SLIDING.
        C
0024        70 FX=CS*SX/(1.-SX)*XIAP*(2.-XIAP)
0025           FY=CALFA*SY/(1.-SX)*XIAP*(2.-XIAP)
0026           GO TO 30
        C
        C.....COMPLETE SLIDING (WHEEL LOCK).
        C
0027        10 XIA=0.
0028           TEMP=SQRT(ABS(SX*CS)**2.+ABS(SY*CALFA)**2.)
0029           FX=CS*MU/TEMP
0030           FY=CALFA*MU*SY/TEMP
```

```
        C
        C.....MULTIPLY BY FZ.
        C
0031       30 X=-FX*FZ
0032          Y=-FY*FZ
0033          Z=0.
0034          MZ=0.
        C
        C     ------------------------------------------------------------
        C
0035          RETURN
0036          END

        ROUTINES CALLED:
        SIN    , COS    , ABS    , SQRT

        OPTIONS = /ON,/OP

        BLOCK        LENGTH
        TMHS1    440     (001560)*
        BLK1     26      (000064)

        **COMPILER ----- CORE**
           PHASE        USED  FREE
        DECLARATIVES 00216 01576
        EXECUTABLES  00639 01153
        ASSEMBLY     00441 03917

DK1:TMHS1,LP:=CR:
```

```
        C      TMHS2          HSRI-II   (DOC.6, REF.4)
        C      ------------------------------------------------------------
        C
0001           SUBROUTINE TMHS2(SX,ALFA,FX,FY,MZ,X,Y,Z,XIA,XIS)
        C
        C      ------------------------------------------------------------
        C
        C      UNIFORM PRESSURE DISTRIBUTION
        C      COMPLETE SLIDING ONLY AT WHEEL LOCK
        C      WHEEL LOCK FORCES ARE COMPUTED.
        C      TRANSITION FROM ADHESION TO SLIDING.
        C
        C      ------------------------------------------------------------
        C
0002           REAL MU,MUO,MZ,MZA,MZT,MZS,MZP,KY,KX,L,MUX,MUY
0003           COMMON/BLK1/MUO,AS,V,MUX,MUY,CS,CALFA,
             +        KX,KY,BX,BY,L,FZ
        C
        C      ------------------------------------------------------------
        C
0004           ALF=ALFA*.0174533
0005           SY=SIN(ALF)/COS(ALF)
        C....SPECIAL CASE FOR SX=SY=0.
0006           IF(ABS(SX).GE..001)GOTO 21
0007           IF(ABS(SY).GE..001)GOTO 21
0008        22 FX=0.
0009           FY=0.
0010           MZ=0.
0011           XIA=1.
0012           XIS=1.
0013           GO TO 40
0014        21 CONTINUE
0015           IF(SX-1.)10,10,20
0016        20 SX=1.
0017        10 SP=SQRT((ABS(SX)**2.)+(ABS(SY)**2.))
0018           VS=V*COS(ALF)*SP
0019           MU=MUO*(1.-AS*VS)
0020           IF(SX-1.)30,15,22
        C
        C.....COMPLETE SLIDING (WHEEL LOCK).
        C
0021        15 XIA=0.
0022           XIS=0.
0023           XIAP=0.
0024           XISP=0.
0025           SXP=0.
0026           SYP=0.
0027           GO TO 100
        C
        C.....DETERMINE ADHESION AND TRANSITION LIMITS.
        C
0028        30 SXP=SX/(1.-SX)
0029           SYP=SY/(1.-SX)
0030           TEMP=SQRT((ABS(SX*CS)**2.)+(ABS(SY*CALFA)**2.))
0031           XIA=.5*MUO*(1.-SX)/TEMP
```

```
      XIS=.5*MU*(1.-SX)*(1./CS+1./CALFA)/SP
      IF(1.-XIA)50,50,60
   50 XIAP=1.
      XISP=1.
      GO TO 100
   60 IF(XIS-XIA)70,70,80
   70 XIAP=XIA
      XISP=XIA
      GO TO 100
   80 IF(1.-XIS)90,90,110
   90 XIAP=XIA
      XISP=1.
      GO TO 100
  110 XIAP=XIA
      XISP=XIS
C
C.....DETERMINE FORCES IN CONTACT PATCH.
C
  100 FXA=CS*SXP*XIAP**2.
      FXT=(CS*SXP*XIAP+.5*MU*SX/SP)*(XISP-XIAP)
      FXS=MU*SX/SP*(1.-XISP)
      FX=FXA+FXT+FXS
C
      FYA=CALFA*SYP*XIAP**2.
      FYT=(CALFA*SYP*XIAP+.5*MU*SY/SP)*(XISP-XIAP)
      FYS=MU*SY/SP*(1.-XISP)
      FY=FYA+FYT+FYS
C
      MZA=(.66666*SXP*(CS-CALFA)*XIAP-.16666*CALFA*(4.*XIAP-3.))*SYP*XIA
     1P**2.
      MZT=.66666*((CS-CALFA)*(SYP*SXP*XIAP*XIAP+.25*(1./CALFA+1./CS)*MU*
     1SYP*SX/SP*XIAP+.25*MU**2./(CS*CALFA)*SX*SY/SP**2.)-.25*(CALFA*SYP*
     2XIAP*(4.*XIAP+2.*XISP-3.)+.5*MU*SY/SP*(2.*XIAP+4.*XISP-3.)))*(XISP
     3-XIAP)
      MZS=.5*MU*SY/SP*(SX/SP*MU*(1./CALFA-1./CS)-XISP)*(1.-XISP)
      MZP=-(BX/KX-BY/KY)*FX*FY
      MZ=MZA+MZT+MZS+MZP/L
C
C.....MULTIPLY BY FZ AND L.
C
   40 X=-FX*FZ
      Y=-FY*FZ
      Z=-MZ*FZ*L
C
C     ----------------------------------------------------------------
C
      RETURN
      END

ROUTINES CALLED:
SIN    , COS    , ABS    , SQRT

OPTIONS = /ON,/OP

BLOCK        LENGTH
TMHS2    824     (003160)*
BLK1     26      (000064)
```

```
        C      TMHS3           HSRI-III  (DOC.6, PAGE 69)
        C      ------------------------------------------------------------
        C
0001           SUBROUTINE TMHS3(SX,ALFA,FX,FY,MZ,X,Y,Z,XIA,XIS)
        C
        C      ------------------------------------------------------------
        C
        C      PARABOLIC PRESSURE DISTRIBUTION
        C      COMPLETE SLIDING OCCURS BEFORE WHEEL LOCK
        C      WHEEL LOCK FORCES ARE COMPUTED.
        C      TRANSITION FROM ADHESION TO SLIDING.
        C
        C      ------------------------------------------------------------
        C
0002           REAL MU,MUO,MZ,MZA,MZT,MZS,MZP,KY,KX,L,MUX,MUY
0003           COMMON/BLK1/MUO,AS,V,MUX,MUY,CS,CALFA,
             +        KX,KY,BX,BY,L,FZ
        C
        C      ------------------------------------------------------------
        C
0004           ALF=ALFA*.0174533
0005           SY=SIN(ALF)/COS(ALF)
        C....SPECIAL CASE FOR SX=SY=0.
0006           IF(ABS(SX).GE..005)GOTO 21
0007           IF(ABS(SY).GE..005)GOTO 21
0008        22 FX=0.
0009           FY=0.
0010           MZ=0.
0011           XIA=1.
0012           XIS=1.
0013           GO TO 999
0014        21 CONTINUE
0015           IF(SX-1.)10,10,22
0016        10 SP=SQRT(SX*SX+SY*SY)
0017           VS=V*COS(ALF)*SP
0018           MU=MUO*(1.-AS*VS)
0019           IF((1.-SX).GE..001)GO TO 30
        C
        C.....COMPLETE SLIDING (WHEEL LOCK).
        C
0020        15 XIA=0.
0021           XIS=0.
0022           XIAP=0.
0023           XISP=0.
0024           SXP=0.
0025           SYP=0.
0026           SX=.999
0027           GO TO 120
        C
        C.....DETERMINE ADHESION AND TRANSITION LIMITS.
        C
0028        30 SXP=SX/(1.-SX)
0029           SYP=SY/(1.-SX)
0030           TEMP=SQRT((ABS(SX*CS)**2.)+(ABS(SY*CALFA)**2.))
0031           XIA=1.-TEMP/(3.*MUO*(1.-SX))
```

