Supplementary Material to "A General Framework for Quantile Estimation with Incomplete Data"

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Table 1: Numerical results for missing-response setting when $\tau = 0.25$ based on $L = 10$ and 1000 replications. The names of the estimators have the form "method-0000", with each digit of the four-digit number, from left to right, indicating if $\pi^1(\boldsymbol{\alpha}^1)$, $\pi^2(\boldsymbol{\alpha}^2)$, $f^1(\boldsymbol{\gamma}^1)$ or $f^2(\boldsymbol{\gamma}^2)$ is used, respectively. The results have been multiplied by 100

	β_1 (intercept)			β_2 (for X_1)			β_3 (for X_2)			β_4 (for X_3)		
	Bias	RMSE	MAE	Bias	RMSE	MAE	Bias	RMSE	MAE	Bias	RMSE	MAE
						$n = 200$						
IPW-1000	1	35	23	0	37	26	0	21	14	-1	41	29
IPW-0100	-9	36	24	-17	41	26	7	21	14	1	40	27
IM-0010	0	29	20	1	32	21	1	16	10	-1	34	22
IM-0001	15	34	22	-8	31	20	-19	26	19	-21	39	27
MR-1000	2	35	23	-1	37	26	0	21	14	-1	41	28
MR-0100	-9	36	24	-17	41	26	7	21	14	1	40	27
MR-0010	-2	33	22	-2	36	23	1	19	12	0	39	26
MR-0001	-5	35	23	-1	36	23	4	21	13	1	40	28
MR-1100	2	35	24	0	37	25	0	21	13	-1	41	28
MR-1010	0	32	22	2	35	23	0	20	13	-1	38	26
MR-1001	2	34	22	0	36	24	0	21	13	-2	40	27
MR-0110	-2	33	22	-2	36	24	1	19	12	0	39	26
MR-0101	-5	35	23	-1	36	24	4	20	13	1	40	28
MR-0011	-2	33	22	0	36	23	1	19	12	0	39	26
MR-1110	0	33	22	2	35	23	0	20	13	-1	39	26
MR-1101	2	34	23	1	36	24	-1	21	13	-2	40	27
MR-1011	0	33	22	2	35	24	0	20	13	-1	39	26
MR-0111	-2	33	22	0	36	24	1	19	13	0	39	27
MR-1111	0	33	22	2	35	24	0	20	13	-1	39	26
						$n = 500$						
IPW-1000	-1	22	15	1	23	16	0	13	8	0	25	16
IPW-0100	-11	25	17	-17	28	20	6	14	10	4	25	17
IM-0010	-1	18	12	1	20	14	0	10	7	0	21	14
IM-0001	14	24	17	-7	20	14	-19	22	19	-19	29	22
MR-1000	0	22	15	1	22	16	0	13	8	0	25	16
MR-0100	-11	25	17	-17	28	20	6	14	10	4	25	17
MR-0010	-2	21	14	-2	23	16	1	12	8	0	24	16
MR-0001	-6	22	15	0	21	15	4	13	9	3	25	16
MR-1100	0	22	14	1	22	15	0	12	8	0	25	16
MR-1010	0	21	14	1	22	15	0	12	8	0	24	16
MR-1001	0	21	14	1	22	15	0	12	8	0	24	16
MR-0110	-2	21	14	-2	22	15	1	12	8	0	24	16
MR-0101	-6	23	15	0	21	15	4	13	9	3	25	16
MR-0011	-2	21	14	0	22	15	1	12	8	1	24	16
MR-1110	0	21	14	2	22	15	0	12	8	0	24	16
MR-1101	0	21	14	1	22	15	0	12	8	0	24	16
MR-1011	0	20	13	1	21	15	0	12	8	0	24	16
MR-0111	-2	21	14	0	22	15	1	12	8	1	24	16
MR-1111	0	21	13	1	22	15	0	12	8	0	24	16

RMSE: root mean square error. MAE: median absolute error. IPW: inverse probability weighting. IM: imputation. MR: multiply robust.

Table 2: Numerical results for missing-response setting when $\tau = 0.5$ based on $L = 10$ and 1000 replications. The names of the estimators have the form "method-0000", with each digit of the four-digit number, from left to right, indicating if $\pi^1(\boldsymbol{\alpha}^1)$, $\pi^2(\boldsymbol{\alpha}^2)$, $f^1(\boldsymbol{\gamma}^1)$ or $f^2(\boldsymbol{\gamma}^2)$ is used, respectively. The results have been multiplied by 100

