Pre-Service Teachers' Understandings and Interpretations of the Common Core State Standards for Mathematical Practice

by

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DEDICATION

To Lydia- may you always dream big!

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ABSTRACT

For at least the past 40 years, the mathematics education community has attempted to characterize important learning goals for students that are not captured completely in specifications of mathematical content objectives. Mathematical processes such as problem solving, reasoning and representation have been prominent in these attempts. The most recent characterization of these learning goals is the Common Core Standards for Mathematical Practice (SMPs). If these important learning goals for students are to be accomplished, it is imperative that teachers understand what is intended to be learned and how their instruction might be tuned to promoting these outcomes.

In this dissertation study, I investigated how pre-service teachers (PSTs) interpret and understand the SMPs. In particular, 17 PSTs, enrolled or recently graduated from an undergraduate or postgraduate elementary teacher education program, completed three tasks that were intended to be approximations of key phases of actual teaching practice: planning for instruction, enacting a lesson, and assessing students. Within each task the PSTs were asked to identify (a) instances during lesson planning where students would be likely to have the opportunity to use or learn to use specific SMPs, (b) instances in a video of a lesson where students engaged in using or learning to use specific SMPs, and (c) instances in assessment items where students had an opportunity to demonstrate their proficiency in using specific SMPs. The PSTs' designations were compared to those of the authors of the curriculum materials. PSTs

were also assessed on measures of their mathematical knowledge for teaching and mathematical beliefs.

Overall, PSTs and the curriculum authors had low levels of agreement in their assignment of SMPs. A close examination of PST responses suggests that they tended to use different criteria to decide when to assign some of the SMPs to mathematical tasks or lessons. For instance, PSTs tended to assign the SMP, "make sense of problems and persevere in solving them," whenever an instructional element involved a mathematics problem. On the other hand, curriculum authors tended to assign this SMP less frequently and appeared to demand more evidence of both sense making and perseverance. There were also features of the materials that prompted PSTs to assign particular SMPs. For instance, PSTs tended to assign the SMP "construct arguments and critique the reasoning of others" whenever an assessment item asked for students to write or show their work regardless of what was actually being asked of the students. This finding suggests that the SMPs may be underspecified as learning goals that can be enacted by teachers and students at this time. The sample of PSTs in this study had high levels of mathematical knowledge for teaching and similar mathematical beliefs, and the lack of variance across the sample did not allow detection of the influence these factors might have had on PSTs' interpretations of SMPs.

Results from this dissertation study help to inform the mathematics teacher educator community of patterns in PSTs' understandings and interpretations of mathematical practices that could help to inform the way beginning teachers are prepared. More broadly, this study suggests that the mathematics education community needs to address the lack of clarity in the SMPs as they are outlined in the Common Core State Standards so that teachers, especially beginning teachers, can consistently implement the SMPs in the everyday work of teaching.

Chapter 1 Introduction

For at least the past 40 years there has been a move towards standards-based reform in the United States. As a part of this movement there have been many attempts to characterize the mathematical skills and knowledge that are crucial for students in school mathematics but are not captured in specifications of mathematical content goals. These have included mathematical processes such as problem solving, reasoning and representation. The most recent characterization of these mathematical processes is the Common Core Standards for Mathematical Practice (SMPs) (NGA & CCSSO, 2010).

My interest in studying the SMPs, and in particular pre-service teachers' (PSTs') understandings and interpretations of these practices, stems from two sources. First, I was involved in two previous studies (Silver & Mortimer, 2015; Silver & Mortimer, in press; Mortimer, 2016) in which we examined mathematics teacher educators' and mathematicians' understandings of the SMPs. In the first study, we had expert raters, two involved with the development of the content standards used in the study and two involved in the development of the assessment used in the study, determine which, if any, of the SMPs were assessed by each assessment question. They completed this individually and agreement calculations were determined between the different pairs of raters. The second study, which resulted in my scholarly paper, was a replication study of Silver and Mortimer (2015) and Silver and Mortimer (in press), however the participants were mathematics teacher educators from a variety of colleges and universities who were not involved in the creation of the content standards or the assessment. One of the most compelling findings from these studies suggests that, though the

SMPs can be seen throughout the assessment used in the study, experts and other mathematics teacher educators were not in agreement about which SMPs were assessed by each assessment item. When probed about their differing assignments, participants in both studies reported that in many cases the SMPs were vaguely described and thus challenging to assign to assessment items. They reported that the limited descriptions available and the very few examples given in the CCSSM materials were not specified enough to determine exactly what most SMPs would look like in assessment items. In other cases, participants reported that the assessment tasks could be solved using multiple methods. These different methods could result in the use of different SMPs and thus it was challenging to know exactly what SMP someone would employ while completing the different assessment tasks.

The lack of specificity given around the descriptions of the SMPs, as judged by the participants, caused them to have difficulty determining which SMPs would be assessed by each assessment item. Even experts in the field only agreed on a small percentage of items. However, practicing teachers are required to both teach and assess the SMPs in their classrooms everyday around the country. If participants in these two studies, who were either experts in the documents being compared or mathematics teacher educators charged with preparing beginning teachers to teach the standards outlined in the CCSSM, cannot agree on what the SMPs look like in the context of assessment items, how are practicing teachers, who likely have less experience in aligning standards with content, instruction and assessment supposed to determine if assessment items assess students' growth in the SMPs? How are teachers supposed to be held responsible for measuring this growth when we as a field do not seem to be in agreement on what is meant by each of the SMPs?

The second source of my interest in PSTs' understandings and interpretations of the SMPs stems from my work with PSTs in teaching courses and as an elementary field instructor. I have taught a mathematics methods course to students seeking postgraduate certification as elementary school teachers as well as another course for undergraduates seeking elementary certification about the importance of listening to and probing student thinking about mathematics. In both of these courses, the SMPs were embedded in the work in which the PSTs engaged. However, the SMPs were not explicitly taught or examined so the PSTs did not gain familiarity or skill in identifying SMPs in the work they were doing. Additionally, in my work as a field instructor, a common practice was to have PSTs include in their lesson plans which standards would be addressed in a lesson prior to teaching it. Tracking on which standards are being taught is a common practice in teaching and helps to ensure that students are exposed to and gain proficiency in all of the required standards. However, when asked to do this, PSTs almost never identified SMPs as standards that were addressed in lessons even in lessons that, in the curriculum materials, were designed to specifically target the SMPs. In many instances the SMPs were listed in the Teacher's Lesson Guide as standards that were targeted in the lesson, but the PSTs simply did not attend to them while examining the lesson. In fact, when I asked PSTs about why they did not include any SMPs in their lesson plans, most of them had never heard of the SMPs despite having already taken their mathematics methods courses.

Though I do not intend to make claims about what exactly should be taught in mathematics methods courses or during field experiences, these encounters made me wonder what we, as mathematics teacher educators, should be thinking about if we hope to produce competent beginning teachers to teach in a country where most states have adopted the CCSSM. If PSTs will be expected to support students in developing proficiency in using the SMPs and be responsible for assessing them on their progress, should planning for, teaching, and assessing students on their growth in utilizing the SMPs be something we should be incorporating in their teacher preparation? What might that look like? What do we need to know about how PSTs understand and interpret the SMPs in order to make these experiences meaningful?

Study Design

In this dissertation I investigated PSTs' understandings and interpretations of the SMPs by having PSTs complete three tasks that were intended to be approximations of key phases of actual teaching practice: planning for instruction, enacting a lesson, and assessing students. Grossman (2009) has suggested the power of using approximations of teaching practice when working with individuals preparing to become teachers. Specific details regarding the tasks that served as approximations of teaching practice in this study are provided in the following two chapters.

I hypothesize that there may be factors that influence the PSTs' understandings and interpretations of the SMPs that could impact the way that the SMPs are approached in different teacher education contexts. For instance, given that the SMPs may be viewed as hallmarks of sophisticated mathematical activity, variations in SMPs' mathematical knowledge might reasonably be expected to be associated with differences in interpretations of the SMPs. By examining and understanding the nuances of PSTs' interpretations of SMPs, as well as the interactions between those interpretations and other factors such as PSTs' mathematical knowledge, mathematics teacher educators should be better positioned to think more precisely about the content and sequencing of learning experiences in preparing competent beginning teachers to teach and assess students in their development in the SMPs.

I conducted this study by having PSTs look at materials from the three tasks, specifically a lesson from the *Everyday Mathematics (EM) Teacher's Lesson Guide*, a video of a teacher teaching a portion of that lesson, and an assessment of the content in the unit from which the lesson comes. For each section or assessment item within each task PSTs were asked to assign as few as zero to as many as three SMPs. They were also asked to provide explanations for why they assigned each SMP to each lesson plan segment, video clip, and assessment item. For each section or assessment item, the authors of the EM curriculum materials also assigned between zero and three SMPs. I calculated the agreement between the PSTs and the EM authors, along with agreement scores after grouping PSTs' by their mathematical knowledge for teaching (MKT), as measured by the Learning Mathematics for Teaching (LMT) instrument (Learning Mathematics for Teaching Project, n.d.), and their beliefs about teaching and learning mathematics, as measured by the Mathematics Beliefs Instrument (MBI) (Peterson, Fennema, Carpenter, & Loef, 1989, as modified by the Cognitively Guided Instruction Project). I also conducted follow-up interviews with the PSTs to learn more about their SMP assignments.

In addition to quantitative agreement calculations, I examined the explanations given by PSTs, both on their scoring sheets as well as through the follow-up interviews. In these explanations they were asked to explain why they assigned particular SMPs to particular lesson sections, video clips, and assessment items. I examined trends and patterns that emerged among the PSTs to try to determine what could be learned about how PSTs, who mostly did not receive explicit instruction on the SMPs, understood and interpreted the SMPs. I also looked at these explanations in light of their MKT and MBI scores.

Contributions of the Study

Teacher education programs strive to prepare beginning teachers to educate diverse students in a variety of contexts around the country. In the past, there was a need for a sort of general teacher education because every state had different standards and thus preparing PSTs too much in any one set of standards had the potential to limit the states in which they were prepared to be successful beginning teachers. However, we now have the CCSSM in place in most states in the United States. According to Ball and Forzani (2011), while discussing the importance of teacher education and the possibilities provided by the CCSSM to improve teacher education, the CCSSM "offer the possibility of a common foundation on which a stronger educational infrastructure could be built" (p. 18). The CCSSM could be seen as an asset in teacher education programs as now we have something quite specific that almost all teachers in the United States will be required to teach. Thus, we have the opportunity to train teachers in targeted ways that were not possible prior to the creation of a common set of standards.

There is much less detail given about the SMPs than there is about the content standards in the CCSSM. In order to prepare teachers to be successful in implementing the SMPs, we need to know as a field what exactly is meant by each SMP, what each SMP looks like when it is being taught by teachers or practiced by students, and what it looks like to assess students' progress in using each of the SMPs. Thus far, there is a lack of consensus on what the SMPs look like in the form of assessment items among experts in the field and among teacher educators (Silver & Mortimer, 2015; Silver & Mortimer, in press; Mortimer, 2016). However, teaching and assessing the SMPs are tasks with which teachers in states that have adopted the CCSSM are charged. How are teachers supposed to complete these tasks with confidence when experts in the field cannot agree on what the SMPs look like in assessment items? This dissertation will bring in other perspectives, those of PSTs as well as those of the authors of the EM curriculum materials, in order to learn more about how PSTs understand and interpret the SMPs in different approximations of practice, as well as how their judgments compare to those of curriculum authors. The results of the study will inform experts in the field of agreements and discrepancies among PSTs and curriculum authors in order to contribute to our collective understanding of what is clear and what needs more specification within the descriptions and examples given of the SMPs in the CCSSM.

Though some have created resources to better specify the SMPs and to provide examples of what the SMPs may look like in practice at different grade levels (e.g., Koestler, Felton, Bieda, & Otten, 2013; O'Connell & SanGiovanni, 2013), little has been done to determine how to connect the SMPs to the work of pre-service teacher education. In order to think more deeply about what knowledge and experiences PSTs may have that could influence their interpretations and understanding of the SMPs, more research needs to be done. Does PSTs' mathematical knowledge influence how they interpret the SMPs? What about their own beliefs about teaching and learning mathematics? Do certain experiences within teacher education programs help to support PSTs in developing competency in teaching and assessing the SMPs? This dissertation will contribute to the body of knowledge on what PSTs bring to teacher education programs that can be leveraged to help them to learn to support students in becoming proficient at utilizing the SMPs to approach mathematics problems.

Organization of Dissertation

This dissertation is organized into five chapters. In Chapter 1 I present the research problem, provide an overview of the study and describe the organization of the dissertation. In Chapter 2 I review and compare several characterizations of mathematical practices, discuss teachers' participatory relationship with curriculum materials and the teacher characteristics that impact that relationship, and discuss approximations of practice. In Chapter 3 I describe the methods used in this study to investigate how PSTs understand and interpret the SMPs. I present the results and my analysis in Chapter 4, digging into both the quantitative agreement calculations and the qualitative analysis of PSTs' explanations for their SMP assignments. To conclude, in Chapter 5 I discuss the results and implications of the study as well as discuss directions for future research.

Chapter 2 Literature Review

Introduction

The SMPs are important for students to be able to use in order to be successful in engaging in complex mathematics. Thus, it is important for teachers to be prepared to teach and assess students in their use of the SMPs. However, as the CCSSM is relatively new, there has not been much research on how teachers interact with and implement the SMPs in their teaching practice. In this chapter I aim to look at what is known about mathematical practices and what influences how teachers interact with curricula and standards.

I begin by discussing different characterizations of mathematical practices that have been developed over the past several decades by different research groups and organizations. I will explore their affordances and limitations leading up to consideration of the most recent iteration included in the CCSSM. Next, I explore a context in which many teachers encounter the SMPs and many new teachers heavily rely on to support their teaching-curriculum materials. I then discuss research that has been done on teachers' relationships with curriculum materials and the factors and teacher characteristics that impact that relationship. Following a discussion of curriculum materials, I share research focused on knowledge and beliefs that may impact the ways in which new teachers understand or take up the CCSSM in their teaching. I focus in on two teacher characteristics that emerge from several research studies: MKT and teachers' beliefs about mathematics and mathematics learning. Finally, I switch from a focus on teachers to a focus on teaching by discussing the use of approximations of practice in the work of preparing PSTs to be competent beginning teachers.

Mathematical Practices

Throughout much of the history of school mathematics standards, standards documents have focused on the mathematical content that is necessary for students to learn rather than focus on preparing students to meaningfully use mathematics in their lives and in their professions. More recent standards documents show evidence of a movement to name and describe practices that span different mathematical content and grade levels (e.g., Kilpatrick, Swafford, & Findell, 2001; NCTM, 1989; NCTM, 2000; NGA & CCSSO, 2010). Rather than solely focusing on a checklist of mathematical content, these standards help to specify the practices that students need in order to solve complex mathematical problems, within and across mathematical content strand areas.

Over the past several decades, there have been several attempts at characterizing these practices. The different characterizations each have affordances and limitations in the sort of detail and guidance given that make them more or less useful for teachers.

Mathematical habits of mind. The concept of *habits of mind,* developed by the Educational Development Center curriculum development team, is used in educational research to describe the way that mathematicians think about mathematics problems: "to give students the tools they will need in order to use, understand, and even make mathematics that does not yet exist" (Cuoco, Goldenberg, & Mark, 1996, p. 376). The researchers suggest focusing high school mathematics on habits of mind, in contrast to the way that mathematics is typically taught-as a set of facts for students to learn. Cuoco et al. (1996) name several habits of mind that they describe as general and applicable to many subjects: students should be pattern sniffers, experimenters, describers, tinkerers, inventers, visualizers, conjecturers, and guessers. Additionally, they discuss some habits of mind more specific to the work of mathematicians: talk big and think small, talk small and think big, use functions, use multiple points of view, mix deduction and experiment, push the language, and use intellectual chants. They also describe habits of mind specific to algebra and geometry. They do not suggest that this list is comprehensive, but use them as a way to describe what they mean by habits of mind. According to Cuoco, Goldenberg, Mark and Hirsch (2010), focusing on the methods used by mathematicians, rather than just the results of those methods, would be a productive way to organize high school curricula.

In addition to the work on high school curricula, this research group developed habits of mind for elementary students: thinking about word meaning; justifying claims and proving conjectures; distinguishing between agreement and logical necessity; analyzing answers, problems, and methods; seeking and using heuristics to solve problems (Goldenberg, Shteingold, and Feurzeig, 2003). The authors note that since language arts is such a focus in elementary school, that the mathematical habits of mind that they suggest have a communication focus. “How to teach a ‘mathematics for all’ becomes clearer when we recognize that mathematics is both a body of facts accumulated over the millennia, and a body of ways of thinking that has allowed people to discover or invent these facts and ideas (Goldenberg et al., 2003, p. 2).

Mathematical process standards. Several groups have endeavored to specify mathematical process standards in order to improve classroom mathematics instruction in the United States.

National Council of Teachers of Mathematics (NCTM) Agenda for Action. In 1980, NCTM came out with a set of recommendations for more robust school mathematics (NCTM, 1980). As a response to a movement towards basic skills, the organization recommended that “problem solving be the focus of school mathematics in the 1980s” (NCTM, 1980, p. 1). The recommendations do not discount the importance of skills in computation but determine that the purpose of acquiring those skills is to be able to use them to solve real world problems. The description includes several recommendations to of how to organize mathematics programs around problem solving, the sort of language to use when teaching problem solving, how teachers can produce classroom environments to facilitate problem solving, what sorts of curriculum materials can best teach problem solving, and the types of research and support needed in order to support this work. The fifth recommendation is focused on evaluation and touches on the evaluation of problem solving, however only gives general statements regarding the fact that problem solving should be assessed and that it may require “innovative techniques” (NCTM, 1980, p. 13). No specific examples are given of what problem solving could look like at different grade levels nor the sorts of problems or activities that could be used to assess students’ problem-solving skills making this characterization not particularly useful for teachers in their everyday practice.

NCTM (1989) Curriculum and Evaluation Standards for School Mathematics. The development of the NCTM (1989) standards rests on students’ need for mathematics in our increasingly mathematical and technical world. The NCTM Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) includes four process standards that span all grade bands and content area strands: (a) mathematics as problem solving, (b) mathematics as communication, (c) mathematics as reasoning, and (d) mathematical connections. For each of the grade bands (i.e., k-4, 5-8 and 9-12) the document dedicates two to three pages to elaborate several sub-standards that are specific to the process standard at that grade band, a more thorough description of what the standard looks like for students in that grade band, as well as several examples of problems in different content areas that would be appropriate for students in each grade band that would allow them to practice that standard.

The four process standards in this characterization, if meant to provide teachers with support in implementing them in their classrooms, are underspecified. First, when reading the descriptions and sample problems for each of the standards, it is challenging to distinguish one from another. For instance, problem solving and reasoning seem quite similar. Many of the problem examples given for both standards emphasize students having strategies to approach mathematical content and to interpret and make sense of their results. Though it may not be problematic for there to be substantial overlap in theory, it makes it challenging for practicing teachers to know if they are indeed supporting students in developing proficiency in any particular standard.

Unique to this characterization, in addition to the four process standards, there is a section of the document devoted specifically to evaluation including 14 evaluation standards. Four of these evaluation standards are dedicated to the four process standards, each with four to five pages dedicated to a description of what evaluation looks like for the standard, as well as two to three examples of assessment problems for students in each grade band. This inclusion provides practicing teachers, as well as those charged with developing mathematics assessments, with at least a few clear examples of what the authors mean by each of the standards. This is an improvement from NCTM's previous characterization of problem solving in the Agenda for Action (1980) in regard to utility for practicing teachers.

NCTM (2000) Principles and Standards for School Mathematics. NCTM revised their recommendations in a characterization of process standards outlined in Principles and Standards for School Mathematics (NCTM, 2000). The purpose of the principles and standards outlined in NCTM (2000) include "setting forth a comprehensive and coherent set of mathematical goals for students in prekindergarten through grade twelve, being a resource for teachers, education leaders, and policy-makers when studying and improving mathematics instructional programs, guiding the development of curriculum frameworks, assessments, and instructional materials, and stimulating ideas and conversations about how to best support students in gaining an understanding of the importance of mathematics" (p. 6). The process standards outlined in this characterization include (a) problem solving, (b) reasoning and proof, (c) communication, (d) connections, and (e) representation. Similar to the process standards outlined in NCTM (1989), these standards are defined in ways that are intended to be applicable across k-12, as opposed to articulating separate process standards for elementary and secondary students and for different areas of mathematics. In the description of the standards the authors group grade levels by preK-2, 3-5, 6-8 and 9-12. For each grade band they devote five to six pages in describing what each of these process standards could look like for each grade level including several example problems as well as a description of the role of the teacher. This characterization includes similar standards to the NCTM (1989) characterization except that instead of "mathematics as reasoning" they have "reasoning and proof" and in addition to the original four standards they have added "representation" which was previously subsumed under "mathematics as communication." Both of these modifications add a bit more specification in that the idea of reasoning and proof, rather than simply reasoning, emphasizes being able to justify one's reasoning rather than just use reasoning to solve a problem, further distinguishing this standard from problem solving. Additionally, separating representation from communication allows for more description of what is meant by representation and how, though it can be used for communication, it is also vital in organizing information to solve problems. Despite these additional specifications, there still exists overlap which makes teaching and assessing these standards challenging for teachers. Additionally, though this characterization devotes more space to example problems and guidance for teachers about how to support students in developing these process standards than those outlined in NCTM (1989), it removes the guidance on how to evaluate students' growth in the process standards.

National Research Council (NRC) strands of mathematical proficiency. In creating the SMPs the authors drew upon both the NCTM (2000) process standards as well as the NRC (Kilpatrick, et al., 2001) strands of mathematical proficiency. Similar to NCTM (2000), the NRC endeavored to compile a comprehensive list of what is needed for students to be successful in learning mathematics. The group drew on research in mathematics education and cognitive psychology, as well as their own conclusions about the mathematical knowledge, understanding, and skill students need. The NRC came up with five strands of mathematical proficiency: (a) adaptive reasoning, (b) strategic competence, (c) conceptual understanding, (d) procedural fluency, and (e) productive disposition. Taken together these strands make up what the authors believe is necessary "for anyone to learn mathematics successfully" (Kilpatrick, et al., 2001, p. 116). The strands are interdependent; the authors use the analogy of intertwined strands as shown below in Figure 2-1. Though many of the strands are quite similar to process standards in other documents, the strand of productive disposition illuminates the fact that research around cognitive science was also integrated into creating these strands. Though it could be argued that this strand is not particularly mathematical in nature, the authors see it as a necessary in order for students to develop proficiency in the other strands.

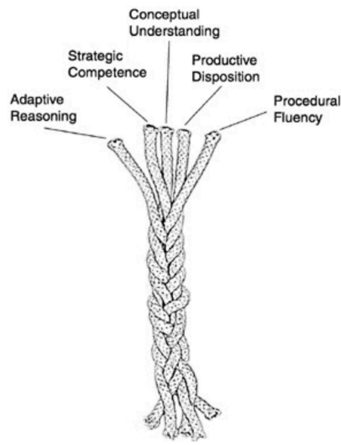


Figure 2-1. Intertwined strands of proficiency (Kilpatrick et al., 2001, p. 117).

The authors devote a chapter of their book to describing the different strands, with each strand given three to five pages. They give a few examples of what it would look like for a student to have or develop proficiency in a particular strand and talk about the ways the strands are interconnected and depend on each other. This dependence, however, makes them less distinct and thus may make it more challenging for teachers to teach and assess these strands. The authors go on to discuss how students in the United States perform on each of the strands, supporting their conclusions with test scores from the National Assessment of Educational Progress (NAEP) and various research studies. The description overall is quite general and likely supports teachers in making sense of the strands but does not give specific suggestions of problems or activities for teachers at different grade levels. Thus, this characterization, as compared to the NCTM (2000) process standards, is less useful for teachers as the strands are less specified than the process standards and there is also less description of what the strands look like in practice for students of various grade levels.

Common Core State Standards for Mathematical Practice (SMPs). The SMPs rest on processes and proficiencies drawn from two of the previously discussed documents: the NCTM process standards (NCTM, 2000) and the strands of mathematical proficiency written by the

NRC (Kilpatrick et al., 2001). These standards outline what mathematics educators should seek to develop in their mathematics students. The same eight standards span all grade levels. The SMPs include:

1. Make sense of problems and persevere in solving them;
2. Reason abstractly and quantitatively;
3. Construct viable arguments and critique the reasoning of others;
4. Model with mathematics;
5. Use appropriate tools strategically;
6. Attend to precision;
7. Look for and make use of structure;
8. Look for and express regularity in repeated reasoning.

In the text of the CCSSM there is a paragraph devoted to each of these practices (see Appendix E). These paragraphs serve as the only description and guidance for teachers in grades k-12 in the CCSSM and is notably the briefest amount of space given to process/practice standards of all of the characterizations discussed in this chapter. In contrast, in the CCSSM there are between 20 and 30 content standards for each grade level k-8 and each of the individual content standards are given a description and at least one example problem. That means that teachers are given a total of eight paragraphs to support them in teaching the practices and 20-30 paragraphs and problems to support them in teaching the content standards. Additionally, the CCSSM, as compared to the other characterizations of process standards, lacks problem examples for each of the practices leaving teachers to wonder what each of these practices look like for students at different grade levels. The lack of specificity in the descriptions of the practices makes it challenging for teachers to interpret and implement them in their classrooms as well as to assess students' development in the practices. Though all of the characterizations described in this section have affordances and limitations, the CCSSM could easily be viewed as a step backwards in that it provides even less guidance for implementation than other characterizations (e.g., Kilpatrick et a., 2001; NCTM 1989; NCTM 2000).

The distinction between content and process standards outlined in the CCSSM has made its way into curriculum materials as more states adopt the CCSSM and curriculum materials companies design materials to align with the CCSSM. For example, the curriculum materials used in this study, EM, which are explained further in the "Methods" section, have labels throughout each lesson that indicate the SMP that the authors of EM intended to be practiced or assessed through each activity and assessment item. Though these labels exist, it is up to the teacher, specifically the relationship between the teacher and the curriculum materials, to leverage the activities to give students opportunities to practice the SMPs and to use assessment items as formative ways to assess students' progress in developing their skills in using the SMPs.

Teachers' Relationships with Curriculum Materials

In a review of literature on teachers' use of mathematics curriculum materials, Remillard (2005) lays out a framework to help illustrate the participatory relationship between teachers and the curriculum as well as how that relationship impacts the enacted curriculum. Remillard's (2005) framework, shown in Figure 2-2, gives a variety of characteristics, both related to the teacher and the curriculum, which could impact the participatory relationship between the teacher and curriculum.

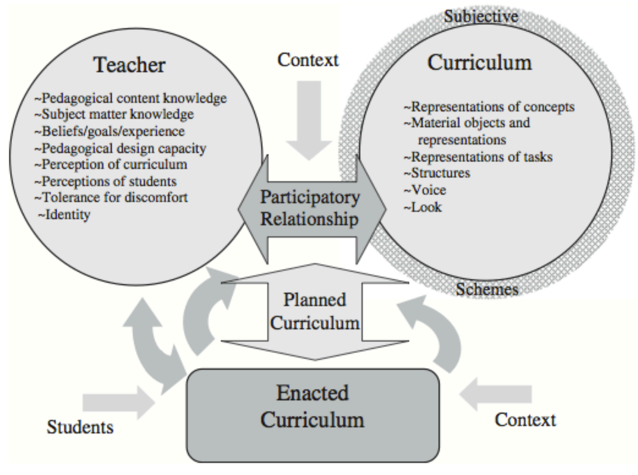


Figure 2-2. Framework of components of teacher-curriculum relationship (Remillard, 2005, p. 235).

Teacher-curriculum participatory relationship. The interaction between teachers and curriculum materials has been studied as different waves of reform have occurred. For example, curriculum materials designed in the post-Sputnik era were created with students in mind but underestimated what influence the teacher would have on the implementation of the curriculum materials (Cohen & Barnes, 1993). Unfortunately, this resulted in many teachers being unprepared to teach a more student-centered curriculum. More recently, with the rollout of the CCSSM, teachers have expressed the need for more professional development in specific content and pedagogical areas in mathematics in order to be prepared to support their students in mastering the content standards and SMPs (Bostic & Matney, 2013). In these instances, the variation in teacher implementation, due to the lack of professional development, led to teachers feeling a lack of confidence in their own abilities to implement the new standards.

Many studies have been conducted to determine just what teacher characteristics influence how a teacher approaches, interprets, and implements curriculum materials. Remillard and Bryans (2004) name this construct *orientation toward curriculum* and define it as:

> A set of perspectives and dispositions about mathematics, teaching, learning, and curriculum that together influence how a teacher engages and interacts with a particular set of curriculum materials and consequently the curriculum enacted in the classroom and subsequent opportunities for student and teacher learning. (p. 364).

There is evidence that teacher characteristics do impact the ways in which teachers approach curriculum materials. In discussing the results of several case studies on teachers' use of curriculum materials (i.e., Heaton, 1992; Prawat, 1992; Prawat, Remillard, Putnam, & Heaton, 1992; Putnam, 1992; Remillard, 1992), Putnam et al. (1992) found that three common categories of knowledge and beliefs emerged that influence teachers' interactions with curriculum materials: (a) knowledge and beliefs about learners, learning and teaching; (b) knowledge of mathematics; and (c) knowledge and beliefs about mathematics. Beliefs about learners and learning manifested themselves in whether or not teachers took a didactic approach to teaching or attempted to foster situations in which students engaged with mathematics in a more meaningful way. Additionally, some teachers held beliefs about the students' enjoyment of mathematics, feeling that math should be fun and engaging for students. Further, some teachers held beliefs that basic facts must be mastered in order for understanding to take place. Teachers' mathematical knowledge, including subject matter knowledge and attention to the mathematics in the curriculum materials, were also factors in the case studies. Teachers' knowledge and beliefs about mathematics included where mathematics comes from, what it is, how it is useful, and sources of mathematical authority. All of these factors impacted the ways that teachers interacted with the curriculum materials.

A different study suggests that teachers' views on authority within a classroom impact how they interact and implement curriculum materials. Lloyd (1999) contrasted two high school mathematics teachers' use of reform curriculum materials. The teachers' own views of authority within the classroom influenced whether they thought the problems and activities contained in the curriculum materials were too constraining for students or too open-ended. These views also impacted the way in which they implemented the lessons contained in the curriculum materials and how faithful they were to the implementation outlined in the lessons.

Taken together, these studies suggest that when studying teachers' interpretations of curriculum materials as implementation of new curriculum materials, it is important to take into account how teachers' knowledge and beliefs may impact their interactions with curriculum materials.

Characteristics Impacting Teachers' Relationships with Curriculum Materials

Though Remillard (2005) lists several teacher characteristics that could impact teachers' relationships with curriculum materials, most of these characteristics seem to fall into two broader categories: teachers' mathematical knowledge and teachers' beliefs.

Teachers' mathematical knowledge. Shulman (1986, 1987) began examining teachers' knowledge and how the ways of knowing content differ between teachers and other professionals. He explained teachers' knowledge as falling into three categories: (a) content knowledge, (b) pedagogical content knowledge, and (c) curriculum knowledge. Although adults in other professions may hold similar mathematical content knowledge, pedagogical content knowledge and curriculum knowledge are uniquely useful to mathematics teachers and are likely less familiar to those who use or study mathematics in other fields. Building on this work, the research group involved in the Study of Instructional Improvement (SII) endeavored to specify the knowledge required for mathematics teachers. They began by examining teaching practice and what teachers actually do in classrooms (Ball & Bass, 2003). This group went on to develop measures to assess teachers' MKT through the use of assessment (e.g., Ball & Bass, 2000).

