# A Personalization of Readmission Risk

Utilizing the prediction model proposed in Helm et al. (2016), one can personalize the time to develop the condition curve. This would be done by first developing a population based model and then applying transfer learning to personalize the predictions to each individual patient (using a Weibull regression to estimate a survival model for time-to-readmission).

The risk factors include socio-demographic, hospital admission and stay characteristics. For the purpose of illustration, we took three personalized risk profiles (high, medium and low) from Helm et al. (2016) and solved for optimal 2-checkup polices. The following table shows the improvements in detection probability upon current practice for high, medium, and low patients from Helm et al. (2016)'s risk profiles. The relative improvements were similar to Table 1, with a better improvement seen on high risk patients.

Table A.1: Comparison against current practice for high, medium, and low risk patients

| Risk of Readmission | Time of First Checkup | Time between Checkups | Detection Probability | | |
|---|---|---|---|---|---|
| | | | Optimal 2-Checkup | Current Practice | Relative Improvement |
| High(72%) | 4.9 | 3.5 | 0.30 | 0.17 | 77.4% |
| Medium(18%) | 6.3 | 4.1 | 0.24 | 0.16 | 51.9% |
| Low(4%) | 6.5 | 4.2 | 0.24 | 0.16 | 49.7% |

# B Model Notation and Parameters

Table B.1: Model Notation and Parameters

| | |
|---|---|
| $\rho$ | The random variable representing time-to-readmission (time between discharge and readmission) given no checkups |
| $g_\rho(\cdot)$ | The probability density function of $\tau$ |
| $D$ | Delay-time, i.e., the length of time prior to $\tau$ that the illness was detectable by a checkup; this is equivalent to the amount of time that a patient is in the ill state |
| $f(\cdot)$ | The probability density function of $D$; accordingly, $F(\cdot)$ is the cumulative distribution function of $D$ |
| $\delta$ | The time when the condition developed, i.e., when the illness is first detectable by a checkup |
| $g_\delta(\cdot)$ | The probability density function of $\delta$ |
| $t_i$ | The time when checkup $i$ is performed |
| $T$ | The latest time following discharge that readmissions are tracked until; thus, this also represents the latest time during which a checkup can be placed |
| $m$ | The number of different checkup methods available |
| $y_{ij} \in \{0,1\}$ | An indicator variable that denotes whether checkup method $j \in \{1,...,m\}$ is used at $t_i$ |
| $r_j \in [0,1]$ | The detection rate of checkup method $j$, i.e. if checkup method $j$ is performed when a patient is ill, then the checkup will detect the illness with probability $r_j$ |
| $r_{(i)} \in [0,1]$ | The detection rate of the checkup employed at $t_i \; \forall \; i \in \{1,...,n\}$ |
| $w_j \in \mathbb{N}$ | The maximum number of times checkup method $j$ can be used |
| $\Pi = (t_1,...,t_n, y_{11},...,y_{nm})$ | A checkup policy |
| $N_i^\Pi \in \{0,1\}$ | An indicator variable that denotes whether or not an illness is detected at $t_i$ given policy $\Pi$ |

# C Solution Approach

**initialization**

Generate 200 solutions[1] according to the recursive construction described in Proposition 1. Each of the 200 initial seeds is generated assuming a deterministic delay-time randomly sampled from the true delay-time distribution; $t \leftarrow 1$

**while** *Not converged* [2] **or** $t \leq 200$ **or** *gradient norm* $\leq 10^{-5}$ **do**

1. Keep the top 25 fittest solutions and eliminate the rest 175 solutions
2. Randomly mate solutions from the 25 solutions to generate 175 offspring solutions
3. Mutate 20 randomly selected solutions by randomly permuting the timing
4. Apply 5 iterations of gradient ascent to each solution
5. $t \leftarrow t + 1$

**end**

Note:

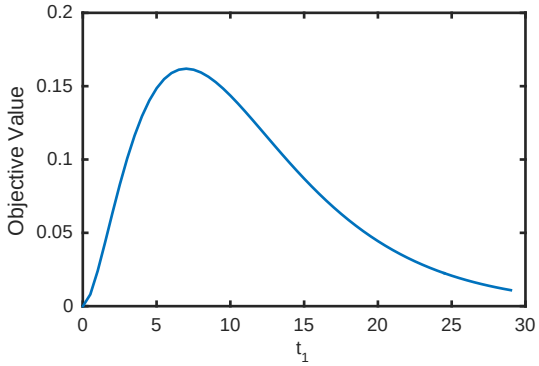
[1] : A solution, for a $n-$checkup problem, is a $n$ dimensional vector. For example, $n = 2$, $(t_1, t_2)$ is a valid solution where $t$ is the timing of checkups. The fitness of a solution is its detection probability (i.e. the objective value).

[2] : We say the algorithm converged if the change in population average fitness is less than $10^{-5}$ from $t$ to $t + 1$.

