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A Latent Variable Approach to Potential Outcomes for Emergency Department Admission Decisions

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Summary

In emergency departments (EDs), care providers continuously weigh admissions against continued monitoring and treatment often without knowing their condition and health needs. To understand the decision process and its causal effect on outcomes, an observational study must contend with unobserved/missing information and a lack of exchangeability between admitted and discharged patients. Our goal was to provide a general framework to evaluate admission decisions from electronic healthcare records (EHR). We describe admission decisions as a decision-making process in which the patient's health needs is a binary latent variable. We estimate latent health needs from EHR with only partial knowledge of the decision process (i.e., initial evaluation, admission decision, length of stay). Estimated latent health needs are then used to understand the admission decision and the decision's causal impact on outcomes. For the latter, we assume potential outcomes are stochastically independent from the admission decision conditional on latent health needs. As a case study, we apply our approach to over 150,000 patient encounters with the ED from the University of Michigan Health System collected from August 2012 through July 2015. We estimate that while admitting a patient with higher latent needs reduces the 30-day risk of revisiting the ED or later being admitted through the ED by over 79%, admitting a patient with lower latent needs actually increases these 30-day risks by 3.0% and 7.6%, respectively.

KEYWORDS:
causal inference, potential outcomes, latent variables, emergency department admission decisions

Introduction

In the United States, hospitalizations account for one-third of healthcare expenditure with over half of admissions originating from the emergency department (ED)[1]. A growing portion of admission decisions to inpatient hospital units are being made in the ED[2]. By ordering more tests or monitoring patients longer, an ED care provider can delay their admission decision to better inform their final decision, but this can delay treatment to other time-critical patients and lead to long waits in already-overcrowded EDs. Alternatively, patients are being increasingly sent to medical short-stay or observation units which allow for extended evaluation for up to 24 to 48 hours to better determine if the patient should be sent home or admitted[3,4]. However, whether delaying or increasing admission decisions improve outcomes, efficiency, or costs is an open-ended and hotly-debated topic with important policy implications[5,6]. Thus, ED care providers must quickly discriminate between who to send home,

continue to treat/monitor, send to a short stay unit, or admit to the hospital, balancing patient outcomes with costs and timely access to care for all patients.

The admission decision process in EDs begins when a patient arrives and proceeds to triage, where they are usually assigned an acuity level based on severity of illness. Acuity is commonly assigned using the emergency severity index (ESI), a five-level triage algorithm designed to facilitate the sorting and streaming of patients. Higher acuity patients (1 or 2) are almost immediately brought to a bed for treatment. Lower acuity patients (3, 4, and 5) wait for treatment until they are brought to a bed. Once in a room/bed, they are visited by providers to determine a plan of care involving a series of examinations, diagnostic testing such as imaging (e.g. radiographs, ultrasound CT scans, MRI), laboratory work, and treatment. After testing and treatment, the patient is either well enough to be discharged home or is admitted to the hospital. This seemingly simple decision-making scenario turns out to be surprisingly complex, with important health and policy implications.

Because admission is one of the most expensive routine decisions made in health care for a patient[7], it is important to determine how the admission decision *causally* impacts patient outcomes such as ED revisits and hospital readmissions. Patients or clinics would ideally be randomized to various treatment groups (e.g. discharge home, admit), but randomized clinical trials are difficult, if not unethical, to conduct for this question. Alternatively, electronic health records (EHR) could be used to leverage large amounts of transactional data on daily ED operations, but then finding an association between admission decisions and an outcome may be insufficient to conclude that the decision caused the outcome. The issue is that the decision to admit a patient or not cannot be reasonably assumed to be independent from their potential outcomes, where potential outcomes are those outcomes that would have been observed if each person could be assigned to all treatment groups. Notably, the severity of a patient's condition is expected to strongly influence both admission decisions and outcomes.

Causal inference methods such as propensity score matching, inverse probability weighting, standardization, g-estimation, and instrument variables (IVs) attempt to overcome this issue. To date, most empirical studies on transfer decisions between hospital units, such as admitting a patient to an inpatient hospital unit, use an IV approach. This approach relies on a variable called an instrument that is correlated with the treatment, but not with the outcomes except perhaps through its association with treatment. Kim et al.[8] examine how congestion impacts ICU admission decisions and patient outcomes. They use congestion as an IV to identify the causal effect of ICU admission decisions on patient outcomes. Chan et al.[9] considers admission to ICU versus step down units (SDUs), an intermediate level of care for semi-critically ill patients who are not sick enough to require intensive care but not stable enough to be treated on a general ward. They use IV approaches to estimate the impact on patient outcomes of routing patients to the SDU from the ED as well as the ICU. Kim et al.[10] study whether ICU occupancy influences ICU admission decisions and patient outcomes in a retrospective study also using IVs. Bartels et al.[11] use IVs to address the potential bias in hospital length-of-stay. Causal inference studies on the ED are more rare. One example is Kuntz et al.[12]. The authors also use IVs to support replacing general hospitals with what they term as "value-adding process clinics" and "solution shop hospitals" for less and more complicated patients, respectively. In these applications, researchers justify their use and choice of IVs as a way to overcome potential bias when *unobserved patient severity/needs* affects both transfer decisions and patient outcomes.

The challenge with IVs is in identifying an appropriate instrument[13,14,15]. When there is a strong backdoor factor (also referred to as endogenous variable or confounder), such as we expect patient severity is for ED admissions and outcomes, the instrument should also be strongly correlated with treatment assignments. This can be extremely challenging. Moreover, an IV approach does not model the backdoor factor, unobserved patient severity, or the process upon which a physician accrues information while treating the patient until an admission decision is finally made. Thus, this approach may not offer insight into the actual decision-making process.

Other methods to making causal inferences try to mimic a randomized trial by compensating for the bias introduced by backdoor factors. These methods include propensity score matching, inverse probability weighting, standardization, and g-estimation; see e.g.,[16,17,18,19] and[20,21,22,23] for comprehensive treatments on these methods. These methods must satisfy certain assumptions: positivity, consistency, and exchangeability, of which exchangeability is arguably the hardest to satisfy. Full exchangeability says that treatment assignments are stochastically independent from potential outcomes. Often too strict, full exchangeability is often replaced with conditional exchangeability which requires treatment assignments are stochastically independent from potential outcomes within strata (e.g., strata defined by sex and age). Additionally, these methods require a conceptual understanding of what factors impact treatment assignments, and this conceptual understanding needs to be formalized mathematically. These methods can fail when an unobserved or latent variable introduces hidden bias that is not taken into account in the model[21]. Latent variable models within causal inference frameworks have been studied in the literature; see e.g.,[24,25,26,27,28,29,30,31,32]. In[27], for instance, the authors consider how to *causally* evaluate a class of probabilistic diagrams when

there are unmeasured, or latent, variables. In [32], the authors use latent variable models to estimate latent confounders (i.e., factors that affect both an intervention and its outcome) and individual-level causal effects from observational data. Applications have been studied in [29,31]. In [29], for instance, the authors study weight loss intervention programs. The authors model unintentional weight loss as a latent variable and use potential outcomes to estimate associations between weight loss and mortality. Motivated by angiography in myocardial infarction, the authors in [31] propose a causal inference method for treatment in a two-arm experimental study with noncompliance in treatment and control arms. Their model includes individual patient covariates and latent variables for unobserved heterogeneity between subjects.

This prior work provides useful models for embedding latent variables in causal inference frameworks. Here, we aim to posit and model a latent variable to capture a care provider's uncertainty related to patient's health needs. Our paper is based on the hypothesis that explicitly modeling a patient's unknown, or *latent*, health needs can help evaluate admission decisions and estimate causal effects of admissions on outcomes. We introduce a model that describes the admission decision as a decision-making process with health needs as a latent variable. We make the assumption that latent health needs provides a natural conditional exchangeability assumption: *admission decisions are stochastically independent from potential outcomes within a group of individuals with similar latent health needs*, and integrate this assumption into the potential outcomes framework to estimate causal effects from observational data. We use this approach to examine data on over 150,000 patient encounters in the ED and inpatient units from the University of Michigan Health System (UMHS) collected from August 2012 through July 2015.

We view our work as making the following contributions:

1. A continuous-time decision making model of the admission decision process in the ED in which health needs are explicitly captured as a latent variable; and

2. A method to estimate causal effects using the latent variable to overcome a lack of exchangeability.

In the process, we impose minimal conditions on EHR data: only an initial observation, final decisions, length of stay, and demographic information are needed, so the method should be applicable to most ED datasets. The rest of the paper is organized as follows: Section 2 describes the modeling framework; Section 3 describes parameter specification and estimation; Section 4 studies estimation via simulation. Section 5 contains our case study and a sensitivity analysis. Section 6 contains a discussion of these results and our conclusions.

Admission Decision Process

Decision-Making Model

We present a model of the decision process for a single patient from the perspective of a care provider (e.g. physician) tasked with making an admission decision. We let random variable $X \in \mathbb{R}^k$ denote patient characteristics known prior to the start of treatment such as age and sex. Let $H \in \{0, 1\}$ denote the *hidden* or *latent* health state. The health state denotes the (unknown) patient's needs for hospital resources (e.g., level of treatment). For this study, we assume H is a binary random variable with $H = 1(0)$ representing higher (lower) needs. We remark that the results presented here also hold in the case when H can take on a finite number of ordered values.

We let $Z \in \{0, 1\}$ be a random variable denoting an initial (noisy) *observation*, or belief, of health needs taken prior to the start of treatment or testing by the medical provider, where $Z = 1(0)$ represents an initial belief of higher (lower) health needs. In practice, the observation Z is usually in the form of ESI or acuity level. Lastly, we define random variables $A \in \{0, 1\}$ to represent the realized admission decision, i.e., admit (1) or discharge (0), and $T \in (0, \infty)$ to represent the time from when treatment begins until the admission decision is made, i.e. *treatment time*. We assume that the random vector (X, H, Z, A, T, Y) are independent and identically distributed, where Y will denote outcomes which we define later.

Putting it all together, immediately after an initial (noisy) assessment, Z, the patient undergoes treatment/testing at time $t = 0$ that yield new (noisy) observations of the patient's health state. The physician views observations collected so far until enough information is collected after a random amount of time T to decide among two mutually-exclusive decisions: discharge patient home; or admit patient to an inpatient hospital unit, with $A = 0, 1$ denoting, respectively, the decision to discharge or admit. Once the physician decides to admit or discharge a patient, a patient no longer undergoes treatment/testing and waits to be sent home or admitted to the hospital.

FIGURE 1 Bayesian network of admission decision model. Patient health needs H influences an initial observation Z. Care providers use this observation and other observations collected so far from treatment and testing until time T, when they make a final decision A on whether to admit or discharge. Dashed circle represents our latent variable, whereas rectangles represent observed variables.