Research suggests that teachers' mathematical knowledge is linked to the quality of their instruction. For example, Hill, Rowan and Ball (2005) studied the relationship between teachers' MKT and students' mathematics achievement and showed that teachers' content knowledge positively impacted student gains in their sample of first and third grade students. Additionally, Hill et al. (2008) studied the link between teacher knowledge and the quality of their mathematical instruction by using measures of MKT and mathematical quality of instruction (MQI) and showed that teacher MKT had a positive impact on MQI scores. Taken together these studies suggest that MKT has a positive impact on teachers' instruction which in turn impacts students' learning.

In Figure 2-3 below, Ball, Thames, and Phelps (2008) outline the domains of MKT. Within the domains there is a distinction between subject matter knowledge and pedagogical content knowledge. Though the domains do not make an explicit distinction between content knowledge and knowledge of mathematical processes, knowledge of both of these, as well as how to support students in learning both of these, is embedded within the domains.

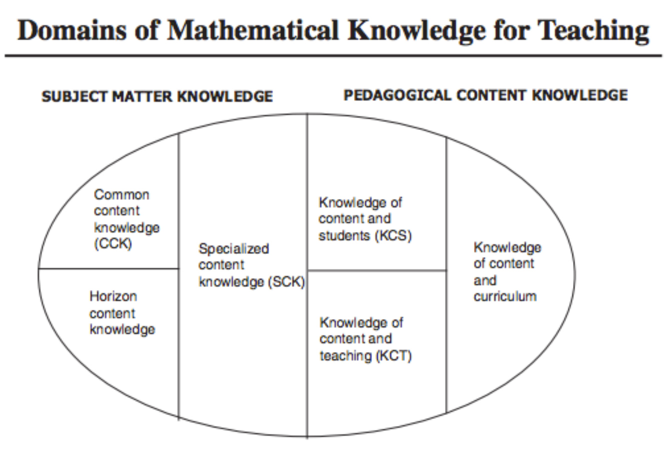


Figure 2-3. The domains of MKT (Ball et al., 2008, p. 403).

Some studies have looked specifically at how teachers' MKT and the curriculum materials, as well as the interaction between teachers' MKT and use of curriculum materials, impact the quality of mathematics instruction (e.g., Charalambous & Hill, 2012; Hill & Charalambous, 2012; Sleep & Eskelson, 2012). Hill and Charalambous (2012) looked at four case studies of teachers and found that high MKT and fidelity to the curriculum materials led to higher MQI scores. Additionally, teachers with high MKT were able to add to the curriculum in useful ways, so the impact of following the curriculum was less pronounced for high MKT teachers. On the other hand, teachers with low MKT had much lower MQI scores if they did not closely follow the curriculum materials. Sleep and Eskelson (2012) looked at how MKT and curriculum materials contributed to classroom instruction in a fractions lesson. The two teachers in their study had differing MKT. However, the teacher with lower MKT scored higher on parts of the MQI related to connecting representations and using student responses in the development of the mathematics whereas the teacher with higher MKT earned higher MQI scores in areas such as use of language and lack of errors. The authors argue that this highlights the importance of teachers' orientations towards mathematics and mathematics teaching as well as their goals for students in producing a teacher-curriculum relationship that results in high quality mathematics instruction. This study and others (e.g., Mcleod & Mcleod, 2002) suggest a need to look at both teachers' cognitive processes and teachers' affective processes, namely teacher beliefs, when considering how the interaction of teacher characteristics and curriculum materials influences mathematics instruction.

Teacher beliefs. Much of the research on teacher beliefs in relation to mathematics education has focused on teachers' beliefs about the nature of mathematics and mathematics teaching and how those beliefs impact their instructional practices. For example, in early work on teacher beliefs, Thompson (1984) studied several teachers and the relationship among their conceptions of mathematics and mathematics teaching, the integrated nature of their conceptions, their reflectiveness on their teaching, the subject matter and their beliefs, and how all of these conceptions interacted with their actual instructional practices. She concluded that "teachers' beliefs, views and preferences about mathematics and its teaching, regardless of whether they are consciously or unconsciously held, play a significant, albeit subtle, role in shaping the teachers' characteristic patterns of instructional behavior" (p. 125). She discusses the complexity of this relationship and how all of these factors interact in order to shape teachers' beliefs and teaching practices.

Other researchers have focused specifically on PSTs' beliefs. For example, Cooney, Shealy and Arvold (1998) studied how PSTs' beliefs change as they encounter and reflect upon experiences in their teacher education programs. This study showed that the beliefs that PSTs had when entering the program, and their willingness to assimilate and accommodate new ideas into their own belief structures impacted how their beliefs changed over the course of their program. Other studies show that a focus on student thinking in mathematics methods courses can support PSTs in changing their beliefs to a more constructivist viewpoint. For instance, Vacc and Bright (1999) closely followed PSTs through their methods course and student teaching experience. The methods course utilized Cognitively Guided Instruction (CGI) to support PSTs in focusing on student thinking. As a result, PSTs' beliefs changed from being teacher-centered to student-centered. Also studying PSTs' beliefs, Swars, Smith, Smith, and Tolar (2007) studied changes in PSTs' beliefs about pedagogy in mathematics, teaching efficacy, mathematics anxiety and specialized content knowledge for teaching mathematics throughout a teacher education program. Their results showed that the PSTs' methods courses caused their beliefs to become more cognitively-aligned and their student teaching experience caused their self-efficacy for themselves as teachers to increase.

In this dissertation study, PSTs engage in tasks that approximate the work of teaching to determine instances where students have the opportunity to practice or be assessed on the SMPs. While completing these approximations of practice PSTs are interacting with curriculum materials and thus that interaction and their resulting interpretations of the SMP could be influenced by their mathematical knowledge and beliefs. As discussed earlier in this chapter, characterizing practices, rather than simply focusing on content standards, suggests that there is more to mathematics than simply memorizing facts. Mathematical practices point to the fact that students must be actively engaged in their own learning rather than passive recipients of knowledge. Thus, in this study, as PSTs are interpreting the SMPs, it seems important to measure PSTs' beliefs because more cognitively-aligned beliefs may suggest a different interpretation of the practices.

Approximations of Practice

Approximations of practice is a term used to describe the sort of work that is involved in the three tasks included in this dissertation. In a study investigating professional education for clergy, teachers, and clinical psychologists, Grossman et al. (2009) identified three concepts for understanding the pedagogies of practice that emerged from these professional programs: representations of practice, decompositions of practice, and approximations of practice. According to Grossman et al., (2009), approximations of practice include opportunities for students to practice or simulate particular aspects of practice. In contrast to the student teaching experience, "simulating certain kinds of practice within the professional education classroom can allow students to try piloting the waters under easier conditions" (Grossman et al., 2009, p. 2076). Approximations may not be entirely authentic, and often are elaborated versions of what professionals will do in the field. For example, PSTs may be required to complete a detailed unit plan with elaborated lesson plans for each day's work. Though a teacher may not actually write out elaborated lesson plans for an entire unit, this exercise gives PSTs the opportunity to engage in the sort of thinking that teachers do when planning for a unit but also makes their thinking visible to instructors.

The use of approximations of practice, or similar activities where PSTs have the opportunity to engage in teaching practices in a low-stakes environment such as a university course, is supported by research on teacher education as well as standards for teacher education programs. For instance, research such as that by Ball, Sleep, Boerst and Bass (2009) describes the importance of practice being the content of teacher education courses. In their description of the courses that they teach and are developing collaboratively, they note that activities "are situated in or around the contexts of teachers' work" (p. 462) and "provide opportunities to practice the kinds of thinking, reasoning, and communicating used in teaching" (p. 462). Approximations of practice fit both of these criteria in that they are opportunities for PSTs to gain practice in the work that teachers actually do. McDonald, Kazemi, and Kavanagh (2013) describe a framework, in the form of a cycle, for orienting the pedagogy of teacher education. In this cycle, activities such as approximations and representations of practice can be used to introduce and learn about different activities and practices that PSTs then go on to enact in their student teaching placements. Approximations of practice additionally are supported by several of the High Leverage Practices developed by Teaching Works at the University of Michigan such as "Designing single lessons and sequences of lessons" and "Selecting and designing formal assessments of student learning" (Teaching Works, 2018).

Other researchers have described the implementation of approximations of practice in their preparation of PSTs in mathematics. For example, Ghousseini and Herbst (2016) studied PSTs' classroom interactions while engaging in representations of practice, decompositions of practice and approximations of practice to determine the opportunities they had to engage in the four components of Hammerness et al.'s (2005) framework for learning to teach. Their study showed that the various pedagogies of practice provided PSTs with the opportunity to "learn not only knowledge of content and students, but also specific techniques and routines to manage that work, elements of the professional vision of teachers, and a sense of the complexities involved in the work of teaching" (Ghousseini & Herbst, 2016, p. 101). Boerst. Sleep, Ball and Bass (2011) describe their work in designing mathematics methods courses and conceptualize the idea of nested practices within teaching. They discuss several approximations of practice that they use to support PSTs in learning how to lead mathematics discussions and also discuss viewing approximations through the lens of assessment.

In addition to the strong research support for the use of approximations of practice, the specific approximations of practice used in this study (i.e., planning for a lesson, enacting a lesson, and assessing students) are also supported by the Council for the Accreditation of Educator Preparation (CAEP) standards, specifically Standard 1: Content and Pedagogical Knowledge. CAEP (2013), as the first sub-point under Standard 1, states that PSTs must demonstrate an understanding of the learner and learning, content, instructional practice, and professional responsibility categories of the Interstate Teacher Assessment and Support Consortium (InTASC) standards (InTASC, 2011). The InTASC (2011) standards draw on planning for instruction, learner development and assessment, all of which relate to the approximations of practice used in this dissertation study.

For this particular study, I focused on the following three approximations of practice in which PSTs will be expected to engage in their work as beginning teachers: planning for instruction, enacting lessons, and assessing students' mastery of mathematics concepts. In each of these tasks, described in detail in Chapter 3, PSTs engage in the sorts of thinking that they will do as beginning teachers. For instance, while planning for instruction teachers look at the given curriculum materials and determine what exactly students will have the opportunity to do and practice. While observing students engage in the work of mathematics, teachers look for evidence that students are practicing the standards and goals that the teacher had intended for the lesson. When designing or preparing to implement an assessment, teachers examine the assessment items to determine what mathematics students will have the opportunity to demonstrate. Though teachers engage in many practices in their work, the three tasks used in this study exemplify much of the work that teachers do when planning, implementing and assessing a lesson designed to target particular content and practice standards.

Research Questions

In order to learn more about PSTs' understandings and interpretations of the SMPs, as well as to consider what characteristics might impact their interpretations, I pose the following questions:

1. Are PSTs able to identify places in the different approximations of practice that they will be expected to use as practicing teachers with the potential to assess or develop students' proficiency in the SMPs? How do they justify their designations?
    a. Are PSTs able to identify places in a third grade EM lesson that provide students with the opportunity to practice an SMP and determine which SMP they have the opportunity to practice? To what degree do they agree with the authors of the curriculum materials?

b. Are PSTs able to identify places in a video of teaching where students are practicing an SMP? To what degree do they agree with the authors of the curriculum materials?

c. Are PSTs able to identify which SMPs have the potential to be assessed by assessment items included in the EM curriculum materials? To what degree do they agree with the authors of the curriculum materials?

d. How does the amount of agreement between the PSTs and the authors of the curriculum materials vary by task?

2. How does PSTs' mathematical knowledge influence their ability to identify and make sense of SMPs in different approximations of practice? How does PSTs' mathematical knowledge impact their agreement with the authors of the curriculum materials? Does it differ by task?

3. How do PSTs' beliefs about mathematics influence their ability to identify and make sense of SMPs in different approximations of practice? How do PSTs' beliefs about mathematics impact their agreement with the authors of the curriculum materials? Does it differ by task?

Chapter 3 Methods

Introduction

In this dissertation study I investigate how 17 PSTs in different stages of elementary teacher preparation programs interpret and understand the SMPs in three approximations of practice: planning for instruction, enacting a lesson, and assessing students' knowledge. I will investigate how PSTs' assignment of SMPs in the different tasks compares to the assignments of the authors of the EM curriculum materials. Additionally, I endeavor to learn more about what PST characteristics impact their understandings and interpretations of the SMPs.

I will begin by describing the materials used in the three tasks. I will then describe the assessments used to measure PSTs' MKT and beliefs about mathematics teaching and learning. Following the descriptions of the materials, I will describe the participants in the study, the teacher education programs in which they are enrolled, and several PST characteristics, such as their year in the program and academic major. Then I will describe the three tasks that the PSTs completed in which PSTs engaged in approximations of practice. Finally, I will outline my analyses. The quantitative analysis includes how I calculated agreement between PSTs and the EM authors and the qualitative analysis includes how I went about analyzing the explanations given by the PSTs in their recording tools when assigning SMPs to the different tasks.

Materials Used in the Tasks

The three tasks each draw from the EM curriculum materials. PSTs assigned SMPs to the five sections of a lesson in the Teacher's Lesson Guide, three video clips of a teacher teaching portions of that lesson, and 19 assessment items from the homework sheet and unit assessment from the curriculum materials associated with the lesson. I intentionally created tasks all from the same lesson so that PSTs did not have to reorient themselves to a new lesson and new mathematical content for each task. For example, if the enacted lesson were a video of a different lesson than the one that they looked at in the Teacher's Lesson Guide, they would likely have to review that lesson plan as well in order to understand what was happening in the clip.

I chose these three approximations of practice because they seemed closely linked to what practicing teachers would do when determining how to teach and assess the SMPs. It is reasonable to think that teachers would consider the opportunities for students to practice using an SMP while planning a lesson, observe and determine if and how students are actually using an SMP while engaging with mathematics problems in an enacted lesson, and consider the opportunities for students to demonstrate their proficiency in using different SMPs while analyzing an assessment provided in the required curriculum materials. Below I describe the SMPs that PSTs were asked to assign to these tasks as well as the EM curriculum materials.

The Common Core Standards for Mathematical Practice (SMPs). The SMPs are part of the CCSSM and "describe varieties of expertise that mathematics educators at all levels should seek to develop in their students" (NGA & CCSSO, 2010). They are not content specific but span all grade levels and mathematical content. The SMPs rest on processes and proficiencies coming from the NCTM process standards (NCTM, 2000) and a report from the NRC (Kilpatrick, et al., 2001). There are eight SMPs listed and described in Appendix E. The CCSS for language arts and mathematics were released in 2010 and 41 states, the District of Columbia and four U.S. territories have adopted the CCSSM at the time of this dissertation study.

The CCSSM does not go into great detail about how these practices should be integrated with the mathematical content standards. As explained in Chapter 2, the practices are underspecified and only given a paragraph of explanation each with the expectation that practicing teachers will be able to interpret what is described in the paragraphs, consider their meaning in relation to the grade level they teach, and teach and assess students in these practices. The only guidance the CCSSM gives for implementing the SMPs into content standards is to target content standards that begin with the word "understand" as they could be potential places for integration (NGA & CCSSO, 2010). The CCSSM authors reason that without the understanding of some mathematical content at each grade level, students are unlikely to be able to engage in the SMPs. Other resources, such as *Putting the Practices into Action: Implementing the Common Core Standards for Mathematical Practice K-8* (O'Connell & SanGiovanni, 2013) provide guidance for teachers on what the SMPs might actually look like for students at different grade levels in elementary classrooms. Additionally, NCTM released a publication linking the SMPs to the NCTM process standards which could also be used as a resource for those more familiar with the process standards (Koestler, Felton, Beida, & Otten, 2013). These supplementary resources, although helpful tools for classroom teachers, are not freely available to all classroom teachers.

Everyday Mathematics (EM). EM, created by the University of Chicago Mathematics Project (UCMP), is a mathematics program for pre-K through 6th grade students and is used in around 220,000 classrooms in the United States at the time of this dissertation study (UCSMP, n.d.). A defining feature of the EM curriculum is that it spirals. Spiraling means "material is revisited repeatedly over months and across grades" (CEMSE, n.d.). The authors use research to support this method and explain that it allows teachers to identify student struggles during the first or second time students visit materials so that these struggles can be addressed in future encounters. Both the physical curriculum materials as well as the online curriculum materials contain ways to track when particular concepts come up in the lesson sequence in order to help teachers keep track of each individual concept as they spiral. The authors make references to many third-party studies that show the success of EM in different contexts (e.g., comparative studies between EM and other curriculum materials in five states using standardized measures (Sconiers, Isaacs, Higgins, McBride, & Kelso, 2003), students' mental computation (Carroll, 1996), and differentiation (Ensign, 2012)).

I chose to use the EM curriculum materials in this study in part because they were already used by all of the districts that PSTs were placed in for their field experiences. I did not want orientation to the curriculum materials to be challenging for the PSTs nor to influence the results of the study. Additionally, the most recent version of the EM program, *Everyday Math-4*, reflects both the content standards contained in the CCSSM and the SMPs. Each lesson contains reference to the specific content standards, SMPs and EM standards that the authors intended to be covered. The EM standards refer to standards that were created by the authors of the curriculum materials. They have included a map of how their standards relate to the CCSSM. Each of the SMPs contains several EM process standards. This makes sense as the SMPs are broad and the EM process standards are more specific in order to give more guidance to teachers when trying to track on students' development in the SMPs while using the EM curriculum materials. For example, for SMP 2 "Reason abstractly and quantitatively," the CCSSM gives the following description:

> Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems

> involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects (NGA & CCSSO, 2010).

The EM Teacher's Lesson Guide (UCSMP, 2015b) lists three goals, or sub-standards, for this SMP: (a) "GMP2.1 Create mathematical representations using numbers, words, pictures, symbols, gestures, tables, graphs, and concrete objects"; (b) "GMP2.2 Make sense of the representations you and others use"; and (c) "GMP2.3 Make connections between representations" (p. EM8). It is unclear as to whether this was an attempt by the authors of the curriculum materials to more clearly specify what they interpret as the intention of each SMP or whether they were trying to superimpose their existing practice standards onto the SMPs.

Included in the EM materials are the authors' designations of which SMPs are covered by each part of the lesson and are assessed by each assessment question. Table 3-1 below shows the authors' designations for the lesson from the Teacher's Lesson Guide and Table 3-2 shows their designations for the Home Link from the lesson and the unit assessment which are used together for the assessment task.

Table 3-1: Author's Designations of SMPs in the Teacher's Lesson Guide

Title of Lesson Section	SMP(s)

Math Message/Calculating Perimeters and Areas	1, 4, 6
Solving the Open Response Problem	1, 4, 6
Getting Ready for Day 2	1, 4, 6
Setting Expectations	4
Reengaging in the Problem/Revising Work	1, 4, 6

Table 3-2: Authors' Designations of SMPs in the Home Link and Unit Assessment

Problem Number	SMP(s)
HL1	6
HL2	6
1	6
2	4
3a	7
3b	7, 8
4a	7
4b	7
5a	None
5b	6
5c	None
6a	4
6b	4
7	4, 6

8	4
9	None
10a	None
10b	None
10c	None

These designations are used for comparison when comparing PSTs' SMP designations with the authors' designations.

For the lesson enactment task, PSTs watched three clips all from a video of a teacher teaching the same lesson that is described in the EM Teacher's Lesson Guide (UCSMP, 2015b) and used in the other tasks. The video came from a website featuring a virtual learning community (UChicago STEM Education, 2017). The video is used as an example of SMP 6. Thus SMP 6 was used as the designation for the EM authors for each of the three clips in the lesson enactment task. Additionally, I had expert raters complete the lesson enactment task for comparison, as explained later in this chapter in the description of the lesson enactment task.

Assessments Used to Gather PSTs' Background Information

Background information about the PSTs was collected in order to provide data that was used to determine how different PST characteristics influence their success in completing the tasks, their agreement with the authors and the expert raters, and their interpretations of the SMPs in the different approximations of practice. As noted in Chapter 2, mathematical knowledge and teacher beliefs were shown to influence how teachers interact with curriculum materials. The measures used for each of these characteristics are described below.

Mathematical knowledge. Stemming from the work of the Study of Instructional Improvement (SII), The Learning Mathematics for Teaching (LMT) project created an assessment to measure school and classroom processes as well as teachers' facility in using mathematical knowledge in their classroom teaching (Learning Mathematics for Teaching Project, n.d.). They elected to develop survey methods, as the large number of teachers in their study made other measures of teacher knowledge less feasible (e.g., observations, interviews). Since their creation, many research groups and individual researchers have used this assessment in their own work with teachers. Several studies have been done to test and validate the items included in the LMT (e.g., Hill, Dean, & Goffney, 2007; Hill, Schilling & Ball, 2004).

The items included in LMT assessments were piloted in four different settings: (a) a teacher professional development program in California, (b) a comparison group who did not attend the professional development, (c) a convenience sample from Quality Educational Data's (QED) database, and (c) a national representative sample (Hill, 2007). After substantial piloting, items were chosen and scaled. The assessment that PSTs took for this study was a subset of these questions. The specific assessment items used in the LMT form in this dissertation study were from the *Number and Operations Content Knowledge* assessment. The items used on the assessment focused on the content domain number and operations as that was the most similar to the content knowledge that PSTs likely employed when completing the tasks.

Beliefs. In order to measure PSTs' beliefs about teaching and learning mathematics, the Mathematics Beliefs Instrument (MBI) was used (Peterson, Fennema, Carpenter, & Loef, 1989, as modified by the Cognitively Guided Instruction Project). The instrument consists of three subscales: (a) Curriculum: the degree to which one believes that mathematics should be taught in relation to problem solving and understanding rather than focusing on facts and memorization;

(b) Learner: the degree to which one believes that students can construct their own mathematical knowledge; (c) Teacher: the degree to which teachers should organize instruction to facilitate children's construction of knowledge (Swars et al., 2007; Swars et al., 2009). The instrument is scored on a five-response Likert scale that, when added together, higher scores indicate that the teacher's beliefs are more aligned with cognitive beliefs. According to Swars et al. 2007 "these subscales have high reliability (Cronbach's alpha= .80 for curriculum, .89 for learner, and .90 for teacher) and represent independent constructs based on confirmatory factor analysis" (p. 329).

Participants

The participants in this study are PSTs from a large Midwestern public university. I sent out a call for participants through email and all participants that were interested in the study were able to participate. A total of 17 participants completed all portions of the study, including the MKT and MBI assessments, the three tasks, and the follow-up interview.

Description of academic programs. Of the 17 participants, 10 were undergraduates and 7 were part of a master's program. All participants were in elementary education programs that included classwork, student teaching experiences, and a teaching certificate upon completion of a state standardized assessment.

Undergraduate program. The undergraduate education program at this university includes students in their third and fourth year of their undergraduate programs. All PSTs in the program are required to have a particular number of credits in humanities, natural science, social science, mathematics, and creative arts prior to graduation. Additionally, PSTs choose a major within elementary education (i.e., language arts, mathematics, integrated science, or social studies). All PSTs in the program will be certified to teach kindergarten-8th grade in self-contained, multi-subject classes upon completing the program and passing the state certification test. Additionally, PSTs are able to take subject-specific tests in their major subject area to be certified to teach in subject-specific classrooms in grades 6-8. It is important to note that undergraduates participating in the study who had just completed their first year in the education program, the third year in their undergraduate program, at the time of data collection had not yet completed all of their coursework or their student teaching semester. Of the ten undergraduate participants, six of them had just completed their first year in the teacher education program, their third year as an undergraduate student, and four had just recently graduated from the program.

Master's program. The particular master's program from which participants were sampled in this study was designed for people who were formerly in careers other than teaching but wanted to earn a master's degree and a teaching certificate to become elementary or middle school teachers. It is a one-year, all-day, intensive program including almost a whole school year in a classroom. In the summer, PSTs work directly with students in a summer school program accompanied by mathematics and language arts methods courses directly supporting them in the work they are doing with students. During the fall semester PSTs are only in the classroom part-time while completing methods courses. In January, they do only coursework and beginning in February they are in the classroom full-time for their student teaching experience. In May and June, they continue coursework and graduate in June. PSTs are expected to complete an undergraduate degree, including required credits in different subject areas, prior to beginning the program. Similar to the undergraduate program, they choose a major area of study (i.e., language arts, mathematics, integrated science, or social studies). All PSTs in the program will be certified to teach kindergarten-8th grade in self-contained, multi-subject classes upon graduating from the program and passing the state certification test. Additionally, PSTs will be able to take subject-specific tests in their major subject area to be certified to teach in subject-specific classrooms in grades 6-8.

Participant characteristics. Of the 17 PSTs participating in this study, the self-reported average number of mathematics courses taken at the undergraduate level was about three. Additionally, three participants were mathematics majors, one was a double major in mathematics and language arts, eight were language arts majors, three were science majors and two were social studies majors. Table 3-3 below shows a coded name for each PST, the number of college-level undergraduate mathematics course that they had taken at the time of data collection and their academic majors. All of this data was self-reported and not taken directly from transcripts or official university documentation. It is likely, particularly for the participants in the master's program, that the number of undergraduate mathematics courses may not be precise, as there may have been a large gap in their enrollment as an undergraduate and their enrollment in their master's program and they likely did not have their undergraduate transcripts in front of them while answering the question. The purpose of this information is just to get an idea of the mathematical experiences of the participants and is not used in any sort of measure of mathematical competency.

Table 3-3: PSTs' Mathematics Courses and Academic Majors

Name	Number of Undergraduate Mathematics Courses	Academic Major
Third-year Undergraduates		
U31	6	Language Arts
U32	5	Mathematics

U33	3	Science
U34	2	Language Arts
U35	3	Language Arts
U36	4	Mathematics
Fourth-year Undergraduates		
U41	4	Social Studies
U42	9	Mathematics and Language Arts
U43	2	Social Studies
U44	8	Mathematics
Master's Participants		
M1	2	Language Arts
M2	2	Language Arts
M3	1	Science
M4	1	Science
M5	3	Language Arts
M6	1	Language Arts
M7	0	Language Arts

The Three Tasks

The three tasks, described below, are approximations of practice for the work of teaching. I chose these particular approximations of practice because they approximate the day-to-day work that teachers do when they are determining how to best teach, support, and assess students in developing proficiency with the SMPs. As the purpose of this study is to better understand how PSTs interpret and understand the SMPs, these approximations seem appropriate and relevant.

Curriculum materials task. While teaching and planning to teach, teachers have the opportunity to consider what opportunities are made available to students to develop their proficiency in using the SMPs. In many cases, teachers are required to use curriculum materials provided by their schools or school districts to teach mathematics. In order to support students' development in the SMPs, it is important for teachers to be able to look at curriculum materials (e.g., daily lessons provided in a Teacher Lesson Guide) and determine where in the materials students have the opportunity to practice using the SMPs. This forethought allows teachers to support the students in these opportunities while implementing the lesson.

For this task, PSTs were given a lesson from the EM Teacher Lesson Guide (UCSMP, 2015b) as well as a copy of the SMPs. Though the authors already included places where they believe students have the opportunity to practice the SMPs, these designations were removed so that there was no indication of which SMPs the authors intended to be practiced in the lesson. The PSTs, working independently, examined the SMPs and the lesson. They went through the lesson and indicated in a recording tool (see Appendix C for an example) each instance in which they thought students had an opportunity to practice an SMP as well as which SMP they had the opportunity to practice. PSTs were asked to explain their judgments through writing in the "Explanation" section of their recording tool. Based on feedback from piloting the data, the recording tool was separated into the five different sections already indicated in the lesson. Each section contained one or two related activities. PSTs were able to identify up to three SMPs that could be practiced by students participating the activities included in the given section. In each section, the authors of the curriculum materials identified between one and three SMPs that are used for comparison.

Lesson enactment task. In addition to the importance of planning, implementation of the plan provides students the opportunity to actually make progress in developing their skills and fluency with using the SMPs. The lesson enactment task helped me to learn about how PSTs are understanding the SMPs when actually seeing students engage with the SMPs in their work on a mathematics task. Though practicing teachers do not always have the opportunity to watch videos of their own teaching, in this task the PSTs focused on how students are practicing SMPs within the context of the lesson. Though having PSTs actually enact a lesson themselves would be a closer approximation of the work that teachers do, it was not possible in the context of this study as the PSTs all had field experiences in different grades and were at different places in their programs. It would have presented a new challenge to either have each of them teach different lessons that were appropriate for the students in their field placements or to have each of them teach the same lesson but to students other than those in their field placements. Results of this task are likely an indication of how PSTs are interpreting what they are seeing when they observe students using SMPs to solve problems in their own classrooms.

PSTs viewed a video of an elementary teacher implementing portions of the mathematics lesson that they examined in the curriculum materials task. In the video, the teacher does not explicitly name the SMPs that she is intending to teach, but the SMPs are embedded in the work that the teacher and students are doing. Using the online video program Edthena, PSTs were asked to identify which SMP or SMPs that they saw students practicing in the video by tagging these instances and explaining their justification for their assignment in writing in the tag. The

PSTs in this sample are already familiar with using Edthena and use it regularly throughout their teacher education program. I choose to use Edthena because I did not want the technology to be a barrier or be problematic for PSTs in completing the task or a limitation in interpreting the results.

Additionally, two advanced graduate students who had experience analyzing videos of teaching and were also familiar with the SMPs completed this task. These raters are referred to as expert raters throughout the dissertation. The creators of EM posted this video on their virtual learning community website (UChicago STEM Education, 2017) for the purpose of being an exemplar of one of the SMPs. They did not, however, go through and identify each instance that an SMP can be seen being practiced by students. Both to be more similar to the task completed by the PSTs as well as to learn more about how PSTs see SMPs in enacted lessons as compared to those who are more expert in analyzing teaching, the two advanced graduate students completed this task together. For each video clip they discussed what they observed and came to consensus on which SMP or SMPs were practiced by students in each clip. Both the designations by the EM authors as well as the designations by the expert raters were used for comparison with the PSTs' designation in Chapter 4.

Assessment task. Teachers are responsible for both teaching the SMPs and assessing students' development in using the SMPs. Teachers are often required to use assessments provided in the curriculum materials used by their district. It is important that teachers are able to look at assessment items and determine where in the items students have the opportunity to demonstrate their proficiency in using the SMPs to solve mathematics problems.

For this task, PSTs examined assessment items from the EM curriculum materials. The assessment that they used is the unit assessment associated with the lesson used in the curriculum materials and lesson enactment tasks. PSTs also completed this task with the two problems on the Home Link homework worksheet included in the lesson as the mathematical content on the Home Link was most related to the lesson that they examined. PSTs, working independently, looked at each item and choose which, if any, SMPs they determined students had the opportunity to demonstrate their proficiency in using. They had the option of determining that no SMP has the potential to be assessed by the item and also the opportunity to say that more than one and up to three SMPs had the potential to be assessed by the item in the recording tool. There was also an "Explanation" section in PSTs' recording sheets to explain why they chose each SMP for each given assessment or Home Link item. The authors of the curriculum materials have identified between zero and three SMPs that they believe are assessed through each assessment item which are used for agreement calculations in the Analysis chapter.

