The University of Michigan
IVHS
Intelligent Vehicle-Highway Systems

Anticipatory Traffic Modeling and Route Guidance: A General Mathematical Formulation

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1. Motivation and assumptions

A fundamental difficulty in anticipatory route guidance is that a Dynamic Router requires information about future flows or travel times on the links composing the network. If we assume that a separate Simulator is performing this forecasting task, then the Simulator and the Dynamic Router are coupled in a feedback loop in the sense that:

(i) the routing decisions of the Dynamic Router depend upon the forecasting decisions of the Simulator; and

(ii) the forecasting decisions of the Simulator depend upon the routing decisions of the Dynamic Router.

This feedback loop is of fundamental importance and one of our objectives is to get a better understanding of the difficulties that it creates. As a first step, we have formulated as a mathematical program the problem of simultaneous performing anticipatory route guidance and travel time forecasting for a traffic network. For simplicity, we have made the following assumptions:

(i) the O/D requests for all times in the time horizon considered are deterministic and known;

(ii) all vehicles are routed by the Dynamic Router and they do not deviate from the routes assigned to them; and

(iii) the network is originally empty.

This initial formulation is presented below after the necessary notation is introduced.

2. Some notation

Network structure: (cf. Fig. 1)

- k denotes an O/D pair in the network, with origin node O_k and destination node D_k. K is the set of all O/D pairs.
- (k,m) denotes path m of O/D pair k. M_k denotes the set of all paths for O/D k.
- ℓ denotes a link in the network from node O_ℓ to node D_ℓ. $\mathcal{L}$ is the set of all links in the network.
- $\underset{(k,m)}{<}$ denotes the (total) ordering of the links composing path (k,m).

Link variables:

- $v_\ell(t)$ denotes the number of vehicles traveling on link ℓ at time t.
- V_ℓ denotes the capacity of link ℓ in terms of number of vehicles.
- $\tau_\ell(t)$ denotes the travel time on link ℓ of a vehicle that arrives at O_ℓ at time t.
  We assume that
  $$\tau_\ell(t) = f_{TE}(\ell, v_\ell(t))$$
  where f_{TE} is a known "impedance" function obtained from the principles of traffic engineering (e.g., Greenshield's model).

- $t_\ell^{(k,m)}(s)$ pertains to path m of O/D k and denotes the time at which a vehicle traveling on (k,m) reaches node O_ℓ assuming that this vehicle left O_k at time s.

  We have that

$$t_\ell^{(k,m)}(s) = s + \sum_{\ell' \underset{(k,m)}{<} \ell} \tau_{\ell'}\left(t_{\ell'}^{(k,m)}(s)\right)$$

  or in recursive form

$$t_{\ell_i}^{(k,m)}(s) = t_{\ell_{i-1}}^{(k,m)}(s) + \tau_{\ell_{i-1}}\left(t_{\ell_{i-1}}^{(k,m)}(s)\right)$$

  with initial condition $t_{\ell_0}^{(k,m)}(s) = s$ (cf. Fig. 1).

Indicator function:

- $I_\ell^{(k,m)}(t,s) := \begin{cases} 1 & \text{if } \ell \in (k,m) \text{ and } t_\ell^{(k,m)}(s) \le t < t_\ell^{(k,m)}(s) + \tau_\ell\left(t_\ell^{(k,m)}(s)\right) \\ 0 & \text{otherwise .} \end{cases}$

  This function is used to track the vehicles that are routed on path (k,m) and that left O_k at time s. It returns 1 whenever these vehicles are on link ℓ of (k,m) at time t. If $t = s$, then this function returns a 1 if and only if ℓ is the first link of (k,m), i.e., $\ell \in (k,m)$ and $O_k = O_\ell$.

Input variables:

- $D^k(t)$ is the (known and deterministic) number of vehicles that join the network at node O_k at time t and whose destination is node D_k.

Output variables:

- $h^{(k,m)}(t)$ is the number of vehicles that are routed to path m of O/D k among all vehicles arriving at O_k at time t and whose destination is D_k. This is the decision variable and once a vehicle is routed to a path, it will stay on that path until it reaches its destination.