**Algorithm 1:** Solution procedure

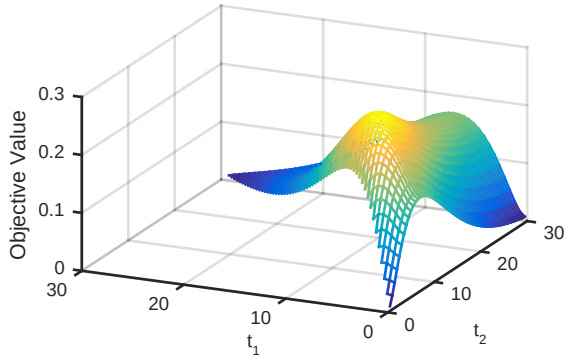
# D Example of Non-concave Objective Function

Figure D.1: Objective value for one checkup




time-to-develop the condition $g_\delta \sim$ gamma$(1.81, 5.08)$, delay-time $D \sim$ exponential$(2.35)$, one checkup with perfect detection rate

Figure D.2: Objective value for two checkups

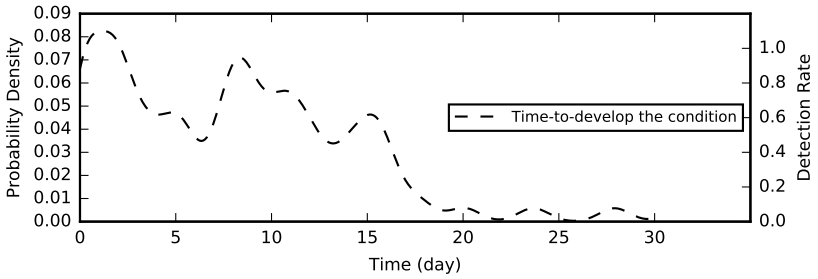



time-to-develop the condition $g_\delta \sim$ gamma$(1.81, 5.08)$, delay-time $D \sim$ exponential$(2.35)$, two checkups with perfect detection rate

# E Multi-modal Distribution

In this appendix, we test the robustness of our model under a multi-modal distribution. We created a counter-factual time-to-develop the condition distribution by simulating the time-to-develop the condition of 63 patients according to the fitted gamma distribution presented in Section 5.1. Then, patients that may have been readmitted on day-12 but were not (possibly due to the current practice of following up with patients on day 12) were added to the cohort. We simulated 24 patients who developed a condition prior to day-12 based on the exponential delay-time distribution. A Gaussian Kernel Density Estimator (KDE) with bandwidth 0.8 was used to fit the time-to-develop the condition distribution curve. The KDE distribution is shown in Figure E.1.

Figure E.1: The counter-factual multi-modal distribution created by a Gaussian Kernel Density Estimator




We studied the sequencing and timing of checkups. Solving for optimal $n$-checkup ($n = 4, ..., 10$) sequencing and timing under a multi-modal distribution, we found that the insights on sequencing and timing developed under the unimodality assumption of Proposition 1 did not hold in the multi-modal case as perfect checkups were no longer placed consecutively. However, the policies were still robust to the sequencing of checkups: the gaps between the worst-case and the best-case sequences for all policies in this test suite (4 to 10 checkups consisting of 3 office visits and 1 to 7 phone calls) were between 0.9% and 1.5%. Moreover, as can be seen in Table G, the optimal detection probabilities of these policies were close to the ones obtained using the original gamma distribution.

Table E.1: Comparison of optimal detection probabilities and gaps between worst and best detection probabilities

| Checkup Policy (P=Phone Call, O=Office Visit) | | 1P3O | 2P3O | 3P3O | 4P3O | 5P3O | 6P3O | 7P3O |
|---|---|---|---|---|---|---|---|---|
| Optimal Detection Probabilities | Original Gamma | 0.40 | 0.43 | 0.46 | 0.48 | 0.50 | 0.52 | 0.54 |
| | KDE | 0.39 | 0.42 | 0.45 | 0.47 | 0.50 | 0.51 | 0.53 |
| Gaps b/t Worst and Best Cases | Original Gamma | 0.2% | 0.3% | 0.4% | 0.5% | 0.4% | 0.3% | 0.4% |
| | KDE | 1.2% | 1.2% | 1.3 % | 1.5% | 1.1% | 0.9% | 1.2% |

As the detection rate increased, we noticed that imperfect checkups were centered around each mode and placed closer together. However, increasing the detection rate did not necessarily widen the overall coverage area. Since the checkups were scattered to cover the prominent modes, the overall coverage area was dictated by the separation of the modes.

