Web-based Supplementary Materials for 'Adaptive Contrast Weighted Learning for Multi-Stage Multi-Treatment Decision-Making'

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S-Table 1: Additional simulation results for Scenario 1 with $\varphi^{(2)}$ (500 replications, $n = 1000$) and fully randomized treatment assignments. $E\{Y^*(g^{opt})\} = 8$.

π	Method	$\varphi^{(2)}$	
		opt%	$\hat{E}\{Y^*(\hat{g}^{opt})\}$
	OWL	75.8 (11.1)	6.91 (0.60)
Correct	ACWL-C_1	89.2 (6.1)	7.63 (0.34)
	ACWL-C_2	87.9 (7.5)	7.39 (0.42)

Additional Simulation 1

This simulation follows Scenario 1 in the main paper but with treatment assignment fully random. Specifically, we have

$$A \sim Multinomial(0.2, 0.2, 0.2, 0.2, 0.2),$$

and

$$Y = \exp[2.06 + 0.2X_3 - |X_1 + X_2|\varphi\{A, g^{opt}(\mathbf{H})\}] + \epsilon,$$

with $\varphi\{A, g^{opt}(\mathbf{H})\}$ taking the form of $\varphi^{(2)} = \{A - g^{opt}(\mathbf{H})\}^2$,

$$g^{opt}(\mathbf{H}) = I(X_1 > -1)\{1 + I(X_2 > -0.4) + I(X_2 > 0.4) + I(X_2 > 1)\}$$

and $\epsilon \sim N(0, 1)$.

The results are shown in S-Table 1.

Additional Simulation 2

This simulation follows Scenario 2 in the main paper but with the treatment models dependent on X_1 and X_2, so that the treatment models and the optimal treatment models are more related than Scenario 2. Specifically, we have $A_1 \sim$ Multinomial$(\pi_{10}, \pi_{11}, \pi_{12})$, with $\pi_{10} = 1/\{1 + \exp(0.5 - 0.5X_1) + \exp(0.5X_2)\}$, $\pi_{11} = \exp(0.5 - 0.5X_1)/\{1 + \exp(0.5 - 0.5X_1) + \exp(0.5X_2)\}$, and $\pi_{12} = 1 - \pi_{10} - \pi_{11}$, and $A_2 \sim$ Multinomial$(\pi_{20}, \pi_{21}, \pi_{22})$, with $\pi_{20} = 1/\{1 + \exp(0.2R_1 - 1) + \exp(0.5X_2)\}$, $\pi_{21} = \exp(0.2R_1 - 1)/\{1 + \exp(0.2R_1 - 1) + \exp(0.5X_2)\}$, and $\pi_{22} = 1 - \pi_{20} - \pi_{21}$.

The outcome models are

$$R_1 = \exp[1.5 - |1.5X_1 + 2|\{A_1 - g_1^{opt}(\mathbf{H}_1)\}^2] + \epsilon_1,$$

with $g_1^{opt}(\mathbf{H}_1) = I(X_1 > -1)\{I(X_2 > -0.5) + I(X_2 > 0.5)\}$ and $\epsilon_1 \sim N(0, 1)$, and

$$R_2 = \exp[1.26 - |1.5X_3 - 2|\{A_2 - g_2^{opt}(\mathbf{H}_2)\}^2] + \epsilon_2,$$

with $g_2^{opt}(\mathbf{H}_2) = I(X_3 > -1)\{I(R_1 > 0.5) + I(R_1 > 3)\}$ and $\epsilon_2 \sim N(0, 1)$.

The results are shown in S-Table 2.

S-Table 2: Additional simulation results based on Scenario 2 with treatment assignment models more related to optimal treatment models (500 replications, $n = 1000$). $E\{Y^*(\mathbf{g}^{opt})\} = 8$.

