

# Improved Internal Wave Spectral Continuum in a Regional Ocean Model

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## Key Points:

- Internal gravity wave spectra in regional models are more realistic as model grid spacing decreases.
- The vertical wavenumber spectra improve less dramatically than the frequency spectra.
- Internal gravity wave consistency relations are applied to modeled spectra.

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**Abstract**

Recent work demonstrates that high-resolution global models forced simultaneously by atmospheric fields and the astronomical tidal potential contain a partial internal (gravity) wave (IW) spectral continuum. Regional simulations of the MITgcm forced at the horizontal boundaries by a global run that carries a partial IW continuum spectrum are performed at the same grid spacing as the global run and at finer grid spacings in an attempt to fill out more of the IW spectral continuum. Decreasing only the horizontal grid spacing from 2 km to 0.25 km greatly improves the frequency spectra and slightly improves the vertical wavenumber spectra of the horizontal velocity. Decreasing only the vertical grid spacing by a factor of 3 does not yield any significant improvements. Decreasing both horizontal and vertical grid spacings yields the greatest degree of improvement, filling the frequency spectrum out to 72 cpd. Our results suggest that improved IW spectra in regional models are possible if they are run at finer grid spacings and are forced at their lateral boundaries by remotely-generated IWs. Additionally, consistency relations demonstrate that improvements in the spectra are indeed due to the existence of IWs at higher frequencies and vertical wavenumbers when remote IW forcing is included and model grid spacings decrease. By being able to simulate an IW spectral continuum to 0.25 km scales, these simulations demonstrate that one may be able to track the energy pathways of IWs from generation to dissipation, and improve the understanding of processes such as IW-driven mixing.

**Plain Language Summary**

Models of internal waves (IWs) may help us to better understand the spatial geography of mixing in the ocean, and are playing an increasingly important role in the planning of satellite missions. Following recent work showing that high-resolution global models contain a partial IW spectrum, this paper describes further improvements in the spectrum seen in a high-resolution regional model forced at the boundaries by a previously-performed global IW simulation. Decreasing only the horizontal grid spacing greatly improves the frequency spectra and slightly improves the vertical wavenumber spectra of velocity. Increasing only the number of vertical levels does not yield any significant improvements. Decreasing both horizontal and vertical grid spacings yields the greatest improvement in both spectra. Our results suggest that regional models can exhibit improved internal wave spectra over global models if two conditions are met—they must have higher

49 horizontal and vertical resolutions, and they must have remotely generated IWs at their  
50 boundaries. Application of the so-called “consistency relations” demonstrates that the  
51 model is indeed carrying a field of high-frequency IWs. Being able to simulate a fuller  
52 IW spectrum demonstrates that one may be able to use these models to improve the un-  
53 derstanding of IW-driven processes and energy pathways.

## 54 **1 Introduction**

55 Internal (gravity) waves (IWs) are a ubiquitous dynamical feature of any density-  
56 stratified fluid. First documented in the ocean by Nansen in his momentous Arctic ex-  
57 pedition (Nansen, 1897), IWs have been at the forefront of both observational and the-  
58 oretical ocean research continuously over the last century. Besides being a common fea-  
59 ture in many in-situ and remote-sensing observations, they contribute to many impor-  
60 tant ocean processes such as mixing and acoustics. They also impact considerations for  
61 operational oceanography and remote-sensing satellite missions, particularly the upcom-  
62 ing Surface Water and Ocean Topography (SWOT) wide-swath altimeter mission (Fu  
63 et al., 2012).

64 As IWs in the linear regime obey well-known dispersion laws, they are often stud-  
65 ied in Fourier space; i.e. in terms of frequency and wavenumber rather than time and  
66 space. Garrett and Munk (1972, 1975) produced an empirical model of the full frequency-  
67 wavenumber spectrum of oceanic IWs assuming a homogeneous and statistically station-  
68 ary ocean. This model of the IW spectral continuum, referred to here as the GM76 spec-  
69 tral model, is still in wide use today, although several improvements have been subse-  
70 quently suggested (e.g., Garrett and Munk, 1979; Levine, 2002).

