Supporting Information for "Measurement error correction and sensitivity analysis in longitudinal dietary intervention studies using external calibration data" by Juned Siddique, Raymond J. Carroll, Michael J. Daniels, Trivellore E. Raghunathan, Elizabeth A. Stuart, and Laurence S. Freedman

A Outlier exclusions

In both PREMIER and OPEN, participants who self-reported extreme energy intakes that fell outside of recommended cutoffs ($<$500 and $<$800 kcal or $>$3,500 and $>$4,000 kcal in women and men, respectively) (Willet 2013, Chapter 13) were removed from our data. Applying this criteria resulted in excluding 25 (5%) participants from OPEN and 10 (1%) participants from PREMIER. In addition, 3 (0.4%) PREMIER participants were missing self-reported sodium at all three time points and were excluded from our analysis.

B Comments on transportability

Here we show that our definition of calibration model transportability implies that selection into the trial does not depend on unobserved characteristics after conditioning on observed characteristics (including those measured with error). Let Z be the true unobserved value of the outcome we wish to measure, and let Y be Z measured with error. Variables $\mathbf{X}$ are background covariates measured without error. Both Y and $\mathbf{X}$ are observed. Let S indicate whether a participant is in the lifestyle intervention ($S = \ell$) or validation study ($S = v$). Define transportability as:

$$f(Z \mid Y, \mathbf{X}, S = \ell) = f(Z \mid Y, \mathbf{X}, S = v). \tag{B.1}$$

Theorem. *Under transportability,*

$$f(S \mid Y, \mathbf{X}, Z) = f(S \mid Y, \mathbf{X}). \tag{B.2}$$

Proof.

$$\begin{aligned} f(S \mid Y, \mathbf{X}, Z) &= \frac{f(Z \mid Y, \mathbf{X}, S) f(S \mid Y, \mathbf{X})}{f(Z \mid Y, \mathbf{X}, S = \ell) f(S = \ell \mid Y, \mathbf{X}) + f(Z \mid Y, \mathbf{X}, S = v) f(S = v \mid Y, \mathbf{X})} \\ &= \frac{f(Z \mid Y, \mathbf{X}, S) f(S \mid Y, \mathbf{X})}{f(Z \mid Y, \mathbf{X}, S)\big\{f(S = \ell \mid Y, \mathbf{X}) + f(S = v \mid Y, \mathbf{X})\big\}} \qquad \text{(via transportability)} \\ &= f(S \mid Y, \mathbf{X}) \end{aligned}$$

Equation B.2 states that whether an individual is assigned to the trial or the validation study depends only on *observed* characteristics Y and $\mathbf{X}$. This is a potentially more realistic assignment mechanism than one in which participants are included or excluded from a study based on *unobserved* characteristics as would be the case if transportability were defined in terms of Y conditional on Z.

C Parameter estimators for imputation regression models

C.1 Estimators for $f(Y_0, Z_0 \mid \mathbf{X}, S = \ell)$

We estimate the parameters of the joint multivariate normal distribution of Y_0 and Z_0 using

$$f(Y_0, Z_0 \mid \mathbf{X}, S = \ell) = f(Z_0 \mid Y_0, \mathbf{X}, S = \ell) f(Y_0 \mid \mathbf{X}, S = \ell).$$

Let $\beta_{0,Y_0\cdot\mathbf{X}}$, $\beta_{1,Y_0\cdot\mathbf{X}}$, and $\sigma^2_{Y_0\cdot\mathbf{X}}$ be the intercept, slope, and residual variance from from $f(Y_0 \mid \mathbf{X}, S = \ell)$ and $\beta_{0,Z_0\cdot Y_0\mathbf{X}}$, $\beta_{1,Z_0\cdot Y_0\mathbf{X}}$, $\beta_{2,Z_0\cdot Y_0\mathbf{X}}$, and $\sigma^2_{Z_0\cdot Y_0\mathbf{X}}$ be the coefficients and residual variance from $f(Z_0 \mid Y_0, \mathbf{X}, S = \ell)$ defined in Equation 7 of the manuscript. The remaining parameters of the joint distribution are,

$$\beta_{0,Z_0\cdot\mathbf{X}} = \beta_{0,Z_0\cdot Y_0\mathbf{X}} + (\beta_{0,Y_0\cdot\mathbf{X}} \times \beta_{1,Z_0\cdot Y_0\mathbf{X}}) \tag{C.1}$$

$$\beta_{1,Z_0\cdot\mathbf{X}} = \beta_{2,Z_0\cdot Y_0\mathbf{X}} + (\beta_{1,Y_0\cdot\mathbf{X}} \times \beta_{1,Z_0\cdot Y_0\mathbf{X}}) \tag{C.2}$$

$$\sigma^2_{Z_0\cdot\mathbf{X}} = \sigma^2_{Z_0\cdot Y_0\mathbf{X}} + \left(\sigma^2_{Y_0\cdot\mathbf{X}} \times \beta^2_{1,Z_0\cdot Y_0\mathbf{X}}\right) \tag{C.3}$$

$$\rho_{Y_0Z_0\cdot\mathbf{X}} = \beta_{1,Z_0\cdot Y_0\mathbf{X}} \times \frac{\sigma_{Y_0\cdot\mathbf{X}}}{\sigma_{Z_0\cdot\mathbf{X}}} \tag{C.4}$$

where $\beta_{0,Z_0\cdot\mathbf{X}}$, $\beta_{1,Z_0\cdot\mathbf{X}}$, and $\sigma^2_{Z_0\cdot\mathbf{X}}$ are the intercept, slope, and residual variance from $f(Z_0|\mathbf{X}, S = \ell)$ and $\rho_{Y_0Z_0\cdot\mathbf{X}}$ is the partial correlation of Y_0 and Z_0 given $\mathbf{X}$ in PREMIER.

C.2 Estimators for $f(Z_0 \mid Y_1, \mathbf{X})$

In Section 4.2 of the manuscript, we specified an informative prior for $\text{corr}(Y_1, Z_0 \mid \mathbf{X})$. Using this parameter, the parameters from $f(Y_1 \mid \mathbf{X})$, and the parameters from $f(Z_0 \mid \mathbf{X})$ (Section C.1) we can identify the parameters of the regression of Z_0 on Y_1, and $\mathbf{X}$. This regression model is written as

$$Z_0 \sim \mathrm{N}\left(\beta_{0,Z_0\cdot Y_1\mathbf{x}} + \beta_{1,Z_0\cdot Y_1\mathbf{x}}Y_1 + \beta_{2,Z_0\cdot Y_1\mathbf{x}}\mathbf{X}, \sigma^2_{Z_0\cdot Y_1\mathbf{x}}\right) \tag{C.5}$$

where

$$\beta_{1,Z_0\cdot Y_1\mathbf{x}} = \frac{\rho_{Y_1Z_0\cdot\mathbf{x}}\sigma_{Z_0\cdot\mathbf{x}}}{\sigma_{Y_1\cdot\mathbf{x}}} \tag{C.6}$$

$$\beta_{0,Z_0\cdot Y_1\mathbf{x}} = \beta_{0,Z_0\cdot\mathbf{x}} - (\beta_{1,Z_0\cdot Y_1\mathbf{x}} \times \beta_{0,Y_1\cdot\mathbf{x}}) \tag{C.7}$$

$$\beta_{2,Z_0\cdot Y_1\mathbf{x}} = \beta_{1,Z_0\cdot\mathbf{x}} - (\beta_{1,Z_0\cdot Y_1\mathbf{x}} \times \beta_{1,Y_1\cdot\mathbf{x}}) \tag{C.8}$$

$$\sigma^2_{Z_0\cdot Y_1\mathbf{x}} = \sigma^2_{Z_0\cdot\mathbf{x}}\left(1 - \rho^2_{Z_0Y_1\cdot\mathbf{x}}\right). \tag{C.9}$$

C.3 Estimators for $f(Z_1 \mid Y_1, Z_0, \mathbf{X})$

In Section 4.2 of the manuscript, we specified $\mathrm{corr}(Y_1, Z_0 \mid \mathbf{X})$ and $\mathrm{corr}(Z_1, Z_0 \mid Y_1, \mathbf{X})$. We obtain $f(Z_1 \mid Y_1, Z_0, \mathbf{X})$ using

$$f(Z_1 \mid Y_1, Z_0, \mathbf{X}) = \frac{f(Z_1, Z_0 \mid Y_1, Z_0, \mathbf{X})}{f(Z_0 \mid Y_1, \mathbf{X})}.$$

This regression model is written as:

$$Z_1 \sim N(\beta_{0,Z_1\cdot Y_1Z_0\mathbf{x}} + \beta_{1,Z_1\cdot Y_1Z_0\mathbf{x}}Y_1 + \beta_{2,Z_1\cdot Y_1Z_0\mathbf{x}}Z_0 + \beta_{3,Z_1\cdot Y_1Z_0\mathbf{x}}\mathbf{X}, \sigma^2_{Z_1\cdot Y_1Z_0\mathbf{x}}) \tag{C.10}$$

where

$$\beta_{2,Z_1\cdot Y_1 Z_0 \mathbf{X}} = \rho_{Z_1 Z_0 \cdot Y_1 \mathbf{X}} \times \frac{\sigma_{Z_1\cdot Y_1 \mathbf{X}}}{\sigma_{Z_0\cdot Y_1 \mathbf{X}}} \tag{C.11}$$

$$\beta_{0,Z_1\cdot Y_1 Z_0 \mathbf{X}} = \beta_{0,Z_1\cdot Y_1 \mathbf{X}} - (\beta_{0,Z_0\cdot Y_1 \mathbf{X}} \times \beta_{2,Z_1\cdot Y_1 Z_0 \mathbf{X}}) \tag{C.12}$$

$$\beta_{1,Z_1\cdot Y_1 Z_0 \mathbf{X}} = \beta_{1,Z_1\cdot Y_1 \mathbf{X}} - (\beta_{1,Z_0\cdot Y_1 \mathbf{X}} \times \beta_{2,Z_1\cdot Y_1 Z_0 \mathbf{X}}) \tag{C.13}$$

$$\beta_{3,Z_1\cdot Y_1 Z_0 \mathbf{X}} = \beta_{2,Z_1\cdot Y_1 \mathbf{X}} - (\beta_{2,Z_0\cdot Y_1 \mathbf{X}} \times \beta_{2,Z_1\cdot Y_1 Z_0 \mathbf{X}}) \tag{C.14}$$

$$\sigma^2_{Z_1\cdot Y_1 Z_0 \mathbf{X}} = \sigma^2_{Z_1\cdot Y_1 \mathbf{X}} \left(1 - \rho^2_{Z_1 Z_0\cdot Y_1 \mathbf{X}}\right). \tag{C.15}$$

The coefficients for the regression of Z_1 on Y_1 and $\mathbf{X}$ were defined in Equation 8 of the manuscript. The coefficients for the regression of Z_0 on Y_1 and $\mathbf{X}$ were defined in Section C.2.

