Electronic Companion – *To share or not to share? Capacity reservation in a shared supplier*

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In the appendix, we prove the lemmas and propositions, and provide additional results when there are three buying firms in the supply network.

A. Proof of results in Sections 3 and 4

Following the assumptions in the paper, we consider the case where $c_{\tau_i} \leq (p-w)\alpha$ where $\tau_i \in \{e, f\}$ throughout the analysis in this section. Otherwise it is obvious that firms will not reserve any capacity above L because the capacity reservation price is greater than the expected profit margin from satisfying the demand when the demand realizes as H. We also assume that the capacity installation cost $\gamma \leq (p-w)\alpha$.

Proof of Lemma 1. The joint distribution follows from the definition of the conditional probabilities. For example, we have $Pr(D_i = H, D_j = H) = Pr(D_i = H|D_j = H) \cdot Pr(D_j = H) = \beta\alpha$. For brevity we skip the detailed derivation for all other cases. Using the joint distribution, we have the mean of demand is $E[D_i] = H\alpha + L(1-\alpha)$, the variance of demand is $Var[D_i] = \alpha(1-\alpha)(H-L)^2$, the covariance between the two demands is $Cov[D_i, D_j] = \alpha(\beta-\alpha)(H-L)^2$, and the correlation between the two demands is

$$\rho = \frac{Cov[D_i, D_j]}{\sqrt{Var[D_i]} \cdot \sqrt{Var[D_j]}} = \frac{\beta - \alpha}{1-\alpha}. \quad (10)$$

For the joint-distribution to be well-defined, we require $\beta \geq \left(\frac{2\alpha-1}{\alpha}\right)^+$. Then the lowest (resp., highest) demand correlation is obtained by setting $\beta = \left(\frac{2\alpha-1}{\alpha}\right)^+$ (resp., $\beta = 1$) in equation (10). $\square$

Proof of Proposition 1. We prove this proposition by deriving firm i's best response function with respect firm j's capacity reservation level under different capacity types and the equilibrium capacity reservation level follows by solving the system of best response functions. We show the derivation of firm 1's best response, and the other case is symmetric.

When firm 2 chooses exclusive capacity, i.e., $\tau_2 = e$, we have firm 1's profit function as follows:

$$\pi_1^{\tau_1 e}(k_1, k_2) = (p - w)E\left[\min\left\{D_1, k_1\right\}\right] - c_{\tau_1} k_1. \tag{11}$$

Following (11), it is immediate that firm 1's optimal capacity reservation level is independent of firm 2's decision, and the best response of firm 1 is that

$$k_1^{\tau_1 e}(k_2) = H. \tag{12}$$

When firm 2 chooses first-priority capacity, i.e., $\tau_2 = f$, we have firm 1's profit function as follows:

$$\pi_1^{\tau_1 f}(k_1, k_2) = (p - w)E\left[\min\left\{D_1, k_1 + (k_2 - D_2)^+\right\}\right] - c_{\tau_1} k_1. \tag{13}$$

Following (13), it is immediate that the best response of firm 1 is that

$$k_1^{\tau_1 f}(k_2) = \begin{cases} H & \text{if } c_{\tau_1} \leq (p - w)\alpha\beta; \\ \min\{H + L - k_2,\ H\} & \text{if } c_{\tau_1} > (p - w)\alpha\beta. \end{cases} \tag{14}$$

We note that firm 2's best response functions are symmetric to those of firm 1's. Solving the system of best response functions of both firms, we obtain the equilibrium capacity reservation levels as follows.

Scenario (i) Both firms reserve exclusive capacity, i.e., $(\tau_1, \tau_2) = (e, e)$. By (12), both firms' best response capacity reservation level is H. Thus, in equilibrium, both firms reserve H units of capacity.

Scenario (ii) One firm, say firm 1, reserves exclusive capacity while the other firm reserves first-priority capacity, i.e., $(\tau_1, \tau_2) = (e, f)$. By (12) and (14), we have the best response functions of the two firms are as follows:

$$k_1^{ef}(k_2) = \begin{cases} H & \text{if } c_e \leq (p - w)\alpha\beta; \\ \min\{H + L - k_2,\ H\} & \text{if } c_e > (p - w)\alpha\beta; \end{cases}$$
$$k_2^{ef}(k_1) = H.$$

Solving the system of equations, we have the equilibrium capacity reservation level k_i^{ef} is as follows:

$$k_1^{ef} = \begin{cases} H & \text{if } c_e \leq (p - w)\alpha\beta, \\ L & \text{if } c_e > (p - w)\alpha\beta; \end{cases}$$

$$k_2^{ef} = H.$$

Scenario (iii) Both firms reserve first-priority capacity, i.e., $(\tau_1, \tau_2) = (f, f)$. By (14), we have the best response functions of the two firms are as follows:

$$k_1^{ff}(k_2) = \begin{cases} H & \text{if } c_f \le (p-w)\alpha\beta; \\ \min\{H + L - k_2,\ H\} & \text{if } c_f > (p-w)\alpha\beta; \end{cases}$$

$$k_2^{ff}(k_1) = \begin{cases} H & \text{if } c_f \le (p-w)\alpha\beta; \\ \min\{H + L - k_1,\ H\} & \text{if } c_f > (p-w)\alpha\beta. \end{cases}$$

Solving the system of equations and selecting the symmetric equilibrium which results in a fair split of profits between the buying firms[1], we have the equilibrium capacity reservation level k_i^{ff} is as follows:

$$k_i^{ff} = \begin{cases} H & \text{if } c_f \le (p-w)\alpha\beta; \\ \frac{H+L}{2} & \text{if } c_f > (p-w)\alpha\beta. \end{cases}$$

□

Proof of Proposition 2. We prove the proposition in two steps. We first discuss the case where $c_e = c_f$, and then discuss the case where $c_e > c_f$.