71 Demonstrating that the GM76 spectrum appears naturally as a ubiquitous feature  
72 of numerical ocean models has remained a primary goal of theoretical research. How-  
73 ever, the spectrum strongly depends on the actions of nonlinear and small-scale dynam-  
74 ics to move energy from the large-scale forcings, e.g., surface winds and tides, to the smaller  
75 scales in which the continuum of spectral energy resides. This inherent complexity presents  
76 an obstacle for simulating a full IW spectrum in global- and basin-scale models.

77 Global modeling of the IW continuum spectrum is still in its infancy, and the avail-  
78 able models have their own advantages and disadvantages. A summary of recent state-  
79 of-the-art global IW continuum modeling efforts, which build upon efforts to include the

80 astronomical tidal potential in high-resolution ocean general circulation models (e.g., Ar-  
81 bic et al., 2010; Rocha et al., 2016) is provided in Arbic et al. (2018). The first demon-  
82 stration that a partial IW continuum spectrum exists in global models was by Müller  
83 et al. (2015), who compared frequency spectra in models to moored observations, and  
84 found that the frequency-horizontal wavenumber spectra of the model’s horizontal ki-  
85 netic energy follows expected IW dispersion curves. The HYbrid Coordinate Ocean Model  
86 (HYCOM; Chassignet et al., 2009) simulations discussed in Müller et al. (2015) have been  
87 vetted against many observations, and have internal tide energy levels that match those  
88 in altimetry reasonably well (e.g., Shriver et al., 2012). The MIT General Circulation  
89 Model (MITgcm; Marshall et al., 1997) simulation employed as the basis for this study,  
90 denoted as LLC4320 (Rocha et al., 2016), has weaker near-inertial kinetic energy and  
91 stronger internal tidal kinetic energy at the ocean surface when compared to drifter ob-  
92 servations (Yu et al., 2019). However, the IW spectral continuum of the LLC4320 sim-  
93 ulation lies closer to observations than does the continuum in the global HYCOM sim-  
94 ulations (Savage et al., 2017).

95 Due to the potential importance of IW models for gaining a better understanding  
96 of ocean mixing, acoustics, and satellite measurements, it is natural to examine the fur-  
97 ther improvements to the IW continuum spectra seen in higher-resolution models. Be-  
98 cause decreasing the grid spacing of the models increases the computational workload  
99 required to integrate the models, modelers often simulate regional domains rather than  
100 the entire globe. One such effort (Mazloff et al., 2020) attempted this feat and found that  
101 their IW spectra are significantly lacking, especially at higher frequencies. It is suspected  
102 that this is due to missing IWs that are generated outside the domain that propagate  
103 to the region of interest, contributing to the IW spectrum inside the region. Therefore,  
104 it is natural to attempt to further improve the IW continuum spectra seen in regional  
105 models by forcing them at their boundaries by the global internal wave models. This manuscript  
106 documents improvements in the IW spectrum of a regional model run with lateral bound-  
107 ary conditions taken from a global IW model, and with horizontal and vertical grid spac-  
108 ings that are finer than can be achieved in today’s state-of-the-art global models.

109 There are many practical applications for models that can better simulate the IW  
110 spectral continuum beyond aiding research in large-scale topics such as acoustics and satel-  
111 lite measurements. For example, the simulation of an IW spectral cascade down to finer  
112 spatial scales may allow one to track the energy pathways of IWs from generation to dis-

113 sipation along the lines of fine-scale parameterizations applied to observations (Polzin  
114 et al., 1995). Ultimately, one may be able to use IW models to study the spatial geog-  
115 raphy of IW-driven mixing, which in turn impacts the large-scale oceanic circulation (e.g.,  
116 Munk & Wunsch, 1998).