Agreement Calculations

In order to determine agreement between the authors and the PSTs I first assigned their designations to an eight-dimensional vector. For example, if the authors assigned SMPs 1, 4, and 6 and the PSTs assigned SMPs 1, 3, and 5, their vectors would be:

Authors: (1, 0, 0, 1, 0, 1, 0, 0)

PSTs: (1, 0, 1, 0, 1, 0, 0, 0)

In order to measure how closely these two vectors are related I found the cosine of the angle between the two vectors, which can be calculated by finding the dot product of the two vectors and dividing their dot product by the product of their norms. I then calculated their agreement as a percent rounded to the nearest whole percent. Here are the calculations using the example above:

$$\frac{\langle A, B \rangle}{||A||\,||B||}$$

$$= \frac{((1,0,0,1,0,1,0,0)(1,0,1,0,1,0,0,0))}{\sqrt{((1^2+1^2+1^2)(1^2+1^2+1^2))}}$$

$$= \frac{1+0+0+0+0+0+0+0}{\sqrt{3}\cdot\sqrt{3}}$$

$$= \frac{1}{3} \approx 33\%$$

Though I examined other ways of determining a method for an agreement calculation, I arrived at this method because it takes into account not just how many instances of agreement exist between the author and the PSTs but also instances where the PSTs or the authors assigned different numbers of SMPs. For example, if the authors and a PST agreed on one SMP, but the authors only assigned one SMP, there would be more agreement if the PST assigned two SMPs than if he or she assigned three SMPs because if the PST assigned two SMPs they really only disagreed with the authors on one designation, whereas if he or she assigned three SMPs they disagreed with the authors on two designations. For example, if the authors assigned only SMP 1 and a PST assigned SMPs 1 and 3, the calculation would be as follows:

$$\frac{((1,0,0,0,0,0,0,0)(1,0,1,0,0,0,0,0))}{\sqrt{((1^2)(1^2+1^2))}}$$

$$= \frac{1+0+0+0+0+0+0+0}{\sqrt{1}\cdot\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} \approx 71\%$$

If, however, an author assigned only SMP 1 and a PST assigned SMP 1, 3, and 5, the authors and the PST still agreed on one SMP however the PST assigned two additional SMPs rather than one resulting in less agreement as shown below:

$$\frac{((1,0,0,0,0,0,0,0)(1,0,1,0,1,0,0,0))}{\sqrt{((1^2)(1^2+1^2+1^2))}}$$

$$=\frac{1+0+0+0+0+0+0+0}{\sqrt{1}\cdot\sqrt{3}}$$

$$=\frac{1}{\sqrt{3}}\approx 58\%$$

I used this method to calculate the agreement between the authors and the PSTs for each of the three tasks and additionally for the agreement between the expert raters and the PSTs on the lesson enactment task.

Qualitative Analysis

In addition to agreement calculations I also examined closely instances where PSTs and authors disagreed on their assignments of SMPs. In order to determine which instances of disagreement to discuss I chose to examine instances where either the authors assigned an SMP and fewer than half of the PSTs assigned that SMP or instances where more than half the PSTs assigned an SMP but the authors did not assign it.

Once these instances were determined I examined the task in which the disagreement occurred. I looked at the assignments of the authors and the PSTs and determined if both or either of the assignments were reasonable. Additionally, I looked at the explanations given by the PSTs. While reading these explanations I looked for patterns in their interpretations of the task as well as their interpretations of the SMPs. I did this for individual items as well as across items to determine if there were trends in the ways that PSTs interpreted and understood the PSTs as well as to determine if there were features of the tasks, such as the mathematical content, that may have impacted how they were interpreted by the PSTs. Though the authors did not give explanations for their designations, in particular cases it was useful to look at the particular EM practice standard that they assigned as they were more specified than the SMPs. These results are discussed in Chapter 4.

Chapter 4 Analysis and Results

Introduction

In this chapter I begin by describing the mathematical knowledge and mathematical beliefs of the participants in this study as measured by the MKT and MBI assessments. In order to determine if agreement is influenced by either mathematical knowledge or mathematical beliefs, I put the PSTs in high, medium, and low groups for each of these characteristics. I describe how I determined these groups as well as the relationship between MKT and MBI scores. I individually compare the PSTs' designations with the authors' through a Spearman correlation as well as compare the PSTs' and authors' designations based on the PSTs' years in the program, MKT and MBI groups.

Following this analysis, I look closely at the frequency of assignment of each SMP by the PSTs and authors and examine instances where PSTs and authors disagree. I then attempt to determine the reason for these discrepancies by investigating the explanations given by the PSTs for their designations. Are the discrepancies due to a lack of mathematical understanding by the PSTs? Were there particular ways in which the PSTs interpreted the SMPs that caused them to assign them differently than the authors? Were there specific features of tasks or problems that increased the likelihood of the assignment of particular SMPs by PSTs? Were there cases where there was disagreement between the PSTs and the authors, but the PSTs were perfectly reasonable in their justification for their assignment? The answers to these questions helps us to better understand how PSTs interpret and understand the SMPs.

Description of Participants

MKT scores. All participants completed the subtest "Elementary Number Concepts and Operations" of the Learning Mathematics for Teaching (LMT) assessment as a measure of their MKT. Table 4-1 below shows the average LMT scores for the PSTs based on the year of the program they were in at the time of the study. The scores are calculated in comparison to the average scores of practicing teachers. This is done because the assessment is not criterion referenced and is only meant for norm referenced comparisons. Each score is given as a standard deviation from the mean of practicing teachers, with negative scores meaning the PSTs' MKT is below average compared to practicing teachers and positive scores meaning the PSTs' MKT is above average compared to practicing teachers. Out of the 17 PSTs, 15 have LMT scores higher than the average practicing teacher, with 10 having scores more than one standard deviation higher than the average practicing teacher.

Table 4-1: Learning Mathematics for Teaching (LMT) Scores

PST Year	Average LMT Score in Standard Deviation
Third-year Undergraduates (n=6)	0.64
Fourth-year Undergraduates (n=4)	1.26
Master's Participants (n=7)	0.72
Average of all Participants (n=17)	0.82

The fourth-year undergraduates, on average, had the highest scores on the LMT assessment and the third-year undergraduates had the lowest. This makes sense as both the fourth-year undergraduates and the master's participants had completed their coursework at the time of this assessment, including mathematics content and methods courses, whereas the third-year undergraduates had completed some mathematics content courses but no mathematics methods courses. They also had fewer opportunities to teach mathematics lessons in their field placements, which could arguably improve their MKT. It is important to notice, however, that all three groups had average LMT scores above average as compared to practicing teachers.

Grouping PSTs by MKT scores. In order to group the PSTs by their MKT scores I looked at their scores on the LMT assessment. The scores ranged from -1.03 to 1.79 with an average of 0.82 and a standard deviation of 0.85. In order to determine groups, I ordered the LMT scores from least to greatest and made groups of five or six PSTs. I determined whether each group would contain five or six PSTs based on where gaps in the scores were most pronounced. The low MKT group has scores ranging from -1.03 to 0.69 with an average score of -0.09 and standard deviation of 0.75. The medium MKT group has scores between 0.83 and 1.24 with an average score of 1.06 and a standard deviation of 0.18. The high MKT group has scores between 1.43 and 1.76 with an average score of 1.61 and standard deviation of 0.14. Though the groups are labeled as low, medium and high, it is important to note that this sample most PSTs had scores greater than average practicing teachers, so the low, medium and high groups are relative to the other PSTs in the sample, not relative to practicing teachers.

MBI scores. All PSTs completed the MBI survey online using Google Forms. For each item, PSTs selected a choice on a five-point scale ranging from strongly agree to strongly disagree. I then assigned numerical values from 1 to 5 to the different responses in order to calculate a numerical score. Table 4-2 below shows the average score of the PSTs sorted by where they were in the program at the time of the study, with possible values between 1 and 5. Higher average scores indicate PST views that are more cognitively-aligned. When looking at the averages of the three groups there is not a lot of variation in the MBI item scores for the different subdomains or in average MBI item scores across subdomains. Across all three subdomains the master's PSTs have slightly lower MBI scores, indicating that as a group they have slightly less cognitively-aligned beliefs about teaching and learning mathematics compared to those in their third or fourth year of the undergraduate program. Between the two undergraduate groups, the third-year PSTs, on average, have higher MBI score for the curriculum and teacher subdomain, whereas the fourth-year undergraduates have higher MBI for the learner subdomain. Though there are these differences, the differences are extremely small.

Table 4-2: Average Item Score on Mathematical Beliefs Instrument (MBI) Scores

PST Year	**Average "Learner" Subdomain Score**	**Average "Curriculum" Subdomain Score**	**Average "Teacher" Subdomain Score**	**Average MBI Score**
Third-year Undergraduates (n=6)	3.41	3.48	3.57	3.49
Fourth-year Undergraduates (n=4)	3.57	3.39	3.54	3.50
Master's Participants (n=7)	3.37	3.27	3.47	3.37
Average of all Participants (n=17)	3.45	3.38	3.53	3.45

Though there is not much variability among groups, there is variability within groups. When looking at individuals there is some notable variability. For example, though the average score for the third-year undergraduate PSTs in the teacher domain is 3.57, the lowest average MBI score within the group is 3.12 whereas the highest is 4.18. Similarly, the lowest average MBI score for the master's PSTs in this subdomain is 2.71 whereas the highest is 3.88. Additionally, when looking at PSTs' ratings on individual items, ratings range from 1 to 5 in every subdomain.

Grouping PSTs by MBI scores. In order to group the PSTs by their MBI scores, I used the PSTs' total MBI scores (see Appendix F). The survey included 48 items and for each item

PSTs put a numerical value from 1 to 5. These values were then totaled across all 48 items with a minimum possible score of 48 and maximum possible score of 240. The PSTs' scores range from 139 to 193, with an average score of 166 and a standard deviation of 14.00. I ordered the PSTs' MBI scores from least to greatest and made groups of five or six, similar to how groups were determined for MKT scores. The group with the lowest MBI scores had a range of scores from 138 to 157 with an average score of 151 and a standard deviation of 5.90. The group with the medium MBI scores had scores ranging from 165 to 168 with an average score of 167 and a standard deviation of 1.64. The high MBI group had scores ranging from 170 to 192 with an average score of 179 and a standard deviation of 9.67.

Relationship between MKT and MBI scores. It is important to determine if there is a correlation between the PSTs' MKT group and MBI group in order to know if agreement calculations for these two characteristics should occur separately or whether they are correlated enough to treat them as one measure. Other studies support that these two scores tend to be correlated, especially at the end of a teacher education program (e.g., Swars et al., 2009). Table 4-3 below shows the frequency of PSTs in each combination of MKT and MBI group. For example, there are three PSTs who fall both in the low MKT and low MBI groups. At first glance it appears that there is little correlation between PSTs' MKT and MBI groups as there are only seven out of the seventeen PSTs whose MKT and MBI groups are the same.

Table 4-3: Frequency of PSTs in Each MKT and MBI Group

		MKT Groups		
MBI Groups		Low	Medium	High
	Low	3	1	2
	Medium	2	2	1

	High	1	3	2

In order to determine if there is significant correlation between PSTs' MKT and MBI scores, I first considered using a Chi-square test. However, because the sample size is so small, the expected value in each group would not be at least five, one of the conditions for using a Chi-square test. Instead, I used Fisher's exact test (Sprent, 2011). Using this test, the MBI and MKT groups are shown to be not significantly correlated ($p=0.844$). This is interesting both because it differs from what Swars et al., (2009) found and also because it means that there is reason to treat these two measures as separate in the analyses.

Agreement Between the PSTs and Authors

I ran a Spearman's correlation to assess the relationship between individual PSTs' MKT and MBI scores and their agreement with the authors on each of the tasks. In Table 4-4 below I list the Spearman correlation coefficient r_s and the statistical significance level (i.e., the p-value). The close r_s values are to ±1 the strong the monotonic relationship. None of the relationships are statistically significant. Scatterplots for each of the relationships can be found in Appendix G.

Table 4-4: Spearman Correlations for Teacher Characteristics and Tasks

Characteristic	Task Agreement	r_s	p-value
MKT Score	Curriculum Materials Task Agreement	-0.2369	0.3600
MKT Score	Lesson Enactment Task Agreement	0.4714	0.0559
MKT Score	Lesson Enactment Task Expert Agreement	0.4557	0.0660
MKT Score	Assessment Task Agreement	-0.4197	0.0935
MBI Score	Curriculum Materials Task Agreement	-0.1477	0.5716
MBI Score	Lesson Enactment Task Agreement	-0.1074	0.6816
MBI Score	Lesson Enactment Task Expert Agreement	0.0273	0.9172
MBI Score	Assessment Task Agreement	0.3459	0.1739

Agreement when PSTs are grouped by program year. All 17 participants responded to all three tasks. Table 4-5 shows the amount of agreement between the authors and the PSTs when the PSTs are grouped by the year of the program that they were in at the time of the study. When looking at the average agreement of the three groups with the authors, the third-year and fourth-year undergraduates had slightly lower average agreement across the tasks at 25% than the master's participants who agreed with the authors on average 29% of the time.

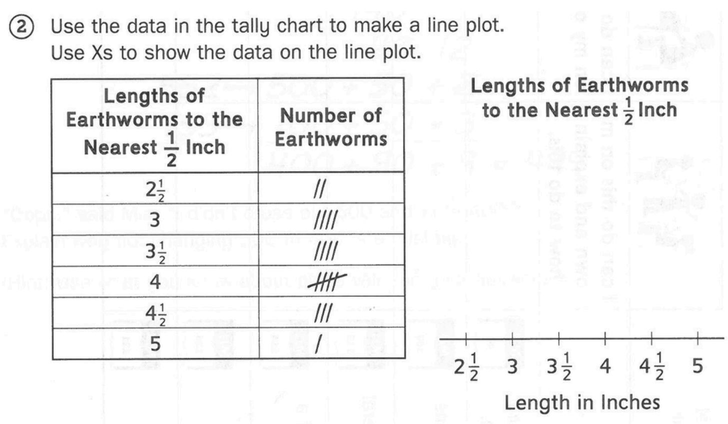
Table 4-5: Agreement Between PSTs Grouped by Program Year and the EM Authors for Each Task

PST Year	Curriculum Materials Task	Lesson Enactment Task	Assessment Task	Average
Third-year Undergraduates (n=6)	28%	14%	32%	25%
Fourth-year Undergraduates (n=4)	29%	27%	19%	25%
Master's Participants (n=7)	38%	24%	25%	29%
Average of all Participants (n=17)	32%	22%	25%	26%

When looking at the average agreement for the different tasks, a couple of results stand out. First, there is a large difference in agreement between the third-year and fourth-year undergraduates for the lesson enactment task. The lesson enactment task consists of analyzing a video of three clips of a teacher teaching and students engaging in an activity during a lesson. This discrepancy seems like a logical result of the many experiences that PSTs have during their fourth year of the undergraduate program including a mathematics methods course in which they learn more about how to teach mathematics as well as analyze videos of teaching and learning mathematics. PSTs additionally experience student teaching during their fourth year. During this experience they spend a substantial amount of time watching a classroom teacher orchestrate mathematics lessons and have an opportunity to teach mathematics themselves as well as view videos of themselves teaching and reflect upon what they are observing. It makes sense that the undergraduates improve in this skill from the third to the fourth year in the program.

Interestingly the master's students fall in between the third- and fourth- year undergraduates. At the time of the study they had completed their mathematics methods course as well as their student teaching, however their time spent in the program is only one year. Due to the organization of the two programs they likely had more experience watching and analyzing teaching than the third-year undergraduates but less experience than the fourth-year undergraduates.

Second, the third-year undergraduates agreed with the EM authors on average more than the fourth-year undergraduates and the master's PSTs on the assessment task. When examining the agreement of each group of PSTs with the authors on individual assessment tasks, there are a few tasks where third-year undergraduates overwhelmingly agreed with the authors and some on which fourth-year undergraduates overwhelmingly disagreed with authors. One example of this phenomenon is in item 2 shown below in Figure 4-1.


② Use the data in the tally chart to make a line plot.
Use Xs to show the data on the line plot.

Lengths of Earthworms to the Nearest $\frac{1}{2}$ Inch	Number of Earthworms
$2\frac{1}{2}$	//
3	////
$3\frac{1}{2}$	////
4	~~////~~
$4\frac{1}{2}$	///
5	/

Figure 4-1. Problem from Everyday Mathematics Grade 3 Unit 4 student assessment. Reprinted from Everyday Mathematics Assessment Handbook, (p. 34), The University of Chicago School Mathematics Project, 2015, Columbus, OH: McGraw-Hill Education. Copyright 2015 by McGraw-Hill Education. Copyright 2015 by McGraw-Hill Education. Reprinted with permission.

The authors assigned SMP 4 "Model with mathematics" to this assessment item. 67% of third year undergraduates agreed with the authors, whereas only 25% of fourth year undergraduates and 14% of master's PSTs agreed with the authors. Many of the fourth year and master's PSTs instead assigned SMPs 5 "Use appropriate tools strategically" and SMP 6 "Attend to precision." The main distinction here is whether the PSTs are interpreting the tally chart and the line plot to be models or tools. It is unclear why third-year undergraduates would interpret these as models and fourth-year and master's students would not.

A similar outcome occurred in item 8 shown in Figure 4-2 below. The authors assigned SMP 4 to this item as well. Of the third-year undergraduates 50% agreed with the authors on their rating, whereas 0% of fourth-year undergraduate and 14% of master's PSTs agreed with the authors. For this item, all of the third-year PSTs that assigned SMP 4 noted that the number sentence is a model of the rectangle, which coincides with the language given in the CCSSM description of the SMP when they provide an example of the SMP: "Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation" (NGA & CCSSO, 2010). SMP 1 "Make sense of problems and persevere in solving them" and SMP 6 "Attend to precision" were the two most frequently assigned SMPs by other PSTs. These practices also make sense for this item.

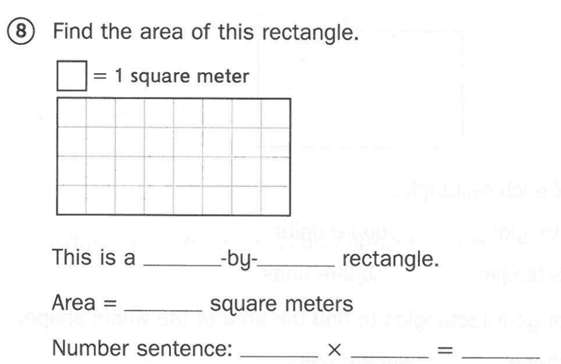


Figure 4-2. Problem from Everyday Mathematics Grade 3 Unit 4 student assessment. Reprinted from Everyday Mathematics Assessment Handbook, (p. 37), The University of Chicago School Mathematics Project, 2015, Columbus, OH: McGraw-Hill Education. Copyright 2015 by McGraw-Hill Education. Reprinted with permission.

As explained in Chapter 4, the lesson enactment task included three groups, the authors, the PSTs and a third group, two expert raters whose professional work involves substantial time watching and evaluating videos of teaching. This group was included because the authors chose this video to be part of the professional learning community associated with the curriculum materials. They chose this video in its entirety to exemplify one the SMPs (i.e., SMP 6 “Attend to precision”). In order to keep the task similar to the other tasks completed by the PSTs, PSTs were able to assign up to three SMPs to each video clip. As the task they completed was quite different than the way the authors went about giving their ratings, the expert raters were used as a more authentic comparison. Table 4-6 below shows the agreement between the PSTs and the expert raters when the PSTs are grouped by their year in the program at the time of the study.

Table 4-6: Agreement Between PSTs Grouped by Program Year and the Expert Raters for the Lesson Enactment Task

PST Year	Lesson Enactment Task Agreement
Third-year Undergraduates (n=6)	53%
Fourth-year Undergraduates (n=4)	55%
Master’s Participants (n=7)	56%
Average of all Participants (n=17)	55%

There are two important things to notice about the agreement when comparing the PSTs and the expert raters. First, the same trend exists as does with the agreement between the PSTs and the EM authors in that the third-year undergraduates have the least agreement with the expert raters, followed by the master’s participants, with the fourth-year undergraduates, those who likely have the most experience observing and reflecting upon teaching, have the highest agreement with the expert raters. It is important to note, however, that the differences in agreement with the expert raters are incredibly small. Second, overall the agreement with the expert raters is higher than the agreement with the authors for this task. This is expected as the expert raters completed a task more similar to the task completed by the PSTs.

Agreement when PSTs are grouped by MKT scores. Table 4-7 below shows the average agreement of each MKT group with the EM authors. The table shows that PSTs with higher MKT, on average, agreed with the authors more than those with low or medium MKT for the lesson enactment task. The opposite trend occurs for the assessment task.

Table 4-7: Agreement for Each Task Between the EM Authors and the PSTs Grouped by MKT

MKT Group	Curriculum Materials Task	Lesson Enactment Task	Assessment Task	Average
Low (n=6)	38%	11%	30%	26%
Medium (n=6)	25%	25%	30%	26%
High (n=5)	35%	29%	17%	27%
Average (n=17)	33%	22%	25%	27%

One reason for this surprising trend for the assessment task is the frequent assignment of SMP 6 “Attend to precision” in cases where the answer required units. For instance, in the first problem the authors determined that SMP 6 fit. Though the authors do not say specifically why they assign an SMP, it is likely both because units are required but also careful, precise calculation. Of the six participants in the low MKT group, four of them agreed with the authors with three of them giving the explanation that SMP 6 is assessed because the PSTs are required to use units. In contrast, only three of the six members of the medium MKT group and two of the high MKT group agreed with the authors on this task. Of the PSTs in the medium MKT group who assigned SMP 6, two said that it was because students had to use units and one said that students needed to do their calculations with precisions. The one student with high MKT who assigned SMP 6 said it was because students need to be precise in counting by twos.

Another reason for this pattern is the interpretation of SMP 7 “Look for and make use of structure.” The authors assigned SMP 7 in several problems, including the problem in Figure 4-3 below. In this case it seems the authors are referring to the structure of the shapes and using those structures to determine a pattern. For this problem, six out of the seven PSTs with low MKT agreed with the authors. However, only one PST with medium MKT and one with high MKT felt that SMP 7 was assessed by this problem. PSTs with medium and high MKT seemed to interpret the problem to be more related to SMP 1 “Make sense of problems and persevere in solving them” and SMP 2 “Reason abstractly and quantitatively.”

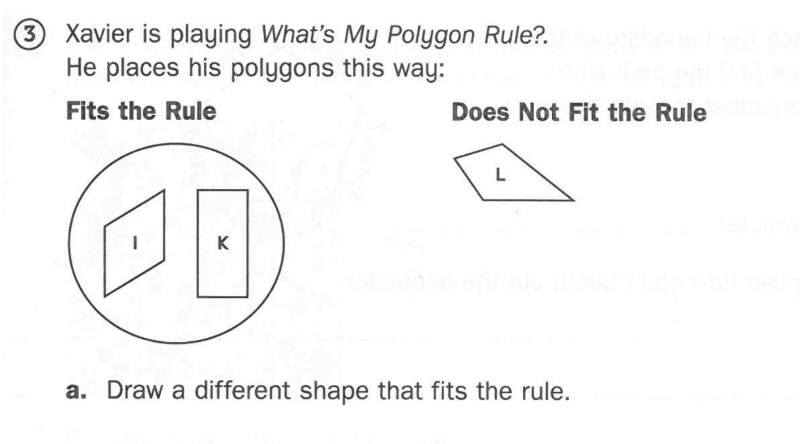


Figure 4-3. Problem from Everyday Mathematics Grade 3 Unit 4 student assessment. Reprinted from Everyday Mathematics Assessment Handbook, (p. 35), The University of Chicago School Mathematics Project, 2015, Columbus, OH: McGraw-Hill Education. Copyright 2015 by McGraw-Hill Education. Reprinted with permission.

Table 4-8 below shows the agreement between the PSTs and the expert raters for the lesson enactment task when the PSTs are grouped by MKT scores. A similar trend exists when comparing the agreement in this table to that of Table 4-7 in that those in the high MKT group agree with the expert raters more than the other groups, however the low and medium MKT groups agree with the expert raters in almost the same percentage of instances. Additionally, there is more agreement total between the PSTs and the expert raters than there is for the PSTs and the authors for this task. This could be because the authors only assigned one SMP to each of the three clips used in the lesson enactment task so there were fewer opportunities for PSTs to agree with the authors. If the PST did not assign that one specific SMP there was 0% agreement. Whereas if a PST missed one of the SMPs assigned by the expert raters there were still other opportunities to agree.

Table 4-8: Agreement Between PSTs Grouped by MKT and the Expert Raters for the Lesson Enactment Task

MKT Group	Lesson Enactment Task Agreement
Low (n=6)	51%
Medium (n=6)	52%
High (n=5)	63%
Average (n=17)	55%

Agreement when PSTs are grouped by MBI scores. Table 4-9 below shows the agreement between the PSTs and the authors based on their MBI groups. As explained in the literature review, it is expected that mathematical beliefs influence the ways that the PSTs interact with curriculum materials and interpret the practice standards.

Table 4-9: Agreement for Each Task Between EM Authors and PSTs Grouped by MBI Score

MBI Group	Curriculum Materials Task	Lesson Enactment Task	Assessment Task	Average
Low (n=6)	35%	21%	20%	25%
Medium (n=5)	29%	26%	27%	27%
High (n=6)	33%	18%	31%	27%

Average (n=17)	32%	22%	26%	27%

On average, PSTs in the medium and high MBI group had the most agreement with the authors, followed by those in low MBI group. There is not a large difference, however, among the three groups. PSTs in the high MBI group had the most agreement with the authors on the assessment task. For the curriculum materials task, all three groups were similar in their agreement with the authors. Interestingly, in the lesson enactment task, the medium MBI group agreed with the authors more than the low and high MBI groups.

Table 4-10 below shows the agreement between the PSTs and the expert raters on the lesson enactment task when the PSTs are grouped by MBI scores. Interestingly, PSTs in the medium MBI group agreed the most with the authors. It is unclear why the trend of PSTs with medium MBI scores agree with the expert raters more than those with higher or lower MBI scores. One explanation is that the expert raters do not have highly cognitively-aligned beliefs about teaching and learning mathematics. The MBI was not administered to the expert raters so it is not possible to know how their scores would compare to the PSTs' scores. It is also important to note here, as when the PSTs were grouped by other characteristics, there is a very small difference among the three groups in agreement with the expert raters.

Table 4-10: Agreement Between PSTs Grouped by MBI Scores and the Expert Raters for the Lesson Enactment Task

MBI Group	Lesson Enactment Task Agreement
Low (n=6)	54%
Medium (n=5)	58%
High (n=6)	53%

Average (n=17)	55%

SMP Frequency

The frequency that the different SMPs were assigned by the authors and the PSTs differs considerably. Table 4-11 below shows the frequency of which the SMPs were assigned by the authors and the PSTs for each of the tasks. The values reported in this table for the PSTs are the total number of times that the PSTs assigned each SMP. Since there are 17 PSTs, the frequencies of SMP assignment for the PSTs is much greater in all categories than the frequency of SMPs assigned by the authors. However, from this table you can see many instances where the authors did not assign an SMP at all for a task, but the PSTs assigned an SMP many times (e.g., for SMP 1 in the assessment task the authors never assigned SMP 1 whereas the PSTs assigned it 103 times). Additionally, there are instances where the authors and PSTs agreed that particular SMPs were present in a task but disagreed on which were most robustly present (e.g., in the curriculum materials task SMP 3 is the most frequently assigned by the PSTs but the authors most frequently assigned SMP 4).

Table 4-11: The Frequency of the SMPs in the Three Tasks as Assigned by the Authors and the PSTs

SMP	Curriculum Materials Task		Lesson Enactment Task		Assessment Task		Total	
	Authors	PSTs	Authors	PSTs	Authors	PSTs	Authors	PSTs
1	4	26	0	12	0	103	4	141
2	0	19	0	8	0	59	0	86
3	0	48	0	26	0	56	0	130
4	5	19	0	16	5	39	10	74
5	0	14	0	23	0	43	0	80
6	4	32	3	16	5	99	12	147
7	0	4	0	2	4	41	4	47
8	0	3	0	3	1	20	1	26

In addition to a comparison between the authors and the PSTs, much can be learned from looking just at the PSTs' designations. SMP 1 "Make sense of problems and persevere in solving them," SMP 3 "Construct viable arguments and critique the reasoning of others," and SMP 6 "Attend to precision" appear the most frequently. After further examination of the explanations given by PSTs in their recording tools, as well as through interviews, it seems that many PSTs were quite liberal in determining what would meet the threshold for these SMPs. Additionally, for SMP 3 and SMP 6 there were particular problem formats and key words that signaled PSTs to choose these SMPs without much further examination of what SMP actually has the potential to be practiced or assessed. This will be discussed more in the following section.

Table 4-12 below shows the same data as Table 4-11 above, however the counts for the frequency of SMPs assigned by the PSTs has been standardized. In this case, the raw frequencies of the SMPs assigned by PSTs reported in Table 4-11 were divided by 17, the number of PSTs in the study. Since it was not necessary to be accurate beyond a whole number in order to compare the PSTs' counts with the authors', I rounded each of these standardized counts to the nearest whole number. As the authors together only gave one rating, the frequencies for the SMPs assigned by the authors are the same in Table 4-11 and 4-12.

Table 4-12: SMP Frequency with PST Standardized Counts

SMP	Curriculum Materials Task		Lesson Enactment Task		Assessment Task		Total	
	Authors	PSTs	Authors	PSTs	Authors	PSTs	Authors	PSTs
1	4	2	0	1	0	6	4	8
2	0	1	0	0	0	3	0	5
3	0	3	0	2	0	3	0	8
4	5	1	0	1	5	2	10	4
5	0	1	0	1	0	3	0	5
6	4	2	3	1	5	6	12	9

7	0	0	0	0	4	2	4	3
8	0	0	0	0	1	1	1	2

When comparing the frequency of the assignment of each SMP by the authors and the PSTs, interesting areas of discrepancy and agreement emerge. For instance, in the curriculum materials task the authors assigned SMPs 1, 4, or 6 100% of the time, with their ratings fairly evenly distributed among the three SMPs (i.e., they assigned SMP 1 four times, SMP 4 five times and SMP 6 four times). The PSTs assigned these three SMPs only 47% of the time. That means that they agreed with the authors slightly less than they disagreed with the authors. For the lesson enactment task, the authors assigned SMP 6 100% of the time whereas the PSTs assigned SMP 6 only 15% of the time. This is a considerable discrepancy. Finally, in the assessment task the authors assigned SMP 4, 6, and 7 in 93% of the instances with their ratings fairly evenly distributed among these three SMPs. I disregarded SMP 8 from this calculation as the authors only assigned it once, so it was not assigned with the robustness of the other three SMPs. For this task the PSTs only assigned SMPs 4, 6, and 7 39% of the time. Further investigation into the nature of these discrepancies sheds light on how the PSTs are interpreting the SMPs and why such discrepancies between the PSTs and the authors may exist.

Examining Discrepancies Between SMP Designations by the PSTs and the Authors

As stated in Chapter 3, the PSTs were required to give an explanation for their designations while filling out the scoring tool for each task. This section is focused on investigating the explanations in order to better understand the nature of the disagreement between the authors and the PSTs. Did the PSTs attend to something that makes mathematical sense but was not attended to by the authors? Were the PSTs generous in their interpretation of a particular SMP? Were there misconceptions that led to the discrepancies?

Curriculum materials task. As stated above, the authors only assigned SMPs 1, 4, and 6 throughout this task, whereas PSTs only chose those three SMPs less than half of the time. Large discrepancies exist most prominently between the authors and the PSTs for SMPs 1, 3 and 4.