(i) Suppose $c_e = c_f = c$. In this case, we show that it is a weakly dominant strategy for firm 1 to choose exclusive capacity. The analysis for firm 2 is similar and omitted for brevity. By Proposition 1, we have $k_2^{e\tau_2} \ge k_2^{f\tau_2}$. Therefore, we have

$$\begin{aligned}
\pi_1^{f\tau_2}(k_1^{f\tau_2}, k_2^{f\tau_2}) &= (p-w)E\left[\min\left\{D_1, k_1^{f\tau_2} + \left(k_2^{f\tau_2} - D_2\right)^+ \mathbb{1}_{\{\tau_2 = f\}}\right\}\right] - ck_1^{f\tau_2} \\
&\le (p-w)E\left[\min\left\{D_1, k_1^{f\tau_2} + (k_2^{e\tau_2} - D_2)^+ \mathbb{1}_{\{\tau_2 = f\}}\right\}\right] - ck_1^{f\tau_2} \\
&\le (p-w)E\left[\min\left\{D_1, k_1^{e\tau_2} + (k_2^{e\tau_2} - D_2)^+ \mathbb{1}_{\{\tau_2 = f\}}\right\}\right] - ck_1^{e\tau_2} \\
&= \pi_1^{e\tau_2}(k_1^{e\tau_2}, k_2^{e\tau_2}).
\end{aligned}$$

The first inequality follows from $k_2^{e\tau_2} \ge k_2^{f\tau_2}$. The second inequality follows from equation (2). Therefore, we have shown that it is a dominant strategy for firm 1 to choose exclusive capacity.

(ii) Suppose $c_e \ge c_f$. In this case, we first show that when the other firm (say firm 1) chooses exclusive capacity, the firm (say firm 2) is better off by choosing first-priority capacity, i.e., $\pi_2^{ee}(k_1^{ee}, k_2^{ee}) \le \pi_2^{ef}(k_1^{ef}, k_2^{ef})$. We note that since $c_e \ge c_f$,

$$\pi_2^{ee}(k_1^{ee}, k_2^{ee}) = (p-w)E\left[\min\left\{D_2, k_2^{ee}\right\}\right] - c_e k_2^{ee} \le (p-w)E\left[\min\left\{D_2, k_2^{ef}\right\}\right] - c_f k_2^{ef} = \pi_2^{ef}(k_1^{ef}, k_2^{ef}). \tag{15}$$

[1] We note that the equilibrium capacity reservation level is not unique when $c_f > (p-w)\alpha\beta$. Solving the system of best response functions, we have the set of all equilibrium capacity reservation levels as $\{(k_1, k_2) : k_1 + k_2 = H + L, k_1 \ge L, k_2 \ge L\}$. In this case, we select the symmetric equilibrium with both firms reserving the same capacity level $\frac{H+L}{2}$ which results in the same expected profit of both firms.

We also note that the inequality is strict when $c_e > c_f$.

We next show that there exists a threshold $\bar{c}_e(c_f;\beta)$ such that when $c_e \le \bar{c}_e(c_f;\beta)$, we have $\pi_1^{ef}(k_1^{ef},k_2^{ef}) \ge \pi_1^{ff}(k_1^{ff},k_2^{ff})$; when $c_e > \bar{c}_e(c_f;\beta)$, we have $\pi_1^{ef}(k_1^{ef},k_2^{ef}) < \pi_1^{ff}(k_1^{ff},k_2^{ff})$. We note that $\pi_1^{ff}(k_1^{ff},k_2^{ff})$ does not change with respect to c_e. Therefore, we need to show that $\pi_1^{ef}(k_1^{ef},k_2^{ef}) \ge \pi_1^{ff}(k_1^{ff},k_2^{ff})$ when $c_e = c_f$, and $\pi_1^{ef}(k_1^{ef},k_2^{ef})$ decreases in c_e. Then we derive the closed-form expression of the threshold.

We note that when $c_e = c_f$, we have

$$\begin{aligned}
\pi_1^{ef}(k_1^{ef},k_2^{ef}) &= (p-w)E\left[\min\left\{D_1, k_1^{ef} + \left(k_2^{ef} - D_2\right)^+\right\}\right] - c_e k_1^{ef} \\
&\ge (p-w)E\left[\min\left\{D_1, k_1^{ff} + \left(k_2^{ef} - D_2\right)^+\right\}\right] - c_e k_1^{ff} \\
&\ge (p-w)E\left[\min\left\{D_1, k_1^{ff} + \left(k_2^{ff} - D_2\right)^+\right\}\right] - c_f k_1^{ff} \\
&= \pi_1^{ff}(k_1^{ff},k_2^{ff}).
\end{aligned}$$

The first inequality follows from $\pi_1^{ef}(k_1^{ef},k_2^{ef}) = \max_{k_1} \pi_1^{ef}(k_1,k_2^{ef})$ and the second inequality follows from $k_2^{ef} \ge k_2^{ff}$ in Proposition 1 and $c_e = c_f$.

We note that $\frac{\partial \pi_1^{ef}(k_1^{ef},k_2^{ef})}{\partial c_e} = -k_1^{ef} \le 0$ when $c_e < (p-w)\alpha\beta$ or $c_e > (p-w)\alpha\beta$ because k_2^{ef} does not change with respect to c_e for a given c_f. We also note that $\pi_1^{ef}(k_1^{ef},k_2^{ef})$ is continuous at $c_e = (p-w)\alpha\beta$. It follows that $\pi_1^{ef}(k_1^{ef},k_2^{ef})$ decreases in c_e.

We next derive the closed-form solution of the threshold $\bar{c}_e(c_f;\beta)$. We note that $\bar{c}_e(c_f;\beta)$ satisfies that when $c_e = \bar{c}_e(c_f;\beta)$, we have $\pi_1^{ef}(k_1^{ef},k_2^{ef}) = \pi_1^{ff}(k_1^{ff},k_2^{ff})$, which gives us the following equation:

$$(p-w)E\left[\min\left\{D_1, k_1^{ef} + \left(k_2^{ef} - D_2\right)^+\right\}\right] - \bar{c}_e(c_f;\beta) k_1^{ef} = \pi_1^{ff}(k_1^{ff},k_2^{ff}). \tag{16}$$

Solving equation (16) for $\bar{c}_e(c_f;\beta)$ using Proposition 1 and the fact that $\bar{c}_e(c_f;\beta) \ge c_f$, we obtain that:

$$\bar{c}_e(c_f;\beta) = \begin{cases} -(p-w)\frac{(H-L)}{2L}\alpha\beta + c_f\frac{H+L}{2L} & \text{if } \beta < \frac{c_f}{(p-w)\alpha}; \\ c_f & \text{if } \frac{c_f}{(p-w)\alpha} \le \beta. \end{cases} \tag{17}$$