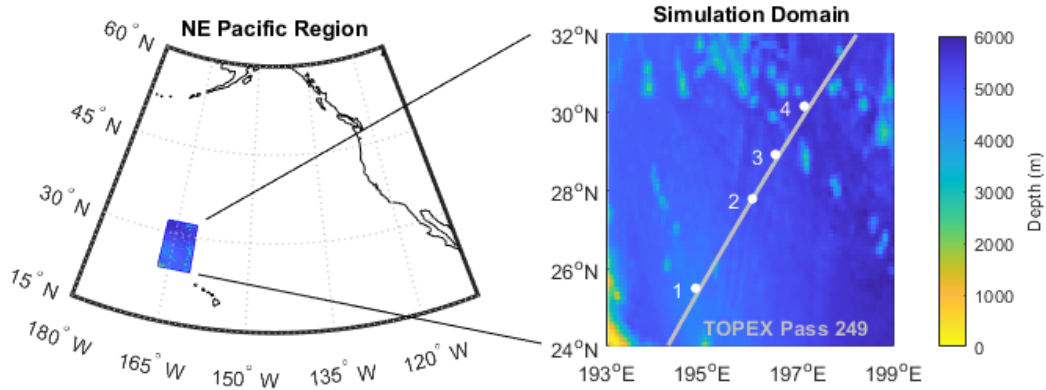
## 117 **2 Observations and Numerical Simulations**

118 This study utilizes observations and MITgcm simulations of a region northwest of  
119 Hawaii bounded by 24°-32°N, 193°-199°E, characterized by an IW beam that is gener-  
120 ated by tidal forcing over a region of shallow (<1000m) topography, the French Frigate  
121 Shoals, in its southwest corner. This domain was partially instrumented in Spring 2006  
122 in the context of the Internal Waves Across the Pacific (IWAP) experiment (Alford et  
123 al., 2007). A collection of observations from six McLane Moored Profilers (MMPs; Mor-  
124 rison III et al., 2001) from IWAP are available; four are within the defined domain. The  
125 MMP observations are described in more detail in Section 2.1. The MITgcm simulations  
126 performed within this domain are described in Section 2.2.

### 127 **2.1 MMP Observations**

128 The MMPs measure pressure, temperature, salinity, and three-dimensional veloc-  
129 ity via a sensor array that travels up and down a moored wire. For the MMP observa-  
130 tions analyzed here, measurements are taken between approximately 90m and 1500m over  
131 a period of about 50 days (depending on the instrument in question) with a sampling  
132 interval of approximately 1.5 hours. The locations of the profilers within the analysis do-  
133 main are displayed in Figure 1, numbered in order from southern-most to northern-most.  
134 The MMP data are calibrated, processed, and linearly interpolated onto standardized  
135 depth levels set 2 dbar apart (Toole, 2001). The high vertical resolution of the MMP data  
136 make them ideal for examining vertical wavenumber spectra. These data have been an-  
137 alyzed in numerous publications (Alford et al., 2007; Zhao et al., 2010; MacKinnon et  
138 al., 2013; Alford et al., 2017; Luecke et al., 2017; Savage et al., 2017).

139 Periods in the MMP observations containing gaps longer than 6 hours are discarded.  
140 The MMP observations are slightly asynchronous in time, so they are linearly interpo-  
141 lated onto even time steps 1.5 hours apart. Any obvious outliers in visual inspections



**Figure 1.** (Left) The Northeast Pacific region showing a colored box at the location of the simulation domain. (Right) A close-up of the simulation domain including the local bathymetry (color), the locations of the MMPs (white dots), and the trajectory of TOPEX-Poseidon satellite pass #249 (gray line).

142 are removed, and any remaining gaps in the data are filled in via linear interpolation.

143 The details of these processed observational records are listed in Table 1.

Profiler (#)	Location (°N, °E)	z Range (m)	z Span (m)	t Range (d.o.y.)	t Span (days)
1	25, 195	88–1398	1310	115–151	36
2	28, 196	94–1396	1302	141–150	9
3	29, 197	88–1402	1314	139–165	26
4	30, 197	94–1400	1306	115–137	22

**Table 1.** Locations in latitude and longitude (°N, °E), depths (z), and times (t) of uninterrupted MMP data availability. d.o.y. denotes day-of-year (for a non-leap year). All values are rounded to the nearest integer.

144 Vertical velocity in the ocean is generally much smaller than platform motion and  
 145 hence immeasurable except from bottom-mounted platforms. The MMP is no exception;  
 146 hence, vertical velocities from the observations are not considered in this work.