SMP 1. SMP 1 "Make sense of problems and persevere in solving them," in this instance is assigned frequently by the authors but very little by the PSTs compared to other SMPs for this task. As explained in the methods section, the authors break up some of the SMPs into smaller grain sizes, so the description of the practice that the authors assigned four times reads as follows: "Reflect on your thinking as you solve your problem" (UCSMP, 2015b, p. 387). They assigned this SMP most frequently to specific questions that were written in the lesson that asked students to reflect on their own thinking, in particular thinking about their errors (e.g., "Did you draw any pens that do not use 24 feet of fence?…How could a drawing that does not have a perimeter of 24 feet help you draw a model that does use 24 feet of fence?" (UCSMP, 2015b, p. 389)), the thinking of others when examining sample responses (e.g., "Look at rectangle A. Why do you think the X is there?" (UCSMP, 2015b, p. 390)), or a comparison of different strategies (For this problem, do you think these rectangles model two different rabbit pens?" (UCSMP, 2015b, p. 390)).

In contrast, very few PSTs assigned SMP 1 to this task. A closer look at the breakdown of the task is important in interpreting these results. Table 4-13 below shows the frequency of SMPs assigned by the authors and the PSTs for each part of the curriculum materials task. Note that the PSTs' scores are not standardized so a one-to-one comparison with the authors is not appropriate. PSTs did not assign SMP 1 as frequently as other SMPs overall for this task, however, they assigned it frequently in the *Math Message and Calculating Perimeters and Area*s section as well as the *Solving the Open Response Problem* section. What is interesting in this case is that in these sections the students are actively engaged in solving a mathematics task. However, in *Getting Ready for Day 2,* the curriculum materials are explaining to the teacher different ways to plan the discussion based on the responses given by students. When this practice was assigned, it was interpreted differently among the PSTs. Though the authors labeled some of the questions that were suggested in the lesson plan with SMP 1, as by asking the questions the teacher would be giving students the opportunity practice SMP 1, PSTs did not feel that this was problem solving as the students were not actually even present for this section of the plan. However, a more substantive discrepancy occurs in the *Reengaging in the Problem and Revising Answers* section. Very few PSTs assigned SMP 1 for this section, though the authors did assign SMP 1. The curriculum materials state "Children reengage in the problem by analyzing and critiquing other children's work in pairs and in a whole-group discussion. Have children discuss with partners before sharing with the whole group" (UCSMP, 2015b, p. 393). The major difference in this section and the sections where PSTs did assign SMP 1 is that in this section there were fewer specific questions outlined for the teacher to ask students and less step-by-step structure for how this section should be enacted. The PSTs were required to think more about what it would look like for students to critique others' work and revise their thinking. It seems reasonable that in this instance more PSTs assigned SMP 3 "Construct viable arguments and critique the reasoning of others."

Table 4-13: Frequency of SMPs Assigned in Each Part of Curriculum Materials Task

SMP	Math Message and Calculating Perimeters and Areas		Solving the Open Response Problem		Getting Ready for Day 2		Setting Expectations		Reengaging in the Problem and Revising Answers	
	Authors	PSTs	Authors	PSTs	Authors	PSTs	Authors	PSTs	Authors	PSTs
1	1	8	1	14	1	1	0	0	1	3
2	0	6	0	8	0	3	0	0	0	2

3	0	5	0	6	0	12	0	11	0	14
4	1	11	1	2	1	3	1	2	1	1
5	0	3	0	6	0	2	0	2	0	1
6	1	7	1	7	1	6	0	3	1	10
7	0	0	0	1	0	0	0	0	0	3
8	0	1	0	0	0	0	0	0	0	2

It is also interesting how the names of the different sections may have cued the PSTs to assign particular SMPs. This is most blatant in the section *Solving the Open Response Problem*, the section where the language of “solving problems” is actually in the title. This is the section where the PSTs most frequently assigned SMP 1.

In addition to the discrepancies between the authors and the PSTs on their assignment of SMP 1, there was variation in the explanations given by the PSTs for why SMP 1 had the potential to be practiced by students in the different sections of this task. For instance, many PSTs simply said that during the *Math Message and Calculating Perimeters and Areas* and *Solving an Open Response* portions of the lesson, students would be working to solve a problem, so that makes this an instance of SMP 1. Others were more specific about what they meant by solving a problem, some attended to the planning of a solution method, and still others talked about working to understand the different parts of the problem. However, none of them actually used the language of the authors: “Reflect on your thinking as you solve your problem” (UCSMP, 2015b, p. 387).

SMP 3. Another SMP with a large discrepancy between the PSTs and the authors was SMP 3 “Construct viable arguments and critique the reasoning of others.” This was the most frequently assigned SMP by the PSTs for this task, however it was never assigned by the authors throughout the task. The PSTs assigned SMP 4 most frequently in the last three sections of the lesson: “Getting Ready for Day 2,” “Setting Expectations” and “Reengaging in the Problem and

Revising Answers." In each of these three sections it seems reasonable that the PSTs assigned SMP 3 and it is unclear why the authors did not.

In the section "Getting Ready for Day 2" the teacher is planning for the second day of the two-day lesson. The lesson provides questions for the teacher to ask students to get them to discuss sample children's work. Some questions are "How could mistakes like this help you draw a correct model for the rabbit pen?" (UCSMP, 2015b, p. 387), "Do you agree that the perimeter of both rectangles is 24 feet? How do you know?" (p. 387) and "What could this child add to make the models easier to understand" (p. 387). These questions seem reasonable to give students the opportunity to critique the sample students' work as well as construct their own arguments.

The next section, "Setting Expectations," simply provides support to the teacher to set expectations for the actual discussion of the sample students' work as well as discussing their own solutions. Two sentence stems are given in this section in order to support students in discussing their own as well as others' explanations: "I think this is a clear and complete explanation because_____." (p. 393) as well as "I think this explanation needs to include_____." (p. 393). Though the students are not yet engaging in the arguing and critiquing, these sentences are setting students up to do so. Students are previewing and possibly practicing the sort of language they will be using to engage with the SMP.

In the final section, "Reengaging in the Problem and Revising Work," students are actually having the discussion about the sample students' work, sharing their own work, and revising the models of their rabbit pens if needed. This seems like a clear opportunity for students to have the opportunity to practice SMP 3.

SMP 4. The authors assign SMP 4 "Model with mathematics" in every section of the curriculum materials task. However, the PSTs assign it very infrequently in all sections except the first, *Math Message and Calculating Perimeters and Areas*. The specific wording the authors used to define SMP 4 is "Model real-world situations using graphs, drawings, tables, symbols, numbers, diagrams, and other representations" (UCSMP, 2015b, p. 387). In the first section students discuss how to find perimeter and area using a worksheet, shown below in Figure 4-4.

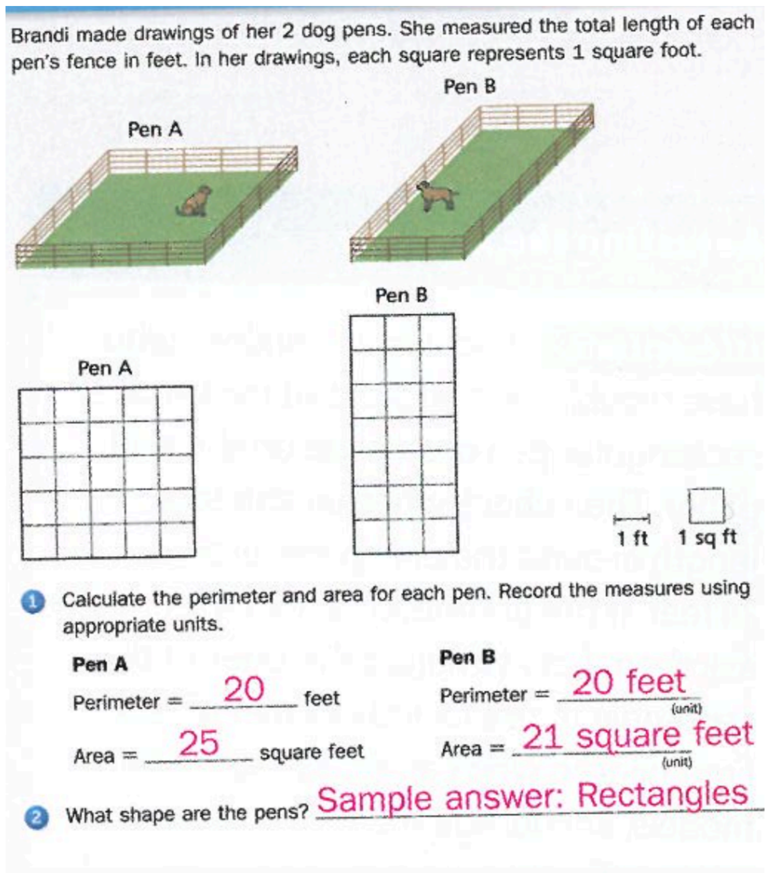
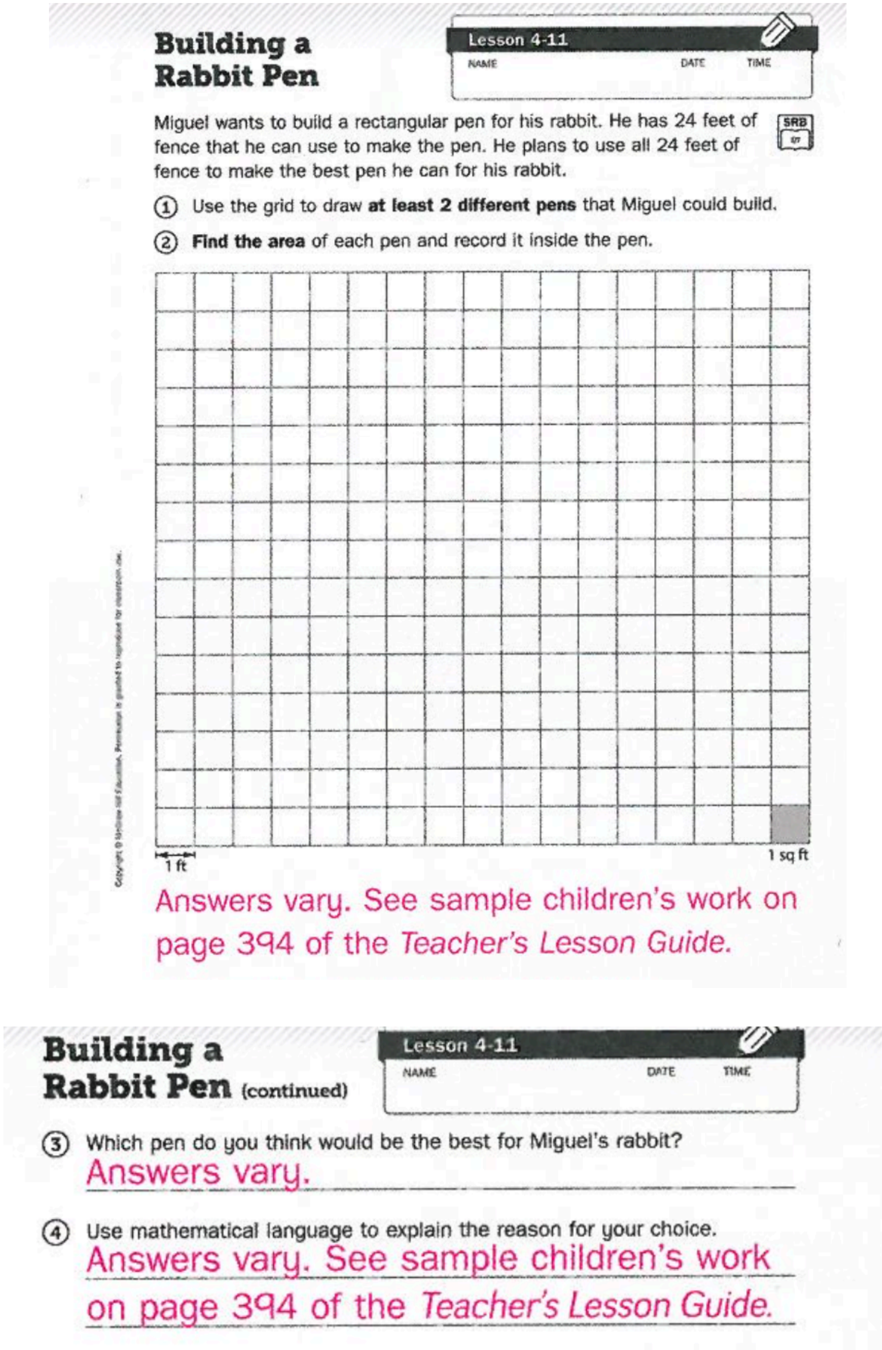


Figure 4-4. Journal page 129 in Everyday Mathematics Grade 3 curriculum materials. Reprinted from Everyday Mathematics Teacher's Lesson Guide Volume 1, (p. 387), The University of Chicago School Mathematics Project, 2015, Columbus, OH: McGraw-Hill Education. Copyright 2015 by McGraw-Hill Education. Reprinted with permission.

The most obvious reason that almost every PST assigned SMP 4 to this section is that the authors explicitly say in the text that the children are using models and provide teachers with the language to support students in understanding how they are modeling:

> Point out to children that they used models of the dog pens to solve the problem. Mathematical models are representations of real-world situations or objects that help solve problems. The drawings of the pens show what the pens look like in the real world, and the rectangles are models that can help children find the areas and perimeters" (UCSMP, 2015b, p. 387).

Interestingly, in the second section of the lesson, students are working on a very similar problem involving building a rabbit pen. The wording that was used by the authors in the previous section as quoted above from p. 387 to explain the modeling that the students are doing is quite similar to what the students are doing in the second section. Figure 4-5 below shows the task that students work on in the second section along with the rest of the lesson. Although there are no drawings of rabbit pens that show what the pens would look like *in the real world*, the students are still creating models to show the different areas that are possible for the pens given the perimeter of 24 feet. However, of the 11 PSTs who assigned SMP 4 in the first section, only two assigned it in the second section.


Building a Rabbit Pen

Lesson 4-11

NAME DATE TIME

Miguel wants to build a rectangular pen for his rabbit. He has 24 feet of fence that he can use to make the pen. He plans to use all 24 feet of fence to make the best pen he can for his rabbit.

1. Use the grid to draw **at least 2 different pens** that Miguel could build.
2. **Find the area** of each pen and record it inside the pen.

1 ft

1 sq ft

Answers vary. See sample children's work on page 394 of the *Teacher's Lesson Guide.*

Building a Rabbit Pen (continued)

Lesson 4-11

NAME DATE TIME

3. Which pen do you think would be the best for Miguel's rabbit?
Answers vary.
4. Use mathematical language to explain the reason for your choice.
Answers vary. See sample children's work on page 394 of the *Teacher's Lesson Guide.*

Figure 4-5. Math Master's page 143-144 in Everyday Mathematics Grade 3 curriculum materials. Reprinted from Everyday Mathematics Teacher's Lesson Guide Volume 1, (p. 389), The University of Chicago School Mathematics Project, 2015, Columbus, OH: McGraw-Hill Education. Copyright 2015 by McGraw-Hill Education. Reprinted with permission.

The explanations given by the PSTs do not help to illuminate the reason for the discrepancy. Why do so many PSTs feel that students had the opportunity to practice SMP 4 in the first section but not in the second? For example, one PST, when explaining why she chose SMP 4 for the first section of the lesson stated that students "…aren't actually building the pen but representing it with mathematics" (PST M4). Another PST said that "Students are using a grid to find the perimeter and area. These can be used in the real world" (PST U42). Still another PST said, "A diagram is used to help conceptualize the problem" (PST U41). In all three of these instances the reasoning given for the PST assigning SMP 4 to the first section of the lesson could also be used to justify assigning it to the second part of the lesson, however none of these PSTs assigned it to the second part of the lesson.

Three explanations seem reasonable for why this discrepancy may have occurred. First, as stated earlier, the authors were very explicit that students were modeling with mathematics while solving the problem in the first section, however they do not explicitly mention modeling in the second section. Second, PSTs may have had different interpretations of what is meant by problems arising in everyday life in the CCSSM definition: "Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace" (NGA & CCSSO, 2010). In the first section of the lesson there are actually drawings of dog pens and the authors refer to these when describing how the students are modeling. In the second section there is no actual picture of a rabbit pen, just a description of a real-world scenario. Perhaps some PSTs read the authors' description and thought that an actual physical picture of a real-world object had to be present in order for a model to be considered a model. This may have been misleading to PSTs. Third, PSTs were only allowed to assign up to three SMPs for each section. When looking across the two sections at the PSTs who assigned SMP 4 in the first section but not the second, of the nine PSTs who fall into this category, five of them assigned three SMPs for the second tasks. This means that perhaps they felt that, even though students were engaging in SMP 4, there were three other SMPs that were more prominently featured in the task.

In the other sections of the lesson PSTs did not assign SMP 4 frequently. It is likely that this was because during the other sections students were discussing their models and revising them, however they had already completed the initial construction of their rabbit pens.

Lesson enactment task. Table 4-14 below shows the ratings given by the authors, expert raters and PSTs for each of the three clips. It is important to note that the raw frequencies should not be compared as the authors gave only one rating and the expert raters came to consensus and gave only three ratings per clip, but no more than one for each SMP for each clip. The PSTs' ratings are the cumulative frequency of the SMPs assigned by all seventeen PSTs.

Table 4-14: Frequency of SMP Assignment in Clips of the Lesson Enactment Task

	Clip 1			**Clip 2**			**Clip 3**		
SMP	Authors	Expert Raters	PSTs	Authors	Expert Raters	PSTs	Authors	Expert Raters	PSTs
1	0	0	3	0	0	3	0	1	6
2	0	0	2	0	1	4	0	0	2
3	0	1	11	0	0	0	0	1	15
4	0	1	8	0	1	5	0	0	3
5	0	0	8	0	1	14	0	0	1
6	1	0	11	1	0	1	1	1	4
7	0	1	0	0	0	1	0	0	1
8	0	0	1	0	0	1	0	0	1

For discussion of the discrepancies for this task I will begin as I did for the curriculum materials task by looking at areas of discrepancy between the authors and the PSTs. The expert raters will be used to further inform the comparison as the authors were not looking at the clips at the same grain size as the expert raters and the PSTs. For this task I considered a difference a discrepancy if the authors assigned an SMP for a clip and fewer than half (i.e., 8 or fewer) of the PSTs assigned the SMP. Similarly, if more than half of the PSTs (i.e., 9 or more) assigned an SMP to a clip and the authors did not assign it, that is also considered a discrepancy. For this task, there are discrepancies for SMPs 3, 5, and 6 in different clips within the task.

SMP 3. The PSTs assigned SMP 3, "Construct viable arguments and critique the reasoning of others," in clip 1 and clip 3 eleven and fifteen times respectively. This means that out of seventeen PSTs almost all of them agreed that students had the opportunity to practice SMP 3 during these clips. Though the authors did not give this rating for these clips, the expert raters agreed that the students had the opportunity to practice this SMP in both of the clips. This is a reasonable assignment as the first clip is a video of students working on the task, including answering the question: "Which pen do you think would be the best for Miguel's rabbit? Use mathematical language to explain the reason for your choice?" (UCSMP, 2015b, p. 389). The third clip is of a class having a discussion about their answers including students sharing their solutions and critiquing others' work.

SMP 5. SMP 5 "Use appropriate tools strategically" was not assigned by the authors but eight PSTs assigned it in clip 1 and fourteen assigned it in clip 2. Expert raters agree with PSTs that SMP 5 was practiced by students in clip 2 but not clip 1.

Though clip 1 was only assigned by eight PSTs, slightly fewer than half, the reason for the discrepancy is important as it exemplifies a common interpretation of SMP 5 by the PSTs.

Clip 1 involved students working on the rabbit pen problem individually. They drew out the pens on graph paper using pencils and some used rulers to make straight lines. PSTs gave a variety of reasons as to why they assigned SMP 5 to this clip. Many PSTs named the physical tools that students were using as they worked (e.g., "the student is using a ruler, paper and pencil to solve the problem" (PST U42)). One PST was quite specific about how the ruler was used and tried to include wording from the SMP in his description to justify why this clip met the threshold for SMP 5:

> At this point in the video, you can see that the student is clearly making his shape more accurate with an appropriate tool, in this case a ruler. They seem to be clear on how to use the tool as it relates to making a decision on when to use the tool, and how to use the tool strategically. (PST U34).

Still other PSTs described the way the student determined the area of the rabbit pens by utilizing the squares on the graph paper (e.g., "the student is using the grid box that is provided to find the area. The student is counting the square units to help find the area. The student is using the strategy to mark the boxes with x's that have been already accounted for in finding the area" (PST U36)).

In the discussion that the expert raters had when coming to consensus on which SMPs were practiced in which clips, they did discuss the presence of SMP 5 in clip 1. Though they agreed that students used tools, it was the idea of using them strategically that they did not believe was present. Students did indeed use rulers, graph paper and writing utensils, however it seemed to the expert raters that they were likely told by teachers to use rulers to make their lines straight and also that since this is not the mathematical use for a ruler, any straightedge could have been used for the same purpose, it should not count as an instance of SMP 5. The expert raters did not discuss the strategic use of the graph paper so it could be that attending more to that tool specifically may have led the expert raters to come to different conclusions.

SMP 6. SMP 6 "Attend to precision" was the SMP that the authors believed was exemplified by the video as a whole (i.e., all three clips plus others put together to make a complete video of the lesson). Interestingly, the PSTs and expert raters are both in disagreement with the authors on different clips. Eleven PSTs agreed with the authors for clip 1 however only one agreed for clip 2 and only four for clip 3. The expert raters only agreed that students practiced SMP 6 in clip 3.

As the authors only assigned one SMP for the whole set of clips, they may not agree that SMP 6 was practiced by students in all three clips. It is interesting, however, that for clip 1 more than half of the PSTs assigned SMP 6 whereas the expert raters did not, and for clip 3 the expert raters assigned SMP 6 but only four PSTs assigned it. For the first clip, the reasoning that PSTs gave for the presence of students attending to precision was pretty equally distributed among the following four student activities: labelling the diagram, using appropriate units, double-checking their work and being careful about measuring and counting. When discussing whether SMP 6 was a good fit for clip 1, the expert raters discussed the possibility of assigning SMP 6, but it did not end up being among the most prominent three SMPs according to the expert raters. According to the expert raters, students were attending to precision, but that it was really just the precision in counting that would have made a difference in the students' answers. For instance, although students were prompted to include units in their answer and the units are arguably important, the problem was not a case where students were doing unit conversions or another type of problem where attending to the units would actually influence the numerical answer.

Assessment task. When looking at the assessment task, the overall frequency of assignment of SMPs as shown in Table 4-11 above shows a large discrepancy between the authors and the PSTs in the assignment of SMP 1, as well as smaller yet still substantial differences in SMPs 2, 3, 4, and 7. In addition, Table 4-15 below shows the breakdown of which SMPs were assigned by the authors and the PSTs for each problem on the assessment. It is important to remember that these are not standardized scores so the highest possible frequency for any SMP on a problem for the authors is 1 and for the PSTs is 17. It is more appropriate to look at the presence or lack thereof of an SMP as assigned by the authors and the strength of the presence for the PSTs. Throughout this section I will go through each of the SMPs that had the greatest discrepancies between the authors and PSTs and examine individual assessment items as well as PSTs' explanations to try to determine the cause of the discrepancy.

Table 4-15: Frequency of Assignment of SMPs for Assessment Items

	Standards for Mathematical Practice (SMPs)							
Problem	1	2	3	4	5	6	7	8
HL1								
Authors	0	0	0	0	0	1	0	0
PSTs	10	5	0	2	1	10	1	0
HL2								
Authors	0	0	0	0	0	1	0	0
PSTs	11	3	4	9	1	5	1	0
1								
Authors	0	0	0	0	0	1	0	0
PSTs	0	1	0	0	16	9	0	0

2								
Authors	0	0	0	1	0	0	0	0
PSTs	4	3	0	6	4	4	0	0
3a								
Authors	0	0	0	0	0	0	1	0
PSTs	5	5	1	2	1	0	7	3
3b								
Authors	0	0	0	0	0	0	1	1
PSTs	2	2	14	0	0	3	2	2
4a								
Authors	0	0	0	0	0	0	1	0
PSTs	1	1	5	1	0	5	8	1
4b								
Authors	0	0	0	0	0	0	1	0
PSTs	2	1	5	1	0	5	8	1
5a								
Authors	0	0	0	0	0	0	0	0
PSTs	5	3	0	2	3	11	0	1
5b								
Authors	0	0	0	0	0	1	0	0
PSTs	2	3	12	0	0	5	0	0
5c								

Authors	0	0	0	0	0	0	0	0
PSTs	4	4	1	2	0	5	0	1
6a								
Authors	0	0	0	1	0	0	0	0
PSTs	8	4	0	3	3	7	1	1
6b								
Authors	0	0	0	1	0	0	0	0
PSTs	8	4	0	2	3	7	2	1
7								
Authors	0	0	0	1	0	1	0	0
PSTs	7	5	14	2	1	2	1	0
8								
Authors	0	0	0	1	0	0	0	0
PSTs	8	4	0	4	4	6	3	3
9								
Authors	0	0	0	0	0	0	0	0
PSTs	9	1	0	2	1	3	3	1
10a								
Authors	0	0	0	0	0	0	0	0
PSTs	4	4	0	0	3	4	1	0
10b								
Authors	0	0	0	0	0	0	0	0

PSTs	6	2	0	1	1	6	2	3
10c								
Authors	0	0	0	0	0	0	0	0
PSTs	7	4	0	0	1	2	1	2

SMP 1. The greatest discrepancy between the authors and the PSTs was in the assignment of SMP 1 “Make sense of problems and persevere in solving them.” This SMP was the most frequently assigned by PSTs whereas the authors never assigned this SMP for this task. When determining the reason for this discrepancy I investigated the explanations given by the PSTs. At least part of the reason for the discrepancy lies in the interpretation of the SMP, in particular the threshold for what qualifies as meeting the expectations of the SMP. The CCSSM text defines SMP 1 in the following way:

> Mathematically proficient students start by explaining to themselves the meaning of a “problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for

regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. (NGA & CCSSO, 2010).

To show the range of types of problems that PSTs overwhelmingly determined had the potential to assess SMP 1, I'll refer to Figure 4-6, which shows item 9, and Figure 4-7 which shows item HL2 (i.e., item 2 on the Home Link). In the problem in Figure 4-6, 9 out of the 17 PSTs determined it assessed SMP 1. In the problem in Figure 4-7, 11 out of the 17 PSTs determined it assessed SMP 1.

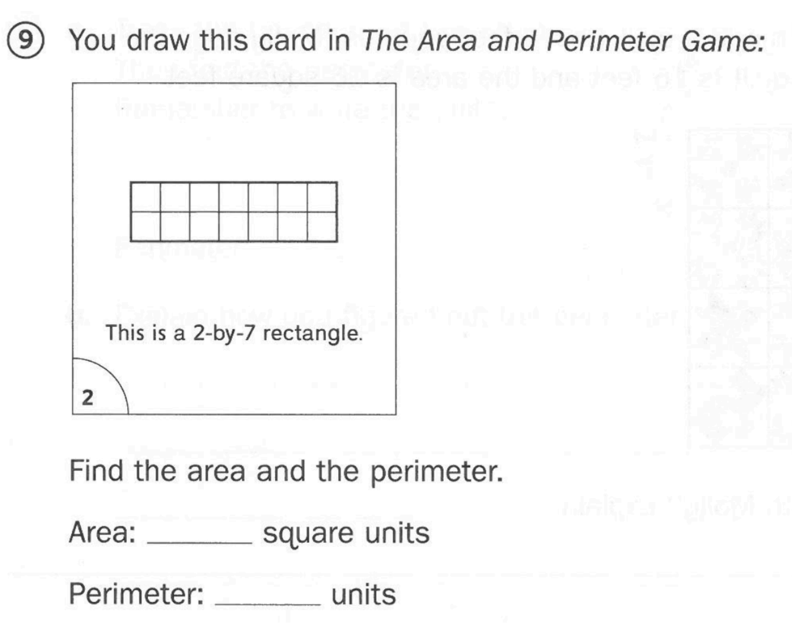


Figure 4-6. Problem from Everyday Mathematics Grade 3 Unit 4 student assessment. Reprinted from Everyday Mathematics Assessment Handbook, (p. 38), The University of Chicago School Mathematics Project, 2015, Columbus, OH: McGraw-Hill Education. Copyright 2015 by McGraw-Hill Education. Reprinted with permission.

② Sue wants to paint the longest wall in her bedroom pink. She measured the wall and found that it is 10 feet long and 8 feet tall. When she went to the hardware store to buy paint, Sue learned that 1 quart of paint can cover 50 square feet.

Sue should buy ______________ of paint.
(unit)

Show how you figured out how much paint Sue will need.

Figure 4-7. Problem from Everyday Mathematics Grade 3 Lesson 4-11 Home Link. Reprinted from Everyday Mathematics Teacher's Lesson Guide Volume 1 (p. 395), The University of Chicago School Mathematics Project, 2015, Columbus, OH: McGraw-Hill Education. Copyright 2015 by McGraw-Hill Education. Reprinted with permission.

Though there may be features in each of these problems that meet the criteria for SMP 1, there is a certainly a difference in the level of problem solving required. For the problem in Figure 4-6, if students know the definition of perimeter and area they are able to solve the problem simply by counting the outside lengths of the squares that comprise the rectangle to determine the perimeter and by counting all of the squares to determine the area. There is very little problem solving required in order to answer the question. However, in order to solve the problem shown in Figure 4-7, students have to read the problem carefully to understand what the problem is asking, determine how to find the area of the wall using multiplication, and then think about the problem context and determine the number of quarts of paint needed. Students must decide if it is possible to buy a fraction of a quart of paint or if it is more reasonable that a paint store would only sell quarts. Additionally, they are asked to show this work.

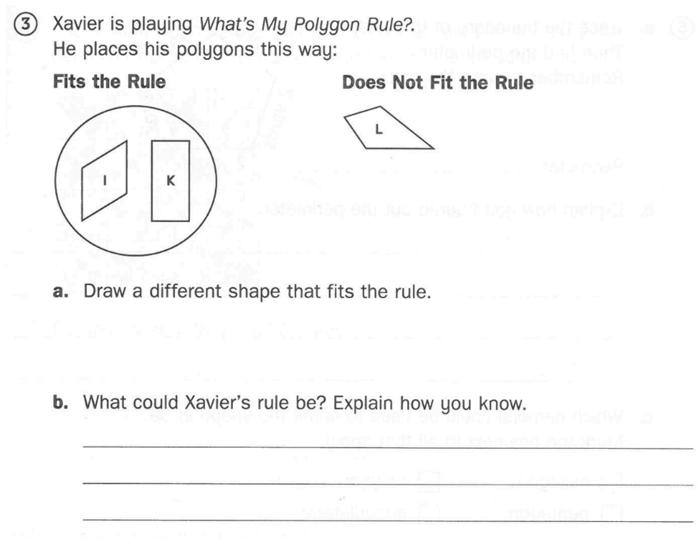
SMP 2. SMP 2 "Reason abstractly and quantitatively" is an interesting SMP in the assessment task as, looking across the different assessment items, there is not an instance where more than five PSTs assigned it to an assessment item. This means that, though the total number of times it was assigned was relatively high, there was never an instance where the majority of the PSTs assigned it to an item.

Of those PSTs who assigned SMP 2 frequently, some assigning it to up to seven items, the most common explanation had to do with quantifying the length of a line or the perimeter of a shape. Though students do abstract a quantity, the wording of the SMP in the CCSSM specifically talks about decontextualizing and contextualizing:

> Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. (NGA & CCSSO, 2010).