FIGURE 2 Structural model of treatment time T and admission decision A is constructed from, respectively, the first-passage time and exit location of Brownian motion B_t. To capture the dependence on patient characteristics X, latent health needs H, and initial observation Z, we assume Brownian motion starts at a point $c(X,Z)b(X)$ and drifts at a speed of $d(H)b(X)$ until reaching the boundary 0 (discharge) or $b(X)$ (admit).

Our model of the admission decision process yields the Bayesian network depicted in Figure 1 , for which nodes are sets of variables and edges link one node to the other whenever variables in the latter node are conditionally dependent on variables in the former node. For example, we see from the network that the admission decision A depends on the initial observation Z, health state H, and patient characteristics X.

Structural model for admission decision & treatment time

We introduce a joint structural model for final decision A and treatment time T to reflect the decision-making process of a care provider. It is a drift-diffusion model that captures similar decision-making scenarios whereby one collects information continuously over time until eventually deciding between two mutually exclusive options after a variable amount of time; see e.g.,[33] for an application in neuroscience of drift-diffusion models to decision-making. Such an approach is also used in threshold regression to jointly model a continuous time-to-event and an event's outcome[34]. Specifically, we introduce parameters $b(X)$, $c(X,Z)$, $d(H)$ and construct a joint model of admission decision A and treatment time T (conditional on X,H and Z) from first-passage locations and times of Brownian motion B_t (Figure 2). Assuming a drift of $d(H)b(X)$ and infinitesimal variance of σ^2, then Brownian motion B_t is a continuous-time stochastic process with stationary and independent increments $B_{t+s} - B_t$ which are normally-distributed with mean $d(H)b(X)s$ and variance $\sigma^2 s$. We model treatment time T as the first-passage time of B_t out of an open interval $(0, b(X))$ starting at some point $B_0 := c(X,Z)b(X)$ with $c(X,Z) \in (0,1)$:

$$T := \inf\{t > 0 : B_t \notin (0, b(X))\},$$

The admission decision A is then captured by whether B_t exits through 0 or through $b(X)$:

$$A := \begin{cases} 0 & B_T = 0 \\ 1 & B_T = b(X). \end{cases}$$

Note the value of B_t does not represent a physical quantity. We can thus scale B_t, σ, and $b(X)$ without changing the distribution of (A,T) given X, H, Z. Hence, it is without loss of generality that we assume $\sigma^2 = 1$ with units 1/time.

Based on the assumptions above, we arrive at a structural model for the joint density function of $(A,T) = (a,t)$:

$$\mathbb{P}(T \in [t, t+dt], A = a | X, H, Z) := g(a, t | b(X), c(X,Z), d(H))\, dt$$

where the differential dt here is informally used to denote an infinitely small (or infinitesimal) change in the value t and $g(a,t|b,c,d)$ denotes the joint density of $(A,T) = (a,t)$ for a given initial point bc, drift rate db, and upper boundary b. Note that to place these parameter functions into the context of the admission decision-making scenario, we assumed that c is a function of patient characteristics X and initial observation Z to reflect that a medical care provider only has Z and X to initially evaluate the patient. We assumed the drift term d is a function of health needs H to reflect that information collected by the care provider in the ED will be determined largely by their health needs H. Lastly, we assumed the boundary b depends on X to reflect that the level of evidence required to make a decision depends on patient characteristics.

Remark 2.1. The function g has two equivalent expressions, both in the form of infinite series (see Appendix B). Following[35], g can be approximated by the truncated version of one series for small t and the truncated version of the other series otherwise. In[35], guidelines are provided for when to use which approximation.

Potential outcomes with latent health needs

We now consider an *outcome* of interest, represented by a random variable $Y \in \mathbb{R}$. The outcome random variable Y may or may not depend on other variables in our admission decision process. Common outcomes of interest in the ED include ED revisits (i.e., return to the ED within, say, 30 days) and hospital admission within 30, 60 or 90 days after the patient is discharged. These two outcomes are often used to measure the quality of hospital care, since it could mean patient follow-up care was not properly organized, or that the patient was not adequately treated before discharge[36,37].

We want to infer the causal impact of *admitting a patient* on the outcome of interest. For causal inference, it is common to consider random *potential* outcomes $Y^a \in \mathbb{R}$ for $a = 0, 1$, where Y^a represents the outcome that would have been observed *if* we always admitted our model patient ($a = 1$) or *if* we always discharged our model patient ($a = 0$). Hence, we assume actual outcomes Y are either Y^0 if the patient is discharged ($A = 0$) or Y^1 if the patient is admitted ($A = 1$), leaving one of the potential outcomes missing.

We want to use actual outcomes Y to provide information on the distribution of potential outcomes Y^a. If admission decisions were randomized, then potential outcomes Y^a would be independent from the admission decision A, and Y^a would have the same distribution as $(Y^a|A = a) = (Y|A = a)$. Put differently, with randomized admission decisions, potential outcomes for admitted patients are *exchangeable* with potential outcomes for discharged patients. In other applications, potential outcomes may reasonably be assumed to be independent from treatment assignments, i.e. *exchangeability* can be a reasonable assumption, even when treatment assignments are not randomized. However, for EDs and other hospital transfer decisions, it should be clear that these decisions cannot be reasonably assumed to be independent from potential outcomes. Admitted patients are necessarily in a poorer health state than discharged patients, and whether a patient is readmitted to an inpatient unit or revisits the ED is likely to be related to a patient's health.

Given this conceptual understanding of the ED admission process, it is then natural to try to account for the impact that severity of patient health has on both admission decisions and outcomes. We address this by exploiting the latent variable H to control for confounding between actual outcomes and admission decisions. Specifically, we make the following assumption

Assumption 2.2 (Conditional Exchangeability)**.** Potential outcomes Y^a ($a = 0, 1$) are independent from the admission decision A conditional on the health state H and patient characteristics X.

Remark 2.3. Conditional exchangeability is a common assumption for causal inference approaches known as standardization and inverse probability weighting (IPW). These approaches assume potential outcomes Y^a are independent from the admission decision A conditional on a subset of observed variables U and then use the fact that $Y^a|U$ has the same distribution as $Y|U, A = a$ to estimate $\mathbb{E}\, Y^a$ for $a = 0, 1$.

Discussion on the decision-making model

Model choice is important for causal inference approaches because estimates generally depend on a model that specifies how variables influence treatment assignments and outcomes. The model should be chosen carefully to recover accurate estimates from causal inference approaches. We thus wanted a model to best reflect the actual admission decision process.

We highlight three important features of the decision-making model. First, patient health needs are latent or hidden (cf., [38]). In the ED, physician's do not always arrive at complete diagnoses for patient symptoms (e.g., chest pain, abdominal pain, syncope, headache, shortness of breath) at the end of the visit. As a result there is variability in how physicians determine the need for hospitalization. Second, patient health needs are elucidated through a series of noisy (i.e. imperfect) observations through diagnostic testing and/or treatment. The noisy assumptions are because a physician's decision to admit a patient involves many unobserved, complex factors. Moreover, diagnostic testing and response to treatment may be ambiguous. Third, decisions to transfer or to continue treatment and testing occur at continuous and variable times throughout a patient's stay in a hospital unit.

To capture these features, we used a particular version of threshold regression[34] based on hitting time of a drift-diffusion process on one of two boundaries. This regression model describes processes of evidence accumulation, such as how humans discriminate between two choices over time[39]. Unlike traditional regression models, it captures many important features: continuous decision times; initial bias in the admission decision process; rate at which information is collected; and the threshold level of information at which point a final decision is made. The underlying assumption is that the individual, in this case, the physician, extracts, per time unit, a constant piece of evidence from the stimulus (drift) which is disturbed by noise (diffusion).

This accumulation stops once enough evidence has been sampled and a decision is made. These features are consistent with the admission decision model.

Another consideration is that our model of the decision-making process is not a standard survival model. Despite being non-standard, threshold regression has been used to model survival processes for many applications, including hospital length of stay and latent health status[40,41,42,43]. In fact, the more commonly-used inverse Gaussian distribution is also a threshold regression model. See Lee and Whitmore for an overview and examples[43]. Moreover, our model also simplified analysis in three main regards. First, we could use one joint model for both the admission decision and treatment time. By contrast, a more conventional survival analysis model (e.g., Weibull regression model) would require two regression models, one model for the admission decision and another for treatment time conditional on the admission decision or one model for the treatment time and another for admission decision conditional on the treatment time. Second, if we were to use two regression models as discussed above, we would require 2 additional parameters to capture main effects (characteristics, acuity, and health state in both models and treatment time or admission decision in the second model) compared to our model.

Third, there was a simple way to capture the dependence of the admission process (A,T) on the initial observation Z and health state Z, because our model was a more realistic model of the admission process than other conventional survival models. The initial observation Z influences the initial level of evidence, and the health state H influences the rate of evidence accumulation. These assumptions can automatically capture treatment times that are longer on average when there was a mismatch between acuity and admission decision, which we observe in our dataset. By contrast, capturing this effect using the two-model approach discussed above would require interaction terms in our regression model, leading to even more coefficients to estimate.

Parameter specification and estimation

To fit the admission decision model to EHR data, we assume that we have data on N visits and that for visit n ($n = 1, 2, 3, \ldots, N$) the following variables are collected: a vector of patient characteristics $x_n \in \mathbb{R}^k$; an initial assessment $z_n \in \{0, 1\}$; admission decision $a_n \in \{0, 1\}$; treatment time $t_n \in (0, \infty)$; and a binary outcome $y_n \in \{0, 1\}$. Although we focus on binary outcomes, results can be extended to continuous outcomes. We assume that the admission decision process is independent and identically-distributed for each visit.

Along with parameter functions $b(X), c(X, Z), d(H)$, we specify the admission decision process with parameter functions:

$$\alpha(X) := \mathbb{P}(H = 1|X) \qquad \text{and} \qquad \beta(X, H) := \mathbb{P}(Z = 1|X, H).$$

We further restrict attention to when

$$\operatorname{logit} \alpha(X),\ \operatorname{logit} \beta(X, H),\ \log |d|(H),\ \operatorname{logit} c(X, Z),\ \log b(X)$$

are linear in their arguments and estimate the set of linear coefficients of these functions. For k-dimensional X, this restriction leads to $8 + 4k$ unknown parameters. The sign of the drift rate $d(H)$ is assumed to be negative for lower health needs ($H = 0$) and positive for higher health needs ($H = 1$) to capture propensity for lower health needs patients to be discharged and higher health needs patients to be admitted. For outcomes, we assume potential outcomes depend linearly on latent health needs:

$$\begin{aligned}\mathbb{P}(Y^a = 1|X, Z, T, H) &= \mathbb{P}(Y = 1|X, Z, T, A = a, H) \\ &:= \mu_1 + (\mu_2 - \mu_1)H + (\mu_3 - \mu_1)a + (\mu_4 - \mu_2 - \mu_3 + \mu_1)Ha\end{aligned}$$

so that

$$\mathbb{E}\left[Y^1 - Y^0|H = 0\right] = \mu_3 - \mu_1 \qquad \text{and} \qquad \mathbb{E}\left[Y^1 - Y^0|H = 1\right] = \mu_4 - \mu_2,$$

where we employ the Conditional Exchangeablility Assumption 2.2 to relate potential outcomes to actual outcomes. Potential outcomes lead to an additional 4 unknown parameters, leading to $12 + 4k$ unknown parameters in total. Note that the latent variable H represents a confounder in the sense that it partitions the population into strata such that, within each stratum, exchangeablity of the $A = 0$ and $A = 1$ populations holds, whereas exchangeability might not hold marginally over H. The latent variable H, however, may also serve as a modifier of the treatment effect of A. Effect modification occurs when the interaction term $(\mu_4 - \mu_2 - \mu_3 + \mu_1)$ is nonzero. Further, we chose to omit variables X, Z, T in the above expression for potential outcomes to reduce the number of parameters and to reflect that patient characteristics, acuity, and treatment time might not have a direct effect on potential outcomes when controlling for patient health needs. However, our framework is sufficiently

flexible that variables X, Z, T may be included if desired, and confidence intervals of any resulting coefficients may be checked to see if they contain 0 in order to justify removing variables.