```
            XIS=1.-CS*CALFA/(CS+CALFA)*SP/(3.*MU*(1.-SX))
      C
            IF(XIS)35,35,40
         35 XIAP=0.
            XISP=0.
            XIA=0.
            XIS=0.
            GO TO 120
      C
         40 IF(XIA)45,45,50
         45 XIAP=0.
            XIA=0.
            GO TO 90
      C
         50 IF(1.-XIA)60,60,70
         60 XIAP=1.
            XISP=1.
            GO TO 120
      C
         70 IF(XIS-XIA)80,80,90
         80 XIAP=XIA
            XISP=XIAP
            GO TO 120
         90 IF(1.-XIS)100,100,110
        100 XIAP=XIA
            XISP=1.
            GO TO 120
        110 XIAP=XIA
            XISP=XIS
      C
      C.....COMPUTATION OF TRACTION FORCE.
      C
        120 FXA=CS*SXP*XIAP**2.
            FXT=(1./3.*CS*SXP*(3.-2.*XIAP-XISP)*XIAP/(1.-XIAP)+
           +MU*SX/SP*(3.-2.*XISP-XIAP)*XISP)*(XISP-XIAP)
            FXS=MU*SX/SP*(1.-3.*XISP**2.+2.*XISP**3.)
            FX= FXA+FXT+FXS
      C
      C.....COMPUTATION OF CORNERING FORCE.
      C
            FYA=CALFA*SYP*XIAP**2.
            FYT=(1./3.*CALFA*SYP*(3.-2.*XIAP-XISP)*XIAP/(1.-XIAP)+
           +MU*SY/SP*(3.-2.*XISP-XIAP)*XISP)*(XISP-XIAP)
            FYS=MU*SY/SP*(1.-3.*XISP**2.+2.*XISP**3.)
            FY= FYA+FYT+FYS
      C
      C.....COMPUTATION OF ALIGNING MOMENT.
      C
            MZA=2./3.*SXP*SYP*(CS-CALFA)*XIAP**3.-1./6.*SYP*CALFA
           +     *(4.*XIAP**3.-3.*XIAP**2.)
            MZT=SY*(CS-CALFA)*(SX/((1.-SX)**2.)*XIAP**2./((1.-XIAP)**2.)
           +*1./15.*(6.*XIAP**2.+3.*XIAP*XISP+XISP**2.-15.*XIAP-5.*XISP+10.))
           +*(XISP-XIAP)
            MZT=MZT+SY*(CS-CALFA)*(SXP*MU/SP*(1./CS+1./CALFA)*XIAP*XISP/(1.-
           +XIAP)*.1*(3.*XISP**2.+3.*XIAP**2.+4.*XIAP*XISP-10.*(XIAP+XISP)+
           +10.))*(XISP-XIAP)
```

```
0068          MZT=MZT+SY*(CS-CALFA)*(SX*MU**2./(SP**2.*CS*CALFA)*.3*(6.*XISP**2.
             ++3.*XIAP*XISP+XIAP**2.-15.*XISP-5.*XIAP+10.))*(XISP-XIAP)
0069          MZT=MZT+SY*CALFA/(1.-SX)*XIAP/(1.-XIAP)*1./6.*(3.-3.*XIAP*(2.-XIAP
             +)-XISP*(3.-XISP)+2.*XISP*XIAP)*(XISP-XIAP)
0070          MZT=MZT+SY*MU*XISP/SP*.5*(3.-3.*XISP*(2.-XIAP)-XIAP*(3.-XIAP)+2.
             +*XIAP*XISP)*(XISP-XIAP)
0071          MZS=MU*SY/SP*(.6*MU*SX/SP*(1./CALFA-1./CS)
             +    *(1.-10.*XISP**3.+15.*XISP**4.-6.*XISP**5.)
             +    -1.5*(XISP**2.-2.*XISP**3.+XISP**4.))
0072          MZP=-(BX/KX-BY/KY)*FX*FY
0073          MZ= MZA+MZT+MZS+MZP/L
        C
        C.....MULTIPLY BY FZ AND L.
        C
0074      999 X=-FX*FZ
0075          Y=-FY*FZ
0076          Z=-MZ*FZ*L
        C
        C     ---------------------------------------------------------------
        C
0077          RETURN
0078          END

        ROUTINES CALLED:
        SIN   , COS   , ABS   , SQRT

        OPTIONS = /ON,/OP

        BLOCK       LENGTH
        TMHS3   1192   (004520)*
        BLK1    26     (000064)

        **COMPILER ----- CORE**
           PHASE       USED  FREE
        DECLARATIVES 00216 01576
        EXECUTABLES  00879 00913
        ASSEMBLY     00673 03685

DK1:TMHS3,LP:=CR:
```

```
C     TMSK1          SAKA1        (DOC.6, REF.6)
C     ------------------------------------------------------------
C
      SUBROUTINE TMSKI(SX,ALFA,FX,FY,MZ,X,Y,Z,XIA)
C
C     ------------------------------------------------------------
C
C     PARABOLIC PRESSURE DISTRIBUTION
C....COMPLETE SLIDING POSSIBLE WITHOUT WHEEL LOCK.
C     WHEEL LOCK (SX=1.) CAN BE SPECIFIED.
C     NO TRANSITION FROM ADHESION TO SLIDING.
C
C     ------------------------------------------------------------
C
      REAL MUO,MUX,MUY,MZ,KY,L,KX
      COMMON/BLK1/MUO,AS,V,MUX,MUY,CS,CALFA,
     +        KX,KY,BX,BY,L,FZ
C
C     ------------------------------------------------------------
C
      ALF=ALFA * 3.1416/180.
      SY=SIN(ALF)/COS(ALF)
      IF(SX-1.)10,17,17
   17 XIA=0.
      SX=1.
      SP=SQRT((ABS(SX)**2.)+(ABS(SY)**2.))
      GO TO 40
C....SPECIAL CASE FOR SX=SY=0.
   10 IF(ABS(SX).GE..001)GOTO 16
      IF(ABS(SY).GE..001)GOTO 16
   15 FX=0.
      FY=0.
      MZ=0.
      XIA=1.
      GO TO 20
   16 CONTINUE
      SXP=SX/(1.-SX)
      SYP=SY/(1.-SX)
      SP=SQRT((ABS(SX)**2.)+(ABS(SY)**2.))
C
C.....DETERMINE ADHESION LIMIT.
C
      XIA=1.-SQRT((ABS(SX*CS)**2.)+(ABS(SY*CALFA)**2.))/(3.*MUO*(1.-SX))
      IF(XIA)40,40,50
   50 IF(1.-XIA)60,60,70
C
C.....COMPLETE ADHESION
C
   60 FX=CS*SXP
      FY=(CALFA+CS*SX)*SYP
      MZ=.1666*SYP*(3.*CS*SX-CALFA)+FX*FY/(KY*L)
      GO TO 20
C
C.....ADHESION AND SLIDING
C
```

```
0029          70 FX=CS*SXP*XIA**2.+MUX*(SX/SP)*(1.-XIA)**2.*(1.+2.*XIA)
0030             FY=(CALFA+CS*SX)*SYP*XIA**2.+MUY*(SY/SP)*(1.-XIA)**2.*(1.+2.*XIA)
0031             MZ=.1666*SYP*(3.*CS*SX+CALFA*(3.-4.*XIA))*XIA**2.
               +    +.5*(SY/SP)*(MUX*SX*(1.+3.*XIA)-3.*MUY*XIA)*(1.-XIA)**2.*XIA
               +    +FX*FY/(KY*L)
0032             GO TO 20
        C
        C.....COMPLETE SLIDING
        C
0033          40 FX=MUX*SX/SP
0034             FY=MUY*SY/SP
0035             MZ=FX*FY/(KY*L)
0036             XIA=0.
        C
        C.....MULTIPLY BY FZ AND L.
        C
0037          20 X=-FX*FZ
0038             Y=-FY*FZ
0039             Z=-MZ*FZ*L
        C
        C        ---------------------------------------------------------------
        C
0040             RETURN
0041             END

        ROUTINES CALLED:
        SIN    , COS    , SQRT   , ABS

        OPTIONS = /ON,/OP

        BLOCK          LENGTH
        TMSKI    605      (002272)*
        BLK1     26       (000064)

        **COMPILER ----- CORE**
           PHASE         USED  FREE
        DECLARATIVES  00216 01576
        EXECUTABLES   00639 01153
        ASSEMBLY      00477 03881