It seems that the authors, as well as most PSTs, did not feel that these types of problems met the threshold for SMP 2.

SMP 3. The assignment of SMP 3 "Construct viable arguments and critique the reasoning of others" was quite different from SMP 2. Looking across the different assessment items, more than half of the PSTs assigned SMP 3 to three items: 3b, 5b, and 7. Fourteen PSTs assigned SMP 3 to item 3b, twelve assigned it to item 5b and fourteen assigned it to item 7. Figures 4-8, 4-9, and 4-10 show these three problems below. It is important to remember that PSTs assigned SMP 3 to only part b of each of problems 3 and 5 although the whole problem is shown in the figures so that the problem can be understood.



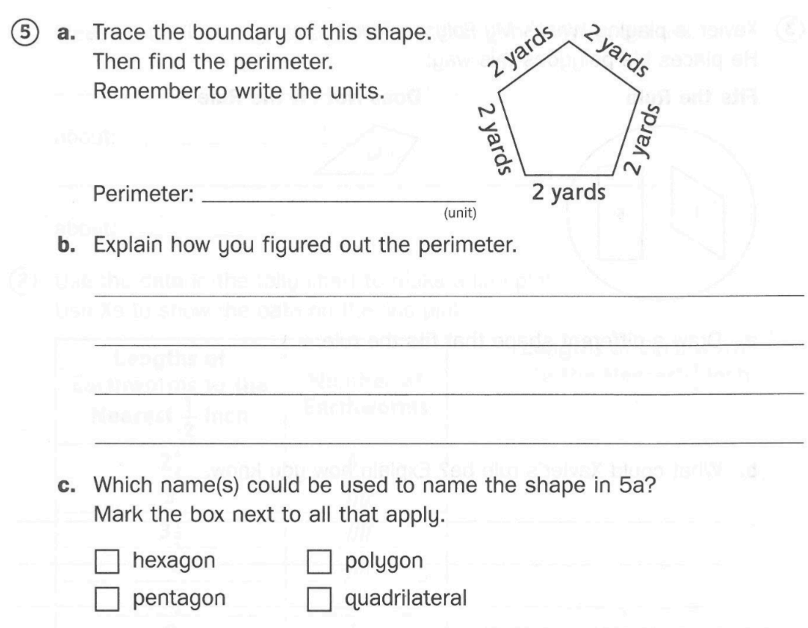
③ Xavier is playing *What's My Polygon Rule?*.
He places his polygons this way:

Fits the Rule **Does Not Fit the Rule**

a. Draw a different shape that fits the rule.

b. What could Xavier's rule be? Explain how you know.

Figure 4-8. Problem from Everyday Mathematics Grade 3 Unit 4 student assessment. Reprinted from Everyday Mathematics Assessment Handbook, (p. 35), The University of Chicago School Mathematics Project, 2015, Columbus, OH: McGraw-Hill Education. Copyright 2015 by McGraw-Hill Education. Reprinted with permission.



⑤ **a.** Trace the boundary of this shape.
Then find the perimeter.
Remember to write the units.

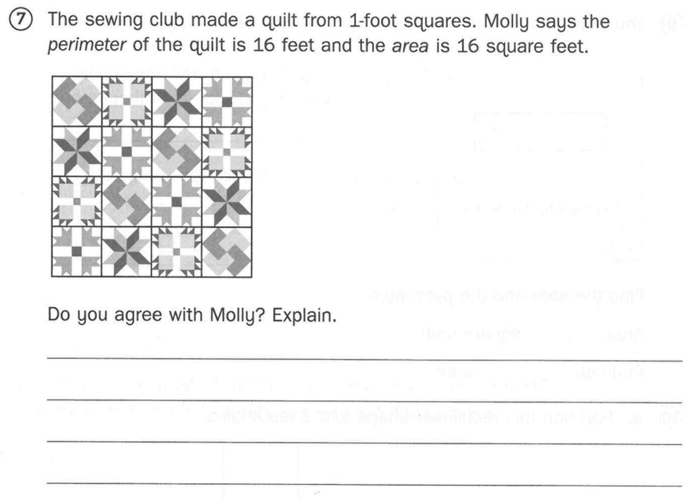
Perimeter: ______________________ (unit)

b. Explain how you figured out the perimeter.

c. Which name(s) could be used to name the shape in 5a?
Mark the box next to all that apply.

☐ hexagon ☐ polygon
☐ pentagon ☐ quadrilateral

Figure 4-9. Problem from Everyday Mathematics Grade 3 Unit 4 student assessment. Reprinted from Everyday Mathematics Assessment Handbook, (p. 36), The University of Chicago School Mathematics Project, 2015, Columbus, OH: McGraw-Hill Education. Copyright 2015 by McGraw-Hill Education. Reprinted with permission.


⑦ The sewing club made a quilt from 1-foot squares. Molly says the *perimeter* of the quilt is 16 feet and the *area* is 16 square feet.

Do you agree with Molly? Explain.

Figure 4-10. Problem from Everyday Mathematics Grade 3 Unit 4 student assessment. Reprinted from Everyday Mathematics Assessment Handbook, (p. 37), The University of Chicago School Mathematics Project, 2015, Columbus, OH: McGraw-Hill Education. Copyright 2015 by McGraw-Hill Education. Reprinted with permission.

In all three of the items students are asked to explain their reasoning in writing. When interviewing the PSTs, many of them stated that when they saw that students had to explain something, or even when they saw several lines available in a problem for a student to write, that signaled them to assign SMP 3 without much consideration for the actual task or what students were being asked to explain. The definition given for SMP 3 is as follows:

> Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments,

distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. (NGA & CCSSO, 2010).

The nature of the explanations required in these three problems are quite different. In item 3b, students are asked to determine the rule that Xavier, a fictional student, used to sort polygons. Then students explain how they knew that Xavier used the rule that they determined. Students in this case are justifying their conclusions and communicating them in writing. For item 5b, students are asked to explain their own strategy for determining the perimeter of a polygon. This likely includes listing out the procedural steps that led them to the perimeter. This does not seem to fit as clearly as an opportunity to practice SMP 3 as they are simply saying what they did procedurally rather than building an argument or justifying why they did those particular steps. It is similar to what was required of students in Figure 4-7, however students are listing their steps in writing rather than simply being asked to show their work. PSTs assigned SMP 3 only four times to item 3b, shown in Figure 4-8, however they assigned it 12 times to item 5b, shown in Figure 4-9, even though the tasks are quite similar, likely due to the requirement of students to write out their answer cuing PSTs to SMP 3. In item 7 students must determine if a student found the correct answer and then explain why they are correct or incorrect. This problem, like item 5b, likely includes listing the steps they used to solve the problem, but then it also requires students to relate their answer back to the context of the problem.

The main challenge with this particular SMP seems to be the presence of cuing features, in this case the presence of lines for students to write their thinking or reasoning rather than simply providing a blank space. Though the word "explain" is used in each of these tasks, it is clear that the nature of the tasks, and what students are asked to explain, is quite different between the tasks. This becomes problematic when PSTs do not take the time to analyze the tasks and discrepancies arise.

SMP 4. SMP 4 "Model with mathematics" was assigned much more by the authors than by the PSTs when looking at the standardized scores (i.e., the authors assigned it five times and the PSTs only two times). Interestingly, in all five items where the authors assigned SMP 4, items 2, 6a, 6b, 7, and 8, fewer than half of the PSTs also assigned SMP 4. In the one instance where more than half of PSTs assigned SMP 4, item HL2, the authors did not assign SMP 4. This means that there were no instances of agreement between the authors and the PSTs on the assignment of SMP 4 in the assessment task.

Figure 4-7 above shows item HL2. In all explanations given by the PSTs they note that students could use a model to explain their thinking, either in the form of a drawing/diagram or an equation. For example, one PST said "They also have to show their work so they can demonstrate their thinking in whatever way they find most useful. This may be in the form of a drawing or even a series of number sentences which means they are providing their own models" (PST U32). Additionally, five out of the nine PSTs who assigned SMP 4 to item HL2 mentioned the presence of a real-world problem, language taken directly from the definition of SMP 4: "Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation" (NGA & CCSSO, 2010). An example of this type of explanation is "Students need to be able to translate the real-world problem into mathematical models to solve this. The additional layer of paint demands a firm grasp of area and the ability to reflect on how their solutions do or don't make sense" (PST M7). Given the explanations provided by the PSTs and their direct connection to the definition given in the CCSSM, it seems reasonable that students would at least have the opportunity to practice SMP 4 when answering this question. It is unclear why the authors did not assign SMP 4 to this item, though upon further examination, and explained further below, in four out of the five items where the authors assigned SMP 4 students were using models rather than creating them. Perhaps the authors felt that students had to create their own models in order for an assessment item to count as an instance of SMP 4.

The authors most frequently assigned SMP 4 in instances where students used models to find answers. For instance, items 6a and 6b, shown below in Figure 4-11, require students to find the perimeter and area of a given rectangle. It is surprising that the authors assign SMP 4 in this instance given their own definition of SMP 4: "Model real-world situations using graphs, drawings, tables, symbols, numbers, diagrams, and other representations" (UCSMP, 2015b, p. 387). As they include the term "real-world" situations specifically, it is surprising that they assigned it here as this item does not include a real-world situation but a context-free rectangle.

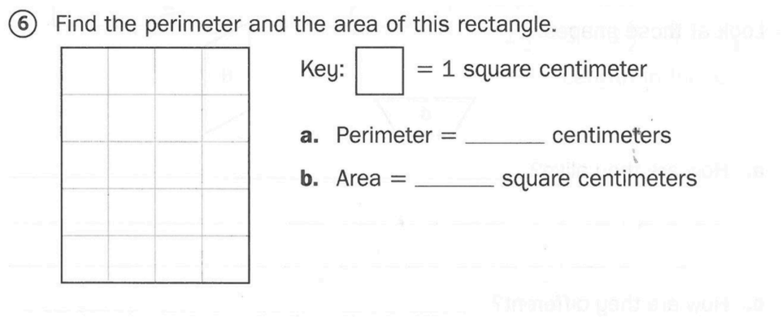


Figure 4-11. Problem from Everyday Mathematics Grade 3 Unit student assessment. Reprinted from Everyday Mathematics Assessment Handbook, (p. 36), The University of Chicago School Mathematics Project, 2015, Columbus, OH: McGraw-Hill Education. Copyright 2015 by McGraw-Hill Education. Reprinted with permission.

Only three PSTs assigned SMP 4 to 6a and only two to 6b. Of those PSTs who assigned SMP 4, most simply stated that the model is provided, whereas one stated that the actual numerical values of the perimeter and area are a model of the rectangle. It is unclear why one PST assigned it in 6a and not in 6b as the tasks are so similar.

Item 2 is an instance where the authors assigned SMP 4 and students are creating, rather than just using a model. Figure 4-1 above shows item 2. Students are asked to look at a table of a real-world situation, the frequency of earthworms of different lengths, and make a line plot to model the data. This fits much more clearly with the authors' definition of SMP 4 given above.

Only six PSTs assigned SMP 4 to item 2, although it was the most frequently assigned SMP by the PSTs for this item. The two SMPs that were assigned the most frequently other than SMP 4 to this item were SMP 5 "Use appropriate tools strategically" and SMP 6 "Attend to precision." In the explanations given it is clear that some PSTs interpreted the task to be a sort of transformation of the data in the table to data in the line plot rather than a model of the data (e.g., "Students have to interpret a tally chart in order to transfer the data onto the line plot. This requires precision" (PST U44)). PSTs also interpreted the data table and the line plot to be tools rather than models. Additionally, some PSTs incorrectly interpreted "real-world situations" to be those that the students themselves were likely to encounter in their current lives as third graders. The PSTs who interpreted it this way said students were unlikely to encounter a situation involving the lengths of earthworms.

SMP 7. The authors assigned SMP 7 “Look for and make use of structure” more frequently than the PSTs. The authors assigned it to items 3a and 3b, shown in Figure 4-8, and 4a, and 4b, shown in Figure 4-12, whereas there are no items where more than half of the PSTs assigned SMP 7. However, there does not seem to be as much discrepancy for this SMP as some others as seven PSTs assigned SMP 7 to 3a, two assigned it to 3b, eight assigned it to 4a and eight assigned it to 4b. Though fewer than half of the PSTs assigned it in each case, for items 3a, 4a, and 4b almost half of the PSTs assigned it. The item with the most discrepancy, 3b as shown in Figure 4-8, asks students to explain why the student in the problem chose the rule that he did. Interestingly, only one PST who assigned SMP 7 in 3a also assigned it in 3b. In further examining PSTs’ explanations for their SMP assignments, it seems that the seven PSTs who assigned SMP 7 to 3a were attending to the actual using of the structure to determine the rule in 3a but then focused more on the fact that students had to communicate their thinking in 3b, assigning SMP 3 “Construct arguments and critique the reasoning of others” and SMP 6 “Attend to precision.”

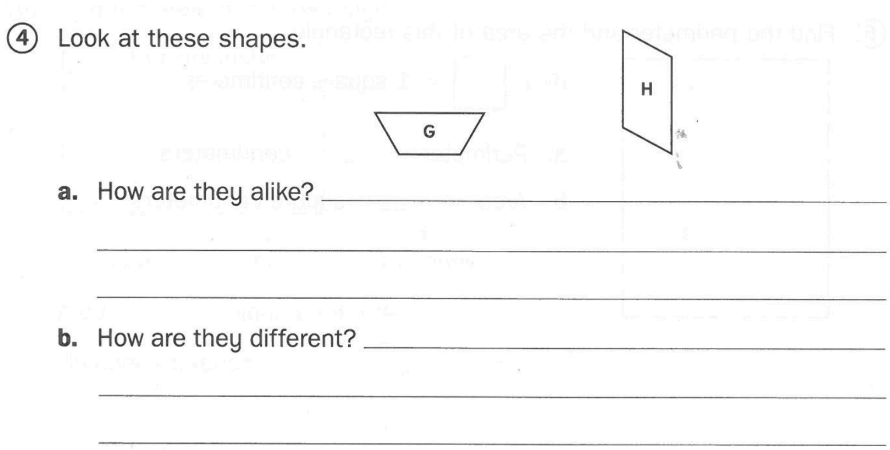


Figure 4-12. Problem from Everyday Mathematics Grade 3 Unit student assessment. Reprinted from Everyday Mathematics Assessment Handbook, (p. 35), The University of Chicago School Mathematics Project, 2015, Columbus, OH: McGraw-Hill Education. Copyright 2015 by McGraw-Hill Education. Reprinted with permission.

Summary

In this chapter I reported results obtained using LMT and MBI instruments to measure PSTs’ MKT and mathematical beliefs. I grouped them in high, medium and low groups for each of these characteristics in order to determine whether PSTs’ MKT or MBI scores influenced their understandings and interpretations of the SMPs by comparing their designations with those of the authors of the EM curriculum materials. I detailed the results of this quantitative analysis and discussed some possible reasons for various discrepancies.

Following this quantitative analysis, I probed further into the explanations given by the PSTs as well as information gained through interviews. Through this investigation I learned more about how PSTs understood the SMPs and identified common patterns that emerged in the ways they interpreted and in some cases misinterpreted SMPs. The implications of these results, the limitations of this study and possibilities for future research are discussed in the next chapter.

Chapter 5 Discussion and Implications

The inclusion of mathematical practices in standards documents represents a major advancement in our thinking about what students need to know in order to engage with complex mathematics. Since we have determined that the most recent iteration of these practices, the SMPs, are important for students to learn, it is also important that teachers have opportunities to learn how to support students in gaining proficiency in using the SMPs as they approach mathematically rigorous problems. However, many preservice and practicing teachers are not adequately prepared to support students in doing this work. As curriculum materials have been created or revised in response to the CCSSM, the main focus for many school districts has remained on the content standards, likely because they are most similar to what teachers were already teaching and what districts were prepared to train teachers to do. Schools of education have a chance to provide opportunities to PSTs to learn ways to support students in using the SMPs to solve mathematics problems. However, if mathematics teacher educators want to graduate well-prepared beginning teachers who have an understanding of the SMPs similar to that of other professionals in the field, there is more work to be done.

Based on the findings in Chapter 2, teachers' MKT and mathematical beliefs were shown to impact teachers' relationships with curriculum materials. This lead me to predict that teachers with higher MKT and more cognitively aligned beliefs about mathematics would agree more with the authors of the curriculum materials and have more sophisticated interpretations of the SMPs. I conducted a study to investigate how PSTs see the SMPs in different materials and in enacted practice and the explanations that they use to justify their claims. I compared these designations to those of the authors of the curriculum materials in order to ascertain if PSTs interpret the SMPs similarly to those designing curriculum materials for use in classrooms. In these comparisons, I included measures of PSTs' MKT and mathematical beliefs in order to determine what PST characteristics influence their interpretations of the SMPs. I then looked closely at the explanations given by PSTs and observed patterns in the ways that they interpreted the SMPs in the different tasks. Together these findings contribute to the knowledge of mathematics teacher educators of how PSTs understand and interpret the SMPs in the context of curriculum materials, enacted practice and assessments and will hopefully help to move the field forward in thinking about how to prepare teachers to support students in gaining proficiency in using the SMPs.

Main Findings and Contributions

Three main findings are prominent in this study.

Impact of mathematical knowledge and beliefs. PSTs' MKT and mathematical beliefs did not have the predicted impact on the frequency with which they agreed with the authors on where they see the SMPs in a mathematics lesson plan, an enactment of that lesson, and a student assessment. This finding is surprising because, based on prior research, I predicted that both MKT and MBI scores would have a positive impact on PSTs' agreement with the authors. It also seems reasonable to hypothesize that PSTs with high MKT would have a more nuanced interpretation of the SMPs as the purpose of the SMPs is to capture the work that one does when actually doing mathematics and these PSTs likely have more proficiency in doing mathematics.

Though the quantitative data show that MKT and MBI scores did not have much of an impact on agreement between the PSTs and authors, there is a bit more nuance to understanding why this occurred in some cases. For instance, there were cases, as described in Chapter 4, where

PSTs in the low MKT group agreed overwhelming with the authors, and the medium and high MKT groups did not, but the explanations given by the PSTs with low MKT showed an over-simplified interpretation of the SMP. For example, PSTs with low MKT assigned SMP 6 “Attend to precision” frequently to assessment items, but often gave the explanation that a problem qualified as an opportunity for students to practice this SMP simply because students had to provide units in their responses. The authors’ rationale seemed to better capture the intention of the SMP. The rationale of the authors used for the assignment of SMP 6 to different assessment items was that the item required “explaining mathematical thinking clearly and precisely” (UCSMP, 2015b, p. 407). So, despite the high agreement between PSTs in the low MKT group and the authors for SMP 6 on assessment items, in most instances they did not agree precisely on what was meant by SMP 6, it just happened that both of their interpretations applied to particular items. Though we cannot know why PSTs in the medium and high MKT groups did not assign SMP 6, it is likely that their thinking about the SMP was more nuanced than the low MKT group and that they interpreted SMP 6 as going beyond simply requiring students to include units in their answers. Though there were cases such as these where further examination of PSTs’ explanations revealed a more complex explanation for the disagreement, overall MKT and MBI scores did not have the predicted impact on agreement between PSTs and the authors.

There are at least two possible interpretations of why MKT and MBI scores did not considerably impact PSTs’ agreement with the authors. First, MKT and MBI may not actually predict PSTs’ understandings and interpretations of the SMPs. Perhaps having a high MKT, for example, has a positive impact on PSTs’ use of curriculum materials to plan for instruction, but does not impact PSTs’ ability to identify instances where students have the potential to practice an SMP in curriculum materials, in practice, or in assessment items. A second interpretation is that the SMPs are under-specified and, regardless of a PST's mathematical knowledge or beliefs, it is challenging for even professionals in the field to come to consensus on what is meant by each SMP and what they look like in the context of teaching and learning in classrooms. This interpretation is supported in the studies that inspired this dissertation (i.e., Silver & Mortimer, 2015; Silver & Mortimer, in press; Mortimer, 2015) where experts in mathematics and mathematics teacher education could not agree on the what the SMPs looked like in an assessment. If this is the case, more work is necessary in the field as a whole in defining the SMPs and recognizing them in the different teaching practices. Once this is established, teacher educators must work to determine the best way to train PSTs and in-service teachers to support students in learning and assessing students' proficiency in using the SMPs.

Though MKT and MBI scores were not highly correlated with agreement with the authors, PSTs' years in the program were positively correlated with their agreement with the authors for the enacted lesson task. PSTs in both the undergraduate and master's programs are given extensive opportunities to observe and participate in classrooms both through video in their methods courses as well as through their pre-student teaching and student teaching experiences. In their field experiences they are often required to record themselves teaching and watch and reflect on their own teaching practice as well as the teaching of their cooperating teachers. This finding suggests that practice-based teacher education provides a foundation for beginning teachers to notice and interpret what is happening in classrooms. It contributes to the argument against those who believe that a college education void of coursework in pedagogy is sufficient for someone to be a successful teacher:

> From the time that schools for children and youth emerged on a wide scale in the nineteenth and twentieth centuries, many critics have asserted teachers do not require

> formal, professional preparation. They need to know the basics of the subjects they teach, how to keep order in the classroom, how to get along with people, and how to abide by administrative regulations. An adequate secondary and college education, so the argument goes, will give them the requisite subject matter knowledge. (Hansen, 2008, p.10)

The fact that, regardless of mathematics knowledge and beliefs, PSTs who had completed their education program, including their field experiences, had higher agreement with the authors on the task which required them to observe and interpret teaching and learning in action shows that the program had a positive impact on their expertise in these skills. Those who were further in the program at the time of this study had judgements more attuned to professionals in the field than those who had high MKT but who had fewer experiences observing and interpreting enacted practice. Teachers who simply study mathematics in a university setting likely would not have had the opportunity to practice the skills involved in noticing and interpreting what happens in a classroom. Taken together, the fact that MKT and MBI scores did not have a substantial impact on PSTs' agreement with curriculum authors but that, for at least one of the tasks, time in the program did impact PSTs' interpretations of the SMPs, suggests that schools of education have a major opportunity to influence PSTs' understanding of the SMPs through experiences provided in their course work and field experiences.

The criteria for an SMP. The second main finding is related specifically to the PSTs' and authors' interpretations of the SMPs. One pattern that emerged was disagreement among the PSTs and between the PSTs and the authors around what "counts" as a place where students have the opportunity to practice or be assessed on an SMP. For instance, the most commonly assigned SMP in the assessment task was SMP 1 "Make sense of problems and persevere in solving them." This SMP includes at least three pieces: sense-making, problem solving, and persevering. Though some PSTs were explicit about how this SMP was assessed through different assessment items, many simply said that because students were solving problems it must be an instance of SMP 1. This shows little nuance in the interpretation of what is included in SMP 1 and a low threshold of what would count of an instance of a student being assessed on this SMP in that an assessment item only needed one of the three criteria to count as an instance of SMP 1. The authors did not assign this SMP at all in the assessment task which shows that they had required more than just the criterion of problem solving in order for an assessment item to be an instance of SMP 1. They only assigned SMP 1 in the curriculum materials task in instances where substantial work needed to be done by the students in order to make sense of and solve the problem. This shows they required all three of the criteria in order for an item to count as an instance of SMP 1.

Another prominent example of a difference in criteria occurred with the assignment of SMP 5 "Use appropriate tools strategically." There were several instances where PSTs were quite generous in assigning this SMP to tasks simply because students were using a pencil or a piece of paper without regard to there being a specific mathematical use for the tool or any sort of strategic thinking or decision-making on the part of the student when selecting a tool for the problem. SMP 5 is intended to require students to make choices about which tools to use to solve problems and it is likely that all students were required to use pencils and paper in these activities. PSTs and the authors disagreed on what criteria was necessary in this instance for students making decisions about tools as the authors did not assign SMP 5 at all in any of the tasks. It was also unclear, in some instances, if a tool in use by a student was used because the student chose to use it or because it was required by the teacher. For instance, when completing the task in the lesson the students who were shown in the lesson enactment task were using graph paper. It was unclear whether all students were using graph paper and it was just the supplies they were given or whether students strategically chose to use graph paper to help them solve the problem. The graph paper, as opposed to using a writing utensil, is an instance where there could have been a strategic decision made by students to help them better solve the mathematics problem. This could impact whether someone determined a video clip as an instance of SMP 5 or not, but many PSTs tended to generously assume students made their own choices while it seems the authors and expert raters did not.

An instance of disagreement about criteria also occurred in several instances of the assignment of SMP 4 "Model with mathematics." In the description given in the CCSSM, the SMP requires that students use models to help them solve problems that emerge from the real world. Interestingly, the central problem to the lesson in the study was about constructing a rabbit pen, a seemingly real-world problem when compared to many mathematics problems which only involve computation and are void of context. However, some PSTs argued that students would likely never have to construct a rabbit pen in their real-world contexts and thus the example did not come from the world in which students' experiences lie. The authors did not have this interpretation of the problem and assigned SMP 4 for several sections of the curriculum materials task. In this instance, the authors were more generous in their interpretation of SMP 4 than the PSTs, interpreting the phrase "real-world" to mean a problem having a context in the world rather than a problem being meaningful in the world of those who will be completing the problem.

The idea of criteria in many instances is really an indication of how thoroughly PSTs read and understood an SMP as well as the level of analysis given to problems or tasks. Schools of education have an important role in supporting PSTs in this sort of work, both in applying content standards and practices to curriculum materials and assessments. Though there were some instances where MKT played a role in PSTs' interpretations, careful work on reading and understanding standards as well as practicing task analysis would help all PSTs become more proficient at identifying and designing opportunities for students to practice and be assessed on content standards and the SMPs.

Signaling task features. A third finding is that there were certain features of tasks or problems that cued PSTs to assign particular SMPs without more careful examination of the task or the SMP. For instance, as discussed earlier, many PSTs assigned SMP 6 "Attend to precision" whenever an assessment question prompted students to include units. In most instances, the accuracy of the unit did not impact the numerical answer on the assessment item, as opposed to, for instance, a unit conversion problem where careful use of units impacts the solution. The problems on the student assessment included a place for units where students were expected to write their answers merely as an exercise at the end of the problem to make sure students were attending, at least superficially, to the context of the problem. The authors did not count these instances as items assessing SMP 6 as students simply had to go back and read the original problem to determine the units rather than make sense of the context of the problem.

Another example of features of a task that cued PSTs to assign a particular SMP was the presence of lines for an explanation in assessment questions. Some PSTs counted this as an instance of SMP 3 "Construct arguments and critique the reasoning of others" in every problem that included lines for writing. However, in some instances student were merely asked to write the steps that they used to determine their answer rather than construct an argument or critique another's reasoning. The fact that PSTs assigned this SMP regardless of the nature of the task, simply because there was space to write, shows that little attention was given to analyzing the task and determining the thinking that was necessary to successfully respond to the prompt.

The use of signaling features in tasks, similar to having differing thresholds, also points to the need for careful attention to tasks. It would be useful for PSTs to have practice analyzing tasks in order to determine the type of thinking that is required of students to compete the tasks. Many of these disagreements were not instances of a lack of mathematical knowledge or understanding, but rather a less than thorough reading and consideration of the assessment tasks or the descriptions of the SMPs. Methods courses, mathematics content courses, as well as field experiences for pre-service elementary school teachers are places where PSTs can be supported in doing this sort of work.

Limitations

There were several features of the sample that limited what could be said about PSTs as a population in general. First, due to the small sample size (n=17) statistically significant data could not be generated. Second, all of the PSTs that participated in the study were enrolled or had recently graduated from the same university. Taken together, these two limitations of the sample mean that the results of this study cannot be generalized to other universities, particularly those that differ in size, program structure, and academic focus. Another feature of the sample is that the PSTs had relatively high MKT scores compared to the average practicing teacher as well as relatively high and also somewhat homogenous MBI scores. This is likely due to the nature of the program in which the PSTs were enrolled as well as the characteristics of students enrolled in the university as a whole. The courses and experiences that master's students had were likely similar to those of the undergraduates as the master's program is also a teacher certification program rather than a program for those already in the teaching field and has many of the same faculty involved in the program and course design for the experiences associated with elementary mathematics. The lack of diversity in MKT and MBI scores means that, given a different sample with more varying scores, the comparisons between the PSTs and authors based on MKT and MBI scores may have yielded different results. For instance, if the sample included a substantial number of PSTs with both well below average and well above average MKT scores, the comparisons with the authors may have been different as well as the patterns that emerged in the PSTs' explanations.

Another limitation is that comparisons with the EM authors were used as a measure of agreement when the curriculum authors are not necessarily experts on the SMPs. Throughout the analysis I tried to be sure I was not putting too much clout in the authors' designations but rather discussing why the areas of disagreement may have occurred. In some cases of disagreement, it was reasonable that the PSTs came to different conclusions than the authors but in others it was less reasonable and illuminated a possible point of misunderstanding or misinterpretation by the PSTs. Careful treatment of this measure of agreement is important in order to not discount completely reasonable designations from the PSTs that may be different from those of the authors.

A final limitation is that the recording tool was prohibitive in that it allowed for PSTs to identify up to three SMPs for each section in the lesson, video segment and assessment item. Though the directions stated that PSTs could assign as few as zero and as many as three SMPs, for some PSTs having the space for three SMPs may have prompted them to assign multiple SMPs even if they initially only thought zero or one "fit." This could have resulted in some of the more generous interpretations of the SMPs as PSTs were searching for SMPs that they could assign in order to fill all of the spaces in the recording tool. For others, there may have been more than three SMPs that they thought were appropriate to assign to a particular item or task but instead were limited to three and had to use their judgment to determine the three that most closely fit. Given more space to identify as many SMPs as PSTs wanted would have likely resulted in at least slightly different findings.

Future Research

Learning more about different populations. As stated in the limitations section, one of the major limitations was the nature and size of the sample used in this study. It was a small sample, all from the same two teacher education programs both at the same university without much variation in MKT and MBI scores. Though I created low, medium and high groups for the MKT and MBI scores, a study with a more diverse sample could help the mathematics education community think more about how mathematical knowledge and beliefs could influence PSTs' interpretations and assignment of standards.

I imagine a next study could take place at a college or university without such a programmatic orientation towards MKT which would likely yield a wider range of MKT scores. This study, using the same three tasks, would help to better understand how MKT may influence PSTs' understandings and interpretations of the SMPs. Additionally, if I sampled from a college or university where PSTs were in the teacher education program for all four years of their undergraduate work rather than the two-year or one-year programs that were attended by the participants in this study, there would likely be more variety in both MKT and MBI scores as well as more points at which to measure the impact of the years in the teacher education program on their designations and explanations. This would allow me to better measure the impacts of all of these characteristics on the PSTs' interpretation and understanding of the SMPs.

It would also be interesting to have a larger sample size. The sample size of seventeen in this study limited what could be said statistically about the group. For instance, there was not a significant relationship between PSTs' MKT and MBI scores in this study, but other studies suggest that the relationship may exist (e.g., Swars et al., 2009). With a larger sample, this relationship may be more prominent. Additionally, a larger sample, particularly if it included a more diverse group of PSTs from a variety of colleges and universities, would provide more generalizable data and a clearer picture of how MKT and mathematics beliefs impact PSTs' interpretations and understanding of the SMPs.