	β_1 (intercept)			β_2 (for X_1)			β_3 (for X_2)			β_4 (for X_3)		
	Bias	RMSE	MAE	Bias	RMSE	MAE	Bias	RMSE	MAE	Bias	RMSE	MAE
						$n = 200$						
IPW-1000	1	35	23	-1	39	24	0	22	14	1	41	28
IPW-0100	-9	34	22	-24	41	28	7	20	13	3	37	24
IM-0010	0	27	18	0	29	19	0	16	10	0	31	21
IM-0001	44	53	43	-23	36	27	-27	33	28	-26	42	31
MR-1000	3	34	23	-2	36	23	0	21	14	1	40	27
MR-0100	-9	34	22	-23	41	28	7	20	13	3	37	24
MR-0010	-1	31	22	-4	34	22	1	19	12	1	36	24
MR-0001	-4	32	22	-7	32	21	2	20	13	3	38	26
MR-1100	3	33	22	-2	35	24	0	21	14	1	40	28
MR-1010	1	31	22	0	34	22	0	19	12	1	37	24
MR-1001	3	33	23	-2	34	22	0	20	14	1	39	26
MR-0110	-1	31	21	-4	33	22	1	19	12	0	36	24
MR-0101	-4	32	22	-6	32	21	2	20	13	3	38	26
MR-0011	0	32	22	-2	33	21	0	19	13	1	37	24
MR-1110	1	31	22	0	34	23	0	19	12	1	37	25
MR-1101	3	33	23	-1	34	22	0	20	14	1	39	27
MR-1011	1	32	23	0	34	22	0	19	13	1	37	24
MR-0111	0	31	22	-2	33	21	0	19	12	1	37	24
MR-1111	1	32	23	0	34	22	0	19	13	1	38	24
						$n = 500$						
IPW-1000	-1	21	14	1	24	15	0	12	9	0	24	16
IPW-0100	-10	22	16	-22	30	22	7	14	9	4	23	15
IM-0010	-1	17	12	0	18	12	0	10	7	0	20	13
IM-0001	42	46	42	-22	28	22	-27	29	27	-25	33	26
MR-1000	0	21	14	0	22	15	0	12	8	1	24	17
MR-0100	-10	22	16	-22	30	22	7	14	9	4	23	15
MR-0010	-2	20	14	-4	21	15	1	11	8	1	23	16
MR-0001	-5	20	14	-5	19	13	3	12	8	2	24	16
MR-1100	0	21	14	0	21	15	0	12	8	1	24	17
MR-1010	0	19	13	0	20	14	0	12	8	0	23	16
MR-1001	0	20	14	0	20	14	0	12	8	0	23	16
MR-0110	-2	19	13	-4	20	14	1	11	8	1	23	16
MR-0101	-6	20	15	-4	20	13	3	12	8	2	24	15
MR-0011	-1	19	13	-2	19	13	0	11	8	0	23	16
MR-1110	0	19	13	0	20	13	0	12	8	0	24	16
MR-1101	0	20	14	0	20	13	0	12	8	0	24	16
MR-1011	-1	19	13	1	20	13	0	12	8	0	23	16
MR-0111	-1	19	13	-2	19	13	0	11	8	0	23	16
MR-1111	-1	19	13	1	20	13	0	12	8	0	24	16

RMSE: root mean square error. MAE: median absolute error. IPW: inverse probability weighting. IM: imputation. MR: multiply robust.

Table 3: Numerical results for missing-response setting when $\tau = 0.75$ based on $L = 10$ and 1000 replications. The names of the estimators have the form "method-0000", with each digit of the four-digit number, from left to right, indicating if $\pi^1(\boldsymbol{\alpha}^1)$, $\pi^2(\boldsymbol{\alpha}^2)$, $f^1(\boldsymbol{\gamma}^1)$ or $f^2(\boldsymbol{\gamma}^2)$ is used, respectively. The results have been multiplied by 100

	β_1 (intercept)			β_2 (for X_1)			β_3 (for X_2)			β_4 (for X_3)		
	Bias	RMSE	MAE	Bias	RMSE	MAE	Bias	RMSE	MAE	Bias	RMSE	MAE
						$n = 200$						
IPW-1000	5	41	26	-7	45	30	2	26	17	0	47	33
IPW-0100	-7	36	24	-29	46	32	7	22	14	3	40	28
IM-0010	1	29	20	-2	31	20	0	16	11	0	32	21
IM-0001	96	103	95	-49	58	50	-45	49	46	-41	56	42
MR-1000	6	40	26	-7	43	30	3	24	16	0	47	32
MR-0100	-7	36	24	-29	46	32	7	22	14	3	40	28
MR-0010	2	38	26	-9	42	27	1	21	15	0	42	28
MR-0001	0	36	25	-14	40	27	1	22	15	-1	42	28
MR-1100	6	40	26	-7	43	29	3	24	17	-1	47	31
MR-1010	2	38	25	-3	42	28	1	22	15	0	44	28
MR-1001	5	39	26	-5	42	27	2	24	15	-2	45	30
MR-0110	1	37	26	-8	42	27	2	21	15	0	41	27
MR-0101	0	37	25	-13	40	27	1	22	15	-1	42	29
MR-0011	2	37	25	-7	40	26	0	22	15	-1	42	28
MR-1110	2	38	25	-3	42	28	1	23	15	0	44	29
MR-1101	5	40	26	-5	42	27	2	24	15	-2	45	30
MR-1011	3	38	25	-4	41	27	1	23	16	-1	44	29
MR-0111	2	37	26	-6	41	27	0	22	15	0	43	29
MR-1111	3	38	26	-3	41	27	1	23	16	-1	44	29
						$n = 500$						
IPW-1000	1	26	17	-3	29	18	1	15	10	0	30	20
IPW-0100	-9	23	16	-28	35	28	7	15	10	4	25	18
IM-0010	0	18	12	-1	18	13	0	10	7	0	21	15
IM-0001	95	98	95	-47	51	48	-46	47	45	-41	47	41
MR-1000	2	25	17	-3	27	17	1	15	10	0	29	20
MR-0100	-9	23	16	-28	35	28	7	15	10	4	25	18
MR-0010	-1	23	15	-8	26	17	2	13	9	1	27	18
MR-0001	-3	23	16	-12	24	17	1	14	9	1	27	18
MR-1100	2	25	16	-4	26	17	1	15	10	0	29	20
MR-1010	1	24	16	-2	25	16	1	14	9	0	28	19
MR-1001	1	24	17	-3	25	16	1	14	10	1	28	18
MR-0110	-1	23	14	-7	25	17	2	13	9	1	27	17
MR-0101	-3	23	16	-11	25	16	1	14	9	1	27	18
MR-0011	0	23	15	-5	23	15	1	14	9	1	28	18
MR-1110	1	24	16	-2	25	16	1	14	9	0	28	19
MR-1101	1	24	17	-3	25	17	1	14	9	1	28	19
MR-1011	0	23	16	-2	25	16	1	14	9	0	28	19
MR-0111	-1	23	15	-5	24	15	1	14	9	1	27	18
MR-1111	0	23	16	-2	25	16	1	14	9	0	28	19