 DK1:SK1,LP:=CR:
```

```
      C     TMGDR           GOODYEAR  (DOC.6, REF.5)
      C     -------------------------------------------------------------
      C
0001        SUBROUTINE TMGDR(SX,ALFA,FX,FY,MZ,X,Y,Z,XIA,XOLD,YOLD,ZOLD)
      C
      C     -------------------------------------------------------------
      C
      C     PARABOLIC PRESSURE DISTRIBUTION
      C     COMPLETE SLIDING OCCURS BEFORE WHEEL LOCK
      C     WHEEL LOCK (SX=1.) SHOULD NOT BE SPECIFIED.
      C     NO TRANSITION FROM ADHESION TO SLIDING.
      C
      C     -------------------------------------------------------------
      C
0002        REAL MUO,MUX,MUY,MZ,KY,L,KX
0003        COMMON/BLK1/MUO,AS,V,MUX,MUY,CS,CALFA,
          +          KX,KY,BX,BY,L,FZ
      C
      C     -------------------------------------------------------------
      C
0004        ALF=ALFA*.0174533
0005        SY=SIN(ALF)/COS(ALF)
0006        IF(SX-1.)10,12,15
0007     15 FX=0.
0008        FY=0.
0009        MZ=0.
0010        XIA=1.
0011        GO TO 20
0012     12 SX=.999
      C....SPECIAL CASE FOR SX=SY=0.
0013     10 IF(ABS(SX).GE..005)GOTO 16
0014        IF(ABS(SY).LE..005)GOTO 15
0015     16 CONTINUE
0016        SXP=SX/(1.-SX)
0017        SYP=SY/(1.-SX)
0018        TEMP=SQRT(ABS(SX*CS)**2.+ABS(SY*CALFA)**2.)
      C
      C.....DETERMINE ADHESION LIMIT.
      C
0019        XIA=1.-TEMP/(MUO*(1.-SX)*3.)
0020        IF(XIA)40,40,50
0021     50 IF(1.-XIA)60,60,70
      C
      C.....COMPLETE ADHESION
      C
0022     60 FX=CS*SXP
0023        FY=CALFA*SYP
0024        MZ=-.16666*CALFA*SYP+.66666*(CS-CALFA)*SXP*SYP
0025        GO TO 20
      C
      C.....ADHESION AND SLIDING
      C
0026     70 TEMP1=1.+XIA+XIA**2.
0027        FX=.33333*CS*SXP*TEMP1
0028        FY=.33333*CALFA*SYP*TEMP1
```

```
0029          MZ=-SYP*XIA**3*CALFA/6.+.066666*(CS-CALFA)*SXP*SYP*(1.+2.*XIA
             +  +3.*XIA**2+4.*XIA**3)
0030          GO TO 20
        C
        C.....COMPLETE SLIDING
        C VALUES BECOMME MEANINGLESS WHEN XIA LE 0; ASSIGN PREV. VALUES
0031       40 FX=XOLD
0032          FY=YOLD
0033          MZ=ZOLD
0034          XIA=0.
        C
        C.....MULTIPLY BY FZ AND L.
        C
0035       20 X=-FX*FZ
0036          Y=-FY*FZ
0037          Z=-MZ*FZ*L
        C
        C     ------------------------------------------------------------------
        C
0038          RETURN
0039          END

        ROUTINES CALLED:
        SIN    , COS    , ABS    , SQRT

        OPTIONS = /ON,/OP

        BLOCK        LENGTH
        TMGDR   474     (001664)*
        BLK1    26      (000064)

        **COMPILER ----- CORE**
           PHASE       USED  FREE
        DECLARATIVES 00216 01576
        EXECUTABLES  00719 01073
        ASSEMBLY     00509 03849

DK1:TMGDR,LP:=CR:
```

```
C           DATA PLOTTING SUBROUTINE
C
      SUBROUTINE TMPLOT(ISW)
      COMMON/BLK2/ALF1(101),SX1(101),FX(101),FY(101),XMZ(101),
     +            X(101),Y(101),Z(101),
     +            SCALX,SCALY,XMIN,YMIN,IPR,ISW1,ISW3,NUMBER,N,M,ITYPE
      REAL*8 LABEL(2,5)
      REAL*8 NAME(5)
      DATA LABEL(1,1),LABEL(2,1)/'LONGITUD','.SLIP SX'/,
     +     LABEL(1,2),LABEL(2,2)/'LATERAL ','FORCE FY'/,
     +     LABEL(1,3),LABEL(2,3)/'LONGITUD','. FORCE '/,
     +     LABEL(1,4),LABEL(2,4)/'ALIGNING',' MOMENT '/,
     +     LABEL(1,5),LABEL(2,5)/'SLIP ANG','LE (DEG)'/
      DATA NAME/'MODEL 1','MODEL 2','MODEL 3','SAKAI   ','GOODYEAR'/
      IF(ISW.NE.0)GOTO(1,2,200,4,100,6,100,8) ISW3
C
C.....FIRST CALL TO PLOT,CHECK FOR VALIDITY OF CALL
      IF(ISW1.EQ.1.AND.ISW3.GE.1.AND.ISW3.LE.5) ISW=ISW3
      IF(ISW1.EQ.2.AND.ISW3.GE.6.AND.ISW3.LE.8)ISW=ISW3
      IF(ISW.NE.0)GOTO(10,20,30,40,50,60,70,75)ISW3
C
C.....ILLEGAL PLOT CALL
      WRITE(IPR,1000)ISW1,ISW3
      ISW3=0
      RETURN
C
C MAKE SURE XMZ(MOMENT) IS WITHIN BOUNDS OF GRAPH
  200 K=N
      GOTO 300
  100 K=M
  300 DO 400 I=1,K
      IF(XMZ(I).LT.-.2)XMZ(I)=-.2
      IF(XMZ(I).GT..2)XMZ(I)=.2
  400 CONTINUE
      GOTO(1,2,3,4,5,6,7)ISW3
C.....LEGAL PLOT CALL
C.....FIND MAX AND MIN POINTS, DRAW A GRID AND LABEL AXES
C
C
   10 XMAX=1.
      XMIN=0.
      YMAX=1.2
      YMIN=0.
      NXDIV=5
      NYDIV=6
      ITYPE=6
      NXSKIP=2
      NYSKIP=1
      NXSIG=1
      NYSIG=1
      NX=1
      NY=3
      GOTO 80
   20 XMAX=1.
      XMIN=0.
```

```
0038          YMAX=1.2
0039          YMIN=0.
0040          NXDIV=5
0041          NYDIV=6
0042          ITYPE=6
0043          NXSKIP=2
0044          NYSKIP=1
0045          NXSIG=1
0046          NYSIG=1
0047          NX=1
0048          NY=2
0049          GOTO 80
0050       30 XMAX=1.
0051          XMIN=0.
0052          YMAX=.2
0053          YMIN=-.2
0054          NXDIV=5
0055          NYDIV=8
0056          ITYPE=2
0057          NXSKIP=2
0058          NYSKIP=1
0059          NXSIG=1
0060          NYSIG=2
0061          NX=1
0062          NY=4
0063          GOTO 80
0064       40 XMAX=1.2
0065          XMIN=-1.2
0066          YMAX=1.2
0067          YMIN=0.
0068          NXDIV=12
0069          NYDIV=6
0070          ITYPE=4
0071          NXSKIP=2
0072          NYSKIP=2
0073          NXSIG=1
0074          NYSIG=1
0075          NX=3
0076          NY=2
0077          GOTO 80
0078       50 XMAX=1.2
0079          XMIN=-1.2
0080          YMAX=.2
0081          YMIN=-.2
0082          NXDIV=12
0083          NYDIV=8
0084          ITYPE=1
0085          NXSKIP=2
0086          NYSKIP=2
0087          NXSIG=1
0088          NYSIG=2
0089          NX=3
0090          NY=4
0091          GOTO 80
0092       60 XMAX=20.
0093          XMIN=0.
```

```
      YMAX=1.2
      YMIN=0.
      NXDIV=5
      NYDIV=6
      ITYPE=6
      NXSKIP=2
      NYSKIP=1
      NXSIG=1
      NYSIG=1
      NX=5
      NY=2
      GOTO 80
   70 XMAX=20.
      XMIN=0.
      YMAX=.2
      YMIN=-.2
      NXDIV=5
      NYDIV=8
      ITYPE=2
      NXSKIP=2
      NYSKIP=1
      NXSIG=1
      NYSIG=2
      NX=5
      NY=4
      GOTO 80
   75 XMAX=25.
      XMIN=0.
      YMAX=2000.
      YMIN=0.
      NXDIV=1
      NYDIV=5
      ITYPE=6
      NXSKIP=0
      NYSKIP=1
      NXSIG=0
      NYSIG=0
      NX=0
      NY=2
      CALL SYMBOL(4.,7.,.2,'CARPET PLOT',0.,11)
C
C.....LABEL WITH THE MODEL NAME
   80 CALL SYMBOL(4.,8., .2,NAME(NUMBER),0.,8)
      CALL GRID(XORG,YORG,XMAX,XMIN,YMAX,YMIN,8.,8.,NXDIV,NYDIV,
     +               SCALX,SCALY,ITYPE)
      CALL GRDNUM(ITYPE,XORG,YORG,XMAX,XMIN,YMAX,YMIN,8.,8.,
     +               NXDIV,NYDIV,NXSKIP,NYSKIP,NXSIG,NYSIG)
      IF(NX.NE.0) CALL PLABEL(ITYPE,1,8.,8.,LABEL(1,NX),16)
      IF(NY.NE.0) CALL PLABEL(ITYPE,2,8.,8.,LABEL(1,NY),16)
      RETURN
C
C
C ....PLOT ONE CURVE OF THE PLOT
    1 CALL PLOTPT(ITYPE,XMIN,YMIN,8.,8.,SCALX,SCALY,SX1(1),FX(1),N)
      RETURN
    2 CALL PLOTPT(ITYPE,XMIN,YMIN,8.,8.,SCALX,SCALY,SX1(1),FY(1),N)
```