## F Proof of Lemmas and Remarks

*Proof of Lemma 1.* An increase in $r$ causes the LHS of Eq. (25) to become smaller than the RHS, so $t_1^*$ is no longer optimal. After simple algebraic manipulation of Eq. (25), $r$ can be expressed in terms of $t_1^*$ as:

$$\begin{aligned} r(t_1^*) &= \frac{g_\delta(t_1^*) - \int_0^{t_1^*} g_\delta(k) f(t_1^* - k)\, dk}{g_\delta(t_1^*)\big[1 - F(t_2^* - t_1^*)\big]} \\ &= \frac{1}{1 - F(t_2^* - t_1^*)} - \frac{\int_0^{t_1^*} g_\delta(k) f(t_1^* - k)\, dk}{g_\delta(t_1^*)[1 - F(t_2^* - t_1^*)]} \end{aligned} \tag{F.1}$$

Differentiating this function with respect to $t_1^*$ yields

$$\begin{aligned} \frac{\partial r}{\partial t_1^*} &= -\frac{f(t_2^* - t_1^*)}{[1 - F(t_2^* - t_1^*)]^2} - \frac{\int_0^{t_1^*} g_\delta(k) f'(t_1^* - k)\, dk + g_\delta(t_1^*) f(0)}{g_\delta(t_1^*)[1 - F(t_2^* - t_1^*)]} \\ &+ \frac{f(t_2^* - t_1^*) \int_0^{t_1^*} g_\delta(k) f(t_1^* - k)\, dk}{g_\delta(t_1^*)[1 - F(t_2^* - t_1^*)]^2} + \frac{g_\delta'(t_1^*) \int_0^{t_1^*} g_\delta(k) f(t_1^* - k)\, dk}{g_\delta(t_1^*)^2 [1 - F(t_2^* - t_1^*)]} \end{aligned} \tag{F.2}$$

Then, notice the following:

$$g_\rho(t_1^*) = \int_0^{t_1^*} g_\delta(k) f(t_1^* - k)\, dk \tag{F.3}$$

$$g_\rho'(t_1^*) = \int_0^{t_1^*} g_\delta(k) f'(t_1^* - k)\, dk + g_\delta(t_1^*) f(0) \tag{F.4}$$

We are interested in the situation where the derivative in Eq. (F.2) is non-positive. Plugging Eq. (F.3) and (F.4) into Eq. (F.2), this is equivalent to saying:

$$0 \geq -\frac{f(t_2^* - t_1^*)}{[1 - F(t_2^* - t_1^*)]^2} - \frac{g_\rho'(t_1^*)}{g_\delta(t_1^*)[1 - F(t_2^* - t_1^*)]} + \frac{f(t_2^* - t_1^*) g_\rho(t_1^*)}{g_\delta(t_1^*)[1 - F(t_2^* - t_1^*)]^2} + \frac{g_\delta'(t_1^*) g_\rho(t_1^*)}{g_\delta(t_1^*)^2 [1 - F(t_2^* - t_1^*)]} \tag{F.5}$$

Combining like terms, we have

$$0 \geq \frac{f(t_2^* - t_1^*)}{[1 - F(t_2^* - t_1^*)]^2} \Big( \frac{g_\rho(t_1^*)}{g_\delta(t_1^*)} - 1 \Big) + \frac{1}{g_\delta(t_1^*)[1 - F(t_2^* - t_1^*)]} \Big( \frac{g_\delta'(t_1^*) g_\rho(t_1^*)}{g_\delta(t_1^*)} - g_\rho'(t_1^*) \Big) \tag{F.6}$$

$$\implies \frac{f(t_2^* - t_1^*)}{[1 - F(t_2^* - t_1^*)]} \Big( 1 - \frac{g_\rho(t_1^*)}{g_\delta(t_1^*)} \Big) \geq \frac{1}{g_\delta(t_1^*)} \Big( \frac{g_\delta'(t_1^*) g_\rho(t_1^*)}{g_\delta(t_1^*)} - g_\rho'(t_1^*) \Big) \tag{F.7}$$

Multiplying both sides by $g_\delta(t_1^*)$ yields

$$\Big( g_\delta(t_1^*) - g_\rho(t_1^*) \Big) \frac{f(t_2^* - t_1^*)}{[1 - F(t_2^* - t_1^*)]} \geq \frac{g_\delta'(t_1^*) g_\rho(t_1^*)}{g_\delta(t_1^*)} - g_\rho'(t_1^*) \tag{F.8}$$

From Eq. (25), it follows that $g_\rho(t_1^*) \leq g_\delta(t_1^*)$. Then, the LHS of Eq. (F.8) is positive. Hence, it is sufficient to show that the RHS of Eq. (F.8) is negative. That is, it is sufficient that

$$\frac{g_\delta'(t_1^*) g_\rho(t_1^*)}{g_\delta(t_1^*)} \leq g_\rho'(t_1^*) \iff \frac{g_\delta'(t_1^*)}{g_\delta(t_1^*)} \leq \frac{g_\rho'(t_1^*)}{g_\rho(t_1^*)} \tag{F.9}$$

The above inequality holds as a result of the delayed readmission log-likelihood inequality, which completes our proof. □