In summary, we have proved that there exists a threshold $\bar{c}_e(c_f;\beta)$ such that when $c_e \le \bar{c}_e(c_f;\beta)$, we have $\pi_1^{ef}(k_1^{ef},k_2^{ef}) \ge \pi_1^{ff}(k_1^{ff},k_2^{ff})$; when $c_e > \bar{c}_e(c_f;\beta)$, we have $\pi_1^{ef}(k_1^{ef},k_2^{ef}) < \pi_1^{ff}(k_1^{ff},k_2^{ff})$. Combining this result with the earlier result in equation (15) that $\pi_2^{ee}(k_1^{ee},k_2^{ee}) \le \pi_2^{ef}(k_1^{ef},k_2^{ef})$, we have shown that when $c_e \le \bar{c}_e(c_f;\beta)$, one firm chooses exclusive capacity while the other chooses first-priority capacity; when $c_e \ge \bar{c}_e(c_f;\beta)$, both firms choose first-priority capacity. $\square$

Proof of Proposition 3. The proof for (ii) follows the proof of (ii) in proposition 2. In what follows we focus on proving (i). That is, a prisoner's dilemma equilibrium occurs, where both firms could have increased their profits if both choosing first-priority capacity, when $c_e = c_f = c$. We first observe that if $c \leq (p-w)\alpha\beta$, we have $\pi_1^{ee}(k_1^{ee}, k_2^{ee}) = \pi_1^{ff}(k_1^{ff}, k_2^{ff})$ by Proposition 1. In what follows, we consider the case where $c > (p-w)\alpha\beta$. In this case, we have

$$\begin{aligned}\pi_1^{ee}(k_1^{ee}, k_2^{ee}) = \pi_1^{ee}(k_1^{ee}, k_2^{ff}) &= (p-w)E\left[\min\left\{D_1, k_1^{ee}\right\}\right] - ck_1^{ee} \\ &\leq (p-w)E\left[\min\left\{D_1, k_1^{ee} + \left(k_2^{ff} - D_2\right)^+\right\}\right] - ck_1^{ee} \\ &\leq \pi_1^{ff}(k_1^{ff}, k_2^{ff}).\end{aligned}$$

The first inequality follows the fact that firm 1 is better off if it can access firm 2's leftover capacity and the second inequality follows that $\pi_1^{ff}(k_1^{ff}, k_2^{ff}) = \max_{k_1} \pi_1^{ff}(k_1, k_2^{ff})$. Therefore, in equilibrium, both firms choose exclusive capacity, but both firms could have been better off if both choose first-priority capacity together. Then we have shown that the prisoner's dilemma occurs. □

Proof of Proposition 4. The result directly follows from the closed-form expression of the threshold $\overline{c}_e(c_f;\beta)$ in equation (17). □

Proof of Proposition 5. To derive the supplier's capacity price decision, we first derive the supplier's subgame-perfect equilibrium profit for given (c_e, c_f) and the choice of capacity types by the suppliers (τ_1, τ_2), denoted by $\pi_s^{\tau_1\tau_2}(c_e, c_f)$. Recall that $\gamma \leq (p-w)\alpha$ and $c_{\tau_i} \leq (p-w)\alpha$. By equation (4) and Proposition 1, we have the following supplier's subgame-perfect equilibrium profits:

$$\pi_s^{ee}(c_e, c_f) = 2w\left[L(1-\alpha) + H\alpha\right] + 2\left(c_e - \gamma\right)H. \tag{18}$$

$$\pi_s^{ef}(c_e, c_f) = \begin{cases} 2w\left[L(1-\alpha) + H\alpha\right] + \left(c_e - \gamma\right)H + \left(c_f - \gamma\right)H & \text{if } c_e \leq (p-w)\alpha\beta; \\ w\left[L(2-2\alpha+\alpha\beta) + H\alpha(2-\beta)\right] + \left(c_e - \gamma\right)L + \left(c_f - \gamma\right)H & \text{if } c_e > (p-w)\alpha\beta. \end{cases} \tag{19}$$

$$\pi_s^{ff}(c_e, c_f) = \begin{cases} 2w\left[L(1-\alpha) + H\alpha\right] + 2\left(c_f - \gamma\right)H & \text{if } c_f \leq (p-w)\alpha\beta; \\ w\left[L(2-2\alpha+\alpha\beta) + H\alpha(2-\beta)\right] + \left(c_f - \gamma\right)(H+L) & \text{if } c_f > (p-w)\alpha\beta. \end{cases} \tag{20}$$

Next, we need to consider the optimal combination of the capacity reservation prices (c_e, c_f) to offer while considering the firms' equilibrium capacity type choice in Proposition 2. Consider the following three scenarios:

(i). In order to induce (e,e), the supplier should offer $c_e^* = (p-w)\alpha$, and $c_f^* = c_e^*$, as $\pi_s^{ee}(c_e, c_f)$ increases in c_e in this range; see (18).

(ii). In order to induce (e,f), the supplier should offer the capacity reservation such that $c_f < c_e \le \overline{c}_e(c_f;\beta)$. As (19) increases in c_f, it is immediate that $\pi_s^{ef}(c_e,c_f) < \pi_s^{ef}(c_e,c_e) \le \pi_s^{ee}(c_e,c_f)$.

(iii). In order to induce (f,f), the supplier should offer the capacity reservation such that $c_f \le \overline{c}_e(c_f;\beta) < c_e$. As (20) increases in c_f, it is immediate that $\pi_s^{ff}(c_e,c_f) < \pi_s^{ff}(c_e,c_e) \le \pi_s^{ee}(c_e,c_f)$.

To summarize, the supplier should offer the capacity reservation prices such that $c_e^* = (p-w)\alpha$, and $c_f^* = c_e^*$ to induce both the buyers to choose exclusive capacity. □

Proof of Lemma 2. Recall that $k_i \le H$, $i = 1,2$. By (5), the probability distribution of the residual demand is as follows:

$$D_r^{\tau_1\tau_2}(k_1,k_2) = \begin{cases} 2H - k_1 - k_2 & \text{w.p. } \alpha\beta; \\ \left[H - k_1 - (k_2 - L)^+ \mathbb{1}_{\{\tau_2 = f\}}\right]^+ + (L - k_2)^+ & \text{w.p. } (1-\beta)\alpha; \\ (L - k_1)^+ + \left[H - k_2 - (k_1 - L)^+ \mathbb{1}_{\{\tau_1 = f\}}\right]^+ & \text{w.p. } (1-\beta)\alpha; \\ \left[L - k_1 - (k_2 - L)^+ \mathbb{1}_{\{\tau_2 = f\}}\right]^+ + \left[L - k_2 - (k_1 - L)^+ \mathbb{1}_{\{\tau_1 = f\}}\right]^+ & \text{w.p. } 1 - 2\alpha + \alpha\beta. \end{cases}$$