Let θ be the vector of unknown parameters and $\Theta := \mathbb{R}^{12+4k}$ be the set of possible parameter vectors. We estimate model parameters by performing maximum likelihood estimation (MLE) using the expectation-maximization (EM) algorithm[44]. For the EM algorithm, we need to specify the complete data likelihood of observing $(H, Z, A, T, Y) = (h, z, a, t, y)$ given parameters θ and patient characteristics $X = x$. Based on our Bayesian network, the complete log-likelihood is expressed as

$$\begin{aligned}(1-a)\left(y\log\left[\mu_1(1-h)+\mu_2 h\right]+(1-y)\log\left[(1-\mu_1)(1-h)+(1-\mu_2)h\right]\right)+\\ a\left(y\log\left[\mu_3(1-h)+\mu_4 h\right]+(1-y)\log\left[(1-\mu_3)(1-h)+(1-\mu_4)h\right]\right)+\\ \log g\left(a,t|b(x),c(x,z),d(h)\right)+z\log\beta(x,h)+(1-z)\log\left[1-\beta(x,h)\right]+\ldots\\ h\log\alpha(x)+(1-h)\log\left[1-\alpha(x)\right],\end{aligned}$$

where we suppressed the dependence on θ in our parameter functions.

Confidence intervals are estimated using a numerical approximation to Oakes Identity[45] used in EM methods to estimate Fisher's information matrix, which when inverted, yields estimates of sampling variances for each parameter. These variances were then used to construct 95% confidence intervals assuming parameter estimates are normally-distributed. Confidence intervals for functions of parameter estimates were estimated using the delta method. For completeness, we consider an alternative estimation approach in the Appendix that first estimates parameters for the admission decision process and then estimate parameters for outcomes. This alternative approach was considered to mirror our dual goals of discovering how a decision process could be explained by latent health needs and then determining how outcomes are influenced by this latent variable; and because estimating latent variables in other settings separately from effects of latent states had also been suggested in[46] and shown numerically in[47] to be more robust to model violations. This alternative approach also does not require distribution assumptions on outcomes.

Simulation

We consider two simulation examples to assess parameter estimation and robustness of our estimates to violations in the Conditional Exchangeability Assumption 2.2. We compared mean difference in potential outcomes $\mathbb{E}\,Y^1 - \mathbb{E}\,Y^0$ for our method to three other methods. Inverse probability weighting was performed by building a logistic regression model to predict admission decisions from patient characteristics X, initial observation Z, and treatment time T. We then estimated mean potential outcomes $\mathbb{E}\,Y^a$ by taking a weighted average of outcomes Y with admission decision $A = a$ where weights are given by one over the probability of admission decision $A = a$ predicted from the logistic regression model. The method of g-estimation was performed by building a logistic regression model to predict admission decision from X, Z, T, and a variable $J := Y - \psi A$ for some parameter ψ. We varied ψ until the fitted regression model had a zero coefficient for J. In order to assess the value of using a latent-variable to adjust for confounding, the last method we compared estimated mean potential outcomes from our model when the dependence on the latent variable H is removed. Specifically, we dropped the dependence on H in the initial observation Z, admission decision process (A, T), and Assumption 2.2. We then adjusted the log drift term $\log|d|(H)$ to be linear in the initial observation Z rather than H and adjusted to the logit of potential outcomes logit $\mathbb{P}(\mathrm{Y^a}|\mathrm{X, Z, T})$ to be linear in patient characteristics X and initial observation Z rather than H. Throughout this section, we assume patient characteristics X is a two-dimensional random vector (X_1, X_2) with X_1 describing a binary characteristic such as gender and X_2 describing a numerical variable such as age.

Example 1: Correct model

We simulated N ED visits by sampling directly from the model of admission decisions and outcomes for a particular choice of 20 unknown parameters (Table E1). We considered both N=1,000 and N=10,000. For comparison, our UMHS dataset has over 150,000 visits. Parameters were estimated along with their confidence intervals and the process repeated for a total of 500 replicates. To arrive at reasonable parameters, we used our UMHS dataset with the admission decisions as a proxy for health needs (see Table E3 for description of UMHS dataset and see Appendix D for justification of parameter choices).

For this model, we checked bias, mean squared error (MSE), and coverage for parameter estimates. Results are summarized in Table E1 . When N=1,000, certain parameters are not estimated accurately. For example, the linear coefficient for the term

$H = 1$ in logit $\beta(X, H)$ is estimated poorly in terms of bias (1.6) and MSE (44.4). Confidence intervals, however, capture this uncertainty with 94% coverage of the true parameter. We also understate estimated outcomes for higher needs patients who are discharged (μ_2) by about 12%. In this case, confidence intervals are associated with only 90% coverage. All estimates, however, improve significantly in terms of bias, MSE, and coverage when N increases to 10,000 data points. Based on this, we hypothesize that estimates will improve when we increase the number of data points to 150,000.

Out of all of the parameters, μ_2 was the most difficult to estimate. MSE decreased from 0.0801 for 1,000 data points to 0.0130 for 10,000 data points, and we expect even more accurate estimates for μ_2 when the number of data points is increased to N=150,000. This parameter corresponds to higher-needs patients that are discharged (i.e., $H = 1$ and $A = 0$). It was estimated from discharged patients, but, by design, the majority of these patients have lower health needs ($H = 0$). By our choice in parameters, this group tended to be the smallest group (out of the four groups divided on health needs and admission decision) in simulated data samples (≈5%), which may explain why it is the most difficult group for which to estimate outcomes.

To connect to other causal inference methods, we also estimated mean difference in potential outcomes:

$$\mathbb{E}\left[Y^1 - Y^0\right]$$

and log odds ratio of potential outcomes

$$\log \frac{\mathbb{E}\, Y^1\, \mathbb{E}\left[1 - Y^0\right]}{\mathbb{E}\, Y^0\, \mathbb{E}\left[1 - Y^1\right]}.$$

Mean difference in potential outcomes has an actual value of −0.12 for this example. When N=1,000, our estimates for this quantity had a bias of 0.019, MSE of 0.014, and 86% coverage. When N=10,000, estimates for this quantity improve leading to a bias of 0.0080, MSE of 0.0017, and 90% coverage. Log odds ratio of potential outcomes has an actual value of −0.63. When N =1,000, our estimates for this log odds ratio had a bias of 0.067, MSE of 0.46, and 86% coverage. When N =10,000, our estimates perform better with a bias of 0.041, MSE of 0.049, and 92% coverage.

By comparison, the other methods (inverse probability weighting, g-estimation, and fitting our model without the latent variable) led to poor estimates of mean difference in potential outcomes which do not improve with sample size. For instance when N=1,000, inverse probability weighting led to an average estimate of mean difference in potential outcomes of 0.053 far from the true value of -0.12. When N=1,000, inverse probability weighting led to a bias in mean difference in potential outcomes 0.173 and MSE of 0.031, whereas g-estimation led to a bias of of 0.166 and MSE of 0.028 and fitting our model without the latent variable led to a bias of 0.176 and MSE of 0.032. When N=10,000, inverse probability weighting led to a bias in mean difference in potential outcomes of 0.168 and MSE of 0.028, whereas g-estimation led to a bias of 0.161 and MSE of 0.026 and fitting our model without the latent variable led to a bias of 0.176 and MSE of 0.031. Critically, all three methods incorrectly suggest admitting a patient carries a higher risk than discharging a patient. These methods are effectively missing the large risk associated with discharging individuals with higher health needs.

Example 2: Violation in Conditional Exchangeability Assumption

We also checked the robustness of our model to violations in our Conditional Exchangability Assumption 2.2. Keeping the rest of the model and parameters as in Example 1, we introduced an independent Bernoulli random variable U with "success" probability 0.05 that influenced the admission decision and the outcome. If $U = 1$ and $A = 1$, so that the patient would be admitted, we then re-defined A to be zero. For these same patients, who also had higher health needs, we set Y to be one. This models the situation when an admitted patient goes home against medical advice and then comes back to the ED if they originally had higher health needs. Results are summarized in Table E2 .

Compared to Example 1, we find that many of the estimates related to latent health needs are more greatly biased. For example, we underestimate the proportion $\alpha(X)$ of higher needs patients. It is important to note that our latent variable is simply a construct to help us understand how care providers in the ED make admission decisions and cannot be measured. Relations between the latent variable and the (measured) variables are specified in a mathematical model to help us explain the statistical properties of the measured variables in terms of the hypothesized latent variable. The implication for Example 2 is that, when multiple endogenous variables share similar relationships between variables (e.g., admission decision and outcomes) then the latent-variable tries to reflect both endogenous variables. Thus, we suspect that this bias reflects that our latent variable captures both the health state H and the second endogenous variable U.

Although our estimates for the latent variable change, we found that we can achieve similar performance to Example 1 for our estimates of mean difference in potential outcomes and log odds ratio of potential outcomes in which the effect of the latent

variable is averaged out. We assume true values are the mean difference in potential outcomes and log odds ratio of potential outcomes *conditional* on $U = 0$, i.e. ignoring the effect of U which is not captured by our model. Our estimate for mean difference in potential outcomes has a bias of −0.0154, MSE of 0.0008, and 92% coverage. Our estimate for log odds ratio of potential outcomes had a bias of −0.0675, MSE of 0.024, and 92% coverage.

As in Example 1, the three other methods led to significantly worse estimates of risk difference. Mean difference in potential outcomes has a bias of 0.152 and MSE of 0.023 for inverse probability weighting, whereas g-estimation led to a bias of 0.142 and MSE of 0.020 and fitting our model without the latent variable led to a bias of 0.162 and MSE of 0.026. Again, both inverse probability and g-estimation incorrectly suggest admitting a patient carries a higher risk than discharging a patient.