D Treatment effects as a function of identified parameters and sensitivity parameters

We calculate the treatment effect in Equation 1 of the manuscript as a function of identified parameters and sensitivity parameters in order to better understand the impact of the sensitivity parameters on the treatment effect.

D.1 Treatment effect under calibration model invariance

Using Equation C.5, the mean of Z_0 for treatment $D = d$ is

$$
\begin{aligned}
E(Z_0 \mid D = d) &= E_{\mathbf{X} \mid D=d}[E_{Y_0 \mid \mathbf{X}, D=d} \{E(Z_0 \mid Y_0, \mathbf{X}, D = d)\}] \\
&= E_{\mathbf{X} \mid D=d} \left\{E_{Y_0 \mid \mathbf{X}, D=d}(\beta_{0,Z_0 \cdot Y_0 \mathbf{X}} + \beta_{1,Z_0 \cdot Y_0 \mathbf{X}} Y_0 + \beta_{2,Z_0 \cdot Y_0 \mathbf{X}} \mathbf{X} \mid D = d)\right\} \\
&= E_{\mathbf{X} \mid D=d} \{\beta_{0,Z_0 \cdot Y_0 \mathbf{X}} + \beta_{1,Z_0 \cdot Y_0 \mathbf{X}}(\beta_{0,Y_0 \cdot \mathbf{X}} + \beta_{1,Y_0 \cdot \mathbf{X}} \mathbf{X}) + \beta_{2,Z_0 \cdot Y_0 \mathbf{X}} \mathbf{X} \mid D = d\} \\
&= \beta_{0,Z_0 \cdot Y_0 \mathbf{X}} + \beta_{1,Z_0 \cdot Y_0 \mathbf{X}}(\beta^{(d)}_{0,Y_0 \cdot \mathbf{X}} + \beta^{(d)}_{1,Y_0 \cdot \mathbf{X}} \mu^{(d)}_{\mathbf{X}}) + \beta_{2,Z_0 \cdot Y_0 \mathbf{X}} \mu^{(d)}_{\mathbf{X}} \\
&= \beta_{0,Z_0 \cdot Y_0 \mathbf{X}} + \beta_{1,Z_0 \cdot Y_0 \mathbf{X}} \mu^{(d)}_{Y_0} + \beta_{2,Z_0 \cdot Y_0 \mathbf{X}} \mu^{(d)}_{\mathbf{X}} \qquad \text{(D.1)}
\end{aligned}
$$

where $\beta^{(d)}_{0,Y_0 \cdot \mathbf{X}}$ and $\beta^{(d)}_{1,Y_0 \cdot \mathbf{X}}$ are the intercept and slope of the regression of Y_0 on $\mathbf{X}$ in treatment group d, respectively, and $\mu^{(d)}_{Y_0}$ and $\mu^{(d)}_{\mathbf{X}}$ are the means of Y_0 and $\mathbf{X}$ in treatment group d, respectively. Using Equations C.5 and C.10, the mean of Z_1 is

$$
\begin{aligned}
E(Z_1) &= E_{\mathbf{X}} E_{Y_1 \mid \mathbf{X}} \left[E_{Z_0 \mid Y_1 \mathbf{X}} \left\{ E(Z_1 \mid Y_1, Z_0, \mathbf{X}) \right\} \right] \\
&= E_{\mathbf{X}} E_{Y_1 \mid \mathbf{X}} \left\{ E_{Z_0 \mid Y_1 \mathbf{X}} (\beta_{0,Z_1 \cdot Y_1 Z_0 \mathbf{X}} + \beta_{1,Z_1 \cdot Y_1 Z_0 \mathbf{X}} Y_1 + \beta_{2,Z_1 \cdot Y_1 Z_0 \mathbf{X}} Z_0 + \beta_{3,Z_1 \cdot Y_1 Z_0 \mathbf{X}} \mathbf{X}) \right\} \\
&= E_{\mathbf{X}} E_{Y_1 \mid \mathbf{X}} \left\{ \beta_{0,Z_1 \cdot Y_1 Z_0 \mathbf{X}} + \beta_{1,Z_1 \cdot Y_1 Z_0 \mathbf{X}} Y_1 + \beta_{2,Z_1 \cdot Y_1 Z_0 \mathbf{X}} (\beta_{0,Z_0 \cdot Y_1 \mathbf{X}} + \beta_{1,Z_0 \cdot Y_1 \mathbf{X}} Y_1 \right. \\
&\left. + \beta_{2,Z_0 \cdot Y_1 \mathbf{X}} \mathbf{X}) + \beta_{3,Z_1 \cdot Y_1 Z_0 \mathbf{X}} \mathbf{X} \right\} \\
&= E_{\mathbf{X}} [\beta_{0,Z_1 \cdot Y_1 Z_0 \mathbf{X}} + \beta_{1,Z_1 \cdot Y_1 Z_0 \mathbf{X}} (\beta_{0,Y_1 \cdot \mathbf{X}} + \beta_{1,Y_1 \cdot \mathbf{X}} \mathbf{X}) + \beta_{2,Z_1 \cdot Y_1 Z_0 \mathbf{X}} \left\{ \beta_{0,Z_0 \cdot Y_1 \mathbf{X}} \right. \\
&\left. + \beta_{1,Z_0 \cdot Y_1 \mathbf{X}} (\beta_{0,Y_1 \cdot \mathbf{X}} + \beta_{1,Y_1 \cdot \mathbf{X}} \mathbf{X}) + \beta_{2,Z_0 \cdot Y_1 \mathbf{X}} \mathbf{X} \right\} + \beta_{3,Z_1 \cdot Y_1 Z_0 \mathbf{X}} \mathbf{X}] \\
&= \beta_{0,Z_1 \cdot Y_1 Z_0 \mathbf{X}} + \beta_{1,Z_1 \cdot Y_1 Z_0 \mathbf{X}} (\beta_{0,Y_1 \cdot \mathbf{X}} + \beta_{1,Y_1 \cdot \mathbf{X}} \mu_{\mathbf{X}}) + \beta_{2,Z_1 \cdot Y_1 Z_0 \mathbf{X}} \left\{ \beta_{0,Z_0 \cdot Y_1 \mathbf{X}} \right. \\
&\left. + \beta_{1,Z_0 \cdot Y_1 \mathbf{X}} (\beta_{0,Y_1 \cdot \mathbf{X}} + \beta_{1,Y_1 \cdot \mathbf{X}} \mu_{\mathbf{X}}) + \beta_{2,Z_0 \cdot Y_1 \mathbf{X}} \mu_{\mathbf{X}} \right\} + \beta_{3,Z_1 \cdot Y_1 Z_0 \mathbf{X}} \mu_{\mathbf{X}} \\
&= \beta_{0,Z_1 \cdot Y_1 Z_0 \mathbf{X}} + \beta_{1,Z_1 \cdot Y_1 Z_0 \mathbf{X}} \mu_{Y_1} + \beta_{2,Z_1 \cdot Y_1 Z_0 \mathbf{X}} (\beta_{0,Z_0 \cdot Y_1 \mathbf{X}} + \beta_{1,Z_0 \cdot Y_1 \mathbf{X}} \mu_{Y_1} \\
&+ \beta_{2,Z_0 \cdot Y_1 \mathbf{X}} \mu_{\mathbf{X}}) + \beta_{3,Z_1 \cdot Y_1 Z_0 \mathbf{X}} \mu_{\mathbf{X}}. \qquad \text{(D.2)}
\end{aligned}
$$

From Equations C.12, C.13, and C.14 we have:

$$\beta_{0,Z_1 \cdot Y_1 \mathbf{X}} - \beta_{0,Z_1 \cdot Y_1 Z_0 \mathbf{X}} = (\beta_{0,Z_0 \cdot Y_1 \mathbf{X}} \times \beta_{2,Z_1 \cdot Y_1 Z_0 \mathbf{X}}) \qquad \text{(D.3)}$$

$$\beta_{1,Z_1 \cdot Y_1 \mathbf{X}} - \beta_{1,Z_1 \cdot Y_1 Z_0 \mathbf{X}} = (\beta_{1,Z_0 \cdot Y_1 \mathbf{X}} \times \beta_{2,Z_1 \cdot Y_1 Z_0 \mathbf{X}}) \qquad \text{(D.4)}$$

$$\beta_{2,Z_1 \cdot Y_1 \mathbf{X}} - \beta_{3,Z_1 \cdot Y_1 Z_0 \mathbf{X}} = (\beta_{2,Z_0 \cdot Y_1 \mathbf{X}} \times \beta_{2,Z_1 \cdot Y_1 Z_0 \mathbf{X}}). \qquad \text{(D.5)}$$

Substituting Equations D.3, D.4, and D.5 into Equation D.2 gives

$$E(Z_1) = \beta_{0,Z_1 \cdot Y_1 \mathbf{X}} + \beta_{1,Z_1 \cdot Y_1 \mathbf{X}} \mu_{Y_1} + \beta_{2,Z_1 \cdot Y_1 \mathbf{X}} \mu_{\mathbf{X}},$$

so that

$$E(Z_1 \mid D = d) = \beta^{(d)}_{0,Z_1 \cdot Y_1 \mathbf{X}} + \beta^{(d)}_{1,Z_1 \cdot Y_1 \mathbf{X}} \mu^{(d)}_{Y_1} + \beta^{(d)}_{2,Z_1 \cdot Y_1 \mathbf{X}} \mu^{(d)}_{\mathbf{X}} \qquad \text{(D.6)}$$

where the superscripts on the regression coefficients indicate treatment condition.