```
0143          RETURN
0144        3 CALL PLOTPT(ITYPE,XMIN,YMIN,8.,8.,SCALX,SCALY,SX1(1),XMZ(1),N)
0145          RETURN
0146        4 CALL PLOTPT(ITYPE,XMIN,YMIN,8.,8.,SCALX,SCALY,FX(1),FY(1),N)
0147          RETURN
0148        5 CALL PLOTPT(ITYPE,XMIN,YMIN,8.,8.,SCALX,SCALY,FX(1),XMZ(1),N)
0149          RETURN
0150        6 CALL PLOTPT(ITYPE,XMIN,YMIN,8.,8.,SCALX,SCALY,ALF1(1),FY(1),M)
0151          RETURN
0152        7 CALL PLOTPT(ITYPE,XMIN,YMIN,8.,8.,SCALX,SCALY,ALF1(1),XMZ(1),M)
0153          RETURN
      C.....SCALE FZ (CURRENTLY STORED AS ISW) RELATIVE TO 2000
      C.....TO BE USED FOR GRAPH ONLY
0154        8 FZ=ISW/2000.
0155          DO 99 I=1,M
0156          FX(I)=-ALF1(I)*FZ
0157       99 WRITE(IPR,2000) FX(I),Y(I)
0158          CALL PLOTPT(ITYPE,XMIN,YMIN,8.,8.,SCALX,SCALY,FX(1),Y(1),M)
0159          RETURN
      C
0160   2000 FORMAT(1X,2F10.3)
0161   1000 FORMAT('1ILLEGAL COMBINATION OF VARIABLE SWEEP (ISW1) AND PLOT',
           +  ' TYPE (ISW3).  THIS SET OF DATA IGNORED.'/'0ISW1=',I3,5X,
           +  'ISW3=',I3)
      C
0162          END

      ROUTINES CALLED:
      SYMBOL, GRID  , GRDNUM, PLABEL, PLOTPT

      OPTIONS = /ON,/OP

      BLOCK        LENGTH
      TMPLOT  1372   (005270)*
      BLK2    1631   (006276)

      **COMPILER ----- CORE**
         PHASE        USED  FREE
      DECLARATIVES 00485 01307
      EXECUTABLES  00858 00934
      ASSEMBLY     00945 03413

 DK1:TMPLOT,LP:=CR:
```

1.5 EXAMPLES

The following three examples were chosen to demonstrate the flexibility of the computer program in producing output from one or more tire models and in sweeping over ranges of s_x and α.

Example 1

Here it is desired to obtain F_y vs. α from tire model HSRI-II only. The output is to be given for α ranging from 1 to 16 degrees in 1 degree increments. The following 8 data cards are needed.

Card A (4I1)

ISW1 = 2	sweep α as indicated on Card C
ISW2 = 1	print output
ISW3 = 0	
ISW4 = 1	type 1 input module

Card B (2F4.0, I3)

[blank]	default (s_x=0.) path is taken

Card C (2F4.0, I3)

ALF11 = 1.	initial α
DALFA = 1.	α increments
M = 16	evaluations

Card D (2F8.3)

CS = 19251.4

CALFA = 9625.7

Card E (4F8.3)

KX = 1000.

KY = 500.

BX = 1.

BY = 1.

Card F (2F8.3)

L = 5.6

FZ = 800.

Card G (5I1)

MODEL(1)=2 exercise HSRI-II only

Card H (5F8.3)

MUO = 1.224

AS = .004

determined by friction data program (see Sec. 2.3)

V = 20.

MUX = 0.

MUY = 0.

relevant only for Sakai model

Punched card listing

```
 21 1

   1.  1. 16
  19251.4  9625.7
    1000.     500.        1.       1.
      5.6     800.
 2
    1.224     .004       20.
```

The program output for Example 1 is reproduced on the next page.

```
  SLIP ANGLE    LONG, SLIP    FORCE-LONG    FORCE-LAT      MOMENT       ADHESION     TRANSITION
   (ALPHA)         (SX)          (FX)          (FY)         (MZ)          (XIA)         (XIS)
   DEGREES       PERCENT          LB.           LB.         LB-IN
  ----------    ----------    ----------    ----------    ----------    ----------    ----------

MODEL 2   FZ= 800.0


      1.0          0.00          0.0         -168.0        156.8         2.914         4.365
      2.0          0.00          0.0         -336.1        313.7         1.457         2.179
      3.0          0.00          0.0         -504.0        469.4         0.971         1.450
      4.0          0.00          0.0         -622.3        511.9         0.727         1.085
      5.0          0.00          0.0         -692.7        484.4         0.581         0.866
      6.0          0.00          0.0         -739.0        444.0         0.484         0.720
      7.0          0.00          0.0         -771.7        404.5         0.414         0.615
      8.0          0.00          0.0         -796.0        369.0         0.362         0.537
      9.0          0.00          0.0         -814.6        337.8         0.321         0.476
     10.0          0.00          0.0         -829.2        310.7         0.288         0.427
     11.0          0.00          0.0         -841.0        287.0         0.262         0.387
     12.0          0.00          0.0         -850.6        266.3         0.239         0.353
     13.0          0.00          0.0         -858.5        247.9         0.220         0.325
     14.0          0.00          0.0         -865.1        231.7         0.204         0.300
     15.0          0.00          0.0         -870.7        217.2         0.190         0.279
     16.0          0.00          0.0         -875.5        204.1         0.177         0.260
```

From previous page

```
 MUG=     1.224
AS=    0.004
V=   20.000
MUX=     0.000
MUY=     0.300
CS=19251.400
CALFA= 9625.700
KX=1000.000
KY= 500.000
BX=    1.000
BY=    1.000
L=    5.600
FZ=    800.000
```

Computer output for Example 1

Example 2

Here it is desired to obtain F_x, F_y, and M_z vs. s_x and α from the HSRI-I, -II, and Sakai tire models. The output is to be given for two α-paths (0° and 2°) where s_x is swept through the six default values. The following 8 data cards are required.

Card A (4I1)

ISW1 = 1	sweep s_x, take α-paths indicated on Card C
ISW2 = 1	print output
ISW3 = 0	
ISW4 = 1	type 1 input module

Card B (2F4.0, I3)

[blank]	use default s_x values

Card C (2F4.0, I3)

ALF11 = 0.	initial α
DALFA = 2.	α increments
M = 2	α-paths

Card D (2F8.3)

CS = 21774.

CALFA = 10887.

Card E (4F8.3)

KX = 1000.

KY = 500.

BX = 1.

BY = 1.

Card F (2F8.3)

L = 7.1

FZ = 1100.

Card G (5I1)

MODEL(1) = 1

MODEL(2) = 2

MODEL(3) = 4

Card H (5F8.3)

MUO = .6

AS = .025

V = 20.

MUX = .37

MUY = .37

Punched card listing

```
11 1

  0.  2.   2
  21774.  10887.
   1000.    500.      1.      1.
     7.1   1100.
124
      .6    .025     20.     .37     .37
```

The resulting computer output is shown on the next page. In examining this output, it should be noted that: (a) the HSRI-I model does not compute aligning moment, (b) the transition region (XIS) is relevant only for the HSRI-II model, (c) the Sakai model, which has parabolic contact pressure, loses adhesion at a low value of longitudinal slip ($s_x < .2$).