Case study

Data

Patient visits to the ED were analyzed using EHR from the University of Michigan Health Systems (UMHS). For each patient visit, we recovered **Demographic information** (age, sex); **Acuity** (emergency severity index); **Treatment Time** (duration between when treatment starts and ends) **Admission Decision** (the decision to discharge patient or admit to them to inpatient unit); **ED Revisit** (a binary outcome [yes/no] specifying whether patient returns to the ED within 30 days of being discharged from any hospital unit, including the ED); **Readmission** (a binary outcome [yes/no] specifying whether patient is admitted in the ED to an inpatient unit within 30 days of being discharged from any hospital unit, *including the ED*). Note we used this broader definition of readmission to be able to define a readmission variable for each visit.

For the analysis, we only included patient visits that met the following criteria: treatment start fell within a three-year period between August 1st, 2012 and August 1st, 2015; the patient was assigned an acuity level of 2 or 3; and the patient was not admitted to an ICU. The latter two criteria were imposed to focus on patient visits with the highest degree of medical uncertainty with respect to needs for longer term acute care need in an inpatient hospital need. A total of 156,720 visits were included in the analysis.

A summary of the data is in Table E3 . Note that each sex, age, and acuity group is well represented in the data. Among the patients included in our data set, there are also a reasonable number of patients that are admitted (approximately 29.4%) and that return to the ED within 30 days after being discharged (approximately 18.7%). The least represented variable is the number of patients that are admitted within 30 days of being discharge from a hospital unit (approximately 7.1%).

Finally, we must contend with missing data. Of the 156,720 visits, 91 (0.06%) had missing data: 58 (0.04%) had a missing treatment time and 33 (0.02%) had a missing admission decision. To handle missing data, we imputed missing variables from one ED visit with corresponding variables of its nearest 'neighbor' (ED visit) with complete data in terms of Euclidean distance[48]. All variables were used for imputation. Age and treatment time were standardized prior to measuring distance in imputation by centering around their respective mean and dividing by their standard deviation. Standardized age was also used in subsequent regression models. Age had a mean of 47.7 years and standard deviation of 19.7 years.

Descriptive statistics of admission decisions and treatment times

We first analyzed treatment times and admission decisions directly to generate questions about latent health needs (Table E4). Sex, age, and acuity are important factors in the admission decision process. Men are admitted at a higher rate than women, suggesting that either admission decisions are biased towards men or that women are more likely to visit the ED with lower latent health needs. The latter would explain why women are more likely to visit the ED (Table E3). Our admission decision model will help us determine if men and women differ on average for similar latent health needs. Similarly, older age groups are admitted at higher rates than younger age groups, which again could suggest bias or differences in latent states at the onset. As for acuity, acuity 2 patients are admitted at higher rates than acuity 3 patients, which is to be expected. It would be natural to attribute these differences to differences in latent health needs.

Analyzing treatment times and admission decisions together further generated questions about latent health needs, while demonstrating that understanding the admission decision process requires working with complex relationships. Treatment times are similar between men and women who are discharged, but differ between men and women who are admitted. Meanwhile, treatment time is similar among age groups who are admitted, but differ among age groups who are discharged. We question whether identifying health needs is more or less difficult in different age and gender groups. Importantly, treatment time

FIGURE 3 Results from parameter estimation: percent with higher latent health needs $H = 1$ (determined by α) and percent with higher initial observation (acuity 2) by latent health needs (determined by β).

decreases with patient acuity among discharged patients, but increases with patient acuity among admitted patients. Are care providers keeping patients longer for whom there is greater uncertainty about their latent health needs? In sum, modeling health needs helps one answer these questions on whether differences in admission rates and treatment times can be attributed to differences in latent health needs.

Fitting the decision-making model to data and estimating causal effects

We fitted the decision-making model and outcomes described in Section 2 to data on age, sex, acuity, admission decisions, and treatment times using the same procedure described in Section 3. Sex and age were considered to be patient characteristics X. Estimates and confidence intervals are reported in the Appendix (Tables E6 – E7). For comparison, we also estimated causal effects by applying inverse probability weighting and g-estimation approaches using the same approach outlined in Section 4.

Inferences from the decision-making model

In fitting the decision-making model, we can use a patient's latent health needs (H) to gain insight into the admission decision process. For this study, we estimated that on average, about 25.3% of patients have higher health needs. Further, men were more likely to have higher health needs than women (Figure 3 ; parameter estimates and 95% confidence intervals are in Table E6 of the Appendix). Based on the estimate of the probability $\alpha(X)$ of having higher health needs, about 23.4% of men had higher health needs at the mean age of 47.0 years, compared to only 19.2% of women, a difference of about 4.2%. This result could largely explain the difference of 5.6% in admission rates between men and women. Similarly, we estimated that older individuals are dramatically more likely to have higher health needs than younger individuals. Only 6.2% of individuals at 18 years of age are estimated to have higher health needs, compared to 44.6% of individuals at 70 years of age, a difference of 38.4%. Again, a difference in health needs can largely explain the difference of 41.3% in admission rates between the 18-24 year old group and the 75+ year old group. Of note, though, is that individuals with higher health needs are smaller in percentage than admitted patients, indicating individuals with lower health needs are often admitted.

An individual's health needs is estimated to disagree with the initial observation, which in the UMHS ED is acuity level, about 25-55% of the time, as determined from the estimate of $\beta(X, H)$ (Figure 3). While this percent mismatch may seem high, it is important to note acuity level does not only target an individual's health needs, but also targets urgency and splits individuals nearly equally between acuity 2 and acuity 3. Mismatch is higher for lower health needs, about 41.6% compared to 31.4% for higher health needs at the mean age of 47.0 years. In other words, acuity level is more sensitive to higher health needs than specific.

The final set of parameters b, c, d describe the admission decision process and its dependence on age, sex, health needs, and initial observation. Parameters b, c, and d are interpreted, respectively, as the threshold of evidence before a final decision is made, the initial level of evidence acquired from patient characteristics and the initial observation, and the rate at which evidence is accrued for the admission decision. Since these parameters indirectly determine the final decision and treatment time, we briefly discuss their estimates. The initial level of evidence is generally below the midpoint of 0.5 and increases significantly with acuity. This result can be interpreted as follows: a care provider tends at the onset towards the decision to discharge a patient over admitting, particularly acuity 3 individuals. This tendency to discharge a patient is slightly higher in men and younger individuals. We also find that the threshold of evidence is significantly higher for women than men and for older age groups, which can be interpreted as care providers are more careful when making a decision with women and older individuals. Lastly, the rate of evidence is determined by an individual's health needs, with information on higher health needs more quickly accrued relative to lower health needs.

Parameters b, c, d on the admission decision process are best examined by assessing their influence on admission rates and treatment times. Figure 4 depicts how admission rates and treatment times are related to health needs, age, and sex. We estimate that care providers take longer with patients with lower health needs than higher health needs. Care providers also take longer when a patient's initial observation does not match their latent health state. So for example, care providers spend the most time with lower-needs individuals who are assigned an acuity 2. This result suggests care providers keep patients longer when there

FIGURE 4 Estimated average treatment times and admission rates by sex, age, acuity, and latent health needs. Higher health needs is accompanied by shorter treatment times and better accuracy in the final decision compared to a lower health state.

FIGURE 5 Risk of ED revisits and readmissions as a function of admission decisions and latent health needs $H = 0$ and $H = 1$.

is uncertainty in their needs. We also estimate that whereas the admission decision tends to agree with health needs, lower-needs individuals have greater disagreement with their ideal final decision (which should be to discharge) than higher-needs individuals (which should be to admit). For example, the proportion of lower-need individuals ($H = 0$) who are discharged ranges from 87% to 97% depending on acuity, sex, and age, compared to 89% to 99% for higher-needs individuals ($H = 1$) who are admitted. Greater disagreement in a lower health state may also translate into keeping patients longer.

As for differences in age and sex, we find that women are kept slightly longer than men and older individuals are kept longer than younger individuals. Put differently, differences in treatment time observed in age and gender groups *cannot* be attributed solely to health needs, highlighting potential bias in the admission decision process. Importantly, we find that keeping women and older individuals longer is accompanied by greater agreement in their final admission decisions, independent of health needs or acuity level. At best, certain higher-needs individuals are correctly admitted 99% of the time. At worst, certain lower-needs individuals are incorrectly admitted 13% of the time.

Impact of latent health state on ED revisits and readmissions

We estimate that admitting an individual in the lower latent state increases the risk of ED revisits and readmissions (Figure 5 ; Tables E6 –E7 in the Appendix). With lower health needs ($H = 0$), an individual has an estimated 21.6% (20.1%, 23.3%) risk of revisiting the ED within 30 days of being discharged if they were admitted, compared to only a 17.3% (17.1%, 17.5%) risk if they were discharged, leading to an estimated risk difference of 4.3%. These individuals also have a 10.6% (9.1%, 12.3%) risk of being admitted though the ED within 30 days after being discharged if they were admitted, compared to 3.0% (2.9%, 3.1%) if they were discharged, leading to an estimated risk difference of 7.6%. That admitting a patient would lead to worse outcomes is contrary to a common assumption that increasing one's level of care means better care.

Admitting an individual with higher health needs ($H = 1$), however, has an opposite effect: significantly decreasing the risk of ED revisits and readmissions (Figure 5 ; Tables E6 –E7 in Appendix). With higher health needs, an individual has an estimated 20.1% (19.6%, 20.6%) risk of revisiting the ED within 30 days of being discharged if they were admitted, compared to 100.0% (100.0%, 100.0%) risk if they were discharged, leading to a risk difference of about -79.9%. These individuals also have a 15.4% (14.9%, 15.8%) risk of being admitted though the ED within 30 days after being discharged if they were admitted, compared to 99.9% (99.5%, 100.0%) if they were discharged, leading to an estimated risk difference of -84.5%. In other words, discharging an individual with higher health needs carries a significant risk.

Marginalizing over the latent health state, we find that an admission carries a lower risk than a discharge, with an estimated risk difference of -17.0% (-18.1%, -15.8%) for revisits and -15.6% (-16.8%, -14.4%) for readmission. In contrast, inverse probability weighting and g-estimation conclude that admission carries a higher risk than a discharge. Inverse probability weighting estimates a 20.3% risk of ED revisits when admitting a patient compared a 18.4% risk when discharging a patient (risk difference of 1.9%), and a 13.7% risk of readmission when discharging a patient compared to a 4.6% risk of readmission (risk difference of 9.1%). G-estimation estimates similar risk differences: 1.5% higher risk of ED revisit and 9.0% higher risk of readmission when admitting a patient over discharging a patient. These estimates would suggest that care providers reduce admissions. If, however, our estimates are correct, then these alternative methods miss the high risk of discharging a high-needs patient.