Given the capacity types (τ_1,τ_2), we note that each of the realized values decreases in k_i, $i = 1,2$. Thus, by definition of the usual stochastic order (Shaked and Shanthikumar 2007), we have for any $0 \le k_1 \le \hat{k}_1$ and $0 \le k_2 \le \hat{k}_2$, the residual demand $D_r^{\tau_1\tau_2}(k_1,k_2) \ge_{st} D_r^{\tau_1\tau_2}(\hat{k}_1,k_2)$ and $D_r^{\tau_1\tau_2}(\hat{k}_1,k_2) \ge_{st} D_r^{\tau_1\tau_2}(\hat{k}_1,\hat{k}_2)$. Thus, we have $D_r^{\tau_1\tau_2}(k_1,k_2) \ge_{st} D_r^{\tau_1\tau_2}(\hat{k}_1,\hat{k}_2)$. □

Proof of Proposition 6 and Observation 1. Solving the decision problem of (6), we have the optimal free capacity $k_s^*(k_1,k_2;\tau_1,\tau_2)$ should be the smallest $k_s \in [0,+\infty)$ such that

$$Pr(D_r^{\tau_1\tau_2}(k_1,k_2) \le k_s) \ge \frac{w-r}{w}. \tag{21}$$

Following Lemma 2, it is immediate that $k_s^*(k_1,k_2;\tau_1,\tau_2)$ decreases in k_1 and k_2.

By (21), if $w < \gamma$, we have $k_s^*(k_1,k_2;\tau_1,\tau_2) = 0$; if $w\alpha\beta > \gamma$, then $k_s^*(k_1,k_2;\tau_1,\tau_2)$ should be the smallest $k_s \in [0,+\infty)$ such that

$$Pr(D_r^{\tau_1\tau_2}(k_1,k_2) \le k_s) \ge \frac{w-r}{w} > 1 - \alpha\beta.$$

Thus, $k_s^*(k_1,k_2;\tau_1,\tau_2) = 2H - k_1 - k_2$. □

B. Proof of results in Section 5

Similar to the analysis in the base model, we consider the case where $c_e \le (p-w)\alpha$ and $c_t \le (p-w)\alpha[1+\hat{t}(1-\beta)]$ when solving for the equilibrium reservation levels. Otherwise it is trivial to show that the firms will not reserve any capacity above the lower bound L when both firms choose exclusive capacity or transferrable capacity.

Proof of Proposition 7. We prove the proposition similar to the proof of Proposition 1. We have buying firm i's profit in equation (8). The capacity reservation level when both firms choose exclusive capacity (e,e) is still the same as in Proposition 1. In what follows we discuss the capacity reservation level in the other two scenarios: (e,t) and (t,t).

We first derive the best response of firm i's capacity reservation level $k_i^{\tau_1\tau_2}(k_j)$ with respect firm j's level k_j, with the capacity transfer price $\hat{t}_i$ and $\hat{t}_j$ respectively. This derivation allows us to obtain the best response functions for both the (e,t) and (t,t) scenarios by setting $\hat{t}_i$ and $\hat{t}_j$ at corresponding appropriate values (specified in each scenario below). There are five cases in total:

Case 1: $k_j \leq 2L-H$. We have

$$k_i^{\tau_1\tau_2}(k_j) = \begin{cases} 2H-k_j & \text{if } c_{\tau_i} \leq (p-w)\hat{t}_i\alpha\beta; \\ H+L-k_j & \text{if } (p-w)\hat{t}_i\alpha\beta < c_{\tau_i} \leq (p-w)\hat{t}_i\alpha(2-\beta); \\ 2L-k_j & \text{if } (p-w)\hat{t}_i\alpha(2-\beta) < c_{\tau_i} \leq (p-w)\hat{t}_i; \\ H & \text{if } (p-w)\hat{t}_i < c_{\tau_i} \leq (p-w)[\alpha+\hat{t}_i(1-\alpha)]; \\ L & \text{if } (p-w)[\alpha+\hat{t}_i(1-\alpha)] < c_{\tau_i} \leq (p-w); \\ 0 & \text{if } (p-w) < c_{\tau_i}. \end{cases}$$

Case 2: $2L-H < k_j \leq L$. We have

$$k_i^{\tau_1\tau_2}(k_j) = \begin{cases} 2H-k_j & \text{if } c_{\tau_i} \leq (p-w)\hat{t}_i\alpha\beta; \\ H+L-k_j & \text{if } (p-w)\hat{t}_i\alpha\beta < c_{\tau_i} \leq (p-w)\hat{t}_i\alpha(2-\beta); \\ H & \text{if } (p-w)\hat{t}_i\alpha(2-\beta) < c_{\tau_i} \leq (p-w)\alpha[1+\hat{t}_i(1-\beta)]; \\ 2L-k_j & \text{if } (p-w)\alpha[1+\hat{t}_i(1-\beta)] < c_{\tau_i} \leq (p-w)[\alpha+\hat{t}_i(1-\alpha)]; \\ L & \text{if } (p-w)[\alpha+\hat{t}_i(1-\alpha)] < c_{\tau_i} \leq p-w; \\ 0 & \text{if } p-w < c_{\tau_i}. \end{cases}$$

Case 3: $L < k_j \leq H$. We have

$$k_i^{\tau_1\tau_2}(k_j) = \begin{cases} 2H-k_j & \text{if } c_{\tau_i} \leq (p-w)\hat{t}_i\alpha\beta; \\ H & \text{if } (p-w)\hat{t}_i\alpha\beta < c_{\tau_i} \leq (p-w)\alpha[\beta+\hat{t}_j(1-\beta)]; \\ H+L-k_j & \text{if } (p-w)\alpha[\beta+\hat{t}_j(1-\beta)] < c_{\tau_i} \leq (p-w)\alpha[1+\hat{t}_i(1-\beta)]; \\ L & \text{if } (p-w)\alpha[1+\hat{t}_i(1-\beta)] < c_{\tau_i} \leq (p-w)[\alpha(2-\beta)+\hat{t}_j(1-2\alpha+\alpha\beta)]; \\ (2L-k_j)^+ & \text{if } (p-w)[\alpha(2-\beta)+\hat{t}_j(1-2\alpha+\alpha\beta)] < c_{\tau_i} \leq p-w; \\ 0 & \text{if } p-w < c_{\tau_i}. \end{cases}$$