Using Equations D.1 and D.6, the change from baseline to follow-up for treatment $D = d$ is

$$E(Z_1 - Z_0 \mid D = d) = (\beta^{(d)}_{0,Z_1\cdot Y_1\mathbf{X}} + \beta^{(d)}_{1,Z_1\cdot Y_1\mathbf{X}}\mu^{(d)}_{Y_1} + \beta^{(d)}_{2,Z_1\cdot Y_1\mathbf{X}}\mu^{(d)}_{\mathbf{X}}) - (\beta_{0,Z_0\cdot Y_0\mathbf{X}} + \beta_{1,Z_0\cdot Y_0\mathbf{X}}\mu^{(d)}_{Y_0} + \beta_{2,Z_0\cdot Y_0\mathbf{X}}\mu^{(d)}_{\mathbf{X}})$$

and the intervention effect (Equation 1 in the manuscript) is thus

$$\begin{aligned}\psi &= E(Z_1 - Z_0 \mid D = 1) - E(Z_1 - Z_0 \mid D = 0)\\ &= \big\{(\beta^{(1)}_{0,Z_1\cdot Y_1\mathbf{X}} + \beta^{(1)}_{1,Z_1\cdot Y_1\mathbf{X}}\mu^{(1)}_{Y_1} + \beta^{(1)}_{2,Z_1\cdot Y_1\mathbf{X}}\mu^{(1)}_{\mathbf{X}}) - (\beta_{0,Z_0\cdot Y_0\mathbf{X}} + \beta_{1,Z_0\cdot Y_0\mathbf{X}}\mu^{(1)}_{Y_0} + \beta_{2,Z_0\cdot Y_0\mathbf{X}}\mu^{(1)}_{\mathbf{X}})\big\}\\ &\quad - \big\{(\beta^{(0)}_{0,Z_1\cdot Y_1\mathbf{X}} + \beta^{(0)}_{1,Z_1\cdot Y_1\mathbf{X}}\mu^{(0)}_{Y_1} + \beta^{(0)}_{2,Z_1\cdot Y_1\mathbf{X}}\mu^{(0)}_{\mathbf{X}}) - (\beta_{0,Z_0\cdot Y_0\mathbf{X}} + \beta_{1,Z_0\cdot Y_0\mathbf{X}}\mu^{(0)}_{Y_0} + \beta_{2,Z_0\cdot Y_0\mathbf{X}}\mu^{(0)}_{\mathbf{X}})\big\}.\end{aligned} \tag{D.7}$$

Assuming that $E(Y_0 \mid D = 1) = E(Y_0 \mid D = 0)$ and $E(\mathbf{X} \mid D = 1) = E(\mathbf{X} \mid D = 0)$ due to randomization, Equation D.7 reduces to

$$\psi = (\beta^{(1)}_{0,Z_1\cdot Y_1\mathbf{X}} + \beta^{(1)}_{1,Z_1\cdot Y_1\mathbf{X}}\mu^{(1)}_{Y_1} + \beta^{(1)}_{2,Z_1\cdot Y_1\mathbf{X}}\mu_{\mathbf{X}}) - (\beta^{(0)}_{0,Z_1\cdot Y_1\mathbf{X}} + \beta^{(0)}_{1,Z_1\cdot Y_1\mathbf{X}}\mu^{(0)}_{Y_1} + \beta^{(0)}_{2,Z_1\cdot Y_1\mathbf{X}}\mu_{\mathbf{X}}). \tag{D.8}$$

Under calibration model invariance with respect to treatment and time we have

$$\begin{aligned}\beta^{(1)}_{0,Z_1\cdot Y_1\mathbf{X}} &= \beta^{(0)}_{0,Z_1\cdot Y_1\mathbf{X}} = \beta_{0,Z_0\cdot Y_0\mathbf{X}}\\ \beta^{(1)}_{1,Z_1\cdot Y_1\mathbf{X}} &= \beta^{(0)}_{1,Z_1\cdot Y_1\mathbf{X}} = \beta_{1,Z_0\cdot Y_0\mathbf{X}}\\ \beta^{(1)}_{2,Z_1\cdot Y_1\mathbf{X}} &= \beta^{(0)}_{2,Z_1\cdot Y_1\mathbf{X}} = \beta_{2,Z_0\cdot Y_0\mathbf{X}}\end{aligned}$$

such that Equation D.8 is now

$$\begin{aligned}\psi &= (\beta_{0,Z_0\cdot Y_0\mathbf{X}} + \beta_{1,Z_0\cdot Y_0\mathbf{X}}\mu^{(1)}_{Y_1}) - (\beta_{0,Z_0\cdot Y_0\mathbf{X}} + \beta_{1,Z_0\cdot Y_0\mathbf{X}}\mu^{(0)}_{Y_1})\\ &= \beta_{1,Z_0\cdot Y_0\mathbf{X}}(\mu^{(1)}_{Y_1} - \mu^{(0)}_{Y_1}).\end{aligned}$$

D.2 Treatment effect under intercept sensitivity parameters

Departures from calibration model invariance with respect to treatment and time in terms of the intercept parameters in Equation D.8 are based on the following reparameterizations in Section 5.1 of the manuscript:

$$\beta^{(d)}_{0,Z_1 \cdot Y_1 \mathbf{x}} = \beta_{0,Z_0 \cdot Y_0 \mathbf{x}} + \Delta^{(d)}_{\beta_0} \tag{D.9}$$

$$\beta^{(d)}_{1,Z_1 \cdot Y_1 \mathbf{x}} = \beta_{1,Z_0 \cdot Y_0 \mathbf{x}} \tag{D.10}$$

$$\beta^{(d)}_{2,Z_1 \cdot Y_1 \mathbf{x}} = \beta_{2,Z_0 \cdot Y_0 \mathbf{x}}. \tag{D.11}$$

Substituting Equations D.9, D.10, and D.11 into Equation D.8 gives

$$\begin{aligned} \psi &= (\beta_{0,Z_0 \cdot Y_0 \mathbf{x}} + \Delta^{(1)}_{\beta_0} + \beta_{1,Z_1 \cdot Y_1 \mathbf{x}} \mu^{(1)}_{Y_1}) - (\beta_{0,Z_1 \cdot Y_1 \mathbf{x}} + \Delta^{(0)}_{\beta_0} + \beta_{1,Z_1 \cdot Y_1 \mathbf{x}} \mu^{(0)}_{Y_1}) \\ &= \big(\Delta^{(1)}_{\beta_0} - \Delta^{(0)}_{\beta_0}\big) + \beta_{1,Z_1 \cdot Y_1 \mathbf{x}} \big(\mu^{(1)}_{Y_1} - \mu^{(0)}_{Y_1}\big). \end{aligned} \tag{D.12}$$

Equation D.12 makes it clear that the intercept sensitivity parameters only influence the treatment effect when the calibration model is treatment-varying, that is $\Delta^{(1)}_{\beta_0} \neq \Delta^{(0)}_{\beta_0}$. Also it is the *difference* in the two sensitivity parameters that drives the treatment effect, not the individual values themselves. This is clear from Figure 1(a) in the manuscript where the effect sizes are constant across values of $\Delta^{(1)}_{\beta_0} - \Delta^{(0)}_{\beta_0}$.

D.3 Treatment effect under slope sensitivity parameters

Departures from calibration model invariance with respect to treatment and time in terms of the slope parameters in Equation D.8 are based on the reparameterizations in Section 5.2

the manuscript:

$$\beta^{(d)}_{0,Z_1\cdot Y_1\mathbf{X}} = \beta_{0,Z_0\cdot Y_0\mathbf{X}} + (1 - \Delta^{(d)}_{\beta_1}) \times \beta_{1,Z_0\cdot Y_0\mathbf{X}} \times \text{Target} \tag{D.13}$$

$$\beta^{(d)}_{1,Z_1\cdot Y_1\mathbf{X}} = \Delta^{(d)}_{\beta_1} \times \beta_{1,Z_0\cdot Y_0\mathbf{X}} \tag{D.14}$$

$$\beta^{(d)}_{2,Z_1\cdot Y_1\mathbf{X}} = \beta_{2,Z_0\cdot Y_0\mathbf{X}} \tag{D.15}$$

where Target represents the intervention target or goal. Before calculating the treatment effect, it is helpful to calculate $E(Z_1|D = d)$ as a function of the slope sensitivity parameters. Substituting Equations D.13, D.14 and D.15 into Equation D.6 gives

$$\begin{aligned} E(Z_1 \mid D = d, \Delta^{(d)}_{\beta_1}) &= \beta_{0,Z_0\cdot Y_0\mathbf{X}} + \big\{(1 - \Delta^{(d)}_{\beta_1}) \times \beta_{1,Z_0\cdot Y_0\mathbf{X}} \times \text{Target}\big\} \\ &+ \big(\Delta^{(d)}_{\beta_1} \times \beta_{1,Z_0\cdot Y_0\mathbf{X}}\mu^{(d)}_{Y_1}\big) + \beta_{2,Z_0\cdot Y_0\mathbf{X}}\mu^{(d)}_{\mathbf{X}} \\ &= \beta_{0,Z_0\cdot Y_0\mathbf{X}} + \big(\beta_{1,Z_0\cdot Y_0\mathbf{X}} \times \text{Target}\big) \\ &+ \big\{\Delta^{(d)}_{\beta_1} \times \beta_{1,Z_0\cdot Y_0\mathbf{X}}(\mu^{(d)}_{Y_1} - \text{Target})\big\} + \beta_{2,Z_0\cdot Y_0\mathbf{X}}\mu^{(d)}_{\mathbf{X}}. \end{aligned} \tag{D.16}$$