Sensitivity to Conditional Exchangeability Assumption

We tested how sensitive our estimates of potential outcomes are to violations in the Conditional Exchangeability Assumption 2.2. In general, the probability density function of Y, A, T given H, X, Z decomposes as

$$\left[f\left(A=1,T|H,X,Z,Y^1\right)\mathbb{P}\left(Y^1|H,X,Z\right)\right]^A\left[f\left(A=0,T|H,X,Z,Y^0\right)\mathbb{P}\left(Y^0|H,X,Z\right)\right]^{1-A},$$

where we use f to denote a general density function. Assumption 2.2 allows us to drop the dependence of the admission decision process A, T on potential outcomes Y^0, Y^1. Alternatively, if we suppose that

$$f\left(A,T|H,X,Z,Y^1\right) \propto \psi_1^{AY^1} f\left(A,T|H,X,Z\right);$$
$$f\left(A,T|H,X,Z,Y^0\right) \propto \psi_0^{(1-A)Y^0} f\left(A,T|H,X,Z\right)$$

then violations in the assumption can be captured by choosing values for ψ_1 and ψ_0 that are not both one.

We thus fit the model to data for ψ_0 and ψ_1 set to either 0.95, 0.975, 1, 1.025, or 1.05 in a factorial design, resulting in 25 comparisons. In all 25 comparisons, we found that estimates on potential outcomes $(\mu_1, \mu_2, \mu_3, \mu_4, \mathbb{E}Y^1 - \mathbb{E}Y^0)$ for either ED revisits or ED readmissions did not differ from their reference value when $\psi_0 = \psi_1 = 1$ by more than 0.022 (Tables E8 –E9). For example, estimates of mean difference in potential outcomes ranged from -0.144 when $\psi_0 = \psi_1 = 0.95$ to -0.168 when $\psi_0 = \psi_1 = 1.05$ in the case of ED readmissions. Thus, these important causal effects were relatively insensitive to the specified violations in Assumption 2.2.

Discussion

We aimed to provide a general framework to evaluate the admission decision process in the ED and to establish causation between admission decisions and outcomes. Our contribution is two-fold: a conceptual model for the ED admission decision process in which a patient's health or needs for resources is latent and a causal inference approach that uses latent health state and observational data to determine to what extent admitting a patient improves outcomes. We evaluated our framework with simulation and with an extensive dataset of over 150,000 patient encounters in the ED from the University of Michigan Health System collected from August 2012 through July 2015. By modeling latent health needs, we could examine variation in the admission decision process due to latent health needs. We could also estimate separate risks of an ED revisit or readmission for individuals with lower needs versus individuals with higher needs.

Our causal inference approach is based on the potential outcomes framework[49] and accounts for the lack of independence between treatment assignment and potential outcomes that arises in an observational study (i.e., when treatment assignments are not random). More broadly in an observational study, potential outcomes from one treatment group are not exchangeable with potential outcomes from another treatment group. A popular strategy is to estimate mean potential outcomes within certain groups or strata, for which it is reasonable to assume potential outcomes are exchangeable between treatment groups[20,21,22,23]. Our approach is similar by estimating mean potential outcomes within similar health needs but differs by using strata that are latent.

By marginalizing over the latent variable, we could still recover estimates of the mean difference in potential outcomes, a common target for causal inference approaches. In simulation, we demonstrate that our method accurately estimates mean difference in potential outcomes when data is generated from our model (Example 1) or closely-generated from our model (Example 2). By contrast, inverse probability weighting, g-estimation, and fitting our model without the latent variable provided significantly worse estimates in both examples to the point of drawing the wrong conclusion. That is, these alternative methods predict an admission carries a higher risk than a discharge, when in fact an admission carries a lower risk than a discharge. These methods did not adequately adjust for the risk of discharging associated with high-needs individual. A similar discrepancy between our method and these alternative methods was found in our case study. While our method predicts an admission carries a lower risk than a discharge, inverse probability weighting and g-estimation predicts an admission carries a higher risk than a discharge. Hence, our method is providing different estimates than other causal inferences.

There are several aspects about our method that should be carefully considered. One consideration is that our causal approach depends on a latent-variable model. Model-dependence is a common concern with all causal inference approaches because estimates generally depend on model choice. For example, we expect estimates of mean difference in potential outcomes to improve with better models of the admission process. Indeed, when data is simulated from our model (Example 1) or closely-simulated from our model (Example 2), we find that our approach yields significantly better estimates of this risk difference than

inverse probability weighting or g-estimation approaches. Model-dependence, however is not a unique concern to our approach. All causal inferences require a conceptual model or hypothesis of the problem, at the very least, to specify to what extent certain variables could influence both treatment assignments and outcomes. For the classic question of whether smoking causes lung cancer, one might hypothesize that poor habits in general (e.g., excessive drinking) could be be associated with increased smoking and lung cancer. Causal methods can be made robust to violations in the mathematical model (c.f., [50]). However, the conceptual model can rarely be validated [22]. In other words, the model should be chosen carefully to recover accurate estimates from causal inference approaches. We thus strove to present a model that best reflects the actual admission decision process.

Even if less robust to model violations, a latent-variable approach could still provide more useful information in certain contexts compared to other causal information approaches. In our case study, for example, we estimated that admitting an individual with lower health needs increased the risk of an ED revisit by 3.0% and readmission by 7.6%, but discharging an individual with higher health needs increased the risk of an ED revisit by 79.9% and readmission by 84.5%. In other words, admitting a patient leads to worse outcomes *only* for lower needs individuals, suggesting efforts to decrease admission rates could be welcome, provided it did not impact higher needs individuals. In contrast, g-estimation and inverse probability weighting estimated that admitting a general individual increased the risk of an ED revisit by about 2% and readmission by about 9%. This result would also support efforts to decrease admission rates, but would not bring the caveat that discharging certain individuals could be disastrous.

Researchers in the clinical community have employed different empirical approaches to similar questions. In the ED, for example, Stowell et al. [51] use a matched pair cluster study to compare quality of care (i.e. length of stay, mortality, hospital readmissions, and rate of transfer to the ICU) for patients outlying in inappropriate wards after admission because of lack of vacant beds in appropriate specialty wards to the care given to non outlying patients. Empirical techniques have similarly been used to assess outcomes from the care process in the ICU (c.f., Suter et al. [52], Azoulay et al. [53], Simchen et al. [54]). Related to the present study are those assessing the impact of length of stay on patient outcomes (c.f. Bueno et al. [55], Williams et al. [56], Nichols et al. [57], Reynolds et al. [58], Kaboli et al. [59]).

We remark that transfer decisions, such as ED admission decisions, are routine in hospital units (c.f., [60]). We contend that our approach is sufficiently broad that it could capture many of these transfer decisions, but specific enough to provide insight to the transfer decision of interest. In addition, and as alluded to, we assume that both observations beyond an initial observation and the number of decisions until the final decision to transfer/discharge are completely missing from the data. This assumption ensures observational data can be analyzed with both minimal restrictions: only an initial observation, final decisions, length of stay, and demographic information are needed; and in general populations.

However, our approach is not without limitations. First is that admission decisions and, in general, transfer decisions between hospital units, may or may not depend on "operational factors" such as congestion, patient home environment, and hospital size (c.f., [61,10,62,63,64,65,66,67]) which need to be accounted for when assessing the causal impact of admission and transfer decisions on patient outcomes. Second is that we used a threshold regression model of the disposition decision. Estimating causal effects using an alternative model of the admission decision and comparing with our model is of clear interest. One alternative model is described in Section 2.4: use a more conventional survival model such as a Weibull regression model for treatment time and an another model for the admission decision conditional on the treatment time. This approach would require 2 additional parameters to capture main effects (characteristics, acuity, and health state in both models and treatment time or admission decision in the second model). More parameters would be needed for interaction terms in order to reproduce the observation in the data whereby a mismatch between acuity and admission decision leads to longer treatment times. Third is that we apply our approach to a general ED population after restricting to those patients with ESI index 2 and 3 and excluding patients admitted to the ICU. EHR, however, may contain additional patient-specific information, such as chief complaint, that can be leveraged at baseline. Fourth, EHR may also contain intermediate information (e.g., vitals, lab tests) that could strengthen causal relationships. However, mapping this information onto observations in the admission decision process is expected to require additional models and assumptions that are specific to a condition and/or hospital. Fifth, we chose a specific form of our parametric model of potential outcomes in which only health needs have a direct effect. However, one may want to include patient characteristics, acuity, and treatment time in this model if these variables are believed to have a direct effect on potential outcomes even when controlling for health state.

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Conflict of interest

The authors declare no potential conflict of interests.

Appendix

Table of parameter estimates for simulation

Table of all parameter estimates for example 1 would go here.

Joint density approximation for admission decision and treatment time

In the main text, we introduced a structural model of the joint distribution of the admission decision A and treatment time T conditional on patient characteristics X, health needs H, and initial observation Z. The density function for $(A, T) = (a, t)$ conditional on X, Z, H was expressed in the form $g(a, t|b(X), c(X, Z), d(H))$ with $b(X), c(X, Z)b(X), d(H)b(X)$ representing respectively a upper boundary, starting point, and drift rate. Dropping the explicit dependence on X, Z, H, the density function $g(a, t|b, c, d)$ can be expressed exactly[35] as either

$$g(a,t|b,c,d) = \exp\left(-(db)bc - \frac{(db)^2 t}{2}\right)\frac{\pi}{b^2}\sum_{k=1}^{\infty} k\exp\left(-\frac{k^2\pi^2 t}{2b^2}\right)\sin(k\pi c)$$

or

$$g(a,t|b,c,d) = \exp\left(-(db)bc - \frac{(db)^2 t}{2}\right)\frac{1}{b^2}\frac{b^3}{\sqrt{2\pi t^3}}\sum_{k=-\infty}^{\infty}(c+2k)\exp\left(-\frac{(c+2k)^2 b^2}{2t}\right).$$

We can thus approximate $g(a, t|b, c, d)$ by truncating either series. However, the number of terms needed for an accurate approximation depends on the value of t. When t is large, g can be accurately approximated with relatively few terms from the second approximation. When t is small, g can be accurately approximated with relatively few terms from the second approximation. We found g could be approximated accurately using the first approximation truncated to 10 terms (from $k = 1$ to $k = 10$) when $t/b^2 \geq 1/10$ and using the second approximation truncated to 21 terms (from $k = -10$ to $k = 10$) when $t/b^2 < 1/10$.

Alternative approach to parameter estimation

We present an alternative approach to estimate unknown parameters. There are three reasons to consider another approach, since MLE yields an asymptotically efficient estimator provided model distributions are correctly specified. First, one may estimate the distribution of latent variables prior to considering its effect on outcomes and then later decide to use these estimate to estimate outcomes. Second, one may not specify a distribution for outcomes. Third, the model may be incorrect, which raises several practical issues, depending on which part of the model is incorrect. Latent variables are hypothetical constructs for the purpose of understanding admission decisions; generally there are no methods for directly measuring them[38]. Therefore, one

may overlook incorrect models for latent variables. An incorrect model is more worrisome when estimating outcomes, since changes in policy will stem from knowledge of how admission decisions causally impact *outcomes*. An incorrect model of outcomes might then influence interpretation of latent variables, if we try to simultaneously estimate latent states and outcomes. We thus propose an alternative approach that disentangles the estimation of the latent variable from the estimation of outcomes. A similar idea of separating estimating latent variables and effects of latent states on outcomes in several steps can be found in[46]. In[47] the authors show numerically that, in certain cases, the approach in[46] can be more robust to model violations.