Case 4: $H < k_j \leq 2H-L$. We have

$$k_i^{\tau_1\tau_2}(k_j) = \begin{cases} H & \text{if } c_{\tau_i} \leq (p-w)\hat{t}_j\alpha; \\ 2H-k_j & \text{if } (p-w)\hat{t}_j\alpha < c_{\tau_i} \leq (p-w)\alpha[\beta+\hat{t}_j(1-\beta)]; \\ L & \text{if } (p-w)\alpha[\beta+\hat{t}_j(1-\beta)] < c_{\tau_i} \leq (p-w)[\alpha\beta+\hat{t}_j(1-\alpha\beta)]; \\ (H+L-k_j)^+ & \text{if } (p-w)[\alpha\beta+\hat{t}_j(1-\alpha\beta)] < c_{\tau_i} \leq (p-w)[\alpha(2-\beta)+\hat{t}_j(1-2\alpha+\alpha\beta)]; \\ (2L-k_j)^+ & \text{if } (p-w)[\alpha(2-\beta)+\hat{t}_j(1-2\alpha+\alpha\beta)] < c_{\tau_i} \leq p-w; \\ 0 & \text{if } p-w < c_{\tau_i}. \end{cases}$$

Case 5: $2H-L<k_j$. We have

$$k_i^{\tau_1\tau_2}(k_j)=\begin{cases}H & \text{if } c_{\tau_i}\le (p-w)\hat{t}_j\alpha;\\ L & \text{if } (p-w)\hat{t}_j\alpha<c_{\tau_i}\le (p-w)\hat{t}_j;\\ 2H-k_j & \text{if } (p-w)\hat{t}_j<c_{\tau_i}\le (p-w)[\alpha\beta+\hat{t}_j(1-\alpha\beta)];\\ (H+L-k_j)^+ & \text{if } (p-w)[\alpha\beta+\hat{t}_j(1-\alpha\beta)]<c_{\tau_i}\le (p-w)[\alpha(2-\beta)+\hat{t}_j(1-2\alpha+\alpha\beta)];\\ (2L-k_j)^+ & \text{if } (p-w)[\alpha(2-\beta)+\hat{t}_j(1-2\alpha+\alpha\beta)]<c_{\tau_i}\le p-w;\\ 0 & \text{if } p-w<c_{\tau_i}.\end{cases}$$

We next derive the equilibrium capacity reservation levels.

Scenario (e,t). We note that under the scenario of (e,t), firm 1's best response functions can be obtained by setting $\hat{t}_1=0$ and $\hat{t}_2=\hat{t}$ in each of the five cases above. Similarly, firm 2's best response functions can be obtained by setting $\hat{t}_1=1$ and $\hat{t}_2=\hat{t}$. Solving for the system of best response functions, we obtain the equilibrium capacity reservation levels in Proposition 7.

Scenario (t,t). We consider the symmetric case where both firms have the same capacity transfer price, i.e., $\hat{t}_1=\hat{t}_2=\hat{t}$. We then obtain the equilibrium capacity reservation levels as shown in Proposition 7 by solving the system of best response functions of both firms. □

Proof of Proposition 8 We prove the proposition in two steps. We first derive the threshold of $\tilde{c}_e(c_t;\beta,\hat{t})$ and then derive the threshold of $\hat{c}_e(c_t;\beta,\hat{t})$.

(i) By Proposition 7, we have that the equilibrium profit of firm 2 under scenario (e,e) as follows:

$$\pi_2^{ee}(k_1^{ee},k_2^{ee})=(p-w)E\left[\min\{D_2,k_2^{ee}\}\right]-c_ek_2^{ee}=(p-w)[L(1-\alpha)+H\alpha]-c_eH.$$

Therefore, we have $\pi_2^{ee}(k_2^{ee},k_2^{ee})$ decreases in c_e.

We also notice that under the scenario of (e,t), the profit of firm 2 for given (k_1,k_2) as follows:

$$\pi_2^{et}(k_1,k_2)=(p-w)E\left[\min\left\{D_2+\hat{t}\min\left\{(D_1-k_1)^+,(k_2-D_2)^+\right\},k_2\right\}\right]-c_tk_2$$

For any given c_t and k_2, we have $\pi_2^{et}(k_1,k_2)$ decreases in k_1, and by Proposition 7 we have k_1^{et} decreases in c_e. It follows that we have $\pi_2^{et}(k_1^{et},k_2^{et})=\max_{k_2}\pi_2^{et}(k_1^{et},k_2)$ increases in c_e.

Therefore, solving for the equation of $\pi_2^{ee}(k_1^{ee},k_2^{ee})=\pi_2^{et}(k_1^{et},k_2^{et})$ considering c_e as the unknown variable, let $c_e^*\ge 0$ denote the solution if it exists and set $c_e^*=0$ if the solution does not exist. Then, we define $\tilde{c}_e(c_t;\beta,\hat{t})\triangleq c_e^*$. It follows that when $c_e\le\tilde{c}_e(c_t;\beta,\hat{t})$, we have $\pi_2^{ee}(k_1^{ee},k_2^{ee})\ge\pi_2^{et}(k_1^{et},k_2^{et})$ and by symmetry $\pi_1^{ee}(k_1^{ee},k_2^{ee})\ge\pi_1^{te}(k_1^{te},k_2^{te})$, which implies that both firms choose exclusive capacity in equilibrium.

(ii) We observe that $\pi_1^{tt}(k_1^{tt},k_2^{tt})$ does not change with respect to c_e. In what follows, we show that either $\pi_1^{et}(k_1^{et},k_2^{et})$ decreases in c_e, or $\pi_1^{et}(k_1^{et},k_2^{et})\le\pi_1^{tt}(k_1^{tt},k_2^{tt})$. We consider the three regions depending on the value of c_t: $c_t \in [0,(p-w)\hat{t}\alpha\beta]$, $c_t \in ((p-w)\hat{t}\alpha\beta,(p-w)\alpha]$, and $c_t \in ((p-w)\alpha,(p-w)\alpha[1+\hat{t}(1-\beta)]]$.