```
          SLIP ANGLE    LONG, SLIP    FORCE-LONG    FORCE-LAT      MOMENT      ADHESION    TRANSITION
           (ALPHA)        (SX)          (FX)          (FY)          (MZ)         (XIA)        (XIS)
           DEGREES       PERCENT        LB.           LB.          LB-IN
          ----------    ----------    ----------    ----------    ----------    ----------    ----------

MODEL 1    FZ=1100.0


               0.0          0.00           0.0           0.0           0.0         1.000         0.000
               0.0          0.20        -577.8           0.0           0.0         0.055         0.000
               0.0          0.40        -523.2           0.0           0.0         0.018         0.000
               0.0          0.60        -460.4           0.0           0.0         0.007         0.000
               0.0          0.80        -395.5           0.0           0.0         0.002         0.000
               0.0          1.00        -330.0           0.0           0.0         0.000         0.000

               2.0          0.00           0.0        -372.0           0.0         0.853         0.000
               2.0          0.20        -574.8         -50.2           0.0         0.054         0.000
               2.0          0.40        -522.3         -22.8           0.0         0.018         0.000
               2.0          0.60        -460.0         -13.4           0.0         0.007         0.000
               2.0          0.80        -395.4          -8.6           0.0         0.002         0.000
               2.0          1.00        -329.9          -5.8           0.0         0.000         0.000
MODEL 2    FZ=1100.0


               0.0          0.00           0.0           0.0           0.0         1.000         1.000
               0.0          0.20        -581.4           0.0           0.0         0.061         0.164
               0.0          0.40        -525.6           0.0           0.0         0.023         0.055
               0.0          0.60        -461.8           0.0           0.0         0.010         0.021
               0.0          0.80        -396.1           0.0           0.0         0.004         0.007
               0.0          1.00        -330.0           0.0           0.0         0.000         0.000

               2.0          0.00           0.0        -372.8         426.0         0.868         1.279
               2.0          0.20        -572.5         -95.3         -42.3         0.060         0.161
               2.0          0.40        -523.2         -44.9         -24.0         0.023         0.054
               2.0          0.60        -460.8         -26.6         -13.5         0.010         0.021
               2.0          0.80        -395.7         -17.2          -7.8         0.004         0.007
               2.0          1.00        -329.8         -11.5          -4.4         0.000         0.000
SAKAI      FZ=1100.0


               0.0          0.00           0.0           0.0           0.0         1.000         0.000
               0.0          0.20        -407.0           0.0           0.0         0.000         0.000
               0.0          0.40        -407.0           0.0           0.0         0.000         0.000
               0.0          0.60        -407.0           0.0           0.0         0.000         0.000
               0.0          0.80        -407.0           0.0           0.0         0.000         0.000
               0.0          1.00        -407.0           0.0           0.0         0.000         0.000

               2.0          0.00           0.0        -287.5         172.4         0.808         0.000
               2.0          0.20        -400.9         -70.0         -56.1         0.000         0.000
               2.0          0.40        -405.5         -35.4         -28.7         0.000         0.000
               2.0          0.60        -406.3         -23.6         -19.2         0.000         0.000
               2.0          0.80        -406.6         -17.7         -14.4         0.000         0.000
               2.0          1.00        -406.8         -14.2         -11.6         0.000         0.000
```

```
 MUO=   0.600
AS=   0.025
V=  20.000
MUX=   0.370
MUY=   0.370
CS=21774.000
CALFA=10887.000
KX=1000.000
KY= 500.000
BX=   1.000
BY=   1.000
L=   7.100
FZ=  1100.000
```

Computer output for Example 2

Example 3

This example illustrates the use of a Type 2 data module for the requirement of obtaining F_y and M_z vs. α (at s_x=0.) from all five tire models. Frictional performance data for each tire model has been calculated by the friction data program described in Section 2.0. The output from each tire model is to be obtained at three speeds (20, 40, and 55 mph). Since these models assume the tire-road friction coefficient to be speed-dependent, this necessitates the use of unique friction data for each model at each speed. A total of 37 data cards are required for the Type 2 data module.

Card A (4I1)

ISW1 = 2	sweep α as indicated on Card C
ISW2 = 1	print output
ISW3 = 0	
ISW4 = 2	type 2 input module

Card B (2F4.0, I3)

[blank]	default (s_x=0.) path is taken

Card C (2F4.0, I3)

ALF11 = 0.	initial α
DALFA = .5	α increments
M = 25	evaluations

Card D (2F8.3)

CS = 21774.

CALFA = 10887.

Card E (4F8.3)

KX = 1000.

KY = 500.

BX = 1.

BY = 1.

Card F (2F8.3)

L = 7.1

FZ = 1100.

Card G (5I1)

MODEL(1) = 1 exercise HSRI-I model

Card H (5F8.3)

$$\left.\begin{array}{l}\text{MUO} = .7089\\ \text{AS} = .0551\\ \text{V} = 20.\end{array}\right\}\quad \text{data for HSRI-I model}$$

Card G (5I1)

MODEL(1) = 2 exercise HSRI-II model

Card H (5F8.3)

$$\left.\begin{array}{l}\text{MUO} = .698\\ \text{AS} = .054\\ \text{V} = 20\end{array}\right\}\quad \text{data for HSRI-II model}$$

.

. (11 G-H pairs)

.

Card G (5I1)

MODEL(1) = 4 exercise Sakai model

Card H (5F8.3)

MUO = .611

AS = 0.

V = 55. data for Sakai model

MUX = 0.

MUY = .286

Card G (5I1)

MODEL(1) = 5 exercise Goodyear model

Card H (5F8.3)

MUO = .3455

AS = 0. data for Goodyear model

V = 55.

[a blank card]

A blank card terminates a Type 2 module which may be followed by a Type 1 module or another Type 2 module. The complete data module for this example is seen in the punched card listing reproduced below.

Listing of the Type 2 data module for Example 3.

```
21 2
  0.  .5 25
  21774.  10887.
   1000.    500.       1.       1.
     7.1   1100.
1
   .7089   .0551     20.
2
    .698    .054     20.
3
    .572    .023     20.
4
    .709             20.               .515
5
   .5440             20.
1
   .6477   .0509     40.
2
    .634    .051     40.
3
    .563    .053     40.
4
    .684             40.               .371
5
   .4364             40.
1
   .5017   .0373     55.
2
    .494    .037     55.
3
    .033    .060     55.
4
    .611             55.               .286
5
   .3455             55.
```

The first two and last two pages of the computer output for Example 3 are reproduced on the next two pages.

```
          SLIP ANGLE    LONG, SLIP    FORCE-LONG    FORCE-LAT      MOMENT      ADHESION     TRANSITION
           (ALPHA)         (SX)          (FX)          (FY)          (MZ)         (XIA)          (XIS)
           DEGREES       PERCENT         LB.           LB.          LB-IN
          ----------    ----------    ----------    ----------    ----------    ----------    ----------

MODEL 1   FZ=1100.0


               0.0          0.00          0.0           0.0           0.0         1.000          0.000
               0.5          0.00          0.0         -95.0           0.0         4.064          0.000
               1.0          0.00          0.0        -190.0           0.0         2.012          0.000
               1.5          0.00          0.0        -285.1           0.0         1.328          0.000
               2.0          0.00          0.0        -380.1           0.0         0.986          0.000
               2.5          0.00          0.0        -452.5           0.0         0.781          0.000
               3.0          0.00          0.0        -498.2           0.0         0.644          0.000
               3.5          0.00          0.0        -528.7           0.0         0.546          0.000
               4.0          0.00          0.0        -549.7           0.0         0.473          0.000
               4.5          0.00          0.0        -564.3           0.0         0.416          0.000
               5.0          0.00          0.0        -574.5           0.0         0.370          0.000
               5.5          0.00          0.0        -581.4           0.0         0.333          0.000
               6.0          0.00          0.0        -586.0           0.0         0.301          0.000
               6.5          0.00          0.0        -588.6           0.0         0.275          0.000
               7.0          0.00          0.0        -589.8           0.0         0.253          0.000
               7.5          0.00          0.0        -589.9           0.0         0.233          0.000
               8.0          0.00          0.0        -589.0           0.0         0.216          0.000
               8.5          0.00          0.0        -587.3           0.0         0.201          0.000
               9.0          0.00          0.0        -585.0           0.0         0.187          0.000
               9.5          0.00          0.0        -582.1           0.0         0.175          0.000
              10.0          0.00          0.0        -578.8           0.0         0.164          0.000
              10.5          0.00          0.0        -575.1           0.0         0.154          0.000
              11.0          0.00          0.0        -571.0           0.0         0.146          0.000
              11.5          0.00          0.0        -566.7           0.0         0.137          0.000
              12.0          0.00          0.0        -562.1           0.0         0.130          0.000


           MUO=    0.709
          AS=    0.055
          V=  20.000
          MUX=    0.000
          MUY=    0.000
          CS=21774.000
          CALFA=10887.000
          KX=1000.000
          KY= 500.000
          BX=    1.000
          BY=    1.000
          L=   7.100
          FZ=  1100.000
```

First two pages of computer output for Example 3.