Under calibration model invariance, Equation D.16 is

$$E(Z_1 \mid D = d, \Delta^{(d)}_{\beta_1} = 1) = \beta_{0,Z_0\cdot Y_0\mathbf{X}} + \beta_{1,Z_0\cdot Y_0\mathbf{X}}\mu^{(d)}_{Y_1} + \beta_{2,Z_0\cdot Y_0\mathbf{X}}\mu^{(d)}_{\mathbf{X}}. \tag{D.17}$$

The difference in the mean of Z_1 in treatment group d under an invariant and varying calibration model in terms of the slope sensitivity parameter is obtained by subtracting Equation D.17 from Equation D.16:

$$E(Z_1 \mid D = d, \Delta^{(d)}_{\beta_1}) - E(Z_1 \mid D = d, \Delta^{(d)}_{\beta_1} = 1) = \beta_{1,Z_0\cdot Y_0\mathbf{X}}\big(1 - \Delta^{(d)}_{\beta_1}\big)\big(\text{Target} - \mu^{(d)}_{Y_1}\big). \tag{D.18}$$

As Equation D.18 makes clear, when Target $> \mu^{(d)}_{Y_1}$, that is, the average intake at follow-up is less than the target intake (as was true in the PREMIER treatment condition), values of $\Delta_{\beta_1}^{(d)} > 1$ will decrease $E(Z_1 \mid D = d)$ as compared to calibration model invariance. When Target $< \mu_{Y_1}^{(d)}$, that is, the average intake at follow-up is greater than the target intake (as was true in the PREMIER control condition), the values of $\Delta_{\beta_1}^{(d)} > 1$ will increase $E(Z_1 \mid D = d)$ as compared to calibration model invariance. The result is that in our sensitivity analysis, measurement error corrected sodium intake at follow-up is smallest for the treatment group when $\Delta_{\beta_1}^{(1)} = 3$ and largest for the control group when $\Delta_{\beta_1}^{(0)} = 3$.

Finally, the treatment effect as a function of identified parameters and slope sensitivity parameters is obtained by substituting Equations D.13, D.14, and D.15 into Equation D.8:

$$\psi = \left\{\Delta_{\beta_1}^{(1)} \times \beta_{1,Z_0 \cdot Y_0 \mathbf{x}}\left(\mu_{Y_1}^{(1)} - \text{Target}\right)\right\} - \left\{\Delta_{\beta_1}^{(0)} \times \beta_{1,Z_0 \cdot Y_0 \mathbf{x}}\left(\mu_{Y_1}^{(0)} - \text{Target}\right)\right\}. \tag{D.19}$$

Thus, when the treatment group achieves the target (i.e. the self-reported mean is less than the target value) and the control group does not achieve the target, differences between treatment groups are largest when $\Delta_{\beta_1}^{(1)}$ and $\Delta_{\beta_1}^{(0)}$ are both greater than 1 or are both less than 1.

Because the treatment effect depends on the sensitivity parameters for both the treatment and control groups as well as whether the self-reported means in each of the treatment groups met the targeted intake, there is no straightforward function as there is for the intercept parameters. However, there are some special cases that can shed light on how the slope sensitivity parameters affect the treatment effect. Under calibration model invariance with respect to treatment ($\Delta_{\beta_1} = \Delta_{\beta_1}^{(1)} = \Delta_{\beta_1}^{(0)}$) Equation D.19 reduces to:

$$\psi = \Delta_{\beta_1} \times \beta_{1,Z_0 \cdot Y_0 \mathbf{x}}\left(\mu_{Y_1}^{(1)} - \mu_{Y_1}^{(0)}\right). \tag{D.20}$$

Relating this expression to the treatment effect under calibration model invariance with respect to treatment and time, the quantity $\Delta_{\beta_1} - 1$ is the proportion increase (or decrease) in the treatment effect under calibration model invariance with respect to treatment as compared to the treatment effect under calibration model invariance with respect to treatment and time.

When the self-reported mean in the treatment group at follow-up is equal to the target intake ($\mu_{Y_1}^{(1)} = Target$), Equation D.19 reduces to:

$$\psi = \Delta_{\beta_1}^{(0)} \times \beta_{1,Z_0 \cdot Y_0 \mathbf{x}} \big(\mu_{Y_1}^{(1)} - \mu_{Y_1}^{(0)} \big). \tag{D.21}$$

As in Equation D.20, the interpretation of $\Delta_{\beta_1}^{(0)} - 1$ in Equation D.21 is the proportion increase (or decrease) in the treatment effect under a time-varying calibration model as compared to the treatment effect under a time-invariant calibration model. Equation D.21, also highlights the fact that the influence of the slope sensitivity parameter for a given treatment condition is a function of how much the self-reported mean at follow-up deviates from the Target.

E Results from sensitivity analyses

In this section we provide greater detail from the sensitivity analyses reported in Section 6.2 of the manuscript including the values that were used to draw the contour plots in Figures 1 and 2 in the manuscript. We also provide additional results from the analyses based on self-reported sodium data in PREMIER.

E.1 Intercept sensitivity analysis results

Table 1 provides—for a range of intercept sensitivity parameters—point estimates for change in log sodium intake from baseline to 6-months for treatment and control groups as well as the difference in sodium reduction between the two treatment groups and its 95% confidence interval. Table 1 also includes the effect sizes and their associated p-values that were plotted in Figure 1 in the manuscript. The first row of Table 1 displays analyses based on the self-reported sodium data.

Negative values of the intercept sensitivity parameter imply less underreporting at follow-up as compared to baseline and result in a larger reduction in sodium intake. Positive values of the intercept sensitivity parameter imply greater undereporting at follow-up as compared to baseline and result in a smaller reduction in sodium intake. Also, as mentioned in Appendix D.2, the difference between treatment groups and the effect size are the same when the difference in treatment and control intercept sensitivity parameters are the same. As was seen in Figure 1 of the manuscript, differences and effect sizes are largest when the differences in treatment and control sensitivity parameters are largest. When $\Delta_{\beta_0}^{(1)} = -0.5$ and $\Delta_{\beta_0}^{(0)} = 0.5$, the difference in sodium reduction is -0.276, with an effect size of -0.949 favoring the treatment condition. In the opposite direction, when $\Delta_{\beta_0}^{(1)} = 0.5$ and $\Delta_{\beta_0}^{(0)} = -0.5$ the difference in sodium reduction is 0.214, with an effect size of 0.739 favoring the control condition.

Table 1: Results from the sensitivity analyses of the PREMIER data across a range of intercept sensitivity parameters. The first row of the table lists results from the analysis of self-reported data.

$\Delta^{(1)}_{\beta_0}$	$\Delta^{(1)}_{\beta_0}$	Tx Change	Ctrl Change	Group Diff	Lower 95% CI	Upper 95% CI	Effect Size	p-value
Self-report		-0.335	-0.105	-0.230	-0.303	-0.158	-0.489	<0.001
-0.5	-0.5	-0.166	-0.136	-0.031	-0.049	-0.012	-0.105	0.004
-0.5	-0.4	-0.166	-0.111	-0.055	-0.074	-0.036	-0.189	<0.001
-0.5	-0.3	-0.166	-0.087	-0.080	-0.099	-0.060	-0.274	<0.001
-0.5	-0.2	-0.166	-0.062	-0.104	-0.125	-0.083	-0.358	<0.001
-0.5	-0.1	-0.166	-0.038	-0.129	-0.152	-0.106	-0.442	<0.001
-0.5	0.0	-0.166	-0.013	-0.153	-0.178	-0.128	-0.527	<0.001
-0.5	0.1	-0.166	0.011	-0.178	-0.205	-0.150	-0.611	<0.001
-0.5	0.2	-0.166	0.036	-0.202	-0.232	-0.172	-0.696	<0.001
-0.5	0.3	-0.166	0.060	-0.227	-0.259	-0.194	-0.780	<0.001
-0.5	0.4	-0.166	0.085	-0.251	-0.287	-0.216	-0.864	<0.001
-0.5	0.5	-0.166	0.109	-0.276	-0.314	-0.237	-0.949	<0.001
-0.4	-0.5	-0.142	-0.136	-0.006	-0.024	0.012	-0.021	0.544
-0.4	-0.4	-0.142	-0.111	-0.031	-0.049	-0.012	-0.105	0.004
-0.4	-0.3	-0.142	-0.087	-0.055	-0.074	-0.036	-0.189	<0.001
-0.4	-0.2	-0.142	-0.062	-0.080	-0.099	-0.060	-0.274	<0.001
-0.4	-0.1	-0.142	-0.038	-0.104	-0.125	-0.083	-0.358	<0.001
-0.4	0.0	-0.142	-0.013	-0.129	-0.152	-0.106	-0.442	<0.001
-0.4	0.1	-0.142	0.011	-0.153	-0.178	-0.128	-0.527	<0.001
-0.4	0.2	-0.142	0.036	-0.178	-0.205	-0.150	-0.611	<0.001
-0.4	0.3	-0.142	0.060	-0.202	-0.232	-0.172	-0.696	<0.001
-0.4	0.4	-0.142	0.085	-0.227	-0.259	-0.194	-0.780	<0.001
-0.4	0.5	-0.142	0.109	-0.251	-0.287	-0.216	-0.864	<0.001
-0.3	-0.5	-0.117	-0.136	0.018	-0.001	0.037	0.064	0.082
-0.3	-0.4	-0.117	-0.111	-0.006	-0.024	0.012	-0.021	0.544
-0.3	-0.3	-0.117	-0.087	-0.031	-0.049	-0.012	-0.105	0.004
-0.3	-0.2	-0.117	-0.062	-0.055	-0.074	-0.036	-0.189	<0.001
-0.3	-0.1	-0.117	-0.038	-0.080	-0.099	-0.060	-0.274	<0.001
-0.3	0.0	-0.117	-0.013	-0.104	-0.125	-0.083	-0.358	<0.001
-0.3	0.1	-0.117	0.011	-0.129	-0.152	-0.106	-0.442	<0.001
-0.3	0.2	-0.117	0.036	-0.153	-0.178	-0.128	-0.527	<0.001
-0.3	0.3	-0.117	0.060	-0.178	-0.205	-0.150	-0.611	<0.001
-0.3	0.4	-0.117	0.085	-0.202	-0.232	-0.172	-0.696	<0.001
-0.3	0.5	-0.117	0.109	-0.227	-0.259	-0.194	-0.780	<0.001
-0.2	-0.5	-0.093	-0.136	0.043	0.023	0.063	0.148	<0.001
-0.2	-0.4	-0.093	-0.111	0.018	-0.001	0.037	0.064	0.082