π	Method	Tree-type DTR	
		opt%	$\hat{E}\{Y^*(\hat{\mathbf{g}}^{opt})\}$
-	Q-learning	54.6 (2.9)	6.10 (0.24)
Correct	BOWL	40.3 (8.2)	4.80 (0.53)
	BOWL-Q	66.0 (10.1)	6.57 (0.53)
	ACWL-C_1	92.5 (3.2)	7.50 (0.13)
	ACWL-C_2	92.7 (3.3)	7.54 (0.12)
Incorrect	BOWL	33.1 (7.9)	4.85 (0.48)
	BOWL-Q	41.4 (9.9)	5.48 (0.58)
	ACWL-C_1	91.6 (3.5)	7.48 (0.12)
	ACWL-C_2	90.9 (3.3)	7.47 (0.11)

Additional Simulation 3

This simulation is for a more complex scenario with 2 stages and 5 treatment options at each stage. Specifically, we have

$$A_1 \sim Multinomial(\pi_{10}/\pi_{1s}, \pi_{11}/\pi_{1s}, \pi_{12}/\pi_{1s}, \pi_{13}/\pi_{1s}, \pi_{14}/\pi_{1s}),$$

with $\pi_{10} = 1$, $\pi_{11} = \exp(0.4 - 0.5X_3)$, $\pi_{12} = \exp(0.5X_4)$, $\pi_{13} = \exp(0.5X_3 - 0.4)$, $\pi_{14} = \exp(-0.5X_4)$, and $\pi_{1s} = \sum_{m=0}^{4} \pi_{1m}$, and

$$A_2 \sim Multinomial(\pi_{20}/\pi_{2s}, \pi_{21}/\pi_{2s}, \pi_{22}/\pi_{2s}, \pi_{23}/\pi_{2s}, \pi_{24}/\pi_{2s}),$$

with $\pi_{20} = 1$, $\pi_{21} = \exp(-0.2R_1)$, $\pi_{22} = \exp(0.5X_3 - 0.4)$, $\pi_{23} = \exp(-0.5X_3)$, $\pi_{24} = \exp(0.2R_1 - 1)$, and $\pi_{2s} = \sum_{m=0}^{4} \pi_{2m}$.

The outcome models are

$$R_1 = \exp[1.5 - |X_1 + X_3|\{A_1 - g_1^{opt}(\mathbf{H}_1)\}^2] + \epsilon_1,$$

with $g_1^{opt}(\mathbf{H}_1) = I(X_1 > -1)\{1 + I(X_4 > -0.4) + I(X_4 > 0.4) + I(X_4 > 1)\}$ and $\epsilon_1 \sim N(0, 1)$, and

$$R_2 = \exp[1.26 - |1.5X_3 - 2|\{A_2 - g_2^{opt}(\mathbf{H}_2)\}^2] + \epsilon_2,$$

S-Table 3: Additional simulation results for two stages and five treatment options at each stage (500 replications, $n = 1000$). $E\{Y^*(\mathbf{g}^{opt})\} = 8$.

π	Method	Tree-type DTR	
		$opt\%$	$\hat{E}\{Y^*(\hat{\mathbf{g}}^{opt})\}$
-	Q-learning	31.7 (3.8)	4.83 (0.32)
Correct	BOWL	15.7 (4.5)	3.53 (0.47)
	BOWL-Q	34.0 (11.3)	4.90 (0.73)
	ACWL-C_1	68.7 (8.7)	6.64 (0.47)
	ACWL-C_2	67.9 (8.7)	6.66 (0.43)
Incorrect	BOWL	9.8 (3.9)	3.04 (0.43)
	BOWL-Q	12.8 (5.9)	3.35 (0.52)
	ACWL-C_1	59.8 (9.9)	6.11 (0.60)
	ACWL-C_2	63.6 (9.2)	6.40 (0.50)

with $g_2^{opt}(\mathbf{H}_2) = I(R_1 > 0)\{1 + I(X_3 > -0.4) + I(X_3 > 0.4) + I(X_3 > 1)\}$ and $\epsilon_2 \sim N(0, 1)$.

The results are shown in S-Table 3.