```
  SLIP ANGLE    LONG. SLIP   FORCE-LONG   FORCE-LAT     MOMENT      ADHESION    TRANSITION
   (ALPHA)         (SX)         (FX)         (FY)         (MZ)        (XIA)        (XIB)
   DEGREES       PERCENT        LB.          LB.          LB-IN
  ----------    ----------   ----------   ----------   ----------   ----------   ----------

GOODYEAR  FZ=1100.0


      0.0          0.00          0.0          0.0          0.0        1.000        0.000
      0.5          0.00          0.0        -87.3         86.6        0.917        0.000
      1.0          0.00          0.0       -160.1        130.1        0.833        0.000
      1.5          0.00          0.0       -219.7        142.3        0.750        0.000
      2.0          0.00          0.0       -267.5        133.2        0.667        0.000
      2.5          0.00          0.0       -304.7        111.5        0.583        0.000
      3.0          0.00          0.0       -332.7         84.2        0.500        0.000
      3.5          0.00          0.0       -352.7         56.7        0.416        0.000
      4.0          0.00          0.0       -366.1         33.1        0.332        0.000
      4.5          0.00          0.0       -374.2         15.6        0.248        0.000
      5.0          0.00          0.0       -378.4          5.0        0.165        0.000
      5.5          0.00          0.0       -379.8          0.6        0.081        0.000
      6.0          0.00          0.0       -379.8          0.6        0.000        0.000
      6.5          0.00          0.0       -379.8          0.6        0.000        0.000
      7.0          0.00          0.0       -379.8          0.6        0.000        0.000
      7.5          0.00          0.0       -379.8          0.6        0.000        0.000
      8.0          0.00          0.0       -379.8          0.6        0.000        0.000
      8.5          0.00          0.0       -379.8          0.6        0.000        0.000
      9.0          0.00          0.0       -379.8          0.6        0.000        0.000
      9.5          0.00          0.0       -379.8          0.6        0.000        0.000
     10.0          0.00          0.0       -379.8          0.6        0.000        0.000
     10.5          0.00          0.0       -379.8          0.6        0.000        0.000
     11.0          0.00          0.0       -379.8          0.6        0.000        0.000
     11.5          0.00          0.0       -379.8          0.6        0.000        0.000
     12.0          0.00          0.0       -379.8          0.6        0.000        0.000




   MUO=    0.345
  AS=    0.000
  V=  55.000
  MUX=    0.000
  MUY=   0.000
  CS=21774.000
  CALFA=10887.000
  KX=1000.000
  KY= 500.000
  BX=    1.000
  BY=    1.000
  L=   7.100
  FZ=  1100.000
```

Final two pages of computer output for Example 3.

2.0 TIRE MODEL DATA PREPARATION

The semi-empirical tire models, whose programs are listed in Section 1.4, require input data which is derived from the results of full-scale tire traction tests made in the laboratory or on the road. There is no way, at this time, to relate tire model input data to tire design characteristics. Thus, the model input data are strictly valid only for a specific tire, at a specific inflation pressure, and carrying a specific load. The frictional performance data (μ_o, A_s, etc.) is further restricted to a specific paved surface and water cover (if any). Notwithstanding the above restrictions, it is frequently possible to estimate changes in input data produced by slight changes in tire design or operating conditions.

The tire models with which this report is concerned all simulate tire tread and carcass elasticity with the same linear elements; specifically, the tread-carcass structure is an array of linear shear springs (tread) attached to a rigid beam which is supported by a linear spring foundation (carcass). The determination of tire structure input data, C_s, C_α, K_x, K_y, common to all of the models, is described in Document 6 (pp. 77-88). Again, it is emphasized that tire structure input data relate to a specific tire at a specific pressure and load (F_z). Changing the load, pressure, or tire, requires consideration of corresponding changes in C_s, C_α, K_x, and K_y in addition to the contact length, L.

The tire models all vary in their schemes for simulating tire-road frictional performance; viz, the generation of the friction-dependent shear force distribution, σ, expressed by

$$\sigma = \mu \cdot p$$

where μ is the tire-road friction potential and p is the normal contact pressure distribution. A pressure distribution, uniform or parabolic, is programmed into each of the tire models; no pressure distribution input data is required. The tire-road friction potential is calculated by the various models with the input data A_s, μ_o, μ_x, μ_y as indicated in Table 3, where V_s is the slip velocity.

TABLE 3

FRICTION PERFORMANCE FACTORS

Model	μ	p
HSRI-I	$\mu_o(1-A_sV_s)$	uniform
HSRI-II	$\mu_o(1-A_sV_s)$	uniform
HSRI-III	$\mu_o(1-A_sV_s)$	parabolic
Goodyear	μ_o	parabolic
Sakai	$\mu_o, \underbrace{\mu_x \text{ and } \mu_y}_{\text{sliding}}$	parabolic

The remainder of this section is concerned with the preparation of frictional performance data.

2.1 FRICTIONAL PERFORMANCE DATA

The frictional performance input data for a particular tire model is determined such that the model will reproduce selected points on measured traction force curves. For example, the HSRI models, which utilize two input parameters (μ_o and A_s), are given parameter values which cause these models to reproduce two points in the high slip region of the measured F_x vs. s_x or F_y vs. α curves. Because of model differences, values of μ_o and A_s for one of the HSRI models will not be optimum for the other two HSRI models.

A digital computer program has been written to calculate frictional performance data, for each of the tire models, from free-rolling F_y vs. α data measured in the laboratory or on the road. The following solution procedures are used.

HSRI-I

μ_o and A_s are obtained by solution of the two simultaneous linear equations which result when the lateral force equation for this model is evaluated at the two data points which are to be reproduced.

$$2x[C_\alpha \tan\alpha(1 - \sqrt{1 + F_y/C_\alpha \tan\alpha})/F_z]_1 + y[V_s]_1 = 1$$

$$2x[C_\alpha \tan\alpha(1 - \sqrt{1 + F_y/C_\alpha \tan\alpha})/F_z]_2 + y[V_s]_2 = 1$$

where $\mu_o = 1/x$, $A_s = y$, and subscripts 1, 2 indicate evaluation at the two data points to be reproduced by the HSRI-I model.

HSRI-II

μ_o and A_s are determined by an iterative solution for these parameters as roots of the following function, $F(\mu_o, A_s)$, which is derived from the lateral force equation for this model.

$$F(\mu_o, A_s) = F_y + \frac{1}{2}\mu_o F_z \left[2 - \frac{\xi_a}{L} - A_s V_s \left(2 - \frac{\xi_a}{L} - \frac{\xi_s}{L}\right)\right]$$

where

$$\frac{\xi_a}{L} = \frac{1}{2}\mu_o F_z / (C_\alpha \tan \alpha)$$

$$\frac{\xi_s}{L} = \frac{1}{2}\mu_o F_z (1 + C_\alpha / C_s) \times (1 - A_s V_s)/(C_\alpha \tan \alpha)$$

HSRI-III

μ_o and A_s are determined by an iterative solution for these parameters as roots of the following function, $F(\mu_o, A_s)$, which is derived from the lateral force equation for this model.

$$F(\mu_o, A_s) = F_y + \mu_o F_z \left\{1 - \left(\frac{\xi_a}{L}\right)^3 - A_s V_s \left[1 - \left(3 - \frac{\xi_a}{L} - \frac{\xi_s}{L}\right)\frac{\xi_a}{L}\frac{\xi_s}{L}\right]\right\}$$

where

$$\frac{\xi_a}{L} = 1 - C_\alpha \tan \alpha / (3\mu_o F_z)$$

$$\frac{\xi_s}{L} = 1 - \left(\frac{C_\alpha C_s}{C_\alpha + C_s}\right) \frac{\tan \alpha}{3\mu_o (1 - A_s V_s) F_z}$$

Goodyear

μ_o is obtained by evaluating the following equation (derived by rewriting the lateral force equation for this model) at the peak datum point of the free-rolling F_y vs. α test data.

$$\mu_o = \frac{C_\alpha \tan \alpha}{6 F_z} \left(\frac{\sqrt{1 + \frac{4}{3}\left[1 - \frac{F_y}{C_\alpha \tan \alpha}\right]} - 1}{1 - \frac{F_y}{C_\alpha \tan \alpha}} \right)$$

Sakai

μ_o and μ_y are determined by an iterative solution for these parameters as roots of the following function, $F(\mu_o,A_s)$, which is derived from the lateral force equation for this model.

$$F(\mu_o, \mu_y) = F_y \left\{ 3\mu_o\left(1 - \frac{\xi_a}{L}\right)\left(\frac{\xi_a}{L}\right)^2 + \mu_y\left[1 - 3\left(\frac{\xi_a}{L}\right)^2 + 2\left(\frac{\xi_a}{L}\right)^3\right] \right\}$$

where

$$\frac{\xi_a}{L} = 1 - C_\alpha \tan \alpha/(3\mu_o F_z)$$

and the valid roots are such that μ_y is less than μ_o.

The iterative solutions required for the HSRI-I, -II, and Sakai models are obtained by the Newton-Raphson method for finding simultaneous roots of two functions, F_1 and F_2, of two variables (μ_o and A_s or μ_y). The two functions are $F(\mu_o, A_s \text{ or } \mu_y)$, written above for the respective models, evaluated at two data points on the F_y vs. α curve which are to be reproduced by the model. The Newton-Raphson method provides corrections, $\Delta\mu_o$ and ΔA_s (or $\Delta\mu_y$), to an estimate of the roots (say μ_o^1 and A_s^1) by solution of the following simultaneous linear equations for these corrections.

$$\frac{\partial F_1}{\partial \mu_o} \Delta\mu_o + \frac{\partial F_1}{\partial A_s} \Delta A_s = - F_1$$

$$\frac{\partial F_2}{\partial \mu_o} \Delta\mu_o + \frac{\partial F_2}{\partial A_s} \Delta A_s = - F_2$$

where

$\frac{\partial F_1}{\partial \mu_o}$, $\frac{\partial F_1}{\partial A_s}$, F_1, etc., are constants calculated from the uncorrected values of μ_o and A_s. The corrected roots

$$\mu_o^2 = \mu_o^1 + \Delta\mu_o$$

$$A_s^2 = A_s^1 + \Delta A_s$$

are then used in repeating the procedure until the corrections become negligible. In most cases, a solution is obtained with a small number of iterations (typically 4-6).

The functions $F(\mu_o, A_s)$ for the HSRI-II, -III models and $F(\mu_o, \mu_y)$ for the Sakai model are highly nonlinear and a comprehensive analysis for the region where valid roots may be found has yet to be performed. In some cases, the program presented below is unable to find roots for one or more of these models. These cases are, usually, a consequence of abnormal data. It is mathematically impossible for certain models to reproduce some pairs of data points in the high-slip region, given C_α determined in the low slip region of the same F_y vs. α test data. Fortunately, these cases seem to be in the minority, at least for the HSRI mobile tire tester and flat bed data used so far.*

2.2 COMPUTER PROGRAM

The program described and listed in this section was developed to compute frictional performance data for the five tire models called by TMMAIN. The program requires the following input data obtained from lateral force vs. slip angle measurements made by a flat bed tire tester (low speed data) or a mobile tire tester (high speed data).

Data Card 1

FZ	tire load, F_z

---

*Wet and dry mobile tire tester data from 9 different tires (including 3 construction types) tested on two surfaces (asphalt and concrete) have been used in testing the frictional performance data program.

CALFA	lateral traction stiffness, C_α
SWITCH	real variable indicating high speed data (=0.) or low speed data (≠0.)
CS	longitudinal traction stiffness, C_s

<u>Data Card 2</u> (FY must have a sign opposite to the sign of ALFA)

ALFA(1)	data point 1
FY(1)	
ALFA(2)	data point 2
FY(2)	
V	traveling velocity

The program finds values of μ_o and A_s which make the HSRI-I, -II, -III models reproduce the two data points on Card 2. Values of μ_o and μ_y are found which make the Sakai model reproduce these two points. A value of μ_o is found which makes the Goodyear model reproduce the data point for which F_y is largest.

Initial values of $\mu_o = 1.$ and $A_s = 0.$ are used to start the Newton-Raphson iterative solution for the HSRI-II model. These starting values have enabled the iteration to converge to a valid solution for the HSRI-II model with all data tried. The initial values of $\mu_o = C_\alpha \tan \alpha/(3F_z) + .1$ and $A_s = 0.$ or $\mu_y = \mu_o/2$ have enabled the iteration to converge to a valid solution for the HSRI-III and Sakai models with most data tried. The initial values are computed as part of the program; they are not read in as input data.

The program detects a nonconvergent solution if the result of an iteration differs from the previous result by more than 5.0, or if no convergence has been achieved after 10 iterations. Convergence is assumed if the result of an iteration differs from the previous result by less than .0001 (μ_o and A_s or μ_y). Non-convergence can usually be traced to an anomaly in the measured data.

The program may fail to converge for the Sakai model for the reason that $\mu_o = \mu_y$ produces the best fit to the selected data points. This result, which occurs in the example given below, is evident on examination of the printed iteration history.

When SWITCH is nonzero, flat bed or other low-speed data are indicated and the $F_y - \alpha$ data point with greatest F_y magnitude is used to determine μ_o for all models. In this case, the velocity input, V, is irrelevant and $A_s = 0$.