To describe the estimation approaches, let θ be a set of model parameters and let Θ be the feasible set for θ. We first assume that we can decompose θ and Θ as (θ_1, θ_2) and $\Theta_1 \otimes \Theta_2$ so that θ_1 fully specifies the part of the model that does not involve outcomes Y:

$$f(z, a, t|x; \theta) = f(z, a, t|x; \theta_1).$$

Second, we assume that *both* the log-likelihood function $\log f(z, a, t, y|x; \theta)$ of data (z, a, t, y) and the log-likelihood function $\log f(z, a, t|x; \theta_1)$ of the data (z, a, t) with outcomes excluded, satisfy the typical regularity conditions needed for MLE, e.g, identifiability and continuity.

A MLE approach would search for $(\theta_1, \theta_2) \in \Theta_1 \otimes \Theta_2$ that maximizes

$$\sum_n \log f(z_n, a_n, t_n, y_n|x_n; \theta_1, \theta_2) = \sum_n \log f(y_n|x_n, z_n, a_n, t_n; \theta_1, \theta_2) + \sum_n \log f(z_n, a_n, t_n|x_n; \theta_1).$$

Under standard regularity conditions and provided the model is correctly specified, we know that the maximum likelihood estimator is a consistent and asymptotically-efficient estimator of the true value of (θ_1, θ_2). These properties can be derived from the theory of estimating functions by noting that the maximum likelihood estimator satisfies

$$\sum_n \begin{bmatrix} \nabla_{\theta_1} \log f(y_n|x_n, z_n, a_n, t_n; \theta_1, \theta_2) + \nabla_{\theta_1} \log f(z_n, a_n, t_n|x_n, \theta_1) \\ \nabla_{\theta_2} \log f(y_n|z_n, a_n, t_n; \theta_1, \theta_2) \end{bmatrix} = 0,$$

where the left hand side is an estimating function with expectation 0.

Our alternative approach is a *two-step* approach whereby MLE is performed once and then generalized estimating equation approach is used next. Step 1 ignores information on outcomes Y and searches for $\theta_1 \in \Theta_1$ that maximizes $\sum_n \log f(z_n, a_n, t_n|x_n; \theta_1)$. Assumed regularity conditions ensure that the maximum likelihood estimator is a consistent and asymptotically-efficient (in the sense when Y is ignored) estimator of the true value of θ_1. These properties also follow from the MLE estimator satisfying

$$\sum_n \nabla_{\theta_1} \log f(z_n, a_n, t_n|x_n, \theta_1) = 0$$

where the left hand side is an estimating equation of θ_1 with mean zero. If $\hat{\theta}_1$ is the resulting estimator, Step 2 then estimates $\hat{\theta}_2$ using a generalized estimating equation. Together, the two-steps amount to solving

$$\sum_n \begin{bmatrix} \nabla_{\theta_1} \log f(z_n, a_n, t_n|x_n, \theta_1) \\ \frac{\partial \mu^T}{\partial \theta_1} V_n(\mu)^{-1} \left(y_n - \mu(x_n, z_n, a_n, t_n; \theta_1, \theta_2)\right) \end{bmatrix} = 0, \tag{C1}$$

where functions μ and $V_n(\mu)$ are defined to respectively approximate the mean and variance of Y given X, Z, A, T and parameters θ_1, θ_2. Only μ needs to be correctly specified for the left hand side of (C1) to be an estimating function of θ_1, θ_2 with mean zero. The two-step MLE/GEE approach will thus also yield a consistent estimator of (θ_1, θ_2).

Handling the latent variable

Finally, we turn to discuss how to handle latent-variables H. It is easiest to define the joint distribution of all the random variables including H. In such a case, $f(y|x, z, a, t; \theta_1, \theta_2)$ and $f(z, a, t|x, \theta_1)$ are not explicitly available, and so, we use

expectation-maximum algorithm to perform MLE[44]. We also note that

$$\begin{aligned}\mathbb{E}(y|x,z,a,t,\theta_1,\theta_2) &= \sum_h \mathbb{E}(y|x,z,a,t,h,\theta_1,\theta_2)\,\mathbb{P}(H=h|x,z,a,t,\theta_1,\theta_2)\\ &= \sum_h \mathbb{E}(y|x,z,a,t,h,\theta_1,\theta_2)\,\mathbb{P}(H=h|x,z,a,t,\theta_1)\\ &= \sum_h \mathbb{E}(y|x,z,a,t,h,\theta_2)\,\mathbb{P}(H=h|x,z,a,t,\theta_1)\\ &= \sum_h \mathbb{E}(y^a|x,z,t,h,\theta_2)\,\mathbb{P}(H=h|x,z,a,t,\theta_1),\end{aligned}$$

where we first drop θ_1 from $\mathbb{E}(y|x,z,a,t,h,\theta_1,\theta_2)$ since we conditioned on $H=h$ along with the other variables; then drop θ_2 in $\mathbb{P}(H=h|x,v,z,a,t,\theta_1)$ because of how parameters θ were decomposed; and finally use Assumption 2.2 to relate estimation of the distribution of outcomes Y with estimation of the distribution of potential outcomes Y^a. These expressions mean that estimation of θ_2 is linked to the estimation of θ_1 only through $\mathbb{P}(H=h|x,z,a,t,\theta_1)$. So even if the latent-variable model or θ_1 is incorrect, we can still estimate θ_2 correctly and thereby make correct conclusions about potential outcomes provided the model for outcomes and $\mathbb{P}(H=h|x,z,a,t,\theta_1)$ are correct. These expressions also motivate choosing $\mu(x,z,a,t;\theta_1,\theta_2)$ in Step 2 in the two-step MLE/GEE approach to be of the form:

$$\mu(x,z,a,t;\theta_1,\theta_2) := \sum_h \nu_{h,a}(x,z,t,\theta_2)\,\mathbb{P}(H=h|x,z,a,t,\theta_1),$$

for functions $\nu_{h,a}(x,z,t,\theta_2)$ that model $\mathbb{E}\left[Y^a|x,z,t,h,\theta_2\right]$.

Simulation results for a two-step approach

We applied our estimation approach to data from the same two simulation examples in the main text with $N = 10,000$. Results can be found in Tables E10 - E11 . In both examples, our two-step approach has similar performance in terms of bias and MSE to the one-step approach for nearly all the estimated parameters. In Example 1, the two-step estimate of μ_2 is arguably the worst when compared to the one-step estimate of μ_2, with bias equal to 0.0667 compared to -0.022. However in Example 2, the two-step estimate of μ_2 has nearly the same bias as the one-step estimate of μ_2, with bias equal to 0.0955 compared to 0.0958.

Justification of parameter choices for simulation

Patient characteristics $X = (X_1, X_2)$ was defined with X_1 describing a binary characteristic such as gender and X_2 describing a numerical variable such as age. We chose an even 50% of simulated individuals to have $X_1 = 1$ and chose X_2 to be a uniformly random variable between -1/2 and 1/2, where for reference about 45% of individuals visiting the ED are female and age is roughly uniformly-distributed from 18 to 75 years among ED visits. We chose parameters in a linear model of logit $\alpha(X)$ such that on average 30% of $X_1 = 0$ individuals have higher health needs ($H = 1$) and 25% of $X_1 = 1$ individuals have higher health needs ($H = 1$) to reflect the approximate 30% admission rates for men and 25% for women. We assumed the linear coefficient for X_2 in logit $\alpha(X)$ was equal to 1 to reflect that older individuals are more likely to have higher health needs ($H = 1$).

For the remaining model components, we assume X has no influence. We chose parameters for logit $\beta(X, H)$ such that 40% of individuals with lower health needs ($H = 0$) have a higher initial observation ($Z = 1$) compared to 80% of individuals with higher health needs ($H = 1$), which when taken with the rate $\alpha(X)$ of latent health needs, reflects that about half of the individuals are assigned the higher acuity level (Acuity 2). We chose parameters for logit $c(X, Z)$ such that $c(X, Z)$ had a value of 0.5 for individuals with a higher initial observation ($Z = 1$) compared to 0.4 for individuals with a lower initial observation ($Z = 0$) to reflect that higher acuity patients are more likely to be admitted (about 42% of acuity 2 patients are admitted compared to 18% of acuity 3 patients). Because the drift rate term $d(H)$ and the boundary $b(X)$ are difficult to relate to the data and we can always re-scale time in the simulation, we simply chose parameters to result in a value of 2 for $d(H)$ for higher health needs ($H = 1$) and -1.5 for lower health needs ($H = 0$) and a constant value of 1 for $b(X)$. Lastly, we assumed that 10% of lower needs patients have a poor outcome $Y = 1$ when discharged ($A = 0$) compared to 20% when admitted ($A = 1$) and that 20% of higher needs patients have a worse outcome $Y = 1$ when admitted ($A = 1$) compared to 90% when discharged

($A = 0$). These values were chosen to capture two potential trends: higher needs patients have worse outcomes than lower needs patients and a mismatch between needs and admission decisions leads to worse outcomes.

Tables of parameter estimates for case study

Tables from Case Study would go here.

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The following tables would be inserted in the main text:

			N=1,000			N=10,000		
		Value	Bias	MSE	Coverage	Bias	MSE	Coverage
$\mathbb{E}\left[Y^1 - Y^0\right]$	n/a	-0.12	0.0192	0.0143	0.86	0.008	0.0017	0.90
$\log \frac{\mathbb{E}\,Y^1\,\mathbb{E}\left[1-Y^0\right]}{\mathbb{E}\,Y^0\,\mathbb{E}\left[1-Y^1\right]}$	n/a	-0.63	0.0674	0.4608	0.86	0.0411	0.0491	0.92

TABLE E1 Performance of parameter estimates when the model is correct (Example 1) in terms of bias, mean square error (MSE), and percent coverage of true parameters for 95% confidence intervals. True parameter values are also reported. Variable X_2 was uniformly distributed between -1/2 and 1/2.