3. The system optimization problem

Given $D^k(t) \quad \forall k \in K, \quad \forall t \in [t_o, T_o]$,

find $h^{(k,m)}(t) \quad \forall m \in M^k, \quad \forall k \in K, \quad \forall t \in [t_o, T_o]$

that minimizes the system's *total travel time*

$$\sum_{s\in[t_o,T_o]} \sum_{k\in K} \sum_{m\in M^k} h^{(k,m)}(s) \cdot \left[\sum_{\ell_i \in (k,m)} \tau_{\ell_i}\left(t_{\ell_i}^{(k,m)}(s)\right) \right]$$

subject to

(1) demand constraints:

$$\sum_{m\in M^k} h^{(k,m)}(t) = D^k(t) \qquad \forall m \in M^k, \quad \forall k \in K, \quad \forall t \in [t_o, T_o]$$

(2) capacity constraints:

$$v_\ell(k) \le V_\ell \qquad \forall \ell \in \mathcal{L}, \quad \forall t \in [t_o, T_o]$$

(3) integer variables and positivity:

$$h^{(k,m)}(t) \in \mathbb{N} \qquad \forall m \in M^k, \quad \forall k \in K, \quad \forall t \in [t_\circ, T_\circ]$$

where

(4) impedance/flow relationship:

$$\begin{aligned} \tau_\ell(t) &= f_{TE}(\ell, v_\ell(t)) \\ v_\ell(t) &= \sum_{k\in K} \sum_{m\in M^k} \sum_{s\leq t} h^{(k,m)}(s) \cdot I_\ell^{(k,m)}(t,s) \\ & \quad \forall \ell \in \mathcal{L}, \quad \forall t \geq t_\circ \ . \end{aligned}$$

4. Discussion

We are currently studying the mathematical program formulated above. Equations (4) highlight the interaction routing – travel time. Essentially, when a vehicle arrives at the origin O_ℓ of link ℓ at time t:

(1) $v_\ell(t)$ is calculated from past and present routing decisions $h^{(k,m)}(s)$, $s \leq t$, using the indicator function $I_\ell^{(k,m)}(t,s)$. This indicator function requires past travel times $\tau_{\ell'}(s)$, $s < t$, on all links ℓ' that may precede ℓ on some path (k,m);

(2) once $v_\ell(t)$ has been calculated, the travel time of the vehicle on link ℓ is determined by

$$\tau_\ell(t) = f_{TE}(\ell, v_\ell(t)) \ .$$

This process is then repeated for the next arrival on any link.

O/D pair k

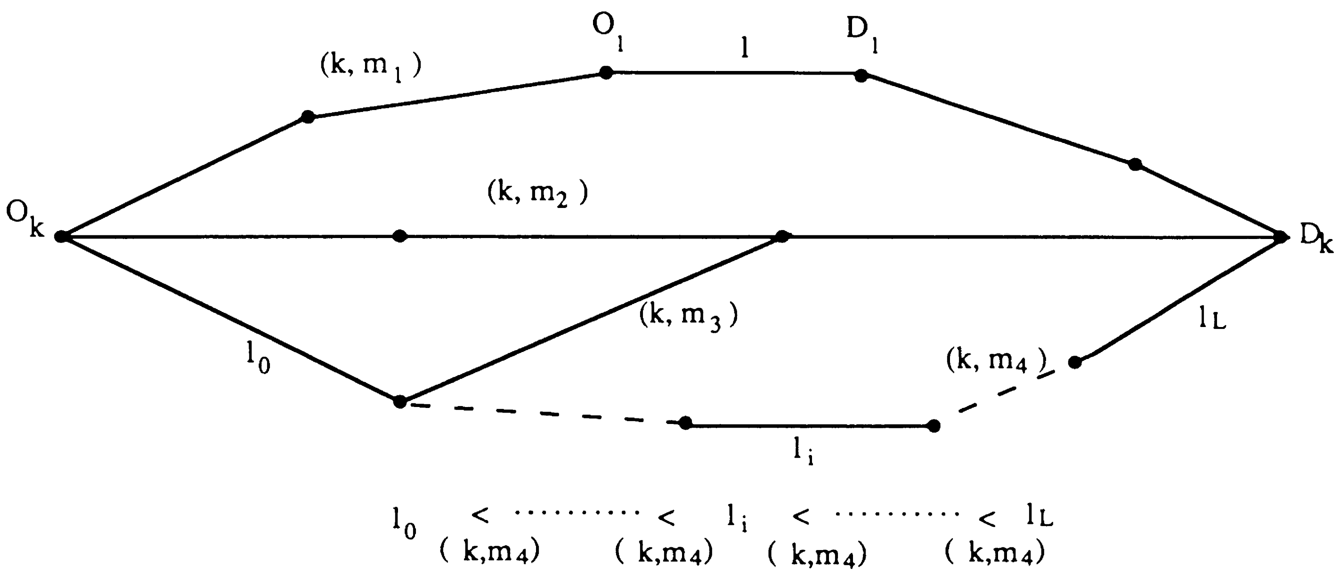

Fig. 1