## 2.2 Model Output

The global circulation model output employed as the basis for this study is the MIT-gcm simulation denoted by MITgcm48 in some previous publications (e.g., Savage et al., 2017) and LLC4320 in others (e.g., Rocha et al., 2016; Qiu et al., 2018). Here, it will be referred to as LLC4320. This simulation was run in the hydrostatic configuration with a nominal  $1/48^\circ$  horizontal grid (2.3km near the equator, 0.75km at high latitudes) and 90 depth levels from the free surface to seafloor depths up to 7000m with thicknesses increasing from 1.0m at the surface level to 480m at the bottom level. The ocean surface is forced with the six-hourly atmospheric fields from the  $0.14^\circ$  European Center for Medium-Range Weather Forecasts (ECMWF) atmospheric operational model analysis. Starting on the first day of 2011, ECMWF output is converted to surface fluxes (Large & Yeager, 2004), providing forcings from atmospheric wind, pressure, and buoyancy. This continues for the yearly duration of the model run. Note that the wind power from these fields may be under-represented in the simulations, as discussed by (Yu et al., 2019) and references therein. A sea ice model (Losch et al., 2010) is used to compute ocean surface fluxes over ice-covered regions. Astronomical tidal forcing, provided by the full luni-solar potential, is applied to MITgcm as an additional atmospheric pressure forcing (Ponte et al., 2015).

The new regional simulations performed for this study are driven by the same atmospheric fields at the ocean surface. A hard boundary condition is applied at the lateral boundaries, in which the model variables (e.g., temperature, salinity, and velocities) at every depth level are set to be directly equal to the LLC4320 hourly output. While simple to implement, a hard boundary condition may cause kinetic energy buildup and wave reflections at and near the boundaries. The hourly time stepping of the boundary conditions from the global model may also create a pulsing signal with a frequency of 24 cpd and its harmonics that will propagate throughout the simulation domain. Visual comparisons of animations from LLC4320 and the regional simulations are performed to ensure that the boundary condition choices do not produce any strong reflections or other anomalous features. All regional simulations are performed on the Niagara super-computing cluster operated by SciNet at the University of Toronto (Ponce et al., 2019).

The first simulation (denoted “One-To-One”) is performed at the same horizontal and vertical grid spacings as the global LLC4320 simulation to determine if the bound-

	LLC4320	One-To-One	Finer- $\Delta z$	Finer- $\Delta x$	Finer-Both
$\Delta x$ (km)	2	2	2	1/4	1/4
# Levels	90	90	270	90	270

**Table 2.** The nominal horizontal grid spacing ( $\Delta x$ ) and number of vertical depth levels of the MITgcm simulations utilized in this work. LLC4320 is a global simulation, and is used to force the boundaries of the other regional simulations.

179 any effects significantly impact the regional simulation. Simulations are then run with  
 180 finer grid spacings, including one at a finer vertical grid spacing of 270 levels (denoted  
 181 “Finer- $\Delta z$ ”), one at a finer horizontal grid spacing of 250m (denoted “Finer- $\Delta x$ ”), and  
 182 one where both horizontal and vertical grid spacings are made finer by these amounts  
 183 (denoted “Finer-Both”). The spatial configurations of these simulations are summarized  
 184 in Table 2.

185 For all but the Finer-Both simulation, a time step size ( $\Delta t$ ) of 25 seconds is used,  
 186 the same as that used in LLC4320. In the Finer-Both simulation, small-scale processes  
 187 cause the simulation to become unstable at this  $\Delta t$  at some point during the integra-  
 188 tion. When this instability is flagged by the model, the simulation is stopped,  $\Delta t$  is de-  
 189 creased, and the simulation is resumed from the beginning of the day in which the in-  
 190 stability occurred. This stopping and restarting of the model integration occurs multi-  
 191 ple times, resulting in a median  $\Delta t$  of 10 seconds and a minimum  $\Delta t$  of 6 seconds. These  
 192  $\Delta t$ ’s are important to keep in mind when exploring internal waves; the wavelengths and  
 193 frequencies of the internal waves that can be simulated by the model are limited by the  
 194 spatial grid spacing and size of the integration time step. Note that in every case the “sam-  
 195 pling” frequency of the model output (one output every 10 minutes, or 144 outputs per  
 196 day) is a multiple of the model’s integration frequency. Additionally, this finer output  
 197 interval (versus hourly from LLC4320) allows for higher frequencies to be studied; a point  
 198 that will become important later in the analyses.