Region 1: $c_t \in [0,(p-w)\hat{t}\alpha\beta]$**.** In this case, it is immediate that $\pi_1^{et}(k_1^{et},k_2^{et})$ decreases in c_e when $c_e \le (p-w)\alpha[\beta+\hat{t}(1-\beta)]$. When $(p-w)\alpha[\beta+\hat{t}(1-\beta)] < c_e \le (p-w)[\alpha\beta+\hat{t}(1-\alpha\beta)])$, we have the equilibrium reservation level as $(L,2H-L)$, and it follows that

$$\begin{aligned}\pi_1^{et}(k_1^{et},k_2^{et}) &= \pi_1^{et}(L,2H-L)\\ &\le (p-w)E\left[\min\left\{D_1, L+\left(1-\hat{t}\right)\min\left\{(D_1-L)^+,(2H-L-D_2)^+\right\}\right\}\right]-(p-w)\alpha[\beta+\hat{t}(1-\beta)]L\\ &= (p-w)[L(1-\alpha)+H\alpha]-(p-w)\alpha\hat{t}H-(p-w)\alpha\beta(1-\hat{t})L\\ &\le (p-w)[L(1-\alpha)+H\alpha]-(p-w)\alpha\beta\hat{t}H \le \pi_1^{tt}(k_1^{tt},k_2^{tt})\end{aligned}$$

When $(p-w)[\alpha\beta+\hat{t}(1-\alpha\beta)]) < c_e \le (p-w)\alpha$, we have the equilibrium reservation level as $(0,2H)$, and it follows that

$$\begin{aligned}\pi_1^{et}(k_1^{et},k_2^{et}) &= (p-w)E\left[\left(1-\hat{t}\right)\min\left\{D_1,(2H-D_2)^+\right\}\right]\\ &= (p-w)[L(1-\alpha)+H\alpha]-(p-w)\alpha\hat{t}H-(p-w)(1-\alpha)\hat{t}L\\ &\le (p-w)[L(1-\alpha)+H\alpha]-(p-w)\alpha\beta\hat{t}H \le \pi_1^{tt}(k_1^{tt},k_2^{tt})\end{aligned}$$

Region 2: $c_t \in ((p-w)\hat{t}\alpha\beta,(p-w)\alpha]$**.** In this case, we have $\pi_1^{et}(k_1^{et},k_2^{et}) = \pi_1^{et}(k_1^{et},H)$. It follows that $\pi_1^{et}(k_1^{et},k_2^{et})$ decreases in c_e.

Region 3: $c_t \in ((p-w)\alpha,(p-w)\alpha[1+\hat{t}(1-\beta)]]$**.** In this case, we have that $\pi_1^{et}(k_1^{et},k_2^{et})$ decreases in c_e when $c_e \in [0,(p-w)\alpha[\beta+\hat{t}(1-\beta)]]$ as the equilibrium capacity reservation levels remain the same. When $c_e = (p-w)\alpha[\beta+\hat{t}(1-\beta)]$, we have

$$\pi_1^{et}(k_1^{et},k_2^{et}) = (p-w)[L(1-\alpha)+H\alpha]-(p-w)\alpha[\beta+\hat{t}(1-\beta)]H \tag{22}$$

We also have that $\pi_1^{et}(k_1^{et},k_2^{et})$ decreases in c_e when $c_e \in ((p-w)\alpha[\beta+\hat{t}(1-\beta)],(p-w)\alpha]$ as the equilibrium reservation levels remain the same. When $c_e \in ((p-w)\alpha[\beta+\hat{t}(1-\beta)],(p-w)\alpha]$, we have

$$\pi_1^{et}(k_1^{et},k_2^{et}) \le (p-w)[L+(H-L)(1-\hat{t})\alpha(1-\beta)]-(p-w)\alpha[\beta+\hat{t}(1-\beta)]L \tag{23}$$

Then we have equation (22)-RHS of equation (23)$=0$. Therefore, we have $\pi_1^{et}(k_1^{et},k_2^{et})$ decreases in c_e.

Summarizing the analysis for the three regions, we have that $\pi_1^{et}(k_1^{et},k_2^{et})$ decreases in c_e, or is either less than $\pi_1^{tt}(k_1^{tt},k_2^{tt})$.

Therefore, solving for the equation of $\pi_1^{et}(k_1^{et}, k_2^{et}) = \pi_1^{tt}(k_1^{tt}, k_2^{tt})$ considering c_e as the unknown variable, let $c_e^\dagger \geq 0$ denote the solution if it exists and set $c_e^\dagger = 0$ if the solution does not exist. Then, we define $\hat{c}_e(c_t;\beta,\hat{t}) \triangleq \max\{c_e^\dagger, \tilde{c}_e(c_t;\beta,\hat{t})\}$. It follows that when $c_e > \hat{c}_e(c_t;\beta,\hat{t})$, both firms choose transferrable capacity. When $\tilde{c}_e(c_t;\beta,\hat{t}) < c_e \leq \hat{c}_e(c_t;\beta,\hat{t})$, one firm chooses exclusive capacity while the other chooses transferrable capacity. □

Proof of Proposition 9 We note that the supplier's profit with given capacity reservation levels (k_1, k_2) and capacity types (τ_1, τ_2) as follows:

$$\pi_s^{\tau_1\tau_2}(k_1, k_2) = \sum_{i,j=1, i\neq j}^{2} \left\{ wE\left[\min\left\{D_i, k_i + (k_j - D_j)^+ \mathbb{1}_{\{\tau_j = t\}}\right\}\right] + (c_{\tau_i} - \gamma)\, k_i \right\}. \tag{24}$$

To derive the supplier's capacity reservation price decisions, we first derive the supplier's subgame-perfect equilibrium profit $\pi_s^{\tau_1\tau_2}(c_e, c_t)$ for given (c_e, c_t) and the capacity types (τ_1, τ_2).

By Proposition 7 and equation (24), we have the following supplier's subgame-perfect equilibrium profits under the scenario (e, e) and (t, t) respectively:

$$\pi_s^{ee}(c_e, c_t) = 2w\left[L(1-\alpha) + H\alpha\right] + 2\,(c_e - \gamma)\,H. \tag{25}$$

$$\pi_s^{tt}(c_e, c_t) = \begin{cases} 2w\left[L(1-\alpha) + H\alpha\right] + 2\,(c_t - \gamma)\,H & \text{if } c_t \leq (p-w)\alpha[\beta + \hat{t}(1-\beta)]; \\ w\left[L(2 - 2\alpha + \alpha\beta) + H\alpha(2-\beta)\right] + (c_t - \gamma)\,(H + L) & \text{if } c_t > (p-w)\alpha[\beta + \hat{t}(1-\beta)]. \end{cases} \tag{26}$$

For (e, t), we note that for the case with the total capacity reservation $2H$, the optimal capacity reservation price bundle should be $c_e \leq (p-w)\alpha$ and $c_t \leq (p-w)\alpha$; see Proposition 7. This case, however, is dominated by the profit under (e, e) when $c_e = (p-w)\alpha$ in equation (25). For the case with total capacity reservation $H + L$, the optimal capacity reservation price bundle should be $c_e = (p-w)\alpha$ and $c_t = (p-w)\alpha[1 + \hat{t}(1-\beta)]$. This case, however, is dominated by the profit under (t, t) when $c_t = (p-w)\alpha[1 + \hat{t}(1-\beta)]$; see (26). Therefore, we can restrict attention to comparing equations (25) and (26).