```
C      FRICTION DATA PREPARATION PROGRAM
C
       REAL MUO,MUY
       DIMENSION F(2),SY(2),FY(2),DFMU(2),DFAS(2),ALFA(2),VS(2)
       DIMENSION A(2),B(2),C(2),DFMUY(2)
C      ASSIGN LOGICAL DFVICE NUMBERS
       IPR=5
       IRD=8
C
C.....READ IN VALUFS
C.....FOR FLATBED DATA, PUT SWITCH=1.,LEAVE ALFA(2),FY(2),V BLANK
C.....FOR OTHER DATA LFAVE SWITCH BLANK OR ENTER 0.
C.....FNTER  FY QUANTITY AS A NEGATIVE NUMBER
  100  READ(IRD,1000,FND=40)FZ,CALFA,SWITCH,CS
       READ(IRD,1000,FND=40)ALFA(1),FY(1),ALFA(2),FY(2),V
       IF(CS.EQ.0.)CS=2.*CALFA
       WRITF(IPR,4000)FZ,CALFA,CS,V,ALFA,FY
C
C
C.....CALCULATE VS AND TAN(ALFA)
       DO 10 I=1,2
       ALFA(I)=ALFA(I)*.0174533
       SY(I)=SIN(ALFA(I))/COS(ALFA(I))
   10  VS(I)=V*SIN(ALFA(I))
C
C
C.....SOLVE FOR GOODYEAR MODFL ROOT OF MUO
C.....PICK THE PEAK FORCF OF THE 2 VALUES GIVEN
C
       MAX=1
       IF(ABS(FY(2)).GT.ABS(FY(1)))MAX=2
       T=1.+FY(MAX)/(CALFA*SY(MAX))
       IF(T.LE..75) MUO=CALFA*SY(MAX)/(6.*FZ)*(1.+SQRT(1.-4./3.*T))/T
C.....IF SLIDING OCCURS BEFORE ALFA(MAX), IE XIA<0, OR T TOO LARGE,
C.....PICK MUO DIFFERENTLY
       IF(3.*MUO*FZ/CALFA.LT.SY(MAX).OR.T.GT..75)MUO=ABS(FY(MAX))/FZ
       WRITE(IPR,6000)MUO
C
C
C.....IF SWITCH TS NOT EQUAL TO 0., FLATBED DATA IS INDICATED;
C.....CALCULATE UNIFORM PRFSSURE MUO FROM HSRI-1 EQN
C.....USING THE PEAK FORCE OF THE 2 GIVEN (ONLY ONF IS NECESSARY)
       IF(SWITCH.FQ.0.)GOTO 1
       T1=CALFA*SY(MAX)
       MUO =2.*T1*(1.-SQRT(1.+FY(MAX)/T1))/FZ
       WRITF(IPR,1500)MUO
       GOTO 100
C
C
C.....SOLVF FOR MODEL1  ROOTS OF AS AND MUO
C
    1  DO 50 I=1,2
       T1=CALFA*SY(I)
       A(I)=2.*T1*(1.-SQRT(1.+FY(I)/T1))/FZ
C.....MANIPULATE ARRAYS FOR SOLUTION OF THE TWO SIMULTANEOUS LINEAR EQN
```

```
      C(I)=1./A(I)
      B(I)=VS(I)/A(I)
   50 CONTINUE
C
      AS=(C(1)-C(2))/(B(1)-B(2))
      MUO=1./(C(1)-AS*B(1))
      WRITE(IPR,3000)MUO,AS
C
C.....SOLVE FOR MODEL 2 ROOTS OF MUO AND AS, USING THE NEWTON-RAPHSON
C.....METHOD AND STARTING WITH MUO=1, AS=0
C
      MUO=1.
      AS=0.
      WRITE(IPR,9000)
      DO 90 KOUNT=1,10
      DO 80 I=1,2
      T1=.5*FZ/(CALFA*SY(I))
      XIA=T1*MUO
      T2=1.-AS*VS(I)
      T3=1.+CALFA/CS
      XIS=T2*T3*XIA
      T=2.-XIA-AS*VS(I)*(2.-XIA-XIS)
      F(I)=FY(I)+.5*MUO*FZ*T
C.....FIND DERIVATIVES OF XIA,XIA AND F WITH RESPECT TO MUO AND AS
      DAMU=T1
      DSMU=T1*T2*T3
      DSAS=-XIA*VS(I)*T3
      DFMU(I)=.5*FZ*T-.5*MUO*FZ*(DAMU-AS*VS(I)*(DAMU+DSMU))
      DFAS(I)=.5*MUO*FZ*VS(I)*(AS*DSAS-(2.-XIA-XIS))
C.....DIVIDE TO SOLVE FOR MUO AND AS
      F(I)=-F(I)/DFMU(I)
      DFAS(I)=DFAS(I)/DFMU(I)
   80 CONTINUE
      DAS=(F(1)-F(2))/(DFAS(1)-DFAS(2))
      DMU=F(1)-DAS*DFAS(1)
      MUO=MUO+DMU
      AS=AS+DAS
      WRITE(IPR,2000)MUO,AS
      IF(ABS(DAS).LT.1.E-4.AND.ABS(DMU).LT.1.E-4)GOTO 11
   90 CONTINUE
   11 CONTINUE
C
C
C
C.....SOLVE FOR MODEL 3 ROOTS OF AS AND MUO, USING THE NEWTON-RAPHSON
C     METHOD,STARTING WITH ESTIMATE OF MUO FOR POSITIVE SLOPE
C
      MUO=.1+CALFA*SY(2)/(3.*FZ)
      AS=0.
      WRITE(IPR,5000)
      WRITE(IPR,2000)MUO,AS
      DO 30 KOUNT=1,10
      DO 20 I=1,2
C.....IF LOOP IS LEFT BEFORE COMPLETION, N IS TO INDICATE NUMBER OF
C.....DATA POINTS OF THE TWO GIVEN THAT ARE IN FULL SLIDING
      N=3-I
```