Continued on next page

Table 1 – continued from previous page

$\Delta^{(1)}_{\beta_0}$	$\Delta^{(1)}_{\beta_0}$	Tx Change	Ctrl Change	Group Diff	Lower 95% CI	Upper 95% CI	Effect Size	p-value
-0.2	-0.3	-0.093	-0.087	-0.006	-0.024	0.012	-0.021	0.544
-0.2	-0.2	-0.093	-0.062	-0.031	-0.049	-0.012	-0.105	0.004
-0.2	-0.1	-0.093	-0.038	-0.055	-0.074	-0.036	-0.189	<0.001
-0.2	0.0	-0.093	-0.013	-0.080	-0.099	-0.060	-0.274	<0.001
-0.2	0.1	-0.093	0.011	-0.104	-0.125	-0.083	-0.358	<0.001
-0.2	0.2	-0.093	0.036	-0.129	-0.152	-0.106	-0.442	<0.001
-0.2	0.3	-0.093	0.060	-0.153	-0.178	-0.128	-0.527	<0.001
-0.2	0.4	-0.093	0.085	-0.178	-0.205	-0.150	-0.611	<0.001
-0.2	0.5	-0.093	0.109	-0.202	-0.232	-0.172	-0.696	<0.001
-0.1	-0.5	-0.068	-0.136	0.067	0.046	0.089	0.233	<0.001
-0.1	-0.4	-0.068	-0.111	0.043	0.023	0.063	0.148	<0.001
-0.1	-0.3	-0.068	-0.087	0.018	-0.001	0.037	0.064	0.082
-0.1	-0.2	-0.068	-0.062	-0.006	-0.024	0.012	-0.021	0.544
-0.1	-0.1	-0.068	-0.038	-0.031	-0.049	-0.012	-0.105	0.004
-0.1	0.0	-0.068	-0.013	-0.055	-0.074	-0.036	-0.189	<0.001
-0.1	0.1	-0.068	0.011	-0.080	-0.099	-0.060	-0.274	<0.001
-0.1	0.2	-0.068	0.036	-0.104	-0.125	-0.083	-0.358	<0.001
-0.1	0.3	-0.068	0.060	-0.129	-0.152	-0.106	-0.442	<0.001
-0.1	0.4	-0.068	0.085	-0.153	-0.178	-0.128	-0.527	<0.001
-0.1	0.5	-0.068	0.109	-0.178	-0.205	-0.150	-0.611	<0.001
0.0	-0.5	-0.044	-0.136	0.092	0.068	0.115	0.317	<0.001
0.0	-0.4	-0.044	-0.111	0.067	0.046	0.089	0.233	<0.001
0.0	-0.3	-0.044	-0.087	0.043	0.023	0.063	0.148	<0.001
0.0	-0.2	-0.044	-0.062	0.018	-0.001	0.037	0.064	0.082
0.0	-0.1	-0.044	-0.038	-0.006	-0.024	0.012	-0.021	0.544
0.0	0.0	-0.044	-0.013	-0.031	-0.049	-0.012	-0.105	0.004
0.0	0.1	-0.044	0.011	-0.055	-0.074	-0.036	-0.189	<0.001
0.0	0.2	-0.044	0.036	-0.080	-0.099	-0.060	-0.274	<0.001
0.0	0.3	-0.044	0.060	-0.104	-0.125	-0.083	-0.358	<0.001
0.0	0.4	-0.044	0.085	-0.129	-0.152	-0.106	-0.442	<0.001
0.0	0.5	-0.044	0.109	-0.153	-0.178	-0.128	-0.527	<0.001
0.1	-0.5	-0.019	-0.136	0.116	0.091	0.142	0.401	<0.001
0.1	-0.4	-0.019	-0.111	0.092	0.068	0.115	0.317	<0.001
0.1	-0.3	-0.019	-0.087	0.067	0.046	0.089	0.233	<0.001
0.1	-0.2	-0.019	-0.062	0.043	0.023	0.063	0.148	<0.001
0.1	-0.1	-0.019	-0.038	0.018	-0.001	0.037	0.064	0.082
0.1	0.0	-0.019	-0.013	-0.006	-0.024	0.012	-0.021	0.544
0.1	0.1	-0.019	0.011	-0.031	-0.049	-0.012	-0.105	0.004
0.1	0.2	-0.019	0.036	-0.055	-0.074	-0.036	-0.189	<0.001

Continued on next page

Table 1 – continued from previous page

$\Delta_{\beta_0}^{(1)}$	$\Delta_{\beta_0}^{(1)}$	Tx Change	Ctrl Change	Group Diff	Lower 95% CI	Upper 95% CI	Effect Size	p-value
0.1	0.3	-0.019	0.060	-0.080	-0.099	-0.060	-0.274	<0.001
0.1	0.4	-0.019	0.085	-0.104	-0.125	-0.083	-0.358	<0.001
0.1	0.5	-0.019	0.109	-0.129	-0.152	-0.106	-0.442	<0.001
0.2	-0.5	0.005	-0.136	0.141	0.113	0.169	0.486	<0.001
0.2	-0.4	0.005	-0.111	0.116	0.091	0.142	0.401	<0.001
0.2	-0.3	0.005	-0.087	0.092	0.068	0.115	0.317	<0.001
0.2	-0.2	0.005	-0.062	0.067	0.046	0.089	0.233	<0.001
0.2	-0.1	0.005	-0.038	0.043	0.023	0.063	0.148	<0.001
0.2	0.0	0.005	-0.013	0.018	-0.001	0.037	0.064	0.082
0.2	0.1	0.005	0.011	-0.006	-0.024	0.012	-0.021	0.544
0.2	0.2	0.005	0.036	-0.031	-0.049	-0.012	-0.105	0.004
0.2	0.3	0.005	0.060	-0.055	-0.074	-0.036	-0.189	<0.001
0.2	0.4	0.005	0.085	-0.080	-0.099	-0.060	-0.274	<0.001
0.2	0.5	0.005	0.109	-0.104	-0.125	-0.083	-0.358	<0.001
0.3	-0.5	0.030	-0.136	0.165	0.135	0.196	0.570	<0.001
0.3	-0.4	0.030	-0.111	0.141	0.113	0.169	0.486	<0.001
0.3	-0.3	0.030	-0.087	0.116	0.091	0.142	0.401	<0.001
0.3	-0.2	0.030	-0.062	0.092	0.068	0.115	0.317	<0.001
0.3	-0.1	0.030	-0.038	0.067	0.046	0.089	0.233	<0.001
0.3	0.0	0.030	-0.013	0.043	0.023	0.063	0.148	<0.001
0.3	0.1	0.030	0.011	0.018	-0.001	0.037	0.064	0.082
0.3	0.2	0.030	0.036	-0.006	-0.024	0.012	-0.021	0.544
0.3	0.3	0.030	0.060	-0.031	-0.049	-0.012	-0.105	0.004
0.3	0.4	0.030	0.085	-0.055	-0.074	-0.036	-0.189	<0.001
0.3	0.5	0.030	0.109	-0.080	-0.099	-0.060	-0.274	<0.001
0.4	-0.5	0.054	-0.136	0.190	0.156	0.223	0.654	<0.001
0.4	-0.4	0.054	-0.111	0.165	0.135	0.196	0.570	<0.001
0.4	-0.3	0.054	-0.087	0.141	0.113	0.169	0.486	<0.001
0.4	-0.2	0.054	-0.062	0.116	0.091	0.142	0.401	<0.001
0.4	-0.1	0.054	-0.038	0.092	0.068	0.115	0.317	<0.001
0.4	0.0	0.054	-0.013	0.067	0.046	0.089	0.233	<0.001
0.4	0.1	0.054	0.011	0.043	0.023	0.063	0.148	<0.001
0.4	0.2	0.054	0.036	0.018	-0.001	0.037	0.064	0.082
0.4	0.3	0.054	0.060	-0.006	-0.024	0.012	-0.021	0.544
0.4	0.4	0.054	0.085	-0.031	-0.049	-0.012	-0.105	0.004
0.4	0.5	0.054	0.109	-0.055	-0.074	-0.036	-0.189	<0.001
0.5	-0.5	0.079	-0.136	0.214	0.178	0.251	0.739	<0.001
0.5	-0.4	0.079	-0.111	0.190	0.156	0.223	0.654	<0.001
0.5	-0.3	0.079	-0.087	0.165	0.135	0.196	0.570	<0.001

Continued on next page

Table 1 – continued from previous page

$\Delta_{\beta_0}^{(1)}$	$\Delta_{\beta_0}^{(1)}$	Tx Change	Ctrl Change	Group Diff	Lower 95% CI	Upper 95% CI	Effect Size	p-value
0.5	-0.2	0.079	-0.062	0.141	0.113	0.169	0.486	$<$0.001
0.5	-0.1	0.079	-0.038	0.116	0.091	0.142	0.401	$<$0.001
0.5	0.0	0.079	-0.013	0.092	0.068	0.115	0.317	$<$0.001
0.5	0.1	0.079	0.011	0.067	0.046	0.089	0.233	$<$0.001
0.5	0.2	0.079	0.036	0.043	0.023	0.063	0.148	$<$0.001
0.5	0.3	0.079	0.060	0.018	-0.001	0.037	0.064	0.082
0.5	0.4	0.079	0.085	-0.006	-0.024	0.012	-0.021	0.544
0.5	0.5	0.079	0.109	-0.031	-0.049	-0.012	-0.105	0.004

E.2 Slope sensitivity analysis results

Table 2 provides—for a range of slope sensitivity parameters—point estimates for change in log sodium intake from baseline to 6-months for PREMIER treatment and control groups as well as the difference in sodium reduction between the two treatment groups and its 95% confidence interval. Table 2 also includes effect sizes and their associated p-values that were plotted in Figure 2 in the manuscript. The first row of Table 2 displays analyses based on the self-reported sodium data.