RMSE: root mean square error. MAE: median absolute error. IPW: inverse probability weighting. IM: imputation. MR: multiply robust.

Table 4: Numerical results for missing-covariate setting based on $n = 500$, $L = 10$ and 1000 replications. The names of the estimators have the form "method-0000", with each digit of the four-digit number, from left to right, indicating if $\pi^1(\boldsymbol{\alpha}^1)$, $\pi^2(\boldsymbol{\alpha}^2)$, $f^1(\boldsymbol{\gamma}^1)$ or $f^2(\boldsymbol{\gamma}^2)$ is used, respectively. The results have been multiplied by 100

	β_1 (intercept)			β_2 (for X_1)			β_3 (for X_2)		
	Bias	RMSE	MAE	Bias	RMSE	MAE	Bias	RMSE	MAE
					$\tau = 0.25$				
IPW-1000	-1	49	33	0	14	10	1	41	27
IPW-0100	287	332	283	-136	143	135	-8	161	75
IM-0010	1	25	16	-1	13	9	0	17	12
IM-0001	-19	35	23	1	13	9	-59	63	60
MR-1000	0	46	30	-1	14	10	0	40	25
MR-0100	186	202	184	-68	70	67	-10	51	34
MR-0010	4	35	23	-1	13	9	2	42	27
MR-0001	6	40	26	0	14	10	-6	39	26
MR-1100	1	46	30	-1	15	10	0	39	25
MR-1010	1	40	25	-1	13	9	2	39	23
MR-1001	0	40	26	-1	13	9	3	40	26
MR-0110	-4	38	24	-1	13	9	-1	48	28
MR-0101	-6	40	26	-2	14	10	-5	48	30
MR-0011	2	36	24	-1	13	9	-2	40	26
MR-1110	1	39	25	-1	13	9	2	39	24
MR-1101	0	40	26	-1	13	9	2	40	25
MR-1011	1	40	26	-1	13	9	3	40	24
MR-0111	-4	39	25	-1	13	9	-4	46	28
MR-1111	1	40	25	-1	13	9	2	40	24
					$\tau = 0.75$				
IPW-1000	-1	39	27	-1	14	9	3	30	20
IPW-0100	736	981	454	-183	217	141	14	211	65
IM-0010	2	26	18	-1	13	9	0	19	13
IM-0001	15	34	23	-3	14	9	-45	50	45
MR-1000	-1	38	26	-1	14	9	2	30	20
MR-0100	119	136	114	-40	43	39	0	38	26
MR-0010	5	31	21	-1	13	9	-5	32	23
MR-0001	11	36	24	-2	14	9	-20	36	25
MR-1100	-1	39	26	-1	14	10	2	30	20
MR-1010	1	35	25	-1	13	9	3	29	18
MR-1001	0	35	24	-1	13	9	3	30	19
MR-0110	2	32	22	-1	13	9	5	33	22
MR-0101	4	36	23	-2	14	9	-5	31	22
MR-0011	7	32	22	-2	13	9	-10	31	21
MR-1110	1	35	25	-1	13	9	3	29	18
MR-1101	0	35	24	-1	13	9	3	29	19
MR-1011	1	35	24	-1	13	9	3	29	19
MR-0111	3	33	22	-1	13	9	-1	31	21
MR-1111	1	35	24	-1	13	9	3	29	19

RMSE: root mean square error. MAE: median absolute error. IPW: inverse probability weighting. IM: imputation. MR: multiply robust.