			N=10,000		
		Value	Bias	MSE	Coverage
logit $\alpha(X)$	Intercept	-0.85	-0.1227	0.0214	0.72
	$X_1 = 1$	-0.25	-0.0055	0.0053	0.94
	X_2	1	0.0339	0.0192	0.92
logit $\beta(X, H)$	$H = 0$	-0.41	0.0185	0.0017	0.94
	$H = 1$	1.39	0.1530	0.0465	0.94
	$X_1 = 1$	0	0.0047	0.0023	0.94
	X_2	0	0.0001	0.0067	0.90
log $\lvert d \rvert(H)$	$H = 0$	0.41	0.0207	0.0009	0.76
	$H = 1$	0.69	0.0042	0.0006	0.90
logit $c(X, Z)$	$Z = 0$	-0.41	-0.0006	0.0006	0.92
	$Z = 1$	0	0.0041	0.0018	0.96
	$X_1 = 1$	0	-0.0143	0.0023	0.96
	X_2	0	-0.0751	0.0084	0.72
log $b(X)$	Intercept	0	-0.0022	0.0001	0.98
	$X_1 = 1$	0	0.0021	0.0001	0.90
	X_2	0	-0.0049	0.0004	0.90
μ_1	Intercept	0.1	0.0076	0.0001	0.70
μ_2	Intercept	0.9	0.0958	0.0095	1.00
μ_3	Intercept	0.2	0.0005	0.0008	0.96
μ_4	Intercept	0.2	-0.0005	0.0002	0.98
$\mathbb{E}\left[Y^1 - Y^0\right]$	n/a	-0.12	-0.0154	0.0008	0.92
$\log \frac{\mathbb{E}\,Y^1\,\mathbb{E}\left[1-Y^0\right]}{\mathbb{E}\,Y^0\,\mathbb{E}\left[1-Y^1\right]}$	n/a	-0.63	-0.0675	0.0240	0.92

TABLE E2 Performance of parameter estimates when the Conditional Exchangeability Assumption is violated (Example 2) in terms of bias, mean square error (MSE), and percent coverage of true parameters for 95% confidence intervals. True parameter values are also reported. Variable X_2 was uniformly distributed between -1/2 and 1/2.

Variable	Count (%)	Mean (SD)
Female	86,814 (55.4)	
Age, years		47.7 (19.7)
Acuity 2	76,033 (48.5)	
Treatment time, hours		5.4 (4.1)
Admitted	46,060 (29.4)	
Revisited ED	29,333 (18.7)	
Readmitted	11,176 (7.1)	

TABLE E3 Composition of UMHS dataset (N=156,720) by demography (age, sex), variables related to ED visit (acuity, treament time, admission decision), and outcomes (30-day ED revisits and readmissions).

Variable	Admitted (%)	Mean (SD) treatment time, hr	
		Discharged	Admitted
Male	22,961 (32.9)	5.6 (4.4)	4.9 (3.3)
Female	23,099 (26.6)	5.6 (4.3)	5.2 (3.4)
Age			
18–24	2,702 (11.0)	4.6 (3.0)	5.0 (3.6)
25–34	3,835 (15.0)	5.0 (3.4)	5.1 (3.4)
35–44	4,499 (20.6)	5.7 (4.3)	5.2 (3.4)
45–54	7,251 (28.9)	6.3 (5.2)	5.2 (3.5)
55–64	10,096 (40.2)	6.4 (5.2)	5.1 (3.4)
65–74	9,105 (50.3)	6.3 (5.1)	5.0 (3.3)
75+	8,571 (52.3)	5.9 (4.3)	5.0 (3.1)
Acuity 3	14,453 (17.9)	5.0 (3.6)	5.6 (3.6)
Acuity 2	31,607 (41.6)	6.5 (5.2)	4.8 (3.2)

TABLE E4 Admission rates and treatment times by sex, age, and acuity.

The following tables would be inserted in the Appendix:

			N=1,000			N=10,000		
		Value	Bias	MSE	Coverage	Bias	MSE	Coverage
logit $\alpha(X)$	Intercept	-0.85	-0.0112	0.0743	0.98	-0.0005	0.0056	0.94
	$X_1 = 1$	-0.25	0.0121	0.0555	0.96	0.0008	0.0049	0.92
	X_2		0.0441	0.2444	0.94	0.0133	0.018	0.94
logit $\beta(X, H)$	$H = 0$	-0.41	-0.0342	0.0620	1.00	-0.0049	0.0016	0.92
	$H = 1$	1.39	1.5758	44.4350	0.94	0.0128	0.0158	0.94
	$X_1 = 1$	0	-0.0056	0.0256	1.00	0.0043	0.0023	0.94
	X_2	0	-0.0628	0.1684	0.98	-0.0003	0.0069	0.92
log $\lvert d \rvert(H)$	$H = 0$	0.41	0.0059	0.0053	0.94	0.0043	0.0005	0.94
	$H = 1$	0.69	-0.0048	0.0061	0.96	0.0011	0.0006	0.90
logit $c(X, Z)$	$Z = 0$	-0.41	-0.0051	0.0053	0.98	0.0001	0.0006	0.90
	$Z = 1$	0	0.0040	0.0190	0.98	-0.0007	0.0019	0.92
	$X_1 = 1$	0	-0.0146	0.0270	0.94	-0.007	0.0025	0.98
	X_2	0	-0.0606	0.1045	0.86	-0.0161	0.0032	0.96
log $b(X)$	Intercept	0	0.0101	0.0013	0.90	0.006	0.0002	0.92
	$X_1 = 1$	0	0.0006	0.0013	0.94	0.001	0.0001	0.90
	X_2	0	-0.0020	0.0040	0.94	-0.0001	0.0004	0.96
μ_1	Intercept	0.1	0.0063	0.0005	0.94	0.0006	0.0001	0.90
μ_2	Intercept	0.9	-0.1152	0.0801	0.94	-0.022	0.013	0.88
μ_3	Intercept	0.2	0.0073	0.0177	0.96	0.0004	0.001	0.96
μ_4	Intercept	0.2	-0.0009	0.0025	0.90	-0.0004	0.0002	0.98

TABLE E5 Performance of parameter estimates when the model is correct (Example 1) in terms of bias, mean square error (MSE), and percent coverage of true parameters for 95% confidence intervals. True parameter values are also reported. Variable X_2 was uniformly distributed between -1/2 and 1/2.

			95% CI	
		Estimate	Lower	Upper
logit $\alpha(X)$	Intercept	-1.294	-1.314	-1.274
	Sex	0.952	0.934	0.969
	Age	-0.254	-0.284	-0.224
logit $\beta(X)$	$H = 0$	-0.331	-0.344	-0.319
	$H = 1$	0.786	0.760	0.812
	Sex	0.193	0.181	0.204
	Age	-0.347	-0.368	-0.326
log $\lvert d \rvert(H)$	$H = 0$	-0.354	-0.363	-0.345
	$H = 1$	-0.054	-0.064	-0.044
logit $c(X, Z)$	$Z = 0$	0.030	0.024	0.036
	$Z = 1$	-0.005	-0.017	0.007
	Sex	0.465	0.456	0.474
	Age	0.921	0.910	0.931
log $b(X)$	Intercept	0.198	0.194	0.202
	Sex	0.099	0.096	0.103
	Age	0.052	0.045	0.059
μ_1	Intercept	0.173	0.171	0.175
μ_2	Intercept	1.000	1.000	1.000
μ_3	Intercept	0.216	0.201	0.233
μ_4	Intercept	0.201	0.196	0.206
$\mathbb{E}\left[Y^1 - Y^0\right]$	n/a	-0.170	-0.181	-0.158
$\log \frac{\mathbb{E}\,Y^1\,\mathbb{E}\left[1-Y^0\right]}{\mathbb{E}\,Y^0\,\mathbb{E}\left[1-Y^1\right]}$	n/a	-0.799	-0.868	-0.729

TABLE E6 Parameter estimates and 95% confidence intervals (CI) for admission decision model and causal effects of admissions on ED revisits. Parameters were expressed as a linear model transformed by a nonlinear link function. Estimates are reported for coefficients in the linear model. Variables were encoded such that male was zero and female was one; acuity 3 was zero and acuity 2 was one. Age was standardized and sex was centered to its mean.

			95% CI	
		Estimate	Lower	Upper
μ_1	Intercept	0.030	0.029	0.031
μ_2	Intercept	0.999	0.995	1.000
μ_3	Intercept	0.106	0.091	0.123
μ_4	Intercept	0.154	0.149	0.158
$\mathbb{E}\left[Y^1 - Y^0\right]$	n/a	-0.156	-0.168	-0.144
$\log \frac{\mathbb{E}\,Y^1\,\mathbb{E}\left[1-Y^0\right]}{\mathbb{E}\,Y^0\,\mathbb{E}\left[1-Y^1\right]}$	n/a	-0.883	-0.998	-0.767

TABLE E7 Parameter estimates and 95% CI of causal effects of admission on ED readmission.

ψ_0	ψ_1	μ_1	μ_2	μ_3	μ_4	$\mathbb{E}\left[Y^1 - Y^0\right]$
0.95	0.95	0.17 (0.17, 0.17)	1.00 (1.00, 1.00)	0.24 (0.22, 0.25)	0.20 (0.19, 0.20)	-0.15 (-0.17, -0.14)
0.95	0.975	0.17 (0.17, 0.17)	1.00 (0.95, 1.00)	0.23 (0.21, 0.24)	0.20 (0.19, 0.20)	-0.16 (-0.17, -0.15)
0.95	1	0.17 (0.17, 0.17)	1.00 (1.00, 1.00)	0.22 (0.20, 0.24)	0.20 (0.20, 0.21)	-0.17 (-0.18, -0.16)
0.95	1.025	0.17 (0.17, 0.17)	1.00 (1.00, 1.00)	0.21 (0.19, 0.22)	0.20 (0.20, 0.21)	-0.18 (-0.19, -0.16)
0.95	1.05	0.17 (0.17, 0.17)	1.00 (1.00, 1.00)	0.20 (0.18, 0.21)	0.21 (0.20, 0.21)	-0.18 (-0.20, -0.17)
0.975	0.95	0.17 (0.17, 0.18)	1.00 (1.00, 1.00)	0.24 (0.22, 0.25)	0.20 (0.19, 0.20)	-0.15 (-0.17, -0.14)
0.975	0.975	0.17 (0.17, 0.18)	1.00 (1.00, 1.00)	0.23 (0.21, 0.24)	0.20 (0.19, 0.20)	-0.16 (-0.18, -0.15)
0.975	1	0.17 (0.17, 0.18)	1.00 (1.00, 1.00)	0.22 (0.20, 0.24)	0.20 (0.20, 0.21)	-0.17 (-0.18, -0.16)
0.975	1.025	0.17 (0.17, 0.18)	1.00 (1.00, 1.00)	0.21 (0.19, 0.22)	0.20 (0.20, 0.21)	-0.18 (-0.19, -0.16)
0.975	1.05	0.17 (0.17, 0.18)	1.00 (1.00, 1.00)	0.20 (0.18, 0.21)	0.21 (0.20, 0.21)	-0.18 (-0.20, -0.17)
1	0.95	0.17 (0.17, 0.18)	1.00 (1.00, 1.00)	0.24 (0.22, 0.25)	0.20 (0.19, 0.20)	-0.15 (-0.17, -0.14)
1	0.975	0.17 (0.17, 0.18)	1.00 (1.00, 1.00)	0.23 (0.21, 0.25)	0.20 (0.19, 0.20)	-0.16 (-0.17, -0.15)
1	1	0.17 (0.17, 0.18)	1.00 (1.00, 1.00)	0.22 (0.20, 0.23)	0.20 (0.20, 0.21)	-0.17 (-0.18, -0.16)
1	1.025	0.17 (0.17, 0.18)	1.00 (1.00, 1.00)	0.21 (0.19, 0.22)	0.20 (0.20, 0.21)	-0.18 (-0.19, -0.16)
1	1.05	0.17 (0.17, 0.18)	1.00 (1.00, 1.00)	0.19 (0.18, 0.21)	0.21 (0.20, 0.21)	-0.19 (-0.20, -0.17)
1.025	0.95	0.17 (0.17, 0.18)	1.00 (0.99, 1.00)	0.24 (0.22, 0.26)	0.20 (0.19, 0.20)	-0.15 (-0.17, -0.14)
1.025	0.975	0.17 (0.17, 0.18)	1.00 (0.99, 1.00)	0.23 (0.21, 0.25)	0.20 (0.19, 0.20)	-0.16 (-0.18, -0.15)
1.025	1	0.17 (0.17, 0.18)	1.00 (1.00, 1.00)	0.22 (0.20, 0.23)	0.20 (0.20, 0.21)	-0.17 (-0.18, -0.16)
1.025	1.025	0.17 (0.17, 0.18)	1.00 (0.99, 1.00)	0.21 (0.19, 0.22)	0.20 (0.20, 0.21)	-0.18 (-0.19, -0.16)
1.025	1.05	0.17 (0.17, 0.18)	1.00 (1.00, 1.00)	0.19 (0.18, 0.21)	0.21 (0.20, 0.21)	-0.19 (-0.20, -0.17)
1.05	0.95	0.17 (0.17, 0.18)	0.99 (0.99, 0.99)	0.24 (0.22, 0.26)	0.20 (0.19, 0.20)	-0.15 (-0.17, -0.14)
1.05	0.975	0.17 (0.17, 0.18)	0.99 (0.99, 0.99)	0.23 (0.21, 0.25)	0.20 (0.19, 0.20)	-0.16 (-0.17, -0.15)
1.05	1	0.17 (0.17, 0.18)	0.99 (0.99, 0.99)	0.22 (0.20, 0.24)	0.20 (0.20, 0.21)	-0.17 (-0.18, -0.15)
1.05	1.025	0.17 (0.17, 0.18)	0.99 (0.99, 0.99)	0.21 (0.19, 0.22)	0.20 (0.20, 0.21)	-0.18 (-0.19, -0.16)
1.05	1.05	0.17 (0.17, 0.18)	0.99 (0.99, 0.99)	0.19 (0.18, 0.21)	0.21 (0.20, 0.21)	-0.18 (-0.20, -0.17)