The sample could also be changed from PSTs to in-service teachers. In-service teachers may have received professional development regarding the CCSSM, but it is likely that these sorts of professional development programs focused more on content standards or on the specific Common Core-aligned curriculum materials that the teachers were being asked to use. In-service teachers have countless opportunities to examine curriculum materials when planning for lessons, observing students in their classrooms working and sharing their thinking (though they likely do not have many opportunities to observe teaching) and designing or at least using assessments to track on students' development on different standards. They are likely more adept at utilizing the resources from different tasks than PSTs. This may provide an interesting comparison with the PSTs and also provide people who run professional development programs with information on how in-service teachers interpret and understand the SMPs.

Pushing the thinking around agreement. In this dissertation study, I used authors of the EM curriculum materials as a comparison group to which to compare PSTs' assignment of SMPs. However, as discussed in the limitations section, the authors of the EM curriculum materials are not necessarily experts on the SMPs. A different group could be used for a similar study, but the comparison group could be those with more expertise in the SMPs, for example people who worked on creating the SMPs or mathematics teacher educators who research and/or teach the SMPs. Previous studies (e.g., Silver & Mortimer, 2015; Silver & Mortimer, in press;

Mortimer, 2016) illuminated the difficulty that even experts have in agreeing on what SMPs look like in the context of assessment items. However, experts in those studies completed the task independently. A study where these experts work together and come to consensus on which SMPs are assessed through assessment items would help to say more about what agreeing with the comparison group means about the PSTs' interpretations of the SMPs.

A study where experts completed the exact same tasks as the PSTs would allow for a more accurate comparison of the authors' and PSTs' ratings. For the three tasks included in this study, the authors gave up to three SMPs for each section in the curriculum materials task and for each assessment item. However, they were not asked specifically to assign up to three SMPs in these instances, they just happened to assign between zero and three SMPs in each instance. Had the task been framed to the curriculum authors the same way that it was to the PSTs, the designations may have been different. Additionally, for the lesson enactment task, I had to bring in additional raters because the task was quite different from what the authors completed. The authors chose the video as a whole as an exemplar of one SMP whereas the PSTs had the opportunity to assign up to three SMPs to each of the three video clips. The agreement calculation would be a more meaningful measure of PSTs' knowledge of the SMPs if the comparison group were made up of expert raters completing the same tasks as the PSTs.

Future work for teacher education. Though this study points to a problem with the ambiguous nature of the way the SMPs are presented to teachers through the CCSSM, the question remains of how to support PSTs in understanding and interpreting the SMPs in ways that help students make progress in their use of the SMPs when approaching complex mathematics problems. This sort of work could take place in mathematics content courses, mathematics methods courses, and in PSTs' field experiences. Future studies could include designing learning experiences, including approximations of practice similar to those used in this study, in order to support PSTs in doing this work and measuring the effectiveness of different learning experiences through a pre- and post-assessment. An example of such a study could be designing field experiences where PSTs must use curriculum materials, determine where students may have the opportunity to practice using different SMPs and watch video of the lesson to determine where students actually did practice different SMPs, all while receiving guidance and feedback from a field instructor throughout the semester. As planning for and implementing lessons, as well as reflecting on lessons through video, is already a part of many teacher education programs, adding the additional task of identifying SMPs and noticing and discussing them through reflection seems like a reasonable addition that would not have to take away from other goals that programs have for PSTs in their field experiences. A pre- and post-assessment could be given using tasks similar to the curriculum materials task and lesson enactment task in this study to determine how PSTs' understanding and interpretation of the different SMPs changed as a result of the experience.

Another example of a possible activity to support PSTs in developing these skills would involve combining task analysis with the SMPs in a mathematics methods course. PSTs would have the opportunity to examine mathematics tasks, discuss the mathematics involved in the task and what strategies are possible for students to use to solve these tasks. Then PSTs would have the opportunity to determine, for different solution methods, the SMPs that students would likely use to solve them. This would help PSTs to be better equipped to design classroom activities as well as assessments to support and assess students in developing their proficiency in using the SMPs to solve problems.

Conclusion

Mathematical practices have been deemed important by the mathematics education community in order to provide k-12 students with the skills necessary to understand and solve complex mathematics problems. Now that mathematical practices are included in the CCSSM, most practicing teachers in the United States are expected to teach and assess students' progress in developing proficiency in using the SMPs. However, as practices have not always been included in state standards, not all teachers are familiar with them and many have never been required to teach and assess them. Despite the change in standards documents, schools of education, in many instances, remain unchanged in the way they are preparing and what they are preparing PSTs to teach despite the fact that they will very likely be expected to teach the SMPs in their future classrooms. If we want to have well-prepared beginning teachers, it is important that we are preparing PSTs to support students in developing their proficiency in using the SMPs.

This dissertation study contributes to the mathematics teacher education community in that it opens a door for mathematics teacher educators to begin to think about whether or not the SMPs need to be explicitly taught and how they should be taught in teacher preparation programs in order for beginning teachers to possess the knowledge and skills to support students in developing their proficiency in the SMPs. The results of the study suggest that there are particular patterns in PSTs' thinking and understanding of the practices that could be entry points in addressing the need for SMP instruction. If we as a community deem it important for K-12 students to gain proficiency in the SMPs, it is imperative that we prepare teachers to both teach and assess the SMPs.

APPENDICES

Appendix A
Lesson 4-11 From Everyday Mathematics Grade 3

Lesson from Everyday Mathematics Grade 3 Teacher's Lesson Guide, Volume 1. Reprinted from Everyday Mathematics Grade 3 Teacher's Lesson Guide, Volume 1, (p. 386-395), The University of Chicago School Mathematics Project, 2015, Columbus, OH: McGraw-Hill Education. Copyright 2015 by McGraw-Hill Education. Reprinted with permission.

Lesson 4-11
Open Response and Reengagement

Building a Rabbit Pen

2-Day Lesson

Overview **Day 1:** Children create and use models of a rabbit pen to solve a problem. **Day 2:** Children compare and discuss their models and explanations and revise their work.

Day 1: Open Response

Before You Begin
Solve the open response problem and identify possible side lengths of rectangles with a perimeter of 24 feet. Have extra copies of *Math Masters*, page TA19 available for children who need additional grid space. If possible, schedule time to review children's work and plan for Day 2 of this lesson with your grade-level team.

Vocabulary
mathematical model • perimeter • area

CCSS Common Core State Standards
Focus Clusters
• Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
• Geometric measurement: recognize perimeter.

	Materials	
1 Warm Up 5 min		
Mental Math and Fluency Children divide using helper facts.	slate	3.OA.7
2a Focus 55–65 min		
Math Message Children find the perimeter and the area of two dog pens.	*Math Journal 1*, p. 129	3.MD.7, 3.MD.7b, 3.MD.8 SMP4, SMP6
Calculating Perimeters and Areas Children discuss strategies for calculating the perimeters and the areas of the pens.	*Math Journal 1*, p. 129; *Student Reference Book*, pp. 174–177 (optional)	3.MD.7, 3.MD.7b, 3.MD.8 SMP1, SMP4, SMP6
Solving the Open Response Problem Children draw at least two rectangles with a fixed perimeter and different areas, and choose one for a rabbit pen.	*Math Masters*, pp. 143–144 and TA19	3.MD.7, 3.MD.7b, 3.MD.8 SMP1, SMP4, SMP6

Getting Ready for Day 2
Review children's work and plan discussion for reengagement. *Math Masters*, p. TA6, p. TA42 (optional); children's work from Day 1

CCSS 3.MD.8 **Spiral Snapshot**
GMC Exhibit rectangles with the same perimeter and different areas or the same area and different perimeters.

4-11 Focus Practice | 5-1 Focus | 6-5 Warm Up | 9-1 Practice

Spiral Tracker Go Online to see how mastery develops for all standards within the grade.

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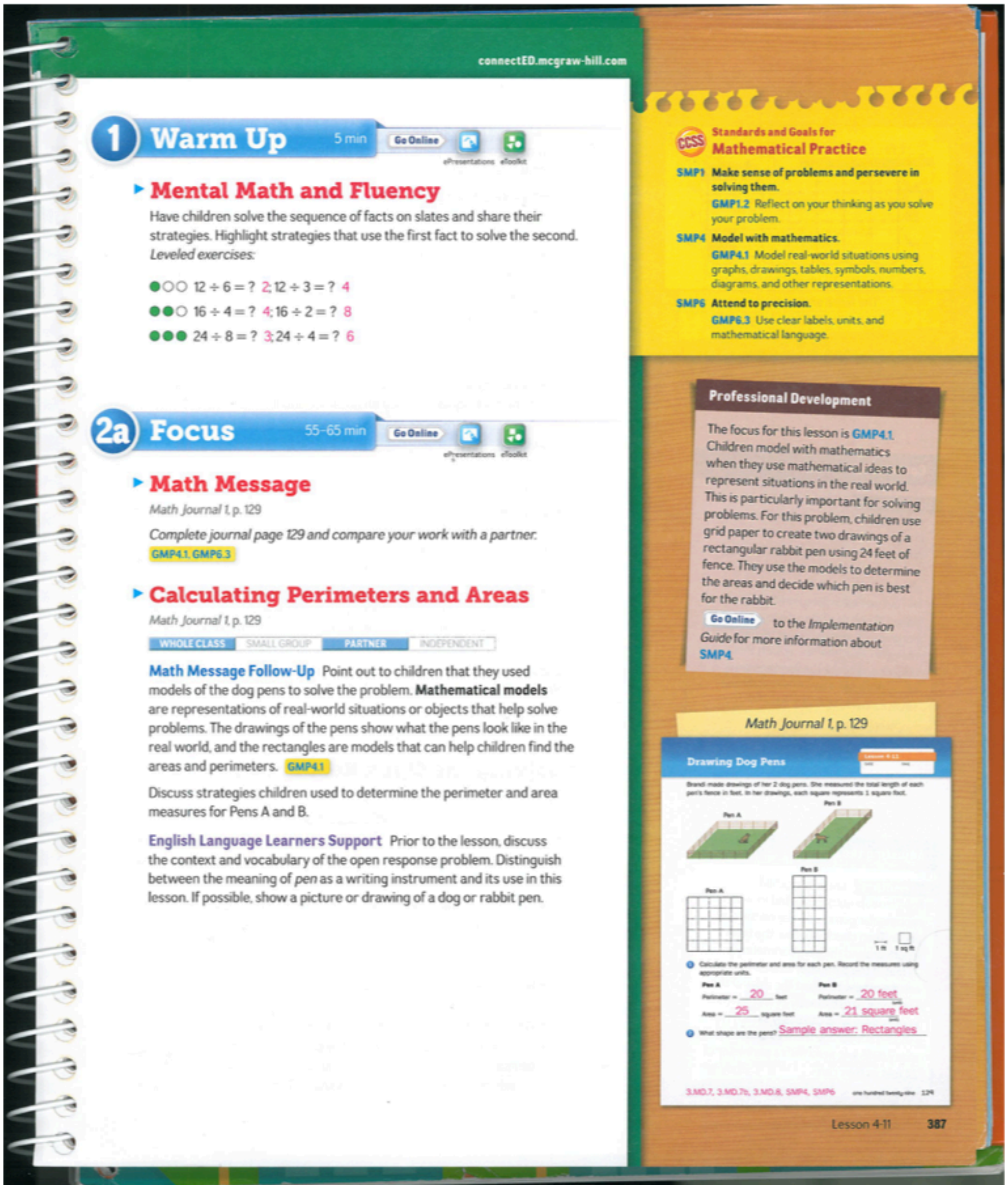
Plan your lessons online with these tools.

ePresentations | Student Learning Center | Facts Workshop Game | eToolkit | Professional Development | Home Connections | Spiral Tracker | Assessment and Reporting | English Learners Support | Differentiation Support

386 Unit 4 | Measurement and Geometry

Figure A-1. Lesson from Everyday Mathematics Grade 3 Teacher's Lesson Guide, Volume 1 (UCSMP, 2015b, p. 386)



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1 Warm Up 5 min

Go Online ePresentations eToolkit

▸ Mental Math and Fluency

Have children solve the sequence of facts on slates and share their strategies. Highlight strategies that use the first fact to solve the second.
Leveled exercises:

●○○ 12 ÷ 6 = ? 2; 12 ÷ 3 = ? 4
●●○ 16 ÷ 4 = ? 4; 16 ÷ 2 = ? 8
●●● 24 ÷ 8 = ? 3; 24 ÷ 4 = ? 6

2a Focus 55–65 min

Go Online ePresentations eToolkit

▸ Math Message

Math Journal 1, p. 129

Complete journal page 129 and compare your work with a partner. GMP4.1, GMP6.3

▸ Calculating Perimeters and Areas

Math Journal 1, p. 129

WHOLE CLASS | SMALL GROUP | PARTNER | INDEPENDENT

Math Message Follow-Up Point out to children that they used models of the dog pens to solve the problem. **Mathematical models** are representations of real-world situations or objects that help solve problems. The drawings of the pens show what the pens look like in the real world, and the rectangles are models that can help children find the areas and perimeters. GMP4.1

Discuss strategies children used to determine the perimeter and area measures for Pens A and B.

English Language Learners Support Prior to the lesson, discuss the context and vocabulary of the open response problem. Distinguish between the meaning of *pen* as a writing instrument and its use in this lesson. If possible, show a picture or drawing of a dog or rabbit pen.

CCSS **Standards and Goals for Mathematical Practice**

SMP1 **Make sense of problems and persevere in solving them.**
GMP1.2 Reflect on your thinking as you solve your problem.

SMP4 **Model with mathematics.**
GMP4.1 Model real-world situations using graphs, drawings, tables, symbols, numbers, diagrams, and other representations.

SMP6 **Attend to precision.**
GMP6.3 Use clear labels, units, and mathematical language.

Professional Development

The focus for this lesson is GMP4.1. Children model with mathematics when they use mathematical ideas to represent situations in the real world. This is particularly important for solving problems. For this problem, children use grid paper to create two drawings of a rectangular rabbit pen using 24 feet of fence. They use the models to determine the areas and decide which pen is best for the rabbit.

Go Online to the *Implementation Guide* for more information about SMP4.

Math Journal 1, p. 129

Drawing Dog Pens

Pen A Pen B

Pen A Pen B

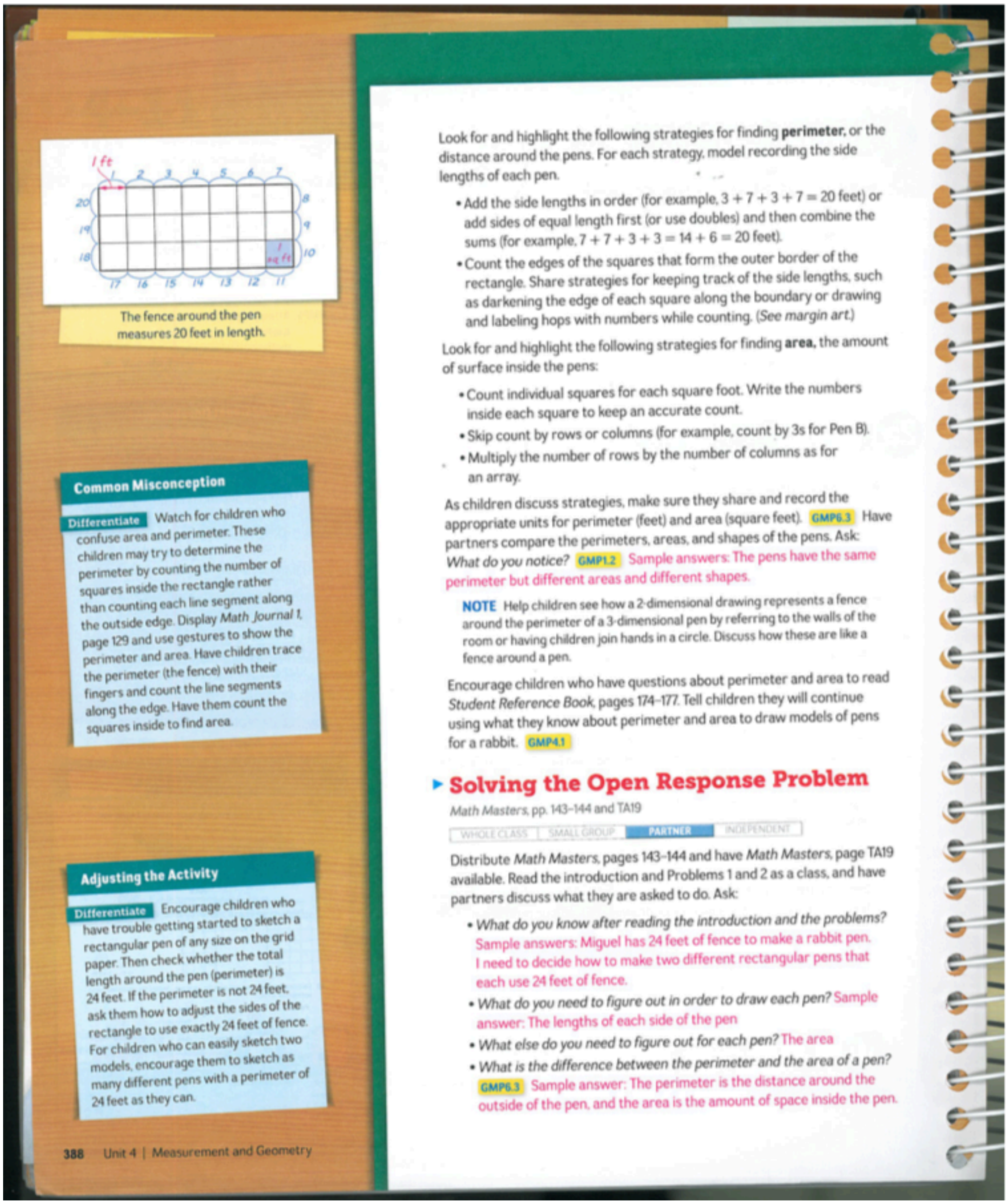
Perimeter = 20 feet Perimeter = 20 feet
Area = 25 square feet Area = 21 square feet

What shape are the pens? Sample answer: Rectangles

3.MD.7, 3.MD.7b, 3.MD.8, SMP4, SMP6 129

Lesson 4-11 387

Figure A-2. Lesson from Everyday Mathematics Grade 3 Teacher's Lesson Guide, Volume 1 (UCSMP, 2015b, p. 387)



Common Misconception

Differentiate Watch for children who confuse area and perimeter. These children may try to determine the perimeter by counting the number of squares inside the rectangle rather than counting each line segment along the outside edge. Display *Math Journal 1*, page 129 and use gestures to show the perimeter and area. Have children trace the perimeter (the fence) with their fingers and count the line segments along the edge. Have them count the squares inside to find area.

Adjusting the Activity

Differentiate Encourage children who have trouble getting started to sketch a rectangular pen of any size on the grid paper. Then check whether the total length around the pen (perimeter) is 24 feet. If the perimeter is not 24 feet, ask them how to adjust the sides of the rectangle to use exactly 24 feet of fence. For children who can easily sketch two models, encourage them to sketch as many different pens with a perimeter of 24 feet as they can.

Look for and highlight the following strategies for finding **perimeter**, or the distance around the pens. For each strategy, model recording the side lengths of each pen.

- Add the side lengths in order (for example, 3 + 7 + 3 + 7 = 20 feet) or add sides of equal length first (or use doubles) and then combine the sums (for example, 7 + 7 + 3 + 3 = 14 + 6 = 20 feet).
- Count the edges of the squares that form the outer border of the rectangle. Share strategies for keeping track of the side lengths, such as darkening the edge of each square along the boundary or drawing and labeling hops with numbers while counting. (*See margin art.*)

Look for and highlight the following strategies for finding **area**, the amount of surface inside the pens:

- Count individual squares for each square foot. Write the numbers inside each square to keep an accurate count.
- Skip count by rows or columns (for example, count by 3s for Pen B).
- Multiply the number of rows by the number of columns as for an array.

As children discuss strategies, make sure they share and record the appropriate units for perimeter (feet) and area (square feet). GMP6.3 Have partners compare the perimeters, areas, and shapes of the pens. Ask: *What do you notice?* GMP1.2 Sample answers: The pens have the same perimeter but different areas and different shapes.

NOTE Help children see how a 2-dimensional drawing represents a fence around the perimeter of a 3-dimensional pen by referring to the walls of the room or having children join hands in a circle. Discuss how these are like a fence around a pen.

Encourage children who have questions about perimeter and area to read *Student Reference Book*, pages 174–177. Tell children they will continue using what they know about perimeter and area to draw models of pens for a rabbit. GMP4.1

Solving the Open Response Problem

Math Masters, pp. 143–144 and TA19

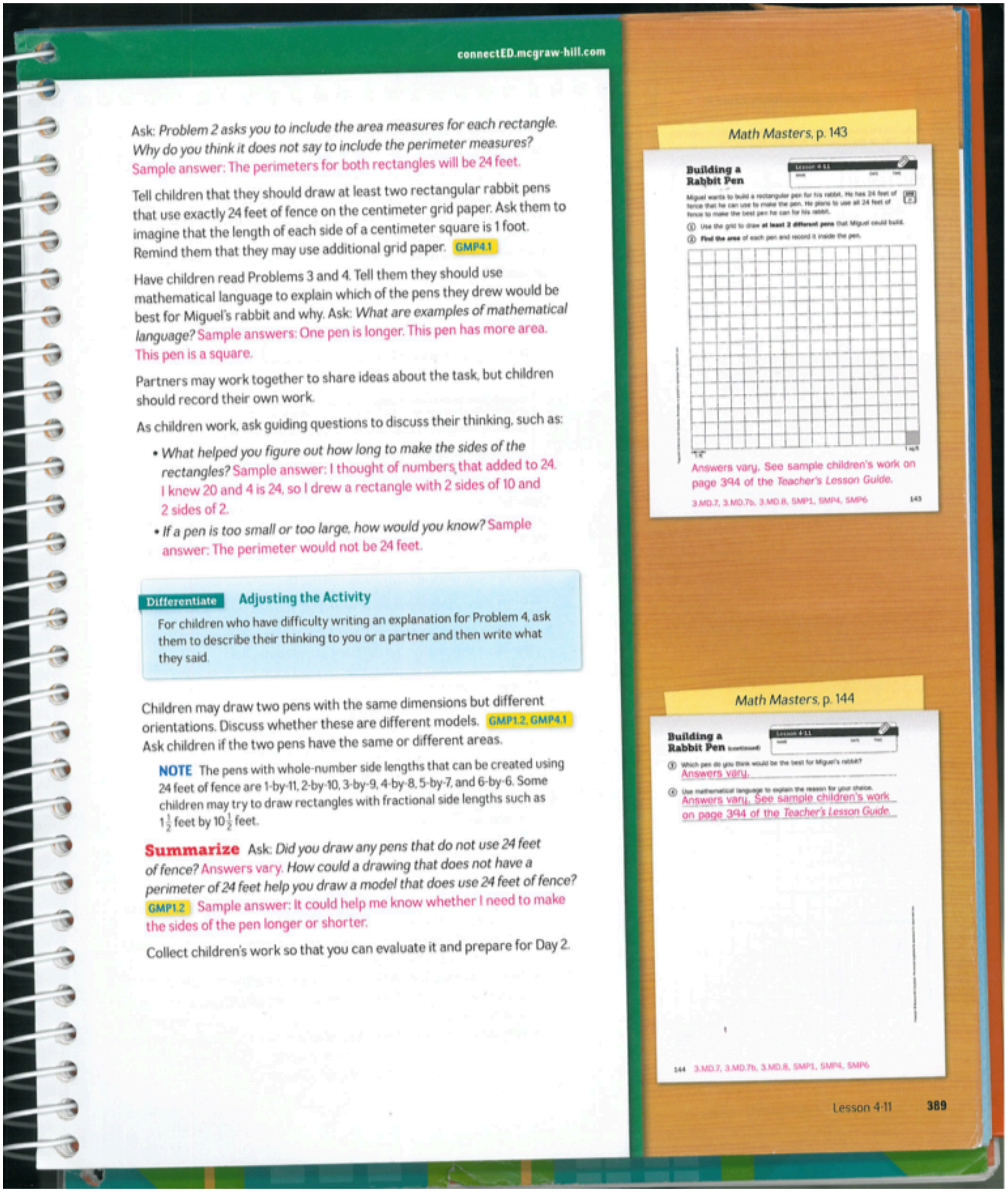
WHOLE CLASS | SMALL GROUP | PARTNER | INDEPENDENT

Distribute *Math Masters*, pages 143–144 and have *Math Masters*, page TA19 available. Read the introduction and Problems 1 and 2 as a class, and have partners discuss what they are asked to do. Ask:

- *What do you know after reading the introduction and the problems?* Sample answers: Miguel has 24 feet of fence to make a rabbit pen. I need to decide how to make two different rectangular pens that each use 24 feet of fence.
- *What do you need to figure out in order to draw each pen?* Sample answer: The lengths of each side of the pen
- *What else do you need to figure out for each pen?* The area
- *What is the difference between the perimeter and the area of a pen?* GMP6.3 Sample answer: The perimeter is the distance around the outside of the pen, and the area is the amount of space inside the pen.

388 Unit 4 | Measurement and Geometry

Figure A-3. Lesson from Everyday Mathematics Grade 3 Teacher's Lesson Guide, Volume 1 (UCSMP, 2015b, p. 388)



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Ask: *Problem 2 asks you to include the area measures for each rectangle. Why do you think it does not say to include the perimeter measures?* Sample answer: The perimeters for both rectangles will be 24 feet.

Tell children that they should draw at least two rectangular rabbit pens that use exactly 24 feet of fence on the centimeter grid paper. Ask them to imagine that the length of each side of a centimeter square is 1 foot. Remind them that they may use additional grid paper. GMP4.1

Have children read Problems 3 and 4. Tell them they should use mathematical language to explain which of the pens they drew would be best for Miguel's rabbit and why. Ask: *What are examples of mathematical language?* Sample answers: One pen is longer. This pen has more area. This pen is a square.

Partners may work together to share ideas about the task, but children should record their own work.

As children work, ask guiding questions to discuss their thinking, such as:

- *What helped you figure out how long to make the sides of the rectangles?* Sample answer: I thought of numbers that added to 24. I knew 20 and 4 is 24, so I drew a rectangle with 2 sides of 10 and 2 sides of 2.
- *If a pen is too small or too large, how would you know?* Sample answer: The perimeter would not be 24 feet.

Differentiate **Adjusting the Activity**

For children who have difficulty writing an explanation for Problem 4, ask them to describe their thinking to you or a partner and then write what they said.

Children may draw two pens with the same dimensions but different orientations. Discuss whether these are different models. GMP1.2, GMP4.1 Ask children if the two pens have the same or different areas.

NOTE The pens with whole-number side lengths that can be created using 24 feet of fence are 1-by-11, 2-by-10, 3-by-9, 4-by-8, 5-by-7, and 6-by-6. Some children may try to draw rectangles with fractional side lengths such as $1\frac{1}{2}$ feet by $10\frac{1}{2}$ feet.

Summarize Ask: *Did you draw any pens that do not use 24 feet of fence?* Answers vary. *How could a drawing that does not have a perimeter of 24 feet help you draw a model that does use 24 feet of fence?* GMP1.2 Sample answer: It could help me know whether I need to make the sides of the pen longer or shorter.

Collect children's work so that you can evaluate it and prepare for Day 2.

Math Masters, p. 143

Building a Rabbit Pen

Lesson 4-11

Answers vary. See sample children's work on page 394 of the *Teacher's Lesson Guide*.

3.MD.7, 3.MD.7b, 3.MD.8, SMP1, SMP4, SMP6 143

Math Masters, p. 144

Building a Rabbit Pen (continued)

Lesson 4-11

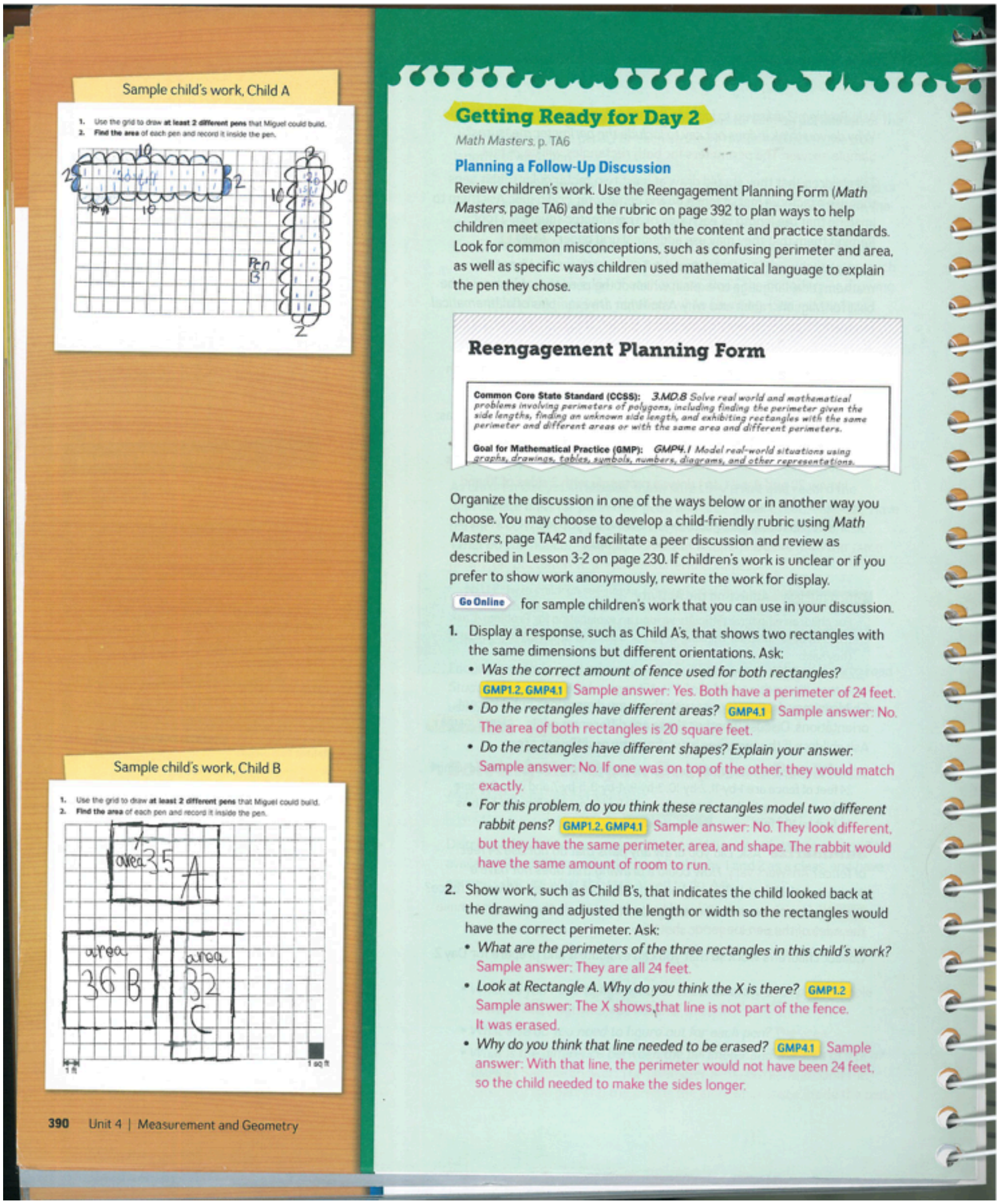
Answers vary.

Answers vary. See sample children's work on page 394 of the *Teacher's Lesson Guide*.