When the capacity types are (e, e), the supplier's profit is maximized at $c_e^* = (p-w)\alpha$. When the capacity types are (t, t), the supplier's profit is maximized either at $c_t^\dagger = (p-w)\alpha[\beta + \hat{t}(1-\beta)]$ with the equilibrium capacity reservation levels (H, H), or $c_t^* = (p-w)\alpha[1 + \hat{t}(1-\beta)]$ with the equilibrium capacity reservation levels $(\frac{H+L}{2}, \frac{H+L}{2})$. The first case is dominated by (e, e), where the capacity reservation price is higher with the same capacity reservation level and the same resulting capacity utilization. Therefore, to understand the supplier'e preference between the two outcomes. we only need to compare the supplier's profit in the second case with the optimal profit under (e, e). Taking the difference of the supplier's profit in these two scenarios, we have:

$$\pi_s^{ee}(c_e^*, c_t^*) - \pi_s^{tt}(c_e^*, c_t^*) = 2w\left[L(1-\alpha) + H\alpha\right] + 2\left[(p-w)\alpha - \gamma\right]H$$

$$-w\left[L(2-2\alpha+\alpha\beta)+H\alpha(2-\beta)\right]-\left\{(p-w)\alpha[1+\hat{t}(1-\beta)]-\gamma\right\}(H+L). \tag{27}$$

We compare equation (27) to 0 and define $\underline{t} \triangleq \frac{(H-L)[(p-w)\alpha+w\alpha\beta-\gamma]}{(H+L)(p-w)\alpha(1-\beta)}$. Then we have if $\hat{t} \leq \underline{t}$, the supplier should set capacity reservation prices such that $c_e^* = (p-w)\alpha$ and $c_e^* \leq \tilde{c}_e(c_t;\beta,\hat{t})$, and both firms choose exclusive capacity. If $\hat{t} > \underline{t}$, the supplier sets capacity reservation prices (c_e^*, c_t^*) such that $c_t^* = (p-w)\alpha[1+\hat{t}(1-\beta)]$ and $c_e^* \geq \hat{c}_e(c_t;\beta,\hat{t})$, and both firms choose transferrable capacity. □

Proof of Proposition 10 We note that Pareto improvement over the equilibrium outcome in Proposition 5 is possible only if the supplier chooses the capacity reservation prices to induce the equilibrium capacity type of (t,t) with reservation level $\left(\frac{H+L}{2}, \frac{H+L}{2}\right)$. By Proposition 9, we have that when $\hat{t} > \underline{t}$, the supplier's profit is higher when the equilibrium capacity type of (t,t) with reservation level $\left(\frac{H+L}{2}, \frac{H+L}{2}\right)$ is induced. We just need to find the condition under which the buying firms' profit is also higher in this scenario.

We have the buying firm's profit difference in these two cases as follows. Recall that $c_e^* = (p-w)\alpha$ and $c_t^* = (p-w)\alpha[1+\hat{t}(1-\beta)]$. We have:

$$\begin{aligned}\pi_1^{tt}(c_e^*, c_t^*) - \pi_1^{ee}(c_e^*, c_t^*) =& (p-w)\left[L\left(1-\alpha+\frac{\alpha\beta}{2}\right)+H\left(\alpha-\frac{\alpha\beta}{2}\right)\right]-(p-w)\alpha[1+\hat{t}(1-\beta)]\frac{H+L}{2} \\ &-(p-w)[L(1-\alpha)+H\alpha]+(p-w)\alpha H \\ =&(p-w)\frac{\alpha(1-\beta)}{2}\left[H-L-\hat{t}(H+L)\right] \end{aligned} \tag{28}$$

Letting equation (28) ≥ 0, we have if $\hat{t} \leq \frac{H-L}{H+L}$, then $\pi_1^{tt}(c_e^*, c_t^*) \geq \pi_1^{ee}(c_e^*, c_t^*)$. Let $\bar{t} \triangleq \frac{H-L}{H+L}$, and then the buying firms are also better off compared to the equilibrium outcome in Proposition 5 if $\hat{t} \leq \bar{t}$.

Finally, to ensure the set $[\underline{t}, \bar{t}]$ is non-empty, we just need

$$\bar{t}-\underline{t} = \frac{H-L}{H+L} - \frac{(H-L)[(p-w)\alpha+w\alpha\beta-\gamma]}{(H+L)(p-w)\alpha(1-\beta)} = \frac{(H-L)(\gamma-p\alpha\beta)}{(H+L)(p-w)\alpha(1-\beta)} \geq 0.$$

Therefore, if $\beta \leq \frac{\gamma}{p\alpha}$, the set is non-empty. The condition also coincides with the condition that the supply chain profit, which is the sum of buying firms' and supplier's profits, is higher under (t,t) for the given (c_e^*, c_t^*). The supply chain efficiency improvement is derived as follows.

When both firms choose transferrable capacity, we have the supply chain profit as

$$\pi_c^{tt}(c_t^*) = p(2-2\alpha+\alpha\beta)L + p\alpha(2-\beta)H - \gamma(H+L).$$

When both firms are induced to choose exclusive capacity, we have the supply chain profit as

$$\pi_c^{ee}(c_e^*) = 2p(1-2\alpha+\alpha\beta)L + 2p\alpha(1-\beta)(L+H) + 2p\alpha\beta H - 2\gamma H.$$

Therefore, we have the efficiency improvement $\Delta\%$ as follows:

$$\Delta\% = \frac{\pi_c^{tt}(c_t^*) - \pi_c^{ee}(c_e^*)}{\pi_c^{tt}(c_t^*)} = \frac{(H-L)(\gamma - p\alpha\beta)}{p(2-2\alpha+\alpha\beta)L + p\alpha(2-\beta)H - \gamma(H+L)}.$$

□

C. Additional analysis with three buying firms

In the paper, we have focused on the scenario with two buying firms to illustrate the key tradeoffs. In this section, we consider the scenario with three buying firms and illustrate the impact of more firms by comparing the equilibrium capacity levels to those when there are two buying firms. We focus on the symmetric equilibria with all firms choosing the same type of capacity reservation and reserving the same capacity levels.