As was shown in Equation D.18, in the treatment group—where the mean sodium intake at follow-up was less than the Target intake, values of the slope sensitivity parameter greater than 1 decrease mean intake at follow-up and result in a greater reduction in sodium intake as compared to calibration model invariance with respect to treatment. In the control group—where the mean sodium intake at follow-up was greater than the Target intake, values of the slope sensitivity parameter greater than 1 increase mean intake at follow-up and result in a smaller reduction in sodium intake as compared to calibration model invariance with respect to time.

For example, the reduction in sodium intake in the treatment condition under calibration model invariance ($\Delta_{\beta_1}^{(1)} = 1$) is -0.045. When ($\Delta_{\beta_1}^{(1)} = 3$), the reduction is -0.072. The reduction in sodium intake in the control condition under calibration model invariance ($\Delta^{(0)}_{\beta_1} = 1$) is -0.014. When ($\Delta^{(0)}_{\beta_1} = 3$), there is an *increase* in sodium intake such that the change from baseline to month=6 is equal 0.013. Thus, the difference in change between the two conditions is largest when $\Delta^{(1)}_{\beta_1} = 3$ and $\Delta^{(0)}_{\beta_1} = 3$. This is displayed in the last row of Table 2 where the effect size is -0.256.

Table 2: Results from the sensitivity analyses of the PREMIER data across a range of slope sensitivity parameters. The first row of the table lists results from the analysis of self-reported data.

$\Delta^{(1)}_{\beta_1}$	$\Delta^{(1)}_{\beta_1}$	Tx Change	Ctrl Change	Group Diff	Lower 95% CI	Upper 95% CI	Effect Size	p-value
Self-report		-0.335	-0.105	-0.230	-0.303	-0.158	-0.489	<0.001
0.3	0.3	-0.036	-0.022	-0.014	-0.034	0.007	-0.047	0.045
0.3	0.4	-0.036	-0.022	-0.015	-0.032	0.003	-0.051	0.036
0.3	0.5	-0.036	-0.020	-0.016	-0.031	-0.001	-0.055	0.026
0.3	0.7	-0.036	-0.018	-0.018	-0.031	-0.005	-0.063	0.016
0.3	0.8	-0.036	-0.016	-0.020	-0.033	-0.007	-0.069	0.012
0.3	1.0	-0.036	-0.014	-0.022	-0.037	-0.008	-0.078	0.007
0.3	1.2	-0.036	-0.010	-0.026	-0.042	-0.010	-0.089	0.005
0.3	1.5	-0.036	-0.007	-0.029	-0.047	-0.011	-0.100	0.004
0.3	2.0	-0.036	-0.001	-0.036	-0.058	-0.013	-0.121	0.003
0.3	2.5	-0.036	0.006	-0.042	-0.070	-0.015	-0.142	0.002
0.3	3.0	-0.036	0.013	-0.049	-0.081	-0.016	-0.161	0.002
0.4	0.3	-0.037	-0.022	-0.015	-0.032	0.003	-0.051	0.036
0.4	0.4	-0.037	-0.022	-0.016	-0.031	-0.000	-0.054	0.029
0.4	0.5	-0.037	-0.020	-0.017	-0.031	-0.003	-0.058	0.021
0.4	0.7	-0.037	-0.018	-0.019	-0.032	-0.006	-0.066	0.013
0.4	0.8	-0.037	-0.016	-0.021	-0.034	-0.007	-0.072	0.010
0.4	1.0	-0.037	-0.014	-0.023	-0.038	-0.009	-0.081	0.006
0.4	1.2	-0.037	-0.010	-0.027	-0.043	-0.010	-0.092	0.004
0.4	1.5	-0.037	-0.007	-0.030	-0.048	-0.012	-0.103	0.003
0.4	2.0	-0.037	-0.001	-0.037	-0.060	-0.014	-0.124	0.002
0.4	2.5	-0.037	0.006	-0.043	-0.071	-0.015	-0.145	0.002
0.4	3.0	-0.037	0.013	-0.050	-0.083	-0.017	-0.164	0.002
0.5	0.3	-0.038	-0.022	-0.016	-0.031	-0.001	-0.055	0.027
0.5	0.4	-0.038	-0.022	-0.017	-0.031	-0.003	-0.059	0.022
0.5	0.5	-0.038	-0.020	-0.018	-0.032	-0.005	-0.063	0.016
0.5	0.7	-0.038	-0.018	-0.020	-0.034	-0.007	-0.071	0.010
0.5	0.8	-0.038	-0.016	-0.022	-0.036	-0.008	-0.077	0.008

Continued on next page

Table 2 – continued from previous page

$\Delta_{\beta_1}^{(1)}$	$\Delta_{\beta_1}^{(1)}$	Tx Change	Ctrl Change	Group Diff	Lower 95% CI	Upper 95% CI	Effect Size	p-value
0.5	1.0	-0.038	-0.014	-0.025	-0.040	-0.010	-0.086	0.005
0.5	1.2	-0.038	-0.010	-0.028	-0.045	-0.011	-0.097	0.004
0.5	1.5	-0.038	-0.007	-0.031	-0.050	-0.012	-0.107	0.003
0.5	2.0	-0.038	-0.001	-0.038	-0.061	-0.014	-0.129	0.002
0.5	2.5	-0.038	0.006	-0.044	-0.073	-0.016	-0.149	0.002
0.5	3.0	-0.038	0.013	-0.051	-0.085	-0.018	-0.168	0.002
0.7	0.3	-0.041	-0.022	-0.018	-0.031	-0.005	-0.063	0.017
0.7	0.4	-0.041	-0.022	-0.019	-0.032	-0.006	-0.066	0.014
0.7	0.5	-0.041	-0.020	-0.020	-0.034	-0.007	-0.071	0.011
0.7	0.7	-0.041	-0.018	-0.023	-0.037	-0.009	-0.079	0.007
0.7	0.8	-0.041	-0.016	-0.024	-0.039	-0.010	-0.084	0.005
0.7	1.0	-0.041	-0.014	-0.027	-0.043	-0.011	-0.093	0.004
0.7	1.2	-0.041	-0.010	-0.030	-0.048	-0.013	-0.104	0.003
0.7	1.5	-0.041	-0.007	-0.034	-0.054	-0.014	-0.115	0.002
0.7	2.0	-0.041	-0.001	-0.040	-0.065	-0.016	-0.136	0.002
0.7	2.5	-0.041	0.006	-0.047	-0.076	-0.017	-0.156	0.002
0.7	3.0	-0.041	0.013	-0.053	-0.088	-0.019	-0.176	0.002
0.8	0.3	-0.042	-0.022	-0.020	-0.033	-0.007	-0.069	0.013
0.8	0.4	-0.042	-0.022	-0.021	-0.034	-0.008	-0.072	0.010
0.8	0.5	-0.042	-0.020	-0.022	-0.036	-0.009	-0.077	0.008
0.8	0.7	-0.042	-0.018	-0.024	-0.039	-0.010	-0.084	0.006
0.8	0.8	-0.042	-0.016	-0.026	-0.041	-0.011	-0.090	0.004
0.8	1.0	-0.042	-0.014	-0.029	-0.045	-0.012	-0.099	0.003
0.8	1.2	-0.042	-0.010	-0.032	-0.051	-0.013	-0.110	0.002
0.8	1.5	-0.042	-0.007	-0.035	-0.056	-0.015	-0.121	0.002
0.8	2.0	-0.042	-0.001	-0.042	-0.067	-0.017	-0.142	0.002
0.8	2.5	-0.042	0.006	-0.049	-0.079	-0.018	-0.162	0.002
0.8	3.0	-0.042	0.013	-0.055	-0.090	-0.020	-0.181	0.002
1.0	0.3	-0.045	-0.022	-0.023	-0.037	-0.009	-0.078	0.009
1.0	0.4	-0.045	-0.022	-0.024	-0.038	-0.009	-0.081	0.007
1.0	0.5	-0.045	-0.020	-0.025	-0.040	-0.010	-0.086	0.006
1.0	0.7	-0.045	-0.018	-0.027	-0.043	-0.012	-0.093	0.004
1.0	0.8	-0.045	-0.016	-0.029	-0.045	-0.012	-0.099	0.003
1.0	1.0	-0.045	-0.014	-0.032	-0.049	-0.014	-0.108	0.003
1.0	1.2	-0.045	-0.010	-0.035	-0.055	-0.015	-0.119	0.002
1.0	1.5	-0.045	-0.007	-0.038	-0.060	-0.016	-0.129	0.002
1.0	2.0	-0.045	-0.001	-0.045	-0.071	-0.018	-0.150	0.002
1.0	2.5	-0.045	0.006	-0.051	-0.083	-0.020	-0.170	0.002
1.0	3.0	-0.045	0.013	-0.058	-0.094	-0.021	-0.189	0.002