144 3.MD.7, 3.MD.7b, 3.MD.8, SMP1, SMP4, SMP6

Lesson 4-11 389

Figure A-4. Lesson from Everyday Mathematics Grade 3 Teacher's Lesson Guide, Volume 1 (UCSMP, 2015b, p. 389)



Sample child's work, Child A

1. Use the grid to draw **at least 2 different pens** that Miguel could build.
2. **Find the area** of each pen and record it inside the pen.

Sample child's work, Child B

1. Use the grid to draw **at least 2 different pens** that Miguel could build.
2. **Find the area** of each pen and record it inside the pen.

390 Unit 4 | Measurement and Geometry

Getting Ready for Day 2

Math Masters, p. TA6

Planning a Follow-Up Discussion

Review children's work. Use the Reengagement Planning Form (*Math Masters*, page TA6) and the rubric on page 392 to plan ways to help children meet expectations for both the content and practice standards. Look for common misconceptions, such as confusing perimeter and area, as well as specific ways children used mathematical language to explain the pen they chose.

Reengagement Planning Form

Common Core State Standard (CCSS): *3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.*

Goal for Mathematical Practice (GMP): *GMP4.1 Model real-world situations using graphs, drawings, tables, symbols, numbers, diagrams, and other representations.*

Organize the discussion in one of the ways below or in another way you choose. You may choose to develop a child-friendly rubric using *Math Masters*, page TA42 and facilitate a peer discussion and review as described in Lesson 3-2 on page 230. If children's work is unclear or if you prefer to show work anonymously, rewrite the work for display.

Go Online for sample children's work that you can use in your discussion.

1. Display a response, such as Child A's, that shows two rectangles with the same dimensions but different orientations. Ask:
   - *Was the correct amount of fence used for both rectangles?* GMP1.2, GMP4.1 Sample answer: Yes. Both have a perimeter of 24 feet.
   - *Do the rectangles have different areas?* GMP4.1 Sample answer: No. The area of both rectangles is 20 square feet.
   - *Do the rectangles have different shapes? Explain your answer.* Sample answer: No. If one was on top of the other, they would match exactly.
   - *For this problem, do you think these rectangles model two different rabbit pens?* GMP1.2, GMP4.1 Sample answer: No. They look different, but they have the same perimeter, area, and shape. The rabbit would have the same amount of room to run.
2. Show work, such as Child B's, that indicates the child looked back at the drawing and adjusted the length or width so the rectangles would have the correct perimeter. Ask:
   - *What are the perimeters of the three rectangles in this child's work?* Sample answer: They are all 24 feet.
   - *Look at Rectangle A. Why do you think the X is there?* GMP1.2 Sample answer: The X shows that line is not part of the fence. It was erased.
   - *Why do you think that line needed to be erased?* GMP4.1 Sample answer: With that line, the perimeter would not have been 24 feet, so the child needed to make the sides longer.

Figure A-5. Lesson from Everyday Mathematics Grade 3 Teacher's Lesson Guide, Volume 1 (UCSMP, 2015b, p. 390)

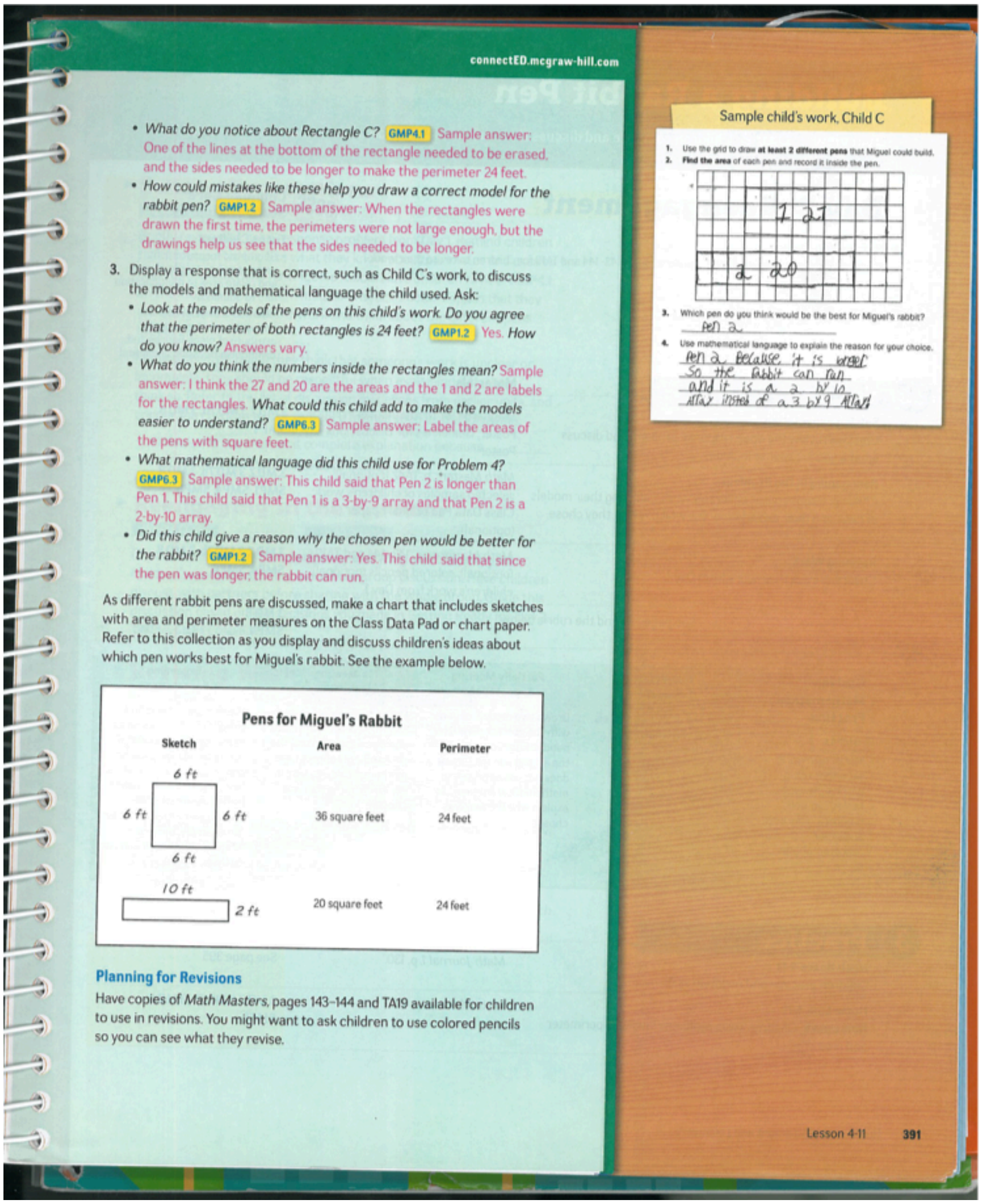
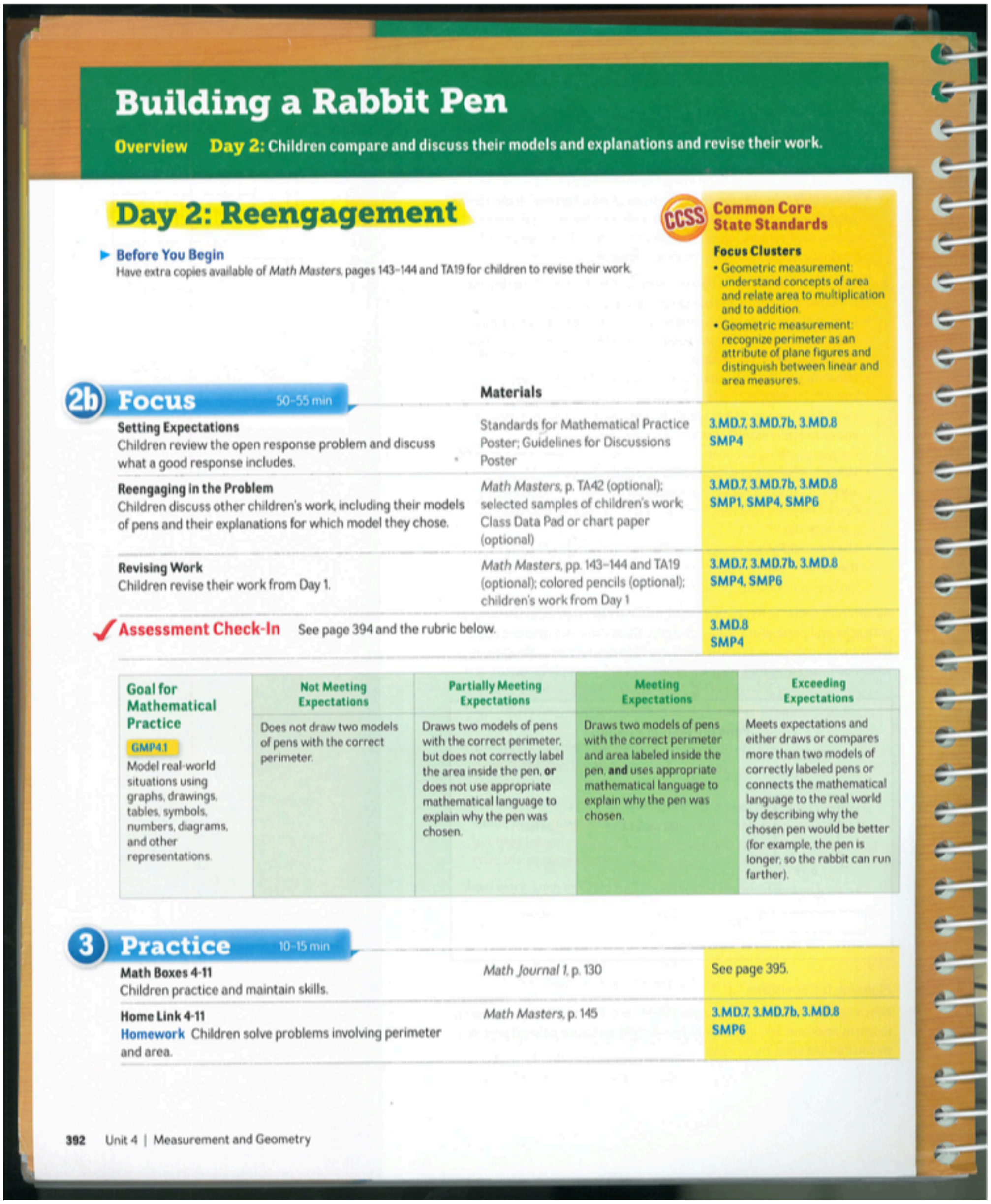


Figure A-6. Lesson from Everyday Mathematics Grade 3 Teacher's Lesson Guide, Volume 1 (UCSMP, 2015b, p. 391)



Building a Rabbit Pen

Overview **Day 2:** Children compare and discuss their models and explanations and revise their work.

Day 2: Reengagement

Before You Begin
Have extra copies available of *Math Masters*, pages 143–144 and TA19 for children to revise their work.

Common Core State Standards

Focus Clusters
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
- Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

2b Focus 50–55 min

	Materials	
Setting Expectations Children review the open response problem and discuss what a good response includes.	Standards for Mathematical Practice Poster; Guidelines for Discussions Poster	3.MD.7, 3.MD.7b, 3.MD.8 SMP4
Reengaging in the Problem Children discuss other children's work, including their models of pens and their explanations for which model they chose.	*Math Masters*, p. TA42 (optional); selected samples of children's work; Class Data Pad or chart paper (optional)	3.MD.7, 3.MD.7b, 3.MD.8 SMP1, SMP4, SMP6
Revising Work Children revise their work from Day 1.	*Math Masters*, pp. 143–144 and TA19 (optional); colored pencils (optional); children's work from Day 1	3.MD.7, 3.MD.7b, 3.MD.8 SMP4, SMP6
Assessment Check-In See page 394 and the rubric below.		3.MD.8 SMP4

Goal for Mathematical Practice	Not Meeting Expectations	Partially Meeting Expectations	Meeting Expectations	Exceeding Expectations
GMP4.1 Model real-world situations using graphs, drawings, tables, symbols, numbers, diagrams, and other representations.	Does not draw two models of pens with the correct perimeter.	Draws two models of pens with the correct perimeter, but does not correctly label the area inside the pen, **or** does not use appropriate mathematical language to explain why the pen was chosen.	Draws two models of pens with the correct perimeter and area labeled inside the pen, **and** uses appropriate mathematical language to explain why the pen was chosen.	Meets expectations and either draws or compares more than two models of correctly labeled pens or connects the mathematical language to the real world by describing why the chosen pen would be better (for example, the pen is longer, so the rabbit can run farther).

3 Practice 10–15 min

Math Boxes 4-11 Children practice and maintain skills.	*Math Journal 1*, p. 130	See page 395.
Home Link 4-11 **Homework** Children solve problems involving perimeter and area.	*Math Masters*, p. 145	3.MD.7, 3.MD.7b, 3.MD.8 SMP6

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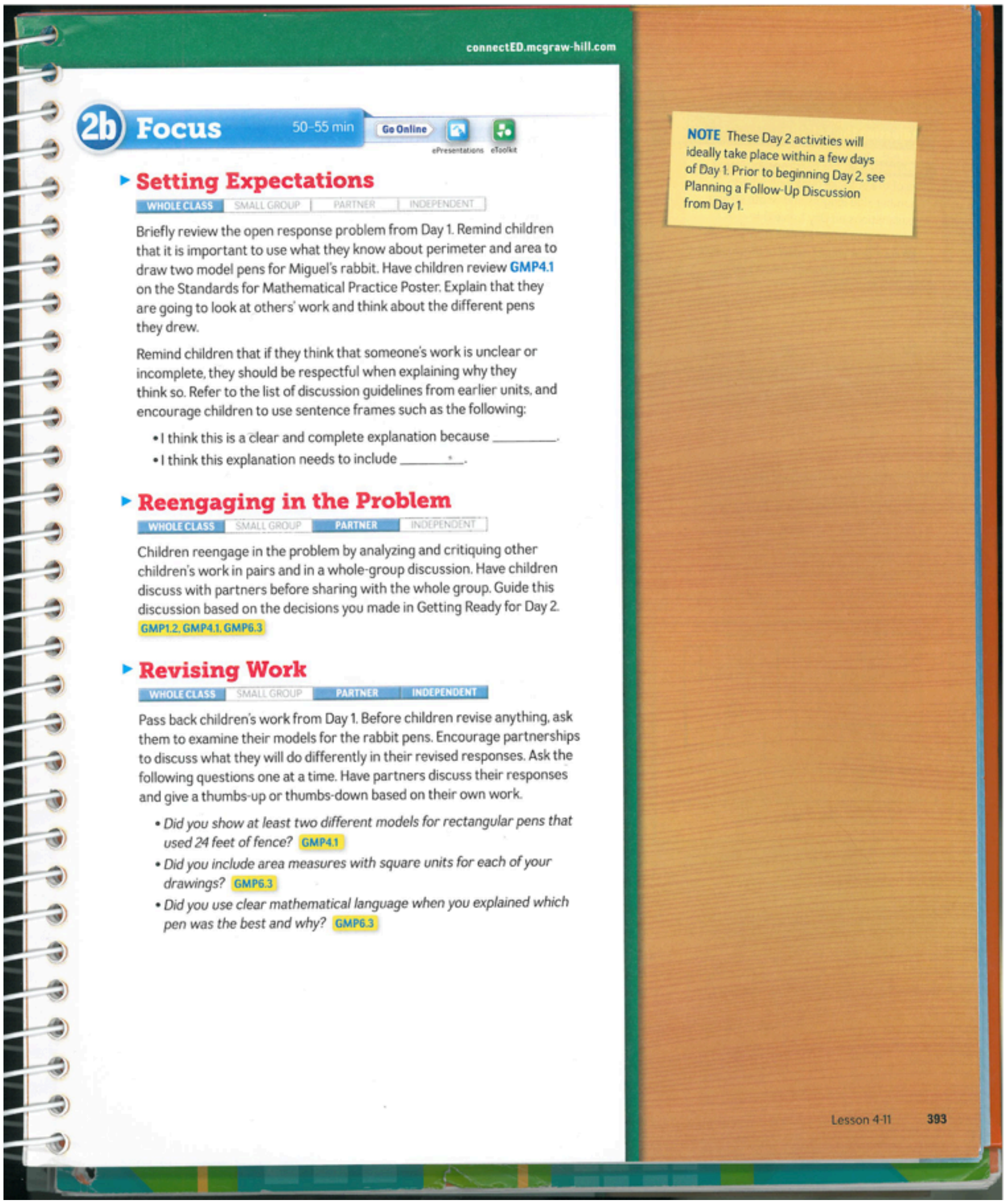
Figure A-7. Lesson from Everyday Mathematics Grade 3 Teacher's Lesson Guide, Volume 1 (UCSMP, 2015b, p. 392)



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2b Focus

50–55 min | Go Online | ePresentations | eToolkit

▸ Setting Expectations

WHOLE CLASS | SMALL GROUP | PARTNER | INDEPENDENT

Briefly review the open response problem from Day 1. Remind children that it is important to use what they know about perimeter and area to draw two model pens for Miguel's rabbit. Have children review **GMP4.1** on the Standards for Mathematical Practice Poster. Explain that they are going to look at others' work and think about the different pens they drew.

Remind children that if they think that someone's work is unclear or incomplete, they should be respectful when explaining why they think so. Refer to the list of discussion guidelines from earlier units, and encourage children to use sentence frames such as the following:

- I think this is a clear and complete explanation because ________.
- I think this explanation needs to include ________.

▸ Reengaging in the Problem

WHOLE CLASS | SMALL GROUP | PARTNER | INDEPENDENT

Children reengage in the problem by analyzing and critiquing other children's work in pairs and in a whole-group discussion. Have children discuss with partners before sharing with the whole group. Guide this discussion based on the decisions you made in Getting Ready for Day 2. GMP1.2, GMP4.1, GMP6.3

▸ Revising Work

WHOLE CLASS | SMALL GROUP | PARTNER | INDEPENDENT

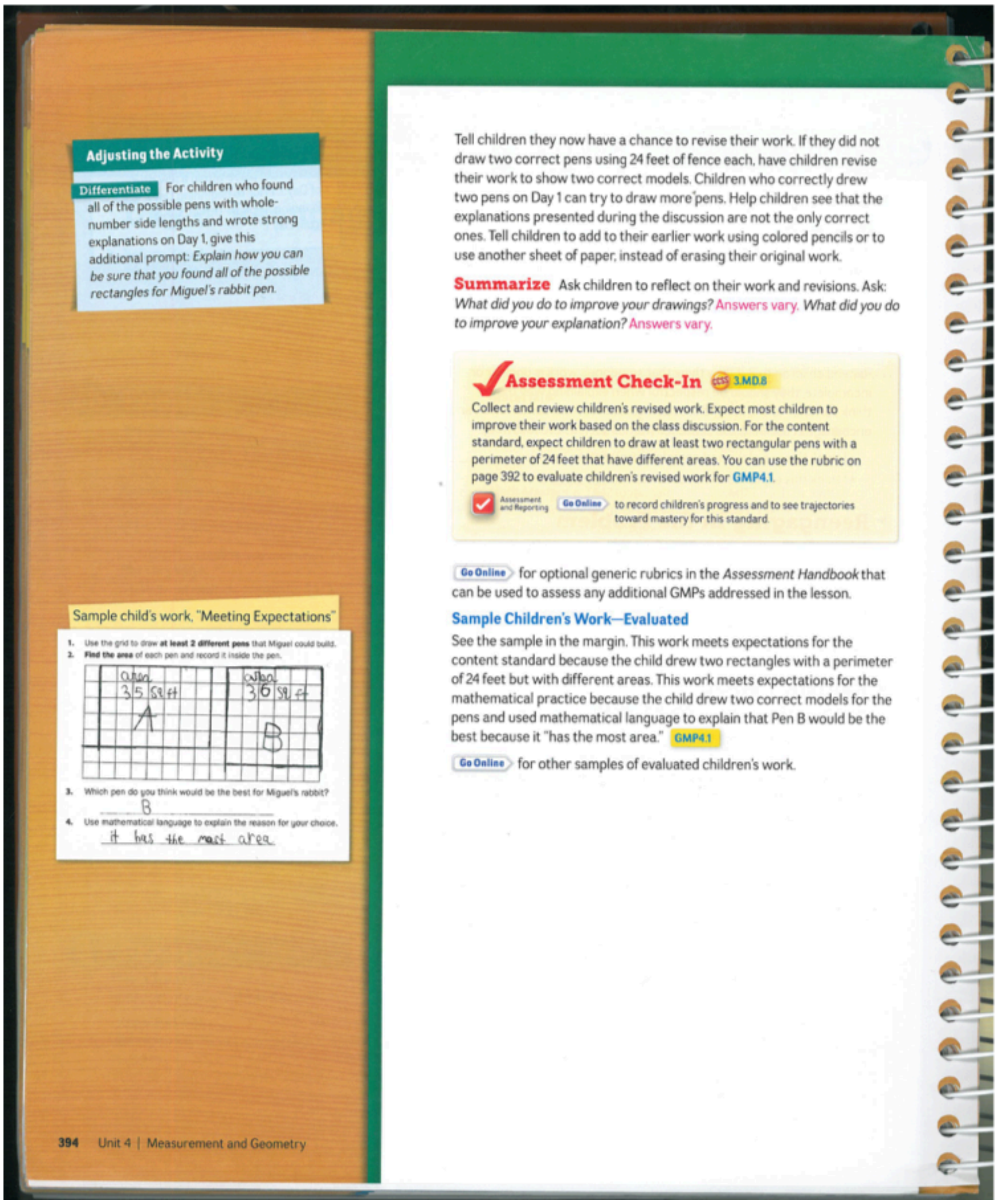
Pass back children's work from Day 1. Before children revise anything, ask them to examine their models for the rabbit pens. Encourage partnerships to discuss what they will do differently in their revised responses. Ask the following questions one at a time. Have partners discuss their responses and give a thumbs-up or thumbs-down based on their own work.

- *Did you show at least two different models for rectangular pens that used 24 feet of fence?* GMP4.1
- *Did you include area measures with square units for each of your drawings?* GMP6.3
- *Did you use clear mathematical language when you explained which pen was the best and why?* GMP6.3

NOTE These Day 2 activities will ideally take place within a few days of Day 1. Prior to beginning Day 2, see Planning a Follow-Up Discussion from Day 1.

Lesson 4-11 393

Figure A-8. Lesson from Everyday Mathematics Grade 3 Teacher's Lesson Guide, Volume 1 (UCSMP, 2015b, p. 393)



Adjusting the Activity

Differentiate For children who found all of the possible pens with whole-number side lengths and wrote strong explanations on Day 1, give this additional prompt: *Explain how you can be sure that you found all of the possible rectangles for Miguel's rabbit pen.*

Sample child's work, "Meeting Expectations"

1. Use the grid to draw **at least 2 different pens** that Miguel could build.
2. **Find the area** of each pen and record it inside the pen.

area 35 sq ft A

area 36 sq ft B

3. Which pen do you think would be the best for Miguel's rabbit? B
4. Use mathematical language to explain the reason for your choice. it has the most area

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Tell children they now have a chance to revise their work. If they did not draw two correct pens using 24 feet of fence each, have children revise their work to show two correct models. Children who correctly drew two pens on Day 1 can try to draw more pens. Help children see that the explanations presented during the discussion are not the only correct ones. Tell children to add to their earlier work using colored pencils or to use another sheet of paper, instead of erasing their original work.

Summarize Ask children to reflect on their work and revisions. Ask: *What did you do to improve your drawings?* Answers vary. *What did you do to improve your explanation?* Answers vary.

Assessment Check-In 3.MD.8

Collect and review children's revised work. Expect most children to improve their work based on the class discussion. For the content standard, expect children to draw at least two rectangular pens with a perimeter of 24 feet that have different areas. You can use the rubric on page 392 to evaluate children's revised work for GMP4.1.

Assessment and Reporting Go Online to record children's progress and to see trajectories toward mastery for this standard.

Go Online for optional generic rubrics in the *Assessment Handbook* that can be used to assess any additional GMPs addressed in the lesson.

Sample Children's Work—Evaluated

See the sample in the margin. This work meets expectations for the content standard because the child drew two rectangles with a perimeter of 24 feet but with different areas. This work meets expectations for the mathematical practice because the child drew two correct models for the pens and used mathematical language to explain that Pen B would be the best because it "has the most area." GMP4.1

Go Online for other samples of evaluated children's work.

Figure A-9. Lesson from Everyday Mathematics Grade 3 Teacher's Lesson Guide, Volume 1 (UCSMP, 2015b, p. 394)

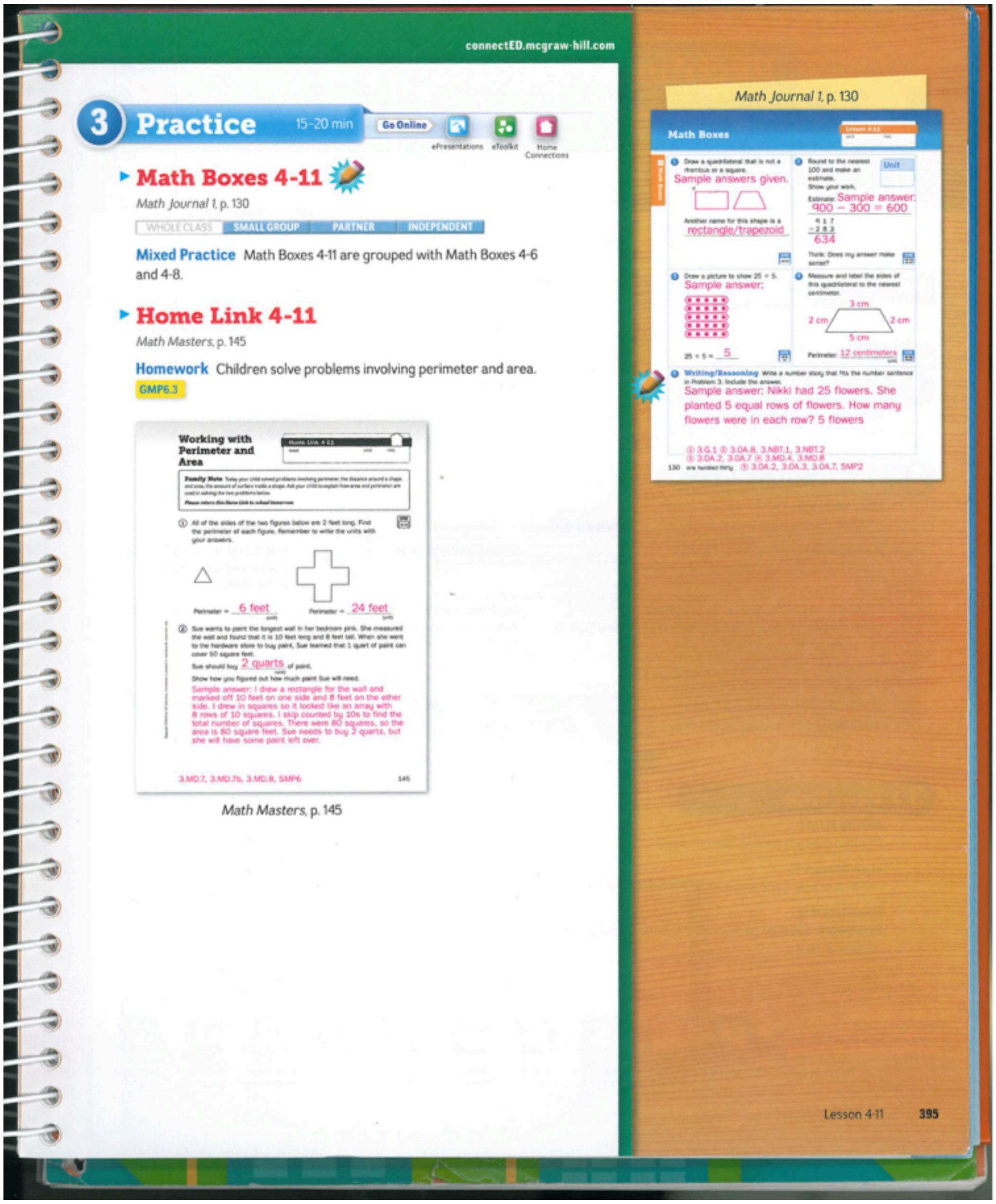
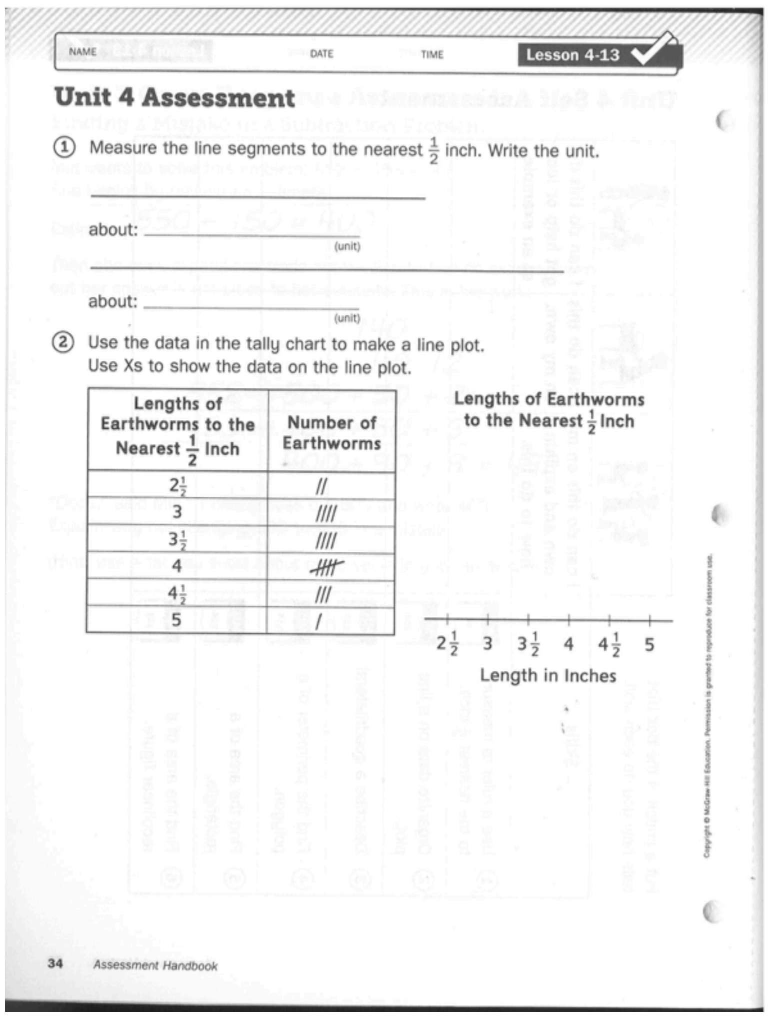


Figure A-10. Lesson from Everyday Mathematics Grade 3 Teacher's Lesson Guide, Volume 1 (UCSMP, 2015b, p. 395)

Appendix B

Unit 4 Assessment from Everyday Mathematics Grade 3

Assessment from Everyday Mathematics Grade 3 Unit student assessment. Reprinted from Everyday Mathematics Assessment Handbook, (p. 34-38), The University of Chicago School Mathematics Project, 2015, Columbus, OH: McGraw-Hill Education. Copyright 2015 by McGraw-Hill Education. Reprinted with permission.



NAME DATE TIME Lesson 4-13

Unit 4 Assessment

① Measure the line segments to the nearest $\frac{1}{2}$ inch. Write the unit.

about: ______ (unit)

about: ______ (unit)

② Use the data in the tally chart to make a line plot.
Use Xs to show the data on the line plot.