TABLE E8 Sensitivity of causal impact of admission on ED revisits to violations in Assumption 2.2, captured when ψ_0 and ψ_1 are not both 1. For each estimate, 95% confidence intervals are provided.

ψ_0	ψ_1	μ_1	μ_2	μ_3	μ_4	$\mathbb{E}\left[Y^1 - Y^0\right]$
0.95	0.95	0.03 (0.03, 0.03)	1.00 (1.00, 1.00)	0.12 (0.11, 0.14)	0.15 (0.15, 0.16)	-0.14 (-0.16, -0.13)
0.95	0.975	0.03 (0.03, 0.03)	1.00 (0.61, 1.00)	0.11 (0.10, 0.13)	0.15 (0.15, 0.16)	-0.15 (-0.16, -0.14)
0.95	1	0.03 (0.03, 0.03)	1.00 (1.00, 1.00)	0.11 (0.09, 0.12)	0.15 (0.15, 0.16)	-0.16 (-0.17, -0.14)
0.95	1.025	0.03 (0.03, 0.03)	1.00 (1.00, 1.00)	0.10 (0.08, 0.11)	0.16 (0.15, 0.16)	-0.16 (-0.17, -0.15)
0.95	1.05	0.03 (0.03, 0.03)	1.00 (1.00, 1.00)	0.09 (0.08, 0.11)	0.16 (0.15, 0.16)	-0.17 (-0.18, -0.16)
0.975	0.95	0.03 (0.03, 0.03)	1.00 (1.00, 1.00)	0.12 (0.11, 0.14)	0.15 (0.15, 0.16)	-0.14 (-0.16, -0.13)
0.975	0.975	0.03 (0.03, 0.03)	1.00 (1.00, 1.00)	0.11 (0.10, 0.13)	0.15 (0.15, 0.16)	-0.15 (-0.16, -0.14)
0.975	1	0.03 (0.03, 0.03)	1.00 (1.00, 1.00)	0.11 (0.09, 0.12)	0.15 (0.15, 0.16)	-0.16 (-0.17, -0.14)
0.975	1.025	0.03 (0.03, 0.03)	1.00 (1.00, 1.00)	0.10 (0.08, 0.11)	0.16 (0.15, 0.16)	-0.16 (-0.17, -0.15)
0.975	1.05	0.03 (0.03, 0.03)	1.00 (1.00, 1.00)	0.09 (0.08, 0.11)	0.16 (0.15, 0.16)	-0.17 (-0.18, -0.16)
1	0.95	0.03 (0.03, 0.03)	1.00 (1.00, 1.00)	0.12 (0.11, 0.14)	0.15 (0.15, 0.16)	-0.14 (-0.16, -0.13)
1	0.975	0.03 (0.03, 0.03)	1.00 (1.00, 1.00)	0.11 (0.10, 0.13)	0.15 (0.15, 0.16)	-0.15 (-0.16, -0.14)
1	1	0.03 (0.03, 0.03)	1.00 (1.00, 1.00)	0.11 (0.09, 0.12)	0.15 (0.15, 0.16)	-0.16 (-0.17, -0.14)
1	1.025	0.03 (0.03, 0.03)	1.00 (1.00, 1.00)	0.10 (0.08, 0.11)	0.16 (0.15, 0.16)	-0.16 (-0.17, -0.15)
1	1.05	0.03 (0.03, 0.03)	1.00 (1.00, 1.00)	0.09 (0.08, 0.11)	0.16 (0.15, 0.16)	-0.17 (-0.18, -0.16)
1.025	0.95	0.03 (0.03, 0.03)	1.00 (1.00, 1.00)	0.12 (0.11, 0.14)	0.15 (0.15, 0.16)	-0.15 (-0.16, -0.13)
1.025	0.975	0.03 (0.03, 0.03)	1.00 (1.00, 1.00)	0.11 (0.10, 0.13)	0.15 (0.15, 0.16)	-0.15 (-0.16, -0.14)
1.025	1	0.03 (0.03, 0.03)	1.00 (1.00, 1.00)	0.11 (0.09, 0.12)	0.15 (0.15, 0.16)	-0.16 (-0.17, -0.15)
1.025	1.025	0.03 (0.03, 0.03)	1.00 (1.00, 1.00)	0.10 (0.09, 0.11)	0.16 (0.15, 0.16)	-0.16 (-0.17, -0.15)
1.025	1.05	0.03 (0.03, 0.03)	1.00 (1.00, 1.00)	0.09 (0.08, 0.11)	0.16 (0.15, 0.16)	-0.17 (-0.18, -0.16)
1.05	0.95	0.03 (0.03, 0.03)	1.00 (1.00, 1.00)	0.12 (0.11, 0.14)	0.15 (0.15, 0.16)	-0.15 (-0.16, -0.13)
1.05	0.975	0.03 (0.03, 0.03)	1.00 (1.00, 1.00)	0.11 (0.10, 0.13)	0.15 (0.15, 0.16)	-0.15 (-0.16, -0.14)
1.05	1	0.03 (0.03, 0.03)	1.00 (1.00, 1.00)	0.11 (0.09, 0.12)	0.15 (0.15, 0.16)	-0.16 (-0.17, -0.15)
1.05	1.025	0.03 (0.03, 0.03)	1.00 (1.00, 1.00)	0.10 (0.08, 0.11)	0.16 (0.15, 0.16)	-0.16 (-0.17, -0.15)
1.05	1.05	0.03 (0.03, 0.03)	1.00 (1.00, 1.00)	0.09 (0.08, 0.11)	0.16 (0.15, 0.16)	-0.17 (-0.18, -0.16)

TABLE E9 Sensitivity of causal impact of admission on ED readmissions to violations in Assumption 2.2, captured when ψ_0 and ψ_1 are not both 1. For each estimate, 95% confidence intervals are provided.

			N=10,000	
			Bias	MSE
logit $\alpha(X)$	Intercept	-0.85	0.0011	0.0056
	$X = 1$	-0.25	0.0016	0.0054
	X_2	1	0.0061	0.0197
logit $\beta(X, H)$	$H = 0$	-0.41	-0.0023	0.0017
	$H = 1$	1.39	0.0014	0.0157
	$X_1 = 1$	0	0.0035	0.0023
	X_2	0	0.0040	0.0073
log $\|d\|(H)$	$H = 0$	0.41	0.0029	0.0005
	$H = 1$	0.69	0.0008	0.0006
logit $c(X, Z)$	$Z = 0$	-0.41	0.0002	0.0006
	$Z = 1$	0	-0.0003	0.0020
	$X_1 = 1$	0	-0.0040	0.0024
	X_2	0	-0.0102	0.0039
log $b(X)$	Intercept	0	0.0072	0.0002
	$X_1 = 1$	0	0.0010	0.0001
	X_2	0	0.0002	0.0004
μ_1	Intercept	0.1	0.0057	0.0001
μ_2	Intercept	0.9	0.0667	0.0057
μ_3	Intercept	0.2	0.0016	0.0010
μ_4	Intercept	0.2	0.0000	0.0002

TABLE E10 Bias and mean square error (MSE) for two-step parameter estimates. True values of parameters are also reported. (Example 1)

			N=10,000	
			Bias	MSE
logit $\alpha(X)$	Intercept	-0.85	-0.1238	0.0216
	$X = 1$	-0.25	0.0036	0.0060
	X_2	1	-0.0111	0.0206
logit $\beta(X, H)$	$H = 0$	-0.41	0.0441	0.0035
	$H = 1$	1.39	0.0515	0.0225
	$X_1 = 1$	0	-0.0028	0.0024
	X_2	0	0.0327	0.0080
log $\|d\|(H)$	$H = 0$	0.41	0.0088	0.0006
	$H = 1$	0.69	-0.0010	0.0006
logit $c(X, Z)$	$Z = 0$	-0.41	-0.0001	0.0006
	$Z = 1$	0	0.0032	0.0019
	$X_1 = 1$	0	0.0033	0.0022
	X_2	0	-0.0036	0.0042
log $b(X)$	Intercept	0	0.0073	0.0002
	$X_1 = 1$	0	0.0013	0.0001
	X_2	0	-0.0015	0.0004
μ_1	Intercept	0.1	0.0081	0.0001
μ_2	Intercept	0.9	0.0955	0.0092
μ_3	Intercept	0.2	0.0017	0.0009
μ_4	Intercept	0.2	0.0000	0.0003

TABLE E11 Bias and mean square error (MSE) for two-step parameter estimates. True values of parameters are also reported. (Example 2)