We next derive the expected profit of the firms given the capacity type choices, and then compare the equilibrium capacity reservation levels to those when there are two buying firms as a first attempt to analyze the more complicated scenario. We denote the expected profit of firm i as $\pi_i^{\tau_1\tau_2\tau_3}(k_1,k_2,k_3)$, where $\tau_i \in \{e,f,t\}$ is the capacity type choice of firm i, and k_i is the capacity reservation level of firm i. We also denote firm i's equilibrium capacity reservation level as $k_i^{\tau_1\tau_2\tau_3}$, given the reservation types of (τ_1,τ_2,τ_3).

If all buying firms choose the exclusive capacity, then firm i's profit is as follows, where $i \in \{1,2,3\}$:

$$\pi_i^{eee}(k_1,k_2,k_3) = (p-w)E\left[\min\{D_i,k_i\}\right] - c_e k_i. \tag{29}$$

If all buying firms choose the first-priority capacity, the issue of capacity allocation arises when two firms need additional capacity while one firm has leftover. We consider the following intuitive allocation rule. In the *capacity allocation* case, the leftover capacity is initially allocated evenly between the other two firms requesting leftover capacity; furthermore, if one firm does not use up the allocated leftover capacity while the other firm needs more, then the rest of leftover from the former can also be used by the latter. We have firm i's profit as follows, where $i,j,k \in \{1,2,3\}$, $i \neq j$, $j \neq k$, and $i \neq k$:

$$\pi_i^{fff}(k_1,k_2,k_3) = (p-w)E\left[\min\{D_i,k_i\}\right]$$

$$+ (p-w)E\left[\min\left\{(D_i-k_i)^+, \begin{array}{l}\frac{(k_j-D_j)^+}{2}+\frac{(k_k-D_k)^+}{2}+\left[\frac{(k_j-D_j)^+}{2}-(D_k-k_k)^+\right]^+ \\ +\left[\frac{(k_k-D_k)^+}{2}-(D_j-k_j)^+\right]^+\end{array}\right\}\right]$$
$$- c_f k_i \qquad (30)$$

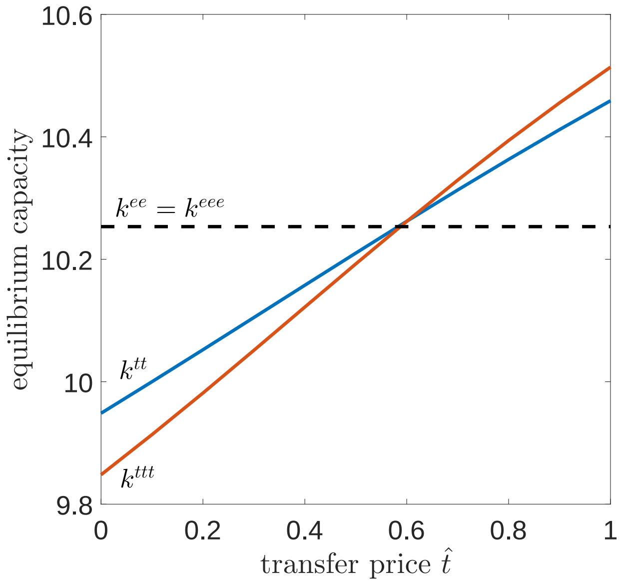
If all buying firms choose the transferrable capacity with the capacity transfer price $\hat{t}$, then the issue of demand allocation also arises when one firm needs additional capacity while two firms have leftover. We consider a similar intuitive allocation rule as in the capacity allocation case. In the *demand allocation* case, the extra demand is initially allocated equally to the other two firms with leftover capacity; furthermore, if one firm's leftover capacity cannot satisfy all the allocated extra demand while the other firm still has leftover capacity after satisfying the allocated extra demand, then the remainder of the extra demand of the former will also be allocated to the latter. In the *capacity allocation* case, we apply the same allocation rule as in the first-priority capacity case discussed above. Then, with the allocation rules specified, we have firm i's profit as follows, where $i,j,k \in \{1,2,3\}$, $i \neq j$, $j \neq k$, and $i \neq k$:

$$\begin{aligned}\pi_i^{ttt}(k_1,k_2,k_3) =& (p-w)E\left[\min\{D_i,k_i\}\right] \\ &+ (p-w)\hat{t}E\left[\min\left\{\begin{array}{l}\frac{(D_j-k_j)^+}{2}+\frac{(D_k-k_k)^+}{2}+\left[\frac{(D_j-k_j)^+}{2}-(k_k-D_k)^+\right]^+ \\ +\left[\frac{(D_k-k_k)^+}{2}-(k_j-D_j)^+\right]^+\end{array}, (k_i-D_i)^+\right\}\right] \\ &+ (p-w)(1-\hat{t})E\left[\min\left\{(D_i-k_i)^+, \begin{array}{l}\frac{(k_j-D_j)^+}{2}+\frac{(k_k-D_k)^+}{2}+\left[\frac{(k_j-D_j)^+}{2}-(D_k-k_k)^+\right]^+ \\ +\left[\frac{(k_k-D_k)^+}{2}-(D_j-k_j)^+\right]^+\end{array}\right\}\right] \\ &- c_t k_i \qquad (31)\end{aligned}$$

In each of the scenarios above, we derive the symmetric equilibrium capacity numerically utilizing the numerical testing bed in Section 6. We use the following default parameters (if not changed as a variable in the analysis): the market price $p=15$, the wholesale price $w=5$, the capacity installation cost $\gamma=7$, the marginal distribution for demand i is normal with mean $\mu_i=10$ and standard deviation $\sigma_i=1$, the demand correlation $\rho=0$, and the capacity reservation cost $c_{\tau_i}=4$ for $\tau_i \in \{e,\ f,\ t\}$.

We make the following observations from Figure 1. First, as the transfer price $\hat{t}$ increases, firms' have a higher valuation of the reserved capacity, and therefore, both the equilibrium capacity k^{tt} and k^{ttt} increase. Second, when the transfer price is low ($\hat{t} \leq 0.6$), the equilibrium capacity k^{tt} is higher than the equilibrium capacity k^{ttt}; when the transfer price is high ($\hat{t} > 0.6$), the equilibrium capacity k^{tt} is lower than the equilibrium capacity k^{ttt}. Thus, although the impact of the transfer

Figure 1 Equilibrium capacity comparison.



Note. k^{ff} (resp., k^{fff}) is the value of k^{tt} (resp., k^{ttt}) when $\hat{t} = 0$. Thus, the plot is omitted from the figure.

price on the equilibrium capacity is qualitatively the same (in terms of the monotonicity of the equilibrium capacity), there could be subtle differences. It could be interesting for future research to investigate the impact with additional firms involved in the capacity reservation games.

References

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