Lengths of Earthworms to the Nearest $\frac{1}{2}$ Inch	Number of Earthworms
$2\frac{1}{2}$	//
3	////
$3\frac{1}{2}$	////
4	~~////~~
$4\frac{1}{2}$	///
5	/

Lengths of Earthworms to the Nearest $\frac{1}{2}$ Inch

$2\frac{1}{2}$ 3 $3\frac{1}{2}$ 4 $4\frac{1}{2}$ 5

Length in Inches

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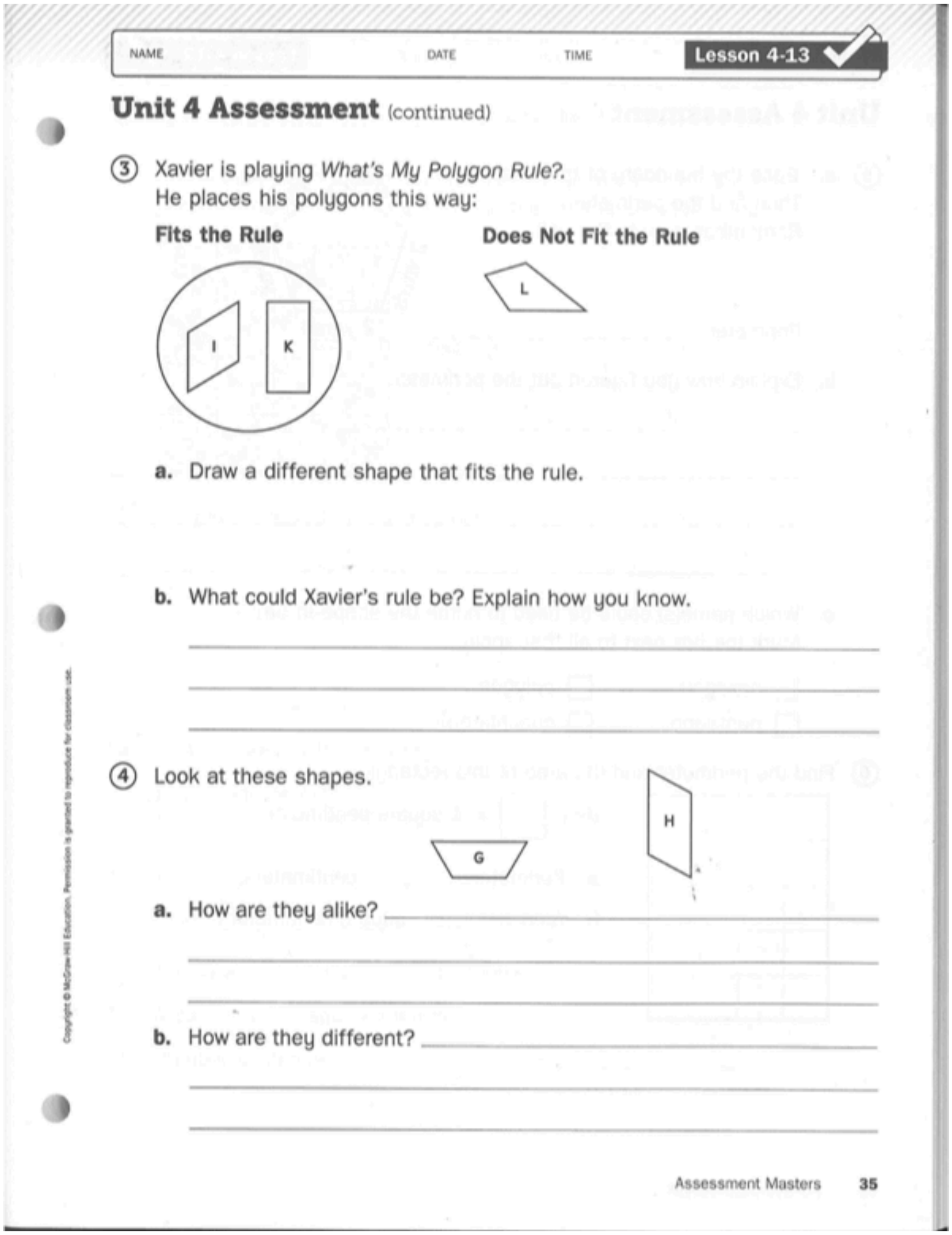
Figure B-1. Assessment from Everyday Mathematics Grade 3 Unit student assessment (UCSMP, 2015a, p. 34)



NAME DATE TIME **Lesson 4-13**

Unit 4 Assessment (continued)

③ Xavier is playing *What's My Polygon Rule?*.
He places his polygons this way:

Fits the Rule (I, K)

Does Not Fit the Rule (L)

a. Draw a different shape that fits the rule.

b. What could Xavier's rule be? Explain how you know.

④ Look at these shapes. (G, H)

a. How are they alike?

b. How are they different?

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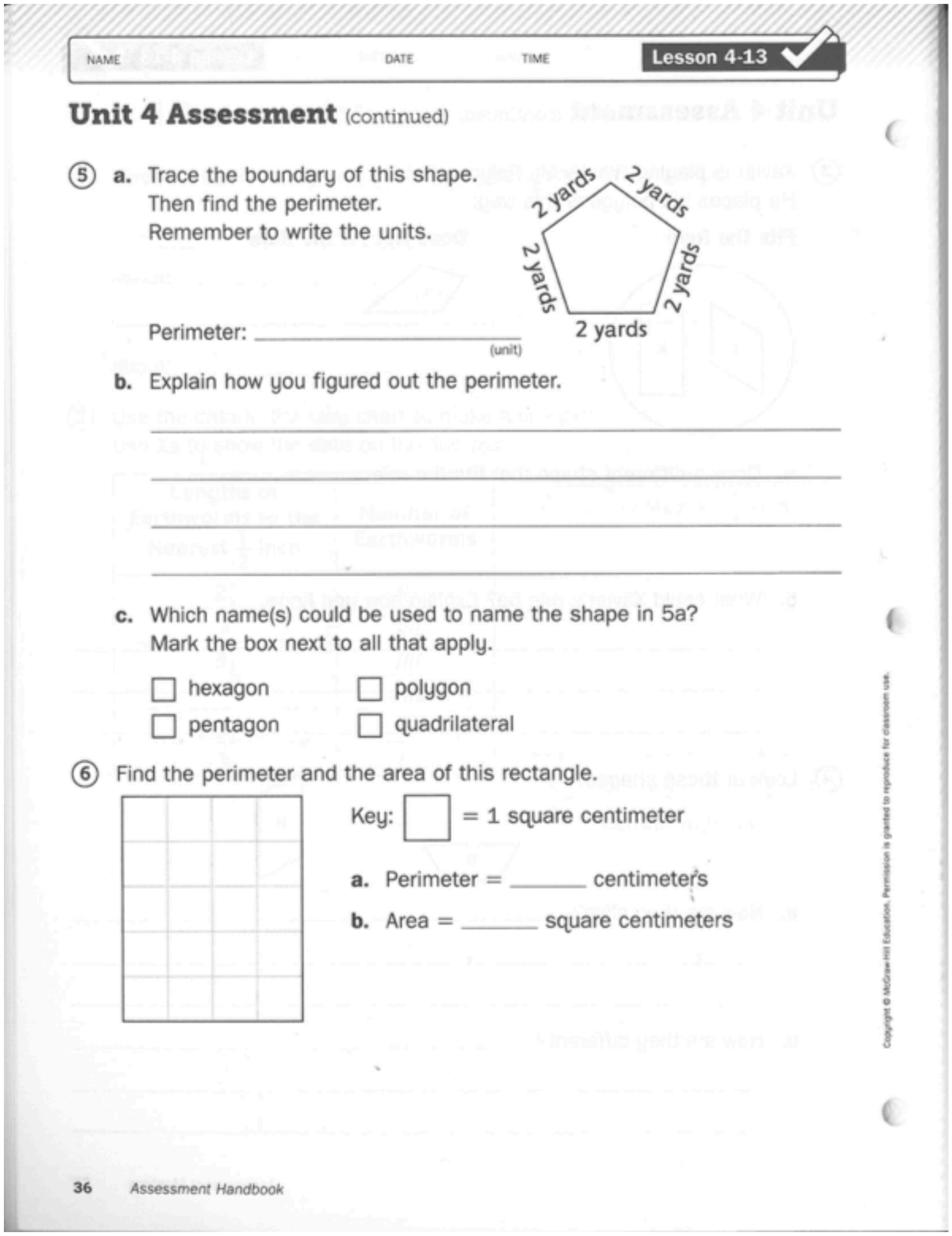
Assessment Masters 35

Figure B-2. Assessment from Everyday Mathematics Grade 3 Unit student assessment (UCSMP, 2015a, p. 35)



NAME DATE TIME **Lesson 4-13**

Unit 4 Assessment (continued)

⑤ **a.** Trace the boundary of this shape.
Then find the perimeter.
Remember to write the units.

2 yards
2 yards
2 yards
2 yards
2 yards

Perimeter: ______________ (unit)

b. Explain how you figured out the perimeter.

____

____

____

____

c. Which name(s) could be used to name the shape in 5a?
Mark the box next to all that apply.

☐ hexagon ☐ polygon
☐ pentagon ☐ quadrilateral

⑥ Find the perimeter and the area of this rectangle.

Key: ☐ = 1 square centimeter

a. Perimeter = _______ centimeters

b. Area = _______ square centimeters

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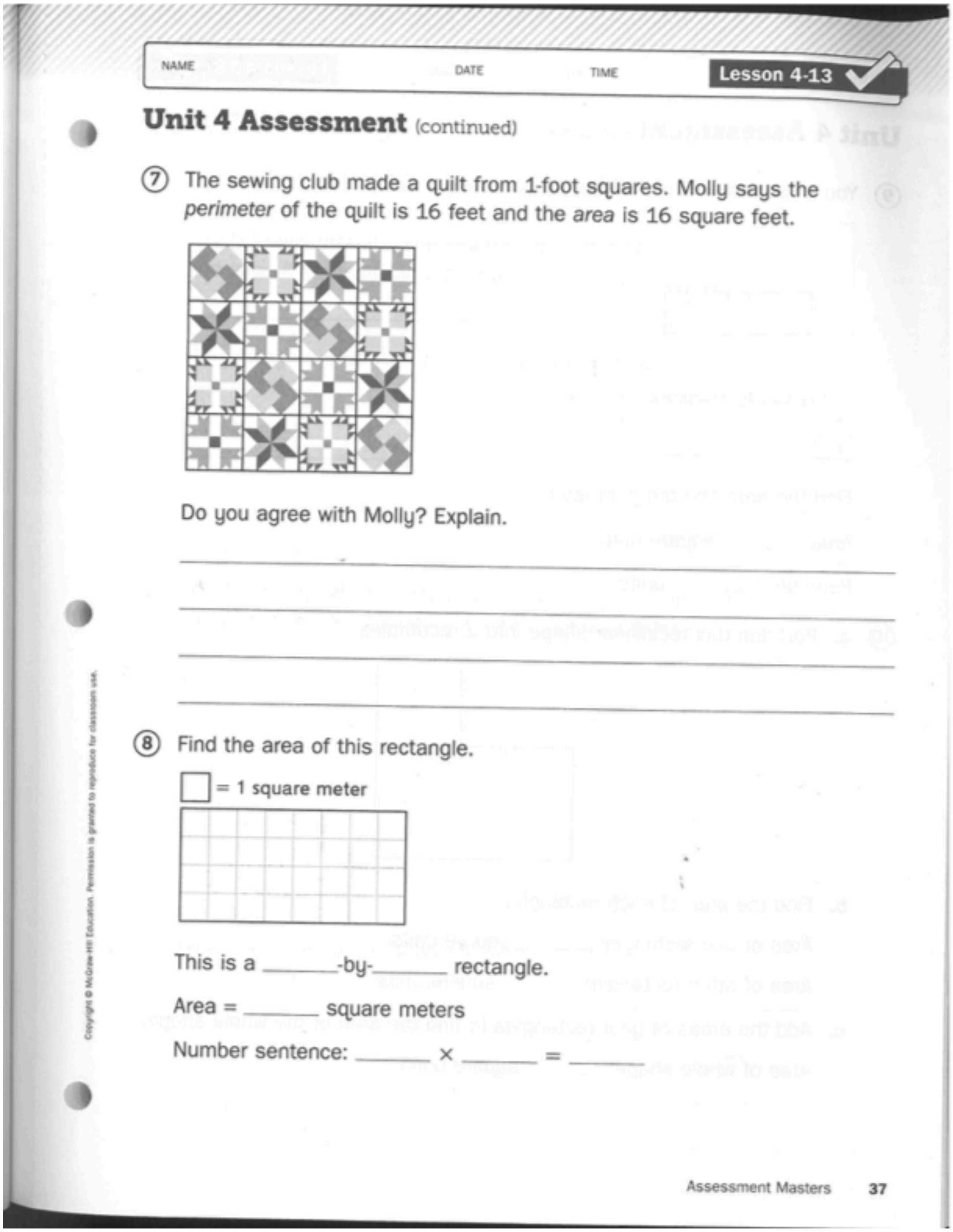
Figure B-3. Assessment from Everyday Mathematics Grade 3 Unit student assessment (UCSMP, 2015a, p. 36)


NAME DATE TIME **Lesson 4-13**

Unit 4 Assessment (continued)

⑦ The sewing club made a quilt from 1-foot squares. Molly says the *perimeter* of the quilt is 16 feet and the *area* is 16 square feet.

Do you agree with Molly? Explain.

____

____

____

____

⑧ Find the area of this rectangle.

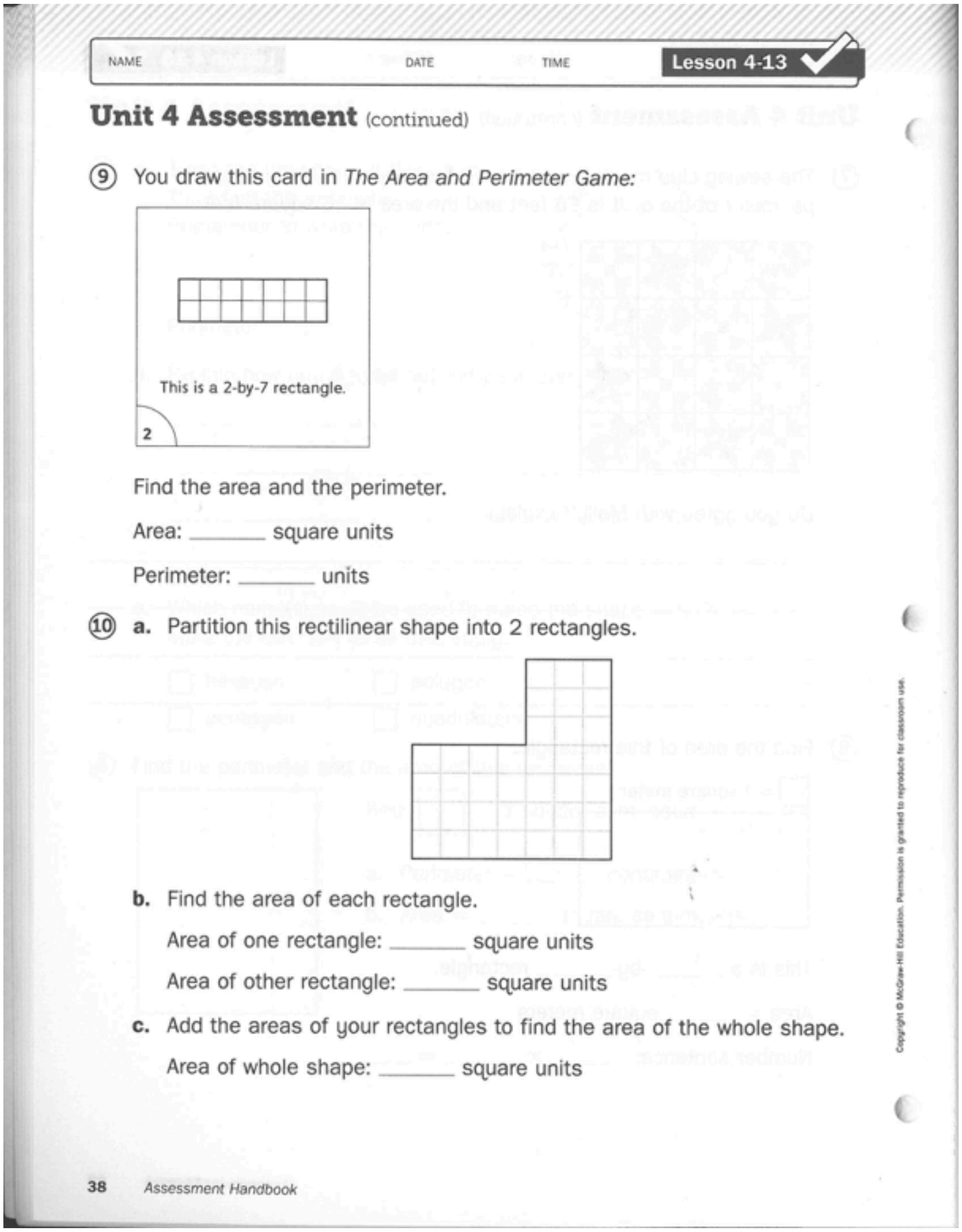
□ = 1 square meter

This is a ______-by-______ rectangle.

Area = ______ square meters

Number sentence: ______ × ______ = ______

Figure B-4. Assessment from Everyday Mathematics Grade 3 Unit student assessment (UCSMP, 2015a, p. 37)



NAME DATE TIME **Lesson 4-13**

Unit 4 Assessment (continued)

⑨ You draw this card in *The Area and Perimeter Game:*

This is a 2-by-7 rectangle.

2

Find the area and the perimeter.

Area: _______ square units

Perimeter: _______ units

⑩ **a.** Partition this rectilinear shape into 2 rectangles.

b. Find the area of each rectangle.

Area of one rectangle: _______ square units

Area of other rectangle: _______ square units

c. Add the areas of your rectangles to find the area of the whole shape.

Area of whole shape: _______ square units

Figure B-5. Assessment from Everyday Mathematics Grade 3 Unit student assessment (UCSMP, 2015a, p. 38)

Appendix C

Sample Recording Tool from Curriculum Materials Task

Section	SMP	SMP	SMP	Explanation
Mental Math and Fluency				
Math Message/Calculating Perimeters and Areas				
Solving the Open Response Problem				
Getting Ready for Day 2				
Setting Expectations				
Reengaging in the Problem/Revising Work				

Figure C-1. Sample recording tool from the curriculum materials task

Appendix D

Interview Protocol

Background Questions

1. Tell me about your experiences with mathematics in your k-12 school years.
2. Tell me about your experiences with mathematics in your college career. How many mathematics or mathematics education courses did you take in college?
3. How confident do you feel in teaching mathematics to elementary school students?

Questions about the 3 Tasks

1. Which task did you find to be the easiest? What made it easy?
2. Which task did you find to be the most difficult? What made it difficult?

Task-Specific Questions

1. When completing the Curriculum Materials Task, what process did you use to determine which SMP fit with the item/activity?
2. When completing the Curriculum Materials Task, were some items/activities more challenging to determine which SMP fit than others?
3. When completing the Assessment Task, what process did you use to determine which SMP fit with the item/activity?
4. When completing the Assessment Task, were some items/activities more challenging to determine which SMP fit than others?
5. When completing the Lesson Enactment Task, what process did you use to determine which SMP fit with the item/activity?
6. When completing the Lesson Enactment Task, were some items/activities more challenging to determine which SMP fit than others?

Appendix E

Common Core Standards for Mathematical Practice (NGA & CCSSO, 2010)

CCSS.MATH.PRACTICE.MP1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

CCSS.MATH.PRACTICE.MP2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

CCSS.MATH.PRACTICE.MP3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

CCSS.MATH.PRACTICE.MP4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

CCSS.MATH.PRACTICE.MP5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

CCSS.MATH.PRACTICE.MP6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

CCSS.MATH.PRACTICE.MP7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

CCSS.MATH.PRACTICE.MP8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Appendix F

Individual PST Information

Table F-1: PSTs' Academic Major, LMT and MBI Scores, and LMT and MBI Groupings

PST	Academic Major	LMT	MBI	LMT Scores	MBI Scores
UG 2018					
U31	Language Arts	Low	Mid	0.16	168
U32	Mathematics	Mid	Mid	0.87	168
U33	Science	Mid	High	1.18	193
U34	Language Arts	Low	Low	-1.03	153
U35	Language Arts	High	Low	1.43	153
U36	Mathematics	Mid	High	1.24	170
UG 2017					
U41	Social Studies	High	High	1.56	188
U42	Mathematics and Language Arts	Low	Low	0.69	151
U43	Social Studies	Mid	Mid	1.01	168
U44	Mathematics	High	Mid	1.76	165
Master's 2017					
M1	Language Arts	High	High	1.56	177
M2	Language Arts	Low	Mid	-1.03	165
M3	Science	Low	High	0.16	170
M4	Science	Mid	Low	1.24	157
M5	Language Arts	Low	Low	0.49	151
M6	Language Arts	Mid	High	0.83	174
M7	Language Arts	High	Low	1.76	139

Table F-2: Agreement Between Individual PSTs and the Authors

	Task			
PST	Curriculum Materials	Lesson Enactment	Assessment	Average
UG 2018				
U31	47%	0%	46%	31%
U32	20%	33%	42%	32%

U33	20%	33%	50%	34%
U34	20%	0%	50%	23%
U35	33%	33%	0%	22%
U36	13%	33%	38%	28%
UG 2017				
U41	47%	33%	46%	42%
U42	40%	33%	8%	27%
U43	20%	33%	35%	29%
U44	13%	67%	15%	32%
Master’s 2017				
M1	33%	33%	15%	27%
M2	40%	67%	38%	48%
M3	47%	0%	50%	32%
M4	19%	67%	8%	31%
M5	27%	0%	35%	21%
M6	33%	33%	23%	30%
M7	47%	67%	31%	48%

Appendix G

Scatterplots of Spearman Correlations Between PSTs' MKT and MBI Scores and Their Agreement with the Authors on the Different Tasks

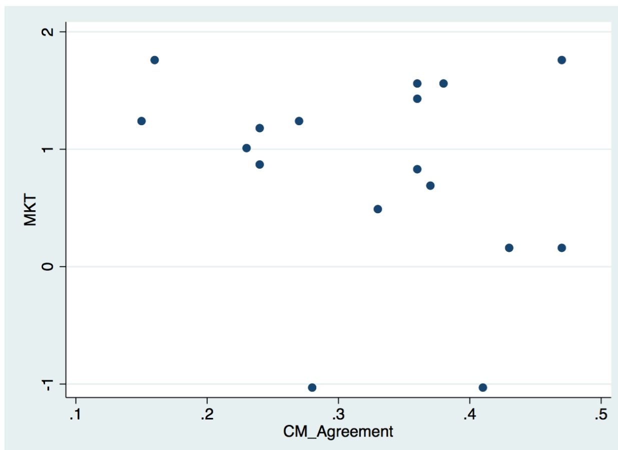


Figure G-1. Spearman correlation between PSTs' MKT scores and agreement with the authors on the curriculum materials task (r_s=-0.2369, p=0.36).

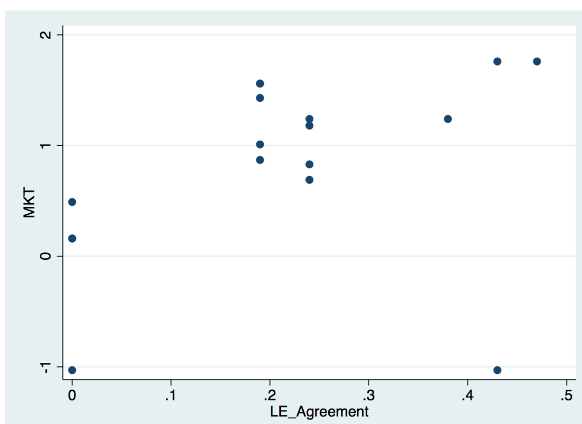


Figure G-2. Spearman correlation between PSTs' MKT scores and agreement with the authors on the lesson enactment task (r_s=0.4714, p=0.0559).

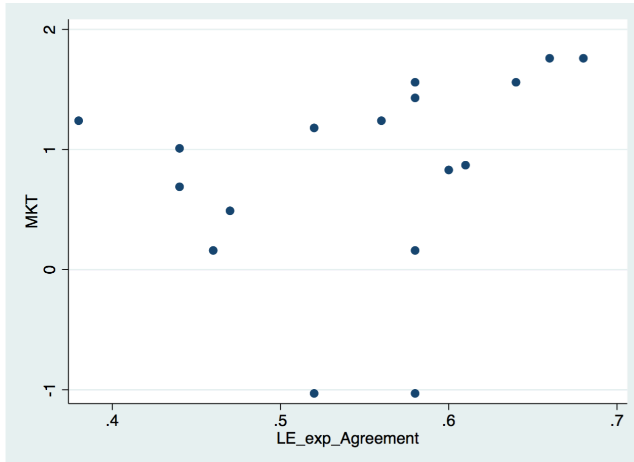


Figure G-3. Spearman correlation between PSTs' MKT scores and agreement with the expert raters on the lesson enactment task (r_s=0.4557, p=0.0660).

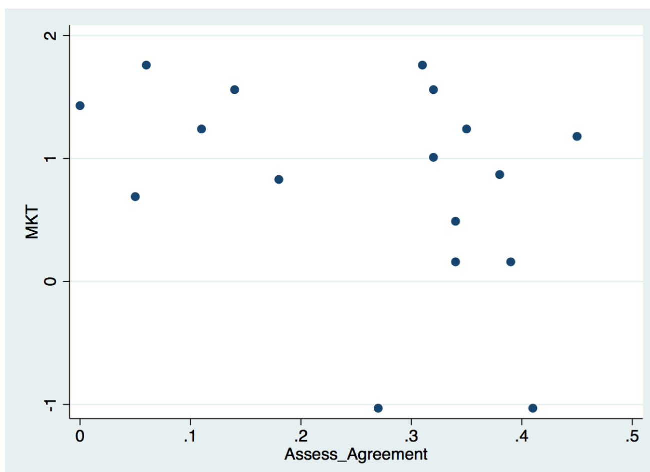


Figure G-4. Spearman correlation between PSTs' MKT scores and agreement with the authors on the assessment task (r_s=-0.4197, p=0.0935).

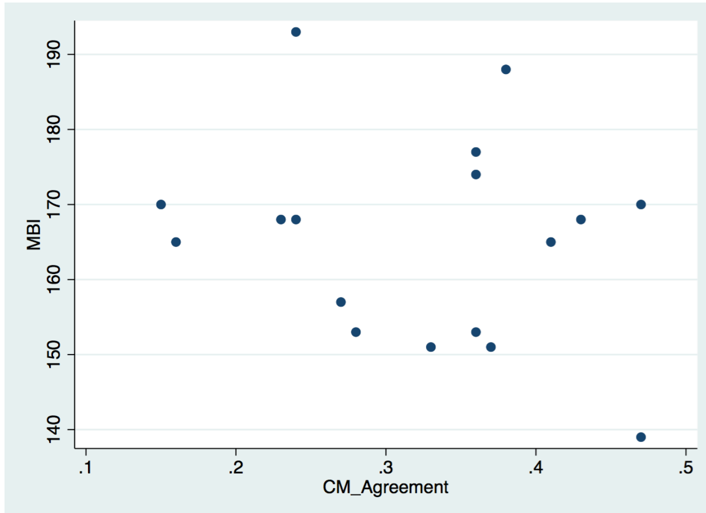


Figure G-5. Spearman correlation between PSTs' MBI scores and agreement with the authors on the curriculum materials task (r_s=-0.1477, p=0.5716).

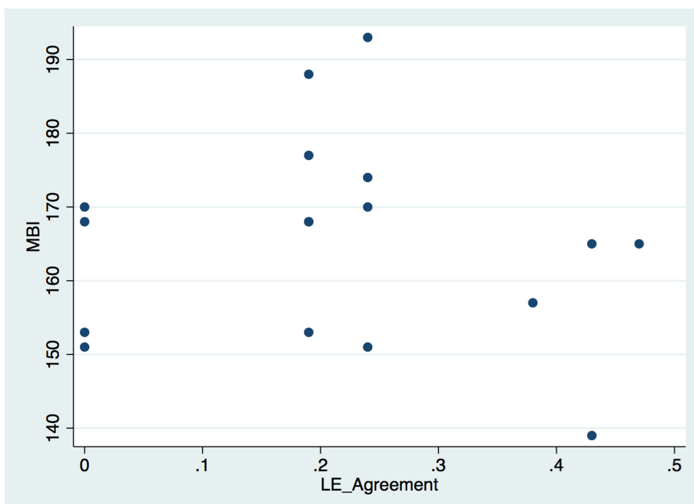


Figure G-6. Spearman correlation between PSTs' MBI scores and agreement with the authors on the lesson enactment task (r_s=-0.1074, p=0.6816).

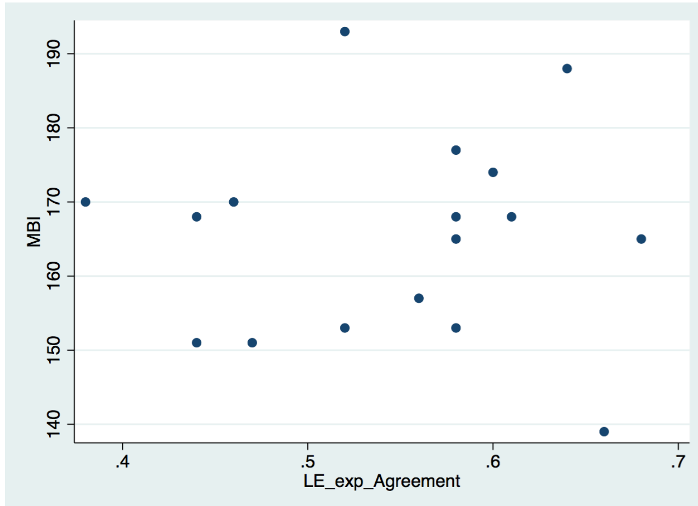


Figure G-7. Spearman correlation between PSTs’ MBI scores and agreement with the expert raters on the lesson enactment task (r_s=0.0273, p=0.9172).

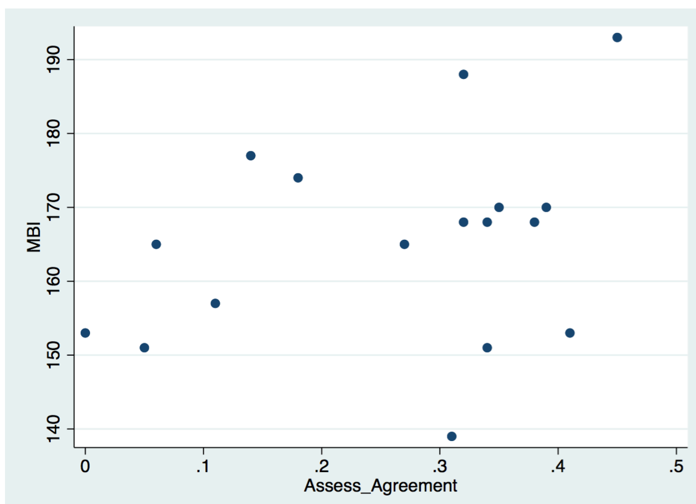


Figure G-8. Spearman correlation between PSTs’ MBI scores and agreement with the authors on the assessment task (r_s=0.3459, p=0.1739).

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