*Proof of Lemma 2.*

$$\frac{\partial}{\partial t}\left(\frac{g_\rho(t)}{g_\delta(t)}\right) = \frac{g_\delta(t)g'_\rho(t) - g_\rho(t)g'_\delta(t)}{g_\delta^2(t)} \tag{F.10}$$

$$= \frac{\frac{g'_\rho(t)}{g_\rho(t)} - \frac{g'_\delta(t)}{g_\delta(t)}}{\frac{g_\delta^2(t)}{g_\rho(t)g_\delta(t)}} \geq 0 \tag{F.11}$$

The last inequality follows from the delayed readmission log-likelihood inequality. □

*Proof of Remark 7.*

For Erlang-exponential distributions, Eq. (25) and (26) (FONCs) become:

$$e^{t_2}(k - t_1) - e^{-t_1}kr = 0 \tag{F.12}$$

$$r{t_1}^k = {t_2}^k - k{t_2}^{k-1} \tag{F.13}$$

Suppose $r$ increases to $r + \epsilon$, from Theorem 1, we know that $t_1$ moves to $t_1 - x, x > 0$.

Suppose $t_2$ moves to $t_2 - y, y > 0$, at the new optimum, Eq. (26) becomes:

$$r{t_1}^k = {t_2}^k - k{t_2}^{k-1} \tag{F.14}$$

$$(r + \epsilon)(t_1 - x)^k = (t_2 - y)^k - k(t_2 - y)^{k-1} \tag{F.15}$$

For $k = 1$, we have

$$(r + \epsilon)x = y - \epsilon t_1 \tag{F.16}$$

We would like to express $x - y$ as a function of $r$ and $\epsilon$ then put lower and upper bounds on it.

Lower bound: One trivial lower bound is $x - y \geq 0$ (result of Theorem 1)

$$(r + \epsilon)x - y = \epsilon t_1 \tag{F.17}$$

$$\Leftrightarrow (r + \epsilon)x - (r + \epsilon)y \geq \epsilon t_1 > 0 \tag{F.18}$$

Upper bound: From Eq. (F.12) we know $t_1 < k = 1$. Also, we know that $x \leq t_1$.

So

$$(r + \epsilon)x - y = \epsilon t_1 \tag{F.19}$$

$$-y = \epsilon t_1 - (r + \epsilon)x \tag{F.20}$$

$$x - y = x - (r + \epsilon)x + \epsilon t_1 \tag{F.21}$$

$$x - y \leq (1 - r - \epsilon)t_1 + \epsilon t_k \leq (1 - r) \tag{F.22}$$

For $k \geq 2$:

$$(t_2 - k)t_2^{k-1} = rt_1 t_1^{k-1} \tag{F.23}$$

Since $t_2 > t_1$ and $k \geq 2$, we have $t_2^{k-1} \geq t_1^{k-1}$. Then

$$t_2 - k \leq rt_1 \leq rk \quad (\text{since } t_1 < k) \tag{F.24}$$

$$t_2 \leq (r+1)k \tag{F.25}$$

Now we bound $t_1$.

$$e^{t_2}(k - t_1) - e^{-t_1}kr = 0 \tag{F.26}$$

$$(k - t_1) - e^{-t_1}kr \leq 0 \tag{F.27}$$

$$(k - t_1) - kr \leq 0 \tag{F.28}$$

$$(1 - r)k - t_1 \leq 0 \tag{F.29}$$

$$t_1 \geq (1 - r)k \tag{F.30}$$

The bounds for $t_1$ and $t_2$ at the new equilibrium (i.e. $t_1 - x$ and $t_2 - y$ are optimal for $t + \epsilon$):

$$t_2 - y \leq (1 + r + \epsilon)k \tag{F.31}$$

$$x - t_1 \leq -(1 - r - \epsilon)k \tag{F.32}$$

Combine the two inequalities, we have

$$x - y - t_1 + t_2 \leq 2(r + \epsilon)k \tag{F.33}$$

Therefore, the desired upper and lower bounds are

$$0 \leq x - y \leq 2(r + \epsilon)k \tag{F.34}$$

□

## G Different Patient Types

In this appendix, we provide an initial analysis on how incorporating both readmitted and non-readmitted could impact the performance of our model. We do acknowledge that due to limited data, our analysis might not represent a more general population. However, we believe that the analysis in this appendix provides further insights on the performance of our model.

At our partner hospital, current practice is to place a phone call on day 2 and an office visit on day 12 after discharge. These checkups could bias the data and results as there could be endogeneity induced by current checkup practice. We considered four types of patients in our chart review cohort: (1) patients who were not going to be readmitted regardless of checkups and intervention (non-readmit-able patients), (2) patients whose 30-day readmissions were detected and prevented by the day-2 and the day-12 checkups, (3) patients whose 30-day readmissions could have been prevented if the checkups were placed on days other than day-2 or day-12, and (4) patients who were going to be readmitted regardless of checkups and intervention (unavoidable readmissions).