Continued on next page

Table 2 – continued from previous page

$\Delta^{(1)}_{\beta_1}$	$\Delta^{(1)}_{\beta_1}$	Tx Change	Ctrl Change	Group Diff	Lower 95% CI	Upper 95% CI	Effect Size	p-value
1.2	0.3	-0.049	-0.022	-0.026	-0.042	-0.011	-0.089	0.006
1.2	0.4	-0.049	-0.022	-0.027	-0.043	-0.011	-0.092	0.005
1.2	0.5	-0.049	-0.020	-0.028	-0.045	-0.012	-0.097	0.004
1.2	0.7	-0.049	-0.018	-0.031	-0.048	-0.013	-0.104	0.003
1.2	0.8	-0.049	-0.016	-0.032	-0.051	-0.014	-0.110	0.003
1.2	1.0	-0.049	-0.014	-0.035	-0.055	-0.015	-0.119	0.002
1.2	1.2	-0.049	-0.010	-0.038	-0.060	-0.016	-0.129	0.002
1.2	1.5	-0.049	-0.007	-0.041	-0.065	-0.018	-0.140	0.002
1.2	2.0	-0.049	-0.001	-0.048	-0.076	-0.020	-0.161	0.001
1.2	2.5	-0.049	0.006	-0.055	-0.088	-0.022	-0.180	0.001
1.2	3.0	-0.049	0.013	-0.061	-0.099	-0.023	-0.199	0.001
1.5	0.3	-0.052	-0.022	-0.029	-0.047	-0.012	-0.100	0.005
1.5	0.4	-0.052	-0.022	-0.030	-0.048	-0.013	-0.103	0.004
1.5	0.5	-0.052	-0.020	-0.032	-0.050	-0.014	-0.107	0.004
1.5	0.7	-0.052	-0.018	-0.034	-0.053	-0.015	-0.115	0.003
1.5	0.8	-0.052	-0.016	-0.036	-0.056	-0.015	-0.120	0.003
1.5	1.0	-0.052	-0.014	-0.038	-0.060	-0.017	-0.129	0.002
1.5	1.2	-0.052	-0.010	-0.042	-0.065	-0.018	-0.140	0.002
1.5	1.5	-0.052	-0.007	-0.045	-0.071	-0.019	-0.150	0.002
1.5	2.0	-0.052	-0.001	-0.051	-0.082	-0.021	-0.171	0.001
1.5	2.5	-0.052	0.006	-0.058	-0.093	-0.023	-0.190	0.001
1.5	3.0	-0.052	0.013	-0.065	-0.104	-0.025	-0.208	0.001
2.0	0.3	-0.059	-0.022	-0.036	-0.057	-0.015	-0.120	0.004
2.0	0.4	-0.059	-0.022	-0.037	-0.059	-0.016	-0.123	0.004
2.0	0.5	-0.059	-0.020	-0.038	-0.061	-0.016	-0.128	0.003
2.0	0.7	-0.059	-0.018	-0.041	-0.064	-0.017	-0.135	0.003
2.0	0.8	-0.059	-0.016	-0.042	-0.067	-0.018	-0.140	0.002
2.0	1.0	-0.059	-0.014	-0.045	-0.071	-0.019	-0.149	0.002
2.0	1.2	-0.059	-0.010	-0.048	-0.076	-0.021	-0.159	0.002
2.0	1.5	-0.059	-0.007	-0.052	-0.081	-0.022	-0.169	0.002
2.0	2.0	-0.059	-0.001	-0.058	-0.092	-0.024	-0.189	0.001
2.0	2.5	-0.059	0.006	-0.065	-0.103	-0.026	-0.208	0.001
2.0	3.0	-0.059	0.013	-0.071	-0.114	-0.028	-0.226	0.001
2.5	0.3	-0.065	-0.022	-0.043	-0.068	-0.018	-0.139	0.004
2.5	0.4	-0.065	-0.022	-0.044	-0.070	-0.018	-0.142	0.004
2.5	0.5	-0.065	-0.020	-0.045	-0.072	-0.019	-0.146	0.003
2.5	0.7	-0.065	-0.018	-0.047	-0.075	-0.020	-0.153	0.003
2.5	0.8	-0.065	-0.016	-0.049	-0.077	-0.021	-0.159	0.002
2.5	1.0	-0.065	-0.014	-0.052	-0.082	-0.022	-0.167	0.002

Continued on next page

Table 2 – continued from previous page

$\Delta_{\beta_1}^{(1)}$	$\Delta_{\beta_1}^{(1)}$	Tx Change	Ctrl Change	Group Diff	Lower 95% CI	Upper 95% CI	Effect Size	p-value
2.5	1.2	-0.065	-0.010	-0.055	-0.087	-0.024	-0.177	0.002
2.5	1.5	-0.065	-0.007	-0.058	-0.092	-0.025	-0.187	0.002
2.5	2.0	-0.065	-0.001	-0.065	-0.103	-0.027	-0.206	0.001
2.5	2.5	-0.065	0.006	-0.072	-0.114	-0.029	-0.225	0.001
2.5	3.0	-0.065	0.013	-0.078	-0.125	-0.032	-0.242	0.001
3.0	0.3	-0.072	-0.022	-0.050	-0.079	-0.020	-0.156	0.004
3.0	0.4	-0.072	-0.022	-0.051	-0.081	-0.021	-0.159	0.004
3.0	0.5	-0.072	-0.020	-0.052	-0.083	-0.021	-0.163	0.003
3.0	0.7	-0.072	-0.018	-0.054	-0.086	-0.023	-0.170	0.003
3.0	0.8	-0.072	-0.016	-0.056	-0.088	-0.023	-0.175	0.003
3.0	1.0	-0.072	-0.014	-0.059	-0.092	-0.025	-0.183	0.002
3.0	1.2	-0.072	-0.010	-0.062	-0.098	-0.026	-0.193	0.002
3.0	1.5	-0.072	-0.007	-0.065	-0.103	-0.028	-0.203	0.002
3.0	2.0	-0.072	-0.001	-0.072	-0.113	-0.030	-0.221	0.002
3.0	2.5	-0.072	0.006	-0.078	-0.124	-0.032	-0.239	0.001
3.0	3.0	-0.072	0.013	-0.085	-0.135	-0.035	-0.256	0.001

F Simulation study

In this section we perform a simulation study to investigate how well our approach for measurement error correction performs with respect to bias, mean square error (MSE), and coverage under a range of varying and invariant calibration models. We examine how well our method does when the calibration model is correctly specified and when it is not. In addition, we investigate the use of non-degenerate priors for the sensitivity parameters rather than only using point-mass priors as was done in the manuscript.

Simulated self-reported sodium intake at two time points (Y_0, Y_1), log BMI, and sex was generated by drawing from a multivariate normal distribution with means

$$\mu^{\mathbf{1}} = \begin{bmatrix} 8.000 \\ 7.130 \\ 3.480 \\ 0.370 \end{bmatrix} \quad \text{and} \quad \mu^{\mathbf{0}} = \begin{bmatrix} 8.000 \\ 8.000 \\ 3.480 \\ 0.370 \end{bmatrix} \tag{F.1}$$

for the treatment and control groups, respectively. Note that the self-reported mean in the treatment group at follow-up (7.13) is less than the target intake at of $\log(2300) = 7.74$, while the self-reported mean at follow-up in the control group (8.00) exceeds the target intake. Sample sizes were 400 participants in each intervention condition. Both groups had the common correlation matrix:

$$\mathbf{R} = \begin{bmatrix} 1.00 & 0.38 & 0.13 & 0.26 \\ & 1.00 & 0.13 & 0.26 \\ & & 1.00 & -0.12 \\ & & & 1.00 \end{bmatrix} \tag{F.2}$$

with variances

$$\sigma = \begin{bmatrix} 0.17 & 0.17 & 0.03 & 0.23 \end{bmatrix}. \tag{F.3}$$

Self-reported sodium intake in the validation sample was drawn using the same means and covariance matrix as the control group with a sample size of 400 participants. The two urinary sodium replicates ($W_j, j = 1, 2$) were drawn from the following distribution:

$$W_{ij} = \beta_0 + \beta_1 Y_{i0} + \beta_2 \log(BMI_i) + \beta_3 MALE_i + b_{0i} + \varepsilon_{ij} \tag{F.4}$$

where $\beta_0 = 5.7$, $\beta_1 = 0.16$, $\beta_2 = 0.39$, $\beta_3 = 0.3$ and b_{0i} and ε_{ij} are independent and both follow a normal distribution with mean 0 and variance equal to 0.05. Based on these values, the mean true sodium intake at baseline is 8.35 for both treatment groups.

F.1 Simulation study of intercept sensitivity parameters

In our simulation study, we generate follow-up data under treatment- and time-varying calibration models. For the simulation study of the intercept sensitivity parameters this corresponds to a treatment group intercept sensitivity parameter $\Delta^{(1)}_{\beta_0} = 0.2$ (treatment group participants underreport more at follow-up as compared to baseline) and a control group intercept sensitivity parameter $\Delta^{(0)}_{\beta_0} = -0.2$ (control group participants underreport less at follow-up as compared to baseline). Based on these values, the mean true sodium intake at follow-up in the control group is 8.30 and the mean true sodium intake at follow-up in the treatment group is 8.25. Thus the difference in change from baseline between treatment groups is -0.05. Note that the intercept sensitivity parameters refer to the percentage of the residual standard deviation of the regression of Z on Y and $\mathbf{X}$ which is set to $\sqrt{0.05}$ in our simulations.

F.1.1 Calibration model assumptions

To reduce the scope of our simulation study, we examine the performance of our method under a limited set of calibration model assumptions. For the control group, we investigate scenarios where participants are as accurate or more accurate in their reporting at follow-up as compared to baseline. This corresponds to intercept sensitivity parameters $\Delta_{\beta_0}^{(0)}$ equal to -0.2 or 0. For the treatment group, we investigate a wider range of scenarios corresponding to treatment group participants either becoming more or less accurate or remaining the same as at baseline. This corresponds to intercept sensitivity parameters $\Delta_{\beta_0}^{(1)}$ equal to -0.2, 0, or 0.2. Note that only 1 of the 6 possible combinations of treatment and control sensitivity parameters corresponds to the the data generating process.