```
      TEMP=CALFA*SY(I)/(3.*MUO*FZ)
      XIS=1.-TEMP*CS/((CALFA+CS)*(1.-AS*VS(I)))
      XIA=1.-TEMP
      T1=3.-XIA-XIS
      T2=XIA*XIS
      T=1.-XIA**3-AS*VS(I)*(1.-T1*T2)
      F(I)=FY(I)+MUO*FZ*T
C     FIND DERIVATIVE OF XIA,XIS AND F WITH RESPECT TO MUO AND AS
      DAMU=TEMP/MUO
      DSMU=DAMU*CS/((CALFA+CS)*(1.-AS*VS(I)))
      DSAS=-DSMU*VS(I)*MUO/(1.-AS*VS(I))
      DFMU(I)=FZ*(T-MUO*(3.*XIA*XIA*DAMU+AS*VS(I)*((DAMU+DSMU)*T2
     +         -T1*(XIA*DSMU+XIS*DAMU))))
      DFAS(I)=-MUO*FZ*VS(I)*(AS*(2.*XIS+XIA-3.)*XIA*DSAS+(1.-T1*T2))
C
C.... MANIPULATE ARRAYS FOR SOLUTION OF DMU AND DAS
      F(I)=-F(I)/DFMU(I)
      DFAS(I)=DFAS(I)/DFMU(I)
   20 CONTINUE
      DAS=(F(1)-F(2))/(DFAS(1)-DFAS(2))
      DMU=F(1)-DAS*DFAS(1)
      MUO=MUO+DMU
      AS=AS+DAS
      WRITE(IPR,2000)MUO,AS
      IF(ABS(DAS).LT.1.E-4.AND.ABS(DMU).LT.1.E-4)GOTO 22
      IF(ABS(DAS).GT.5..OR.ABS(DMU).GT.5.)GOTO 55
   30 CONTINUE
   55 WRITE(IPR,2500)N
   22 CONTINUE
C
C.....SOLVE FOR SAKAI ROOTS OF MUY AND MUO, USING THE NEWTON-RAPHSON
C ....METHOD, STARTING WITH ESTIMATE OF MUO AND MUY FOR STABILITY
C
      MUO=.1+CALFA*SY(2)/(3.*FZ)
      MUY=MUO/2.
      WRITE(IPR,7000)
      WRITE(IPR,8000)MUO,MUY
      DO 70 KOUNT=1,10
      DO 60 I=1,2
C.....IF LOOP IS LEFT BEFORE COMPLETION, N IS TO INDICATE NUMBER OF
C.....DATA POINTS OF THE TWO GIVEN THAT ARE IN FULL SLIDING
      N=3-I
      TEMP=CALFA*SY(I)/(3.*MUO*FZ)
      XIA=1.-TEMP
      T1=1.-XIA
      T2=XIA*XIA
      T3=XIA*T2
      T=1.-3.*T2+2.*T3
      F(I)=FY(I)+FZ*(3.*MUO*T1*T2+MUY*T)
C.....FIND DERIVATIVE OF XIA AND F WITH RESPECT TO MUO AND MUY
      DAMU=TEMP/MUO
      DFMU(I)=6.*FZ*(1.-MUY/MUO)*T1*T1*XIA
      DFMUY(I)=FZ*T
C.....DIVIDE TO SOLVE FOR MUY AND MUO
      F(I)=-F(I)/DFMU(I)
      DFMUY(I)=DFMUY(I)/DFMU(I)
```

```
   60 CONTINUE
      DMUY=(F(1)-F(2))/(DFMUY(1)-DFMUY(2))
      DMU=F(1)-DMUY*DFMUY(1)
      MUY=MUY+DMUY
      MUO=MUO+DMU
      WRITE(IPR,8000)MUO,MUY
      IF(ABS(DAS).LT.1.E-4.AND.ABS(DMU).LT.1.E-4)GOTO 44
      IF(ABS(DMUY).GT.5..OR.ABS(DMU).GT.5.)GOTO 33
   70 CONTINUE
   33 WRITE(IPR,2500)N
   44 CONTINUE
      GOTO 100
   40 CALL EXIT
C
C
 1000 FORMAT(5F10.3)
 1500 FORMAT('0UNIFORM PRESSURE MODEL MUO=',F9.3)
 2000 FORMAT(' MUO=',F9.3,10X,'AS=',F9.3)
 2500 FORMAT(' DOES NOT SEEM TO CONVERGE.  TRY',I2,' OTHER DATA POINT(S)
     +WITH SMALLER SLIP ANGLES')
 3000 FORMAT('0MODEL1 YIELDS MUO=',F10.4,10X,'AS=',F10.4)
 4000 FORMAT('1FZ=',F10.4,/' CALFA=',F10.4/' CS=',F10.4/' V=',F10.4/
     +       ' ALFA=',F8.1,',',F8.1/' FY=',F10.4,',',F10.4//)
 5000 FORMAT('0MODEL3 YIELDS:')
 6000 FORMAT('0GOODYEAR MODEL YIELDS MUO=',F8.4)
 7000 FORMAT('0SAKAI MODEL YIELDS:')
 8000 FORMAT(' MUO=',F9.3,10X,'MUY=',F9.3)
 9000 FORMAT('0MODEL2 YIELDS:')
      END

ROUTINES CALLED:
SIN   , COS   , ABS   , SQRT  , EXIT

OPTIONS = /ON,/OP

BLOCK       LENGTH
MAIN.   2043   (007766)*

**COMPILER ----- CORE**
   PHASE       USED  FREE
DECLARATIVES 00216 01576
EXECUTABLES  00778 01014
ASSEMBLY     01148 03210

DK1:NEWTON,LP:=CR:
```

2.3 EXAMPLE

In this example, μ_o (MUO), A_s (AS), and μ_y (MUY) are found for their respective models using the following two data points.

α (deg)	F_y (lb)
11.2	-844*
15.2	-873

The following two data cards are needed for this example.

Card 1 (5F10.3)

FZ = 800.

CALFA = 9625.7

SWITCH = 0.

CS = 19251.4

Card 2 (5F10.3)

ALFA(1) = 15.2

FY(1) = -873.

ALFA(2) = 11.2

FY(2) = -844.

V = 20.

---

*In accordance with SAE sign convention, negative F_y corresponds to positive α.

Punched card listing

```
800.         9625.7       0.           19251.4
15.2         -873.        11.2         -844.        20.
```

Program Output

```
FZ=  800.0000
CALFA= 9625.7002
CS=19251.4004
V=   20.0000
ALFA=     15.2,     11.2
FY= -873.0000, -844.0000



GOODYEAR MODEL YIELDS MUO=  1.0912

MODEL1 YIELDS MUO=     1.2263             AS=     0.0038

MODEL2 YIELDS:
MUO=     1.214             AS=     0.003
MUO=     1.224             AS=     0.004
MUO=     1.224             AS=     0.004

MODEL3 YIELDS:
MUO=     0.895             AS=     0.040
MUO=     1.004             AS=    -0.018
MUO=     1.044             AS=    -0.008
MUO=     1.046             AS=    -0.008
MUO=     1.046             AS=    -0.008

SAKAI MODEL YIELDS:
MUO=     0.895             AS=     0.447
MUO=     0.856             MUY=     1.073
MUO=     0.948             MUY=     1.067
MUO=     1.075             MUY=     1.079
MUO=     5.017             MUY=     1.090
MUO=   -41.763             MUY=    59.501
DOES NOT SEEM TO CONVERGE.  TRY 1 OTHER DATA POINT
```

Although convergence is not detected for the Sakai model, the approximate solution $\mu_o = \mu_y = 1.077$ is evident from the iteration history printed above.