To include all four types of patients, we went back to the chart review data set, which contained 327 cystectomy patients who underwent cystectomy at our collaborating hospital. We believe that the cohort of 327 patients included the four types of patients. Out of the 327 patients, 63 developed post-surgical conditions that lead to a 30-day readmission. The 63 patients included in our original analyses included type 3 and type 4 patients. The remaining $327 - 63 = 264$ patients included type-1 and type 2 patients, which were not readmitted and therefore not included in our original analysis.

Of the remaining 264 patients, 236 of them developed a condition at some point in their post-discharge recovery. The $264 - 236 = 28$ patients that never developed a condition were considered to be type 1 (not going to be readmitted regardless of monitoring policy). Of the 236 patients that developed a condition at some point, 24 patients were found to have had a condition detected on either the day-2 or the day-12 checkups as recorded on the medical chart. These 24 patients could have either (1) developed a non-readmission causing condition (reason 1) or (2) could have developed a readmission-causing condition that was mitigated by the checkups (reason 2). However, we do not have sufficient data to distinguish between the two reasons. Let $q$ denote the proportion of reason 2 patients among the 24 patients. These patient could be considered as type 2 patients. We could estimate this proportion by looking at those $236 - 24 = 212$ patients who developed a condition but were not detected on day 2 or day 12 by the current follow-up protocol. Out of those 212 patients, 63 patients ($63/212 = 30\%$) were readmitted. This means that if we assume that the characteristics of those 24 patients are same as the population (212 patients), and that the checkups are perfect inspections that can prevent readmissions with probability one, then $q$ can be estimated to be 30%. In reality, $q$ may be smaller than 30% (if the checkups are not perfect) and it may not prevent readmissions

with probability one; or $q$ could be greater than 30% if patients who are found sick on day-2 and day-12 are more likely to be readmitted than the population average is. Another way to estimate $q$ is to use the national average readmission rate of cystectomy (which was observed to be 24% in the SID database). We conducted sensitivity analyses around the proportion of type 2 patients at $q = 25\%, 50\%, 75\%$, and 100%. Gamma distributions were fitted to these cohorts with $q = 25\%, 50\%, 75\%$, and 100% of the 24 patients added.

Finally, we took the checkup policies obtained using the original gamma distribution (types 3 and 4 only) and computed their objective values (suboptimal) by plugging the computed policies into the gamma distributions that included patients (simulated by adding $q = 25\%, 50\%, 75\%$, and 100% of the 24 patients) type 2 patients. We then computed the difference in objective values between the suboptimal objective values and the optimal objective values (using the distribution that included type 2 patients as our testbed) for checkup policies consisting of 1 to 3 office visits and 1 to 7 phone calls. As seen in the following table, by ignoring types 1 and 2 patients, the detection probabilities degraded by at most 3.5%. The most likely value of $q$, according to our estimation, would be around 24%, which shows at worst a very small difference of 0.54% between the original checkup policy (from our simpler model containing only types 3 and 4 patients) and the true optimal. We believe that the small observed differences are sufficient to demonstrate that the results from our simpler analysis with only types 3 and 4 patients should still be valid.

Table G.1: Difference in detection probabilities

| | $q$ | 1 Phone Call | 2 Phone Calls | 3 Phone Calls | 4 Phone Calls |
|---|---|---|---|---|---|
| 1 Office Visit | 25% | −0.42% | −0.46% | −0.48% | −0.49% |
| | 50% | −1.16% | −1.25% | −1.24% | −1.24% |
| | 75% | −1.96% | −2.15% | −2.20% | −2.24% |
| | 100% | −2.64% | −2.90% | −2.98% | −3.05% |
| 2 Office Visits | 25% | −0.51% | −0.54% | −0.54% | −0.51% |
| | 50% | −1.34% | −1.45% | −1.40% | −1.27% |
| | 75% | −2.34% | −2.51% | −2.47% | −2.33% |
| | 100% | −3.17% | −3.39% | −3.33% | −3.16% |
| 3 Office Visits | 25% | −0.49% | −0.53% | −0.52% | −0.53% |
| | 50% | −1.17% | −1.32% | −1.26% | −1.26% |
| | 75% | −2.22% | −2.41% | −2.37% | −2.39% |
| | 100% | −3.04% | −3.28% | −3.22% | −3.25% |