### 3 Methods

#### 3.1 Data Processing

The depth levels in the global MITgcm simulations were historically chosen to obtain the best representation of the mixed-layer and thermocline, and IWs were not a primary concern. The increasing thickness of the vertical levels at greater depths may produce aliasing, damping, or reflecting of high vertical wavenumber motions as they propagate downwards into deeper regions with layer thicknesses longer than the associated wavelengths. To mimic the effects of the coarse vertical discretization in the model, the observations are processed in three separate ways; at the full observational resolution of 2 dbar, averaged within the global model depth levels (90 levels), and averaged within the finer-grid regional model depth levels (270 levels). The model-level-averaged observations were computed by averaging all the observations at depths between the upper and lower bounds of each depth level. These processed observations are referred to as Obs.-Coarse and Obs.-Fine respectively.

From the model outputs, time series of temperature, salinity, and three-dimensional velocity are taken at the MMP locations and truncated to the time span of the corresponding observations taken at that location. It should be noted that these time spans, while being equal in length, are not contemporaneous with the observational records. In the vertical, the model outputs are truncated to the depth levels that are fully contained within the vertical limits of the corresponding observational record. In both the observations and truncated model outputs, temperature and salinity are used to compute potential density and buoyancy frequency using the Thermodynamic Equation of Seawater 2010 (McDougall & Barker, 2011).

To account for changes in stratification in depth and time, the data may be Wentzel-Kramers-Brillouin-Jeffreys (WKBJ) scaled (Leaman & Sanford, 1975). This scaling transforms the data into a coordinate system in which the mean and variability of the stratification, and consequently the buoyancy frequency  $N$ , is constant in depth. WKBJ scaling is based upon the use of the dimensionless factor  $n(z) = \sqrt{\overline{N^2(z)}/N_0^2}$ , in which  $\overline{N^2(z)}$  denotes the all-time mean of  $N^2(z, t)$  at depth  $z$  and  $N_0$  is a constant reference value. While the choice of  $N_0$  is arbitrary, a value of 72 cpd is used for convenience. Other possible choices of  $N_0$  do not drastically affect the results (Alford & Whitmont, 2007), and a constant value for all four locations is employed.

231 Given this definition of  $n(z)$ , the zonal, meridional, and vertical velocities ( $U$ ,  $V$ ,  
 232 and  $W$  respectively) are scaled by dividing the horizontal velocities by  $\sqrt{n(z)}$ , and by  
 233 multiplying the vertical velocities by this same factor. The depths are also WKBJ-stretched  
 234 using  $z' = \int n(z) dz$  with the upper limit set to 100m at all locations. For the rest of  
 235 this paper, all variables and results are assumed to be WKBJ-scaled unless otherwise noted.  
 236 Also, note that extra care should be taken in interpreting these spectra as they are no  
 237 longer in Cartesian coordinates.

### 238 3.2 Spectral Calculations

239 Power spectral densities (shortened to “spectra” in this work) are computed us-  
 240 ing the classical Fourier transform method in time and using the Lomb-Scargle periodogram  
 241 method (Lomb, 1976; Scargle, 1982) in depth; the latter accounts for uneven vertical sam-  
 242 pling and/or uneven WKBJ stretching. The Nyquist wavelength was chosen to be twice  
 243 the smallest vertical spacing of the associated data. This procedure is performed for both  
 244 the complex horizontal rotary velocity  $\mathcal{Z}(z, t) = U(z, t) + iV(z, t)$  and the vertical ve-  
 245 locity  $W$ . The form of the Lomb-Scargle periodogram for complex series is derived in  
 246 Bretthorst (2003), which simplifies to a linear-least-squares fit of sinusoids since  $U$  and  
 247  $V$  are sampled simultaneously. Spectra are computed over 6.85-day windows with 50%  
 248 overlap; a non-integer-day window length was chosen to limit the possibility of aliasing  
 249 of near-inertial and tidal signals with frequencies of an integer or near-integer multiple  
 250 of 1 cpd. No windowing is done in depth due to the limited number of vertical levels in  
 251 the simulations that span the depth range of the observations; about 30 and 90 for the  
 252 90-level and 270-level configurations respectively.