F.1.2 Uncertainty Assumptions

In practice, the analyst is unlikely to correctly specify the sensitivity parameters that correspond to the process that generated the data. Sensitivity parameters were drawn from prior distributions with varying amounts of precision to incorporate this uncertainty. Intercept parameters Δ_{β_0} were drawn from Normal distributions with standard deviations equal to 0 (point mass prior), 0.05, and 0.10.

F.1.3 Results

Table 3 reports the results from our simulation study of intercept sensitivity parameters. The shaded row in Table 3 indicates the simulation scenario where the intercept sensitivity parameters correspond to the true calibration model. Here, bias is 0 and coverage is close to the nominal level. The two rows below the shaded row are scenarios where the intercept sensitivity parameters correspond to the true calibration model but the sensitivity parameters have been drawn from non-degenerate prior distributions. The result is that while bias is again equal to 0, the additional uncertainty incorporated into the sensitivity parameters has resulted in coverage that meets or exceeds the nominal level and confidence interval width is wider than the scenario where sensitivity parameters are drawn with no uncertainty.

The remaining rows in Table 3 are simulation scenarios in which the sensitivity parameters do not correspond to the true calibration model. Here, incorporating uncertainty into the sensitivity parameters can result in improved coverage. For example, the last three rows of Table 3 are the simulation scenarios where the calibration model in the control group is assumed to be time invariant. This misspecified assumption results in a biased treatment effect and when sensitivity parameters are drawn with no uncertainty (using point-mass priors), coverage is only 71%. However, incorporating uncertainty into the sensitivity parameters improves coverage such that in the last row of the table, coverage is near the nominal level. Note also that as mentioned in the manuscript and in Appendix C, treatment effects are constant when the difference between $\Delta_{\beta_0}^{(1)}$ and $\Delta_{\beta_0}^{(0)}$ is the same. Thus the results when $\Delta_{\beta_0}^{(1)} = -0.2$ and $\Delta_{\beta_0}^{(0)} = -0.2$ are the same as under calibration model invariance with respect to treatment and time: $\Delta_{\beta_0}^{(1)} = 0$ and $\Delta_{\beta_0}^{(0)} = 0$

Table 3: Simulation results for intercept sensitivity parameters. The shaded row indicates the simulation scenario where the sensitivity parameters were correctly specified. Drawing sensitivity parameters from proper prior distributions incorporates uncertainty regarding the correct sensitivity parameter and improves coverage when sensitivity parameters are misspecified.

$\Delta_{\beta_0}^{(1)}$	$\Delta_{\beta_0}^{(0)}$	SD	Bias	RMSE	Cvg	CI Width
-0.2	-0.2	0.00	-0.09	0.10	0.17	0.13
-0.2	-0.2	0.05	-0.09	0.10	0.23	0.14
-0.2	-0.2	0.10	-0.09	0.09	0.51	0.18
-0.2	0.0	0.00	-0.14	0.14	0.00	0.13
-0.2	0.0	0.05	-0.14	0.14	0.02	0.14
-0.2	0.0	0.10	-0.13	0.14	0.07	0.18
0.0	-0.2	0.00	-0.05	0.06	0.71	0.13
0.0	-0.2	0.05	-0.04	0.05	0.86	0.14
0.0	-0.2	0.10	-0.04	0.05	0.92	0.18
0.0	0.0	0.00	-0.09	0.10	0.16	0.13
0.0	0.0	0.05	-0.09	0.10	0.23	0.14
0.0	0.0	0.10	-0.09	0.09	0.51	0.18
0.2	-0.2	0.00	0.00	0.03	0.93	0.13
0.2	-0.2	0.05	0.00	0.03	0.95	0.14
0.2	-0.2	0.10	0.01	0.03	1.00	0.18
0.2	0.0	0.00	-0.05	0.06	0.71	0.13
0.2	0.0	0.05	-0.04	0.05	0.86	0.14
0.2	0.0	0.10	-0.04	0.05	0.92	0.18

$\Delta_{\beta_0}^{(1)}$: Treatment intercept sensitivity parameter prior mean
$\Delta_{\beta_0}^{(0)}$: Control intercept sensitivity parameter prior mean
SD: Intercept sensitivity parameter prior standard deviation
RMSE: Root mean squared error
Cvg: Coverage
CI: Confidence interval

F.2 Simulation study of slope sensitivity parameters

For the simulation study of the slope sensitivity parameters we generated follow-up data using a treatment group slope sensitivity parameter $\Delta_{\beta_1}^{(1)} = 0.5$ (treatment group participants underreport more at follow-up as compared to baseline) and a control group intercept sensitivity parameter $\Delta_{\beta_0}^{(0)} = 0.5$ (control group participants underreport less at follow-up as compared to baseline). Based on these values, the mean true sodium intake at follow-up in the control group is 8.33 and the mean true sodium intake at follow-up in the treatment group is 8.26. Thus the difference in change from baseline between treatment groups is -0.07.

F.2.1 Calibration model assumptions

To reduce the scope of our simulation study, we again examine the performance of our method under a limited set of calibration model assumptions. For the control group, we investigate scenarios where participants are as accurate or more accurate in their reporting at follow-up as compared to baseline. This corresponds to slope sensitivity parameters $\Delta_{\beta_1}^{(0)}$ equal to 0.5 or 1.0. For the treatment group, we investigate a wider range of scenarios corresponding to treatment group participant either becoming more or less accurate or remaining the same as at baseline. This corresponds to slope sensitivity parameters $\Delta_{\beta_1}^{(1)}$ equal to 0.5, 1.0, or 2.0.

F.2.2 Uncertainty Assumptions

Sensitivity parameters were drawn from prior distributions with varying amounts of precision to incorporate uncertainty. Slope sensitivity parameters Δ_{β_1} were drawn from a lognormal distribution where $\log(\Delta_{\beta_1})$ had standard deviations equal to 0, 0.17, or 0.35.

F.2.3 Results

Table 4 reports the results from our simulation study of slope sensitivity parameters. The shaded row in Table 4 indicates the simulation scenario where the slope sensitivity parameters correspond to the true calibration model. Here, bias is 0 and coverage is close to the nominal level. The two rows below the shaded row are scenarios where the slope sensitivity parameters correspond to the true calibration model but the sensitivity parameters have been drawn from non-degenerate prior distributions. The result is that while bias is again equal to 0, the additional uncertainty incorporated into the sensitivity parameters has resulted in coverage that meets or exceeds the nominal level and confidence interval width is wider than the scenario where sensitivity parameters are drawn with no uncertainty.

The remaining rows in Table 4 are simulation scenarios in which the sensitivity parameters do not correspond to the true calibration model. Here, as with the intercept sensitivity parameters, incorporating uncertainty into the sensitivity parameters can result in improved coverage. For example, the three rows in Table 4 where $\Delta_{\beta_1}^{(1)} = 1.0$ and $\Delta_{\beta_1}^{(0)} = 0.5$ are the simulation scenarios where the sensitivity parameter in the treatment group is misspecified. This misspecified assumption results in a biased treatment effect and when sensitivity parameters are drawn with no uncertainty (using point-mass priors), coverage is only 55%. However, by incorporating uncertainty into the sensitivity parameters coverage can exceed the nominal level.

Table 4: Simulation results for slope sensitivity parameters. The shaded row indicates the simulation scenario where the sensitivity parameters were correctly specified. Drawing sensitivity parameters from proper prior distributions incorporates uncertainty regarding the correct sensitivity parameter and improves coverage when sensitivity parameters are misspecified.

$\Delta_{\beta_1}^{(1)}$	$\Delta_{\beta_1}^{(0)}$	SD	Bias	RMSE	Cvg	CI Width
0.5	0.5	0.00	0.00	0.02	0.92	0.06
0.5	0.5	0.17	0.00	0.02	0.95	0.07
0.5	0.5	0.35	-0.01	0.02	0.99	0.11
0.5	1.0	0.00	-0.02	0.03	0.86	0.08
0.5	1.0	0.17	-0.02	0.03	0.89	0.09
0.5	1.0	0.35	-0.03	0.04	1.00	0.13
0.5	2.0	0.00	-0.07	0.07	0.46	0.12
0.5	2.0	0.17	-0.07	0.07	0.53	0.14
0.5	2.0	0.35	-0.07	0.08	0.89	0.19
1.0	0.5	0.00	-0.05	0.06	0.55	0.11
1.0	0.5	0.17	-0.05	0.06	0.69	0.13
1.0	0.5	0.35	-0.06	0.07	0.98	0.20
1.0	1.0	0.00	-0.09	0.10	0.16	0.13
1.0	1.0	0.17	-0.09	0.10	0.18	0.15
1.0	1.0	0.35	-0.10	0.11	0.61	0.21
1.0	2.0	0.00	-0.11	0.12	0.19	0.16
1.0	2.0	0.17	-0.12	0.12	0.23	0.19
1.0	2.0	0.35	-0.12	0.13	0.57	0.26
2.0	0.5	0.00	-0.15	0.16	0.12	0.19
2.0	0.5	0.17	-0.16	0.16	0.16	0.24
2.0	0.5	0.35	-0.17	0.18	0.79	0.38
2.0	1.0	0.00	-0.17	0.18	0.10	0.21
2.0	1.0	0.17	-0.18	0.19	0.15	0.26
2.0	1.0	0.35	-0.19	0.20	0.58	0.39
2.0	2.0	0.00	-0.21	0.22	0.07	0.25
2.0	2.0	0.17	-0.22	0.23	0.08	0.29
2.0	2.0	0.35	-0.23	0.24	0.33	0.43

$\Delta_{\beta_1}^{(1)}$: Treatment slope sensitivity parameter prior mean
$\Delta_{\beta_1}^{(0)}$: Control slope sensitivity parameter prior mean
SD: Slope sensitivity parameter prior standard–deviation (lognormal distribution)
RMSE: Root mean squared error
Cvg: Coverage
CI: Confidence interval

References

Willet, W. (2013). *Nutritional Epidemiology, Third Edition.* Oxford University Press, New York.