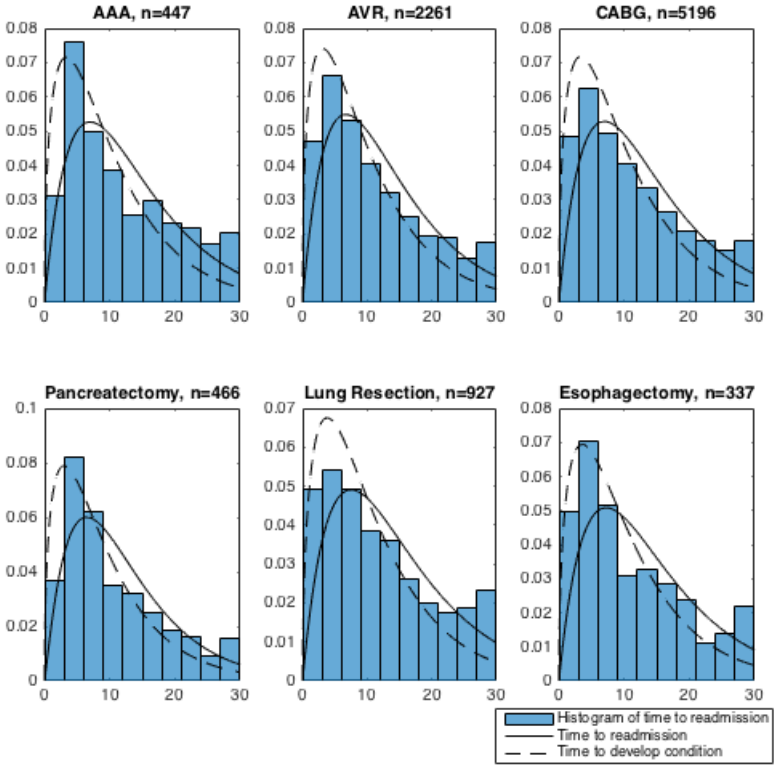
Table continued

| | $q$ | 5 Phone Calls | 6 Phone Calls | 7 Phone Calls |
|---|---|---|---|---|
| 1 Office Visit | 25% | −0.49% | −0.49% | −0.50% |
| | 50% | −1.22% | −1.18% | −1.24% |
| | 75% | −2.24% | −2.22% | −2.28% |
| | 100% | −3.04% | −3.02% | −3.09% |
| 2 Office Visits | 25% | −0.51% | −0.53% | −0.52% |
| | 50% | −1.24% | −1.34% | −1.29% |
| | 75% | −2.31% | −2.42% | −2.38% |
| | 100% | −3.14% | −3.27% | −3.23% |
| 3 Office Visits | 25% | −0.49% | −0.49% | −0.54% |
| | 50% | −1.11% | −1.14% | −1.38% |
| | 75% | −2.20% | −2.22% | −2.49% |
| | 100% | −3.02% | −3.04% | −3.37% |

# H Unimodality Assumption in Six Other Major Surgeries

From the State Inpatient Databases (SID), we extracted the readmission records of patients who had the following common abdominal and chest surgeries in 2009 and 2010: Abdominal Aortic Aneurysm Repair (AAA), Esophagectomy, Pancreatectomy, Aortic Calve Replacement (AVR), Coronary Artery Bypass Grafting (CABG), and Lung Resection. Note that these probability density curves are parameterized using readmitted patient only, and they appear to be unimodal.

Figure H.1: Time-to-readmission and time-to-develop the condition distributions for six major abdominal and chest surgeries



# I Inverse Laplace Transform

Clinical data used to parametrize the delay-time models is limited in the fact that time-to-develop the condition is currently not recorded in any databases known to the authors. The historical data most readily available is the time-to-readmission. To obtain data on the delay-time, which is also to the best of our knowledge not recorded in any major clinical data bases, we conducted a study of 327 medical records and extracted data on how long the patient had been feeling ill before returning to the hospital based on triage notes upon readmission. However, given the time-to-readmission distribution and the delay-time distribution, we can obtain the time-to-develop the condition probability density on larger databases by applying the inverse Laplace transform.

Recall that the time-to-readmission, $\rho$, is the summation of the time-to-develop the condition $\delta$ and the delay-time $D$, i.e., $\rho = \delta + D$. Since $\delta$ and $D$ are assumed to be independent, the Laplace transform of $\rho$, $\mathcal{L}\{g_\rho(x)\}(s)$, is equivalent to the product of the Laplace transforms of $\delta$ and $D$, i.e., $\mathcal{L}\{g_\delta(x)\}(s)$ and $\mathcal{L}\{f(x)\}(s)$.

$$\mathcal{L}\{g_\delta(x)\}(s)\mathcal{L}\{f(x)\}(s) = \mathcal{L}\{g_\rho(x)\}(s) \tag{I.1}$$

Dividing both sides by $\mathcal{L}\{f(x)\}(s)$, we get the following expression for the Laplace transform of the time-to-develop the condition, denoted by $\mathcal{G}(s)$:

$$\mathcal{L}\{g_\delta(x)\}(s) = \frac{\mathcal{L}\{g_\rho(x)\}(s)}{\mathcal{L}\{f(x)\}(s)} =: \mathcal{G}(s) \tag{I.2}$$