253 According to linear wave theory, the spectra of rotary velocity are expected to be  
 254 red in both frequency and vertical wavenumber. Since Fourier transforms expect the data  
 255 to be a sum of sinusoids, this redness can introduce a bias when these sinusoids are fit  
 256 to the data. This bias, also known as spectral leakage, is accounted for here by apply-  
 257 ing pre-whitening to the spectral computations of  $\mathcal{Z}$  by multiplying each Fourier coef-  
 258 ficient by the factor  $\omega m \text{sinc}(\omega dt) \text{sinc}(mdz)$ , where the frequency  $\omega$  and vertical wavenum-  
 259 ber  $m$  are in units of cycles per unit time and unit depth respectively,  $dt$  is the time step  
 260 size, and  $dz$  is the smallest vertical spacing. The pre-whitening is done individually for  
 261 each time window. The resulting spectra are averaged together and then recolored by  
 262 dividing the averaged spectra by the factor  $|(2 - 2 \cos(\pi \omega dt))(2 - 2 \cos(\pi mdz))|$ . These

263 spectra, which have units of squared velocity per unit frequency and vertical wavenum-  
 264 ber, are henceforth referred to as kinetic energy spectra. Upper and lower 95% confidence  
 265 bounds are computed using traditional means and subjected to the same pre-whitening  
 266 procedure.

### 267 3.3 Consistency Relations

268 As first detailed in Fofonoff (1969), for an arbitrary field of freely-propagating lin-  
 269 ear waves, there exists a complete set of linearly independent spectral and cross-spectral  
 270 relations called consistency relations. In statistical tests, the consistency relations are  
 271 used to quantify the confidence that the variability in a set of data can be explained by  
 272 a wave field with the corresponding relations. A summary of the consistency relations  
 273 for an oceanic IW field is detailed in Table 1 of Müller and Siedler (1976) as well as in  
 274 Table 3 of Lien and Müller (1992). An application of these consistency relations to ob-  
 275 servational data is recorded in Müller et al. (1978). Since these relations are derived from  
 276 linear IW theory, they are independent of any spectral model choice, GM76 or otherwise.

277 While there are many consistency relations that can be tested, for the sake of sim-  
 278 plicity, only two are used in this work. These are the ratio of the counter-clockwise (CCW)  
 279 to clockwise (CW) horizontal kinetic energy spectra, and the ratio of the vertical to hor-  
 280 izontal kinetic energy spectra:

$$\frac{S^+}{S^-} = \left( \frac{\omega - f}{\omega + f} \right)^2 \quad (1)$$

$$\frac{N^2}{\omega^2} \frac{S^0}{S^- + S^+} = \frac{\omega^2 - f^2}{\omega^2 + f^2} \quad (2)$$

281 where  $f$  and  $N$  are the local inertial and buoyancy frequencies,  $S^+/S^-$  is the frequency  
 282 ( $\omega$ ) spectra of  $\mathcal{Z}$  at positive/negative values of  $\omega$ , corresponding to CCW/CW rotation  
 283 in time, and  $S^0$  is the frequency spectrum of vertical velocity. Equation 2 is written in  
 284 such a way that the value of the consistency relation approaches unity for  $f \ll \omega \ll$   
 285  $N$ . These consistency relations are written for a hydrostatic ocean; for a non-hydrostatic  
 286 ocean, Equation 2 has an additional multiplicative factor of  $N^2/(N^2 - \omega^2)$  on the right-  
 287 hand side.

288 In this work, to avoid complications of a varying stratification with depth, the fre-  
 289 quency spectra used to evaluate these consistency relations are computed individually  
 290 at each depth using the non-WBKJ-scaled time series (a windowing of 6.85 days with

50% overlap is still employed).  $\overline{N^2(z)}$  at the respective depth is used in computing the theoretical value of the consistency relation in Equation 2.

### 3.4 A Note on the Dispersion Relation versus Model Grid Spacing

The simulated IW spectrum is strongly dependent on the model grid spacing. For a given  $\Delta x$ , decreasing  $\Delta z$  in turn increases the upper limit of the  $\omega$  that can be simulated. For example, for a vertical wavelength of 100m, the model configuration with  $\Delta x = 2000\text{m}$  can only simulate IWs up to a limit of  $\omega \approx 36$  cpd according to the dispersion relation assuming a buoyancy frequency equal to  $N_0 = 72$  cpd. When  $\Delta x = 250\text{m}$ , this limit increases to  $\omega \approx 134$  cpd, which is greater than the time-mean value of  $N$  at nearly all depths in the observations and simulations within the domain used in this work.