Applying the inverse Laplace transform $\mathcal{L}^{-1}\{\cdot\}$ to both sides of Eq. (I.2), we obtain the probability density function of the time-to-develop the condition:

$$g_\delta(x) = \mathcal{L}^{-1}\{\mathcal{G}(s)\}(x) \tag{I.3}$$

The inverse Laplace transform yields closed-form solutions for certain $g_\rho(\cdot)$-$f(\cdot)$ pairs such as Erlang-exponential and normal-normal. Given arbitrary $g_\rho(\cdot)$ and $f(\cdot)$, a closed-form solution may not exist. In such cases, numerical algoyourithms for inverse Laplace transform (Avdis and Whitt (2007), Rizzardi (1995), Lyness and Giunta (1986)) can be implemented.

# J Checkup Quantity vs Quality

In this appendix, we present the computations used to compare, in 10-checkup policies, the effectiveness of increasing the phone call detection rate vs. the effectiveness of upgrading one existing phone call to an office visit.

We start by computing the detection probability as a function of the detection rate of the phone calls. Table J.1 shows the detection probabilities. Notice that the values in Table J.1 correspond to points in Figure 9.

Table J.1: Detection probabilities of 10 checkup policies with 0-3 office visits, detection rate $= 0.2, 0.4, 0.6, 0.8, 1$

| # of Office Visits \ Phone Call Detection Rate | 0.2 | 0.4 | 0.6 | 0.8 | 1 |
|---|---|---|---|---|---|
| 0 | 0.22 | 0.30 | 0.37 | 0.43 | 0.64 |
| 1 | 0.37 | 0.41 | 0.45 | 0.49 | 0.64 |
| 2 | 0.48 | 0.50 | 0.52 | 0.54 | 0.64 |
| 3 | 0.56 | 0.57 | 0.58 | 0.59 | 0.64 |

We then estimate the improvement in detection probability achieved by upgrading an existing phone call to an office visit. Results are shown in Table J.2.

Table J.2: Improvement in detection probabilities achieved by upgrading a phone call to an office visit

| # of Office Visits \ Phone Call Detection Rate | 0.2 | 0.4 | 0.6 | 0.8 |
|---|---|---|---|---|
| $0 \to 1$ | 0.14 | 0.11 | 0.08 | 0.06 |
| $1 \to 2$ | 0.11 | 0.09 | 0.07 | 0.05 |
| $2 \to 3$ | 0.09 | 0.08 | 0.06 | 0.05 |

Next, we estimate the improvement in detection probability achieved by increasing the detection rate of the phone calls. Results are shown in Table J.3. Notice that the values in Table J.3 should be interpreted as the improvement achieved per 20% increase.

Table J.3: Improvement of detection probabilities when increasing detection rates by 20%

| # of Office Visits \ Phone Call Detection Rate | $0.2 \to 0.4$ | $0.4 \to 0.6$ | $0.6 \to 0.8$ |
|---|---|---|---|
| 0 | 0.08 | 0.07 | 0.06 |
| 1 | 0.05 | 0.04 | 0.03 |
| 2 | 0.02 | 0.02 | 0.02 |

In both Tables J.2 and J.3, greater marginal benefits are observed when the detection rate of phone calls is low and the number of office visits is small.

Finally, we compute the relative effectiveness of increasing the phone call detection rate by 20% with respect to upgrading an existing phone call to an office visit. A relative effectiveness of 100% means that increasing the phone call detection rate by 20% is as effective as upgrading a phone call to an office visit. Table J.4 shows the result.

Table J.4: Relative effectiveness of increasing phone call detection rate with respect to replacement of a phone call with an office visit

| # of Office Visits \ Phone Call Detection Rate | $0.2 \to 0.4$ | $0.4 \to 0.6$ | $0.6 \to 0.8$ |
|---|---|---|---|
| $0 \to 1$ | 58% | 63% | 70% |
| $1 \to 2$ | 41% | 46% | 50% |
| $2 \to 3$ | 35% | 29% | 31% |

It is worth noting that as the number of office visits increases, we need a greater improvement in phone call detection rate to match the effect of upgrading from an existing phone call to an office visit. This is, in part, because we keep the total number of checkups fixed at 10, thus more office visits means fewer phone calls and therefore the impact of increasing phone call detection rates is muted. On average, increasing the phone call detection rate by 20% is as effective as upgrading 0.47 existing phone calls to an office visit in terms of detection probability.

# K Description of Validation Method 2

This appendix describes Method 2 to validate the optimal checkup policies involving the use of actual readmission times for patients from the out-sample data set and calculating the probability that their delay-time was long enough to be detected by a checkup that occurred before the actual readmission time. This approach assumes independence between delay-time and time-to-readmission. Let $T$ be the time that the patient was actually readmitted (in the data). The detection probability, $\hat{\mathcal{D}}$, can then be calculated using the following formula. Let $N = \text{argmax}_n : t_n \leq T$.

$$\begin{aligned}
\hat{\mathcal{D}} =& r_1 \cdot (1 - F(T - t_1)) && \text{(K.1)}\\
&+ \sum_{\alpha=2}^{N} \left( r_\alpha \cdot \left( \prod_{\beta=1}^{\alpha-1} (1 - r_\beta) \right) \cdot (1 - F\left(T - t_1\right)) \right) && \text{(K.2)}\\
&+ \sum_{\alpha=3}^{N} \left( \sum_{\beta=2}^{\alpha-1} \left( r_\alpha \cdot \prod_{\gamma=\beta}^{\alpha-1} (1 - r_\gamma) \cdot (F(T - t_{\beta-1}) - F(T - t_\beta)) \right) \right) && \text{(K.3)}\\
&+ \sum_{\alpha=2}^{N} \left( r_\alpha \cdot (F(T - t_{\alpha-1}) - F(T - t_\alpha)) \right) && \text{(K.4)}
\end{aligned}$$

where the four summands collectively represent the total probability of detecting the patient as ill during every scheduled checkup. In particular, the first summand (K.1) represents the probability that the patient enters the ill state before the first checkup and is successfully identified as ill during the first checkup. The second summand (K.2) represents the probability that the patient becomes ill before the first checkup, but checkups 1 through $(\alpha - 1)$ fail to properly detect the patient condition and checkup $\alpha \in \{2, ..., N\}$ successfully identifies the patient as ill. The third summand (K.3) represents the probability that the patient becomes ill between checkups $(\beta - 1)$ and $\beta$ for $\beta \in \{2, ..., (\alpha - 1)\}$, but is not properly identified as being ill until checkup $\alpha \in \{3, ..., N\}$. The fourth summand (K.4) represents the probability that the patient enters the ill state between checkups $(\alpha - 1)$ and $\alpha$, and is immediately identified as being ill during checkup $\alpha \in \{2, ..., N\}$.

# L Validation Results Using Method 1

The following table shows the absolute optimality gaps of the 2010 policies applied on 2009 patients using Method 1.

Table L.1: Absolute optimality gap for 2009 SID patients using n-checkup policies with 0-3 perfect checkups that were parameterized using 2010 SID patients

| # of Office Visits \ Total # of Checkups | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1.4% | 2.1% | 2.4% | 2.7% | 2.9% | 2.6% | 2.9% | 2.9% | 2.7% | 2.9% |
| 1 | 2.4% | 2.7% | 2.9% | 3.0% | 2.8% | 2.9% | 2.8% | 2.9% | 2.8% | 2.8% |
| 2 | N/A | 3.0% | 3.1% | 3.0% | 3.1% | 3.1% | 3.1% | 3.1% | 3.0% | 2.6% |
| 3 | N/A | N/A | 3.2% | 3.0% | 3.2% | 3.1% | 2.9% | 2.9% | 2.8% | 2.6% |

# M Switching Training and Testing Set

This appendix shows the results when the training and testing sets are switched (i.e., 2009 SID for training, 2010 SID for testing).

Figure M.1: Fitted time-to-readmission and recovered time-to-develop the condition for 2009 SID patients

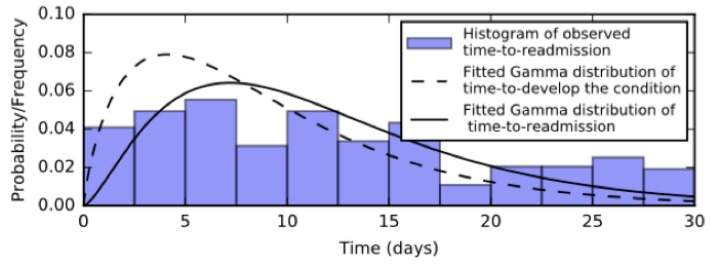



Figure M.2: Detection probability of checkup policies with 0-3 perfect checkups for 2009 SID patients. This figure would replace Figure 8 if training and testing sets were switched

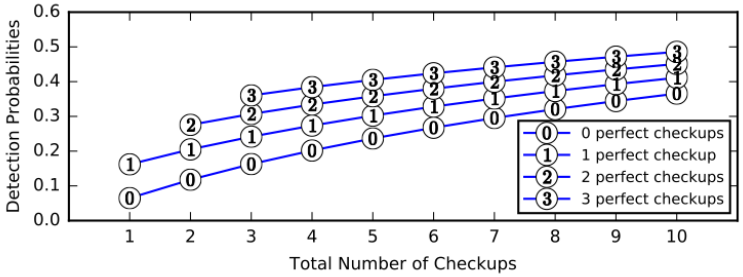



Figure M.3: Detection probability of checkup policies with 0-3 perfect checkups (developed using 2009 SID patients) tested on 2010 SID patients (Method 1). This figure would replace Figure 10 if training and testing sets were switched