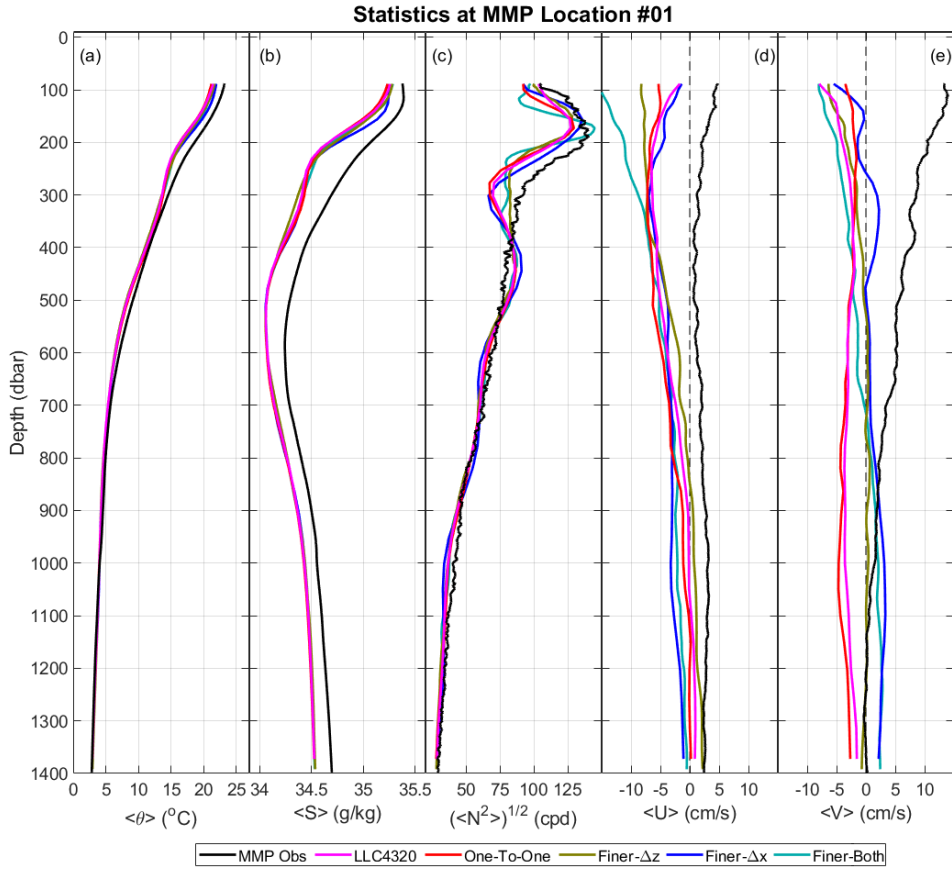
Another way to explore this issue of relating  $\Delta x$  and  $\Delta z$  is in the context of orthogonal vertical modes. For any given vertical mode, there is a prescribed horizontal and vertical wavelength associated with that mode. At high modal numbers, if the grid spacing in one dimension is not sufficient to resolve a given mode, it is unclear how the dynamics associated with that mode in the other dimension are affected. This concept was recently explored in the context of atmospheric modeling (Stewart et al., 2017), which found that there should be at least  $2\pi$  vertical layers between the zero crossings of a given vertical mode in order for that mode to be accurately simulated. Stewart et al. (2017) then used this concept to prescribe the number of vertical layers in their model given its horizontal resolution using the relationship between the horizontal and vertical wavelength of a given vertical mode as previously mentioned. This concept was not initially considered when choosing the model grid spacings used in this work, but, as will be seen, such constraints on model grid spacing may need to be more carefully followed in future high-resolution oceanic IW simulations.

## 4 Results & Discussions

### 4.1 First Look

To get an idea of the dynamical properties at the southern-most MMP location, and to get an early insight into the model-data agreement, record-length means of temperature, salinity, buoyancy frequency, and velocity are shown in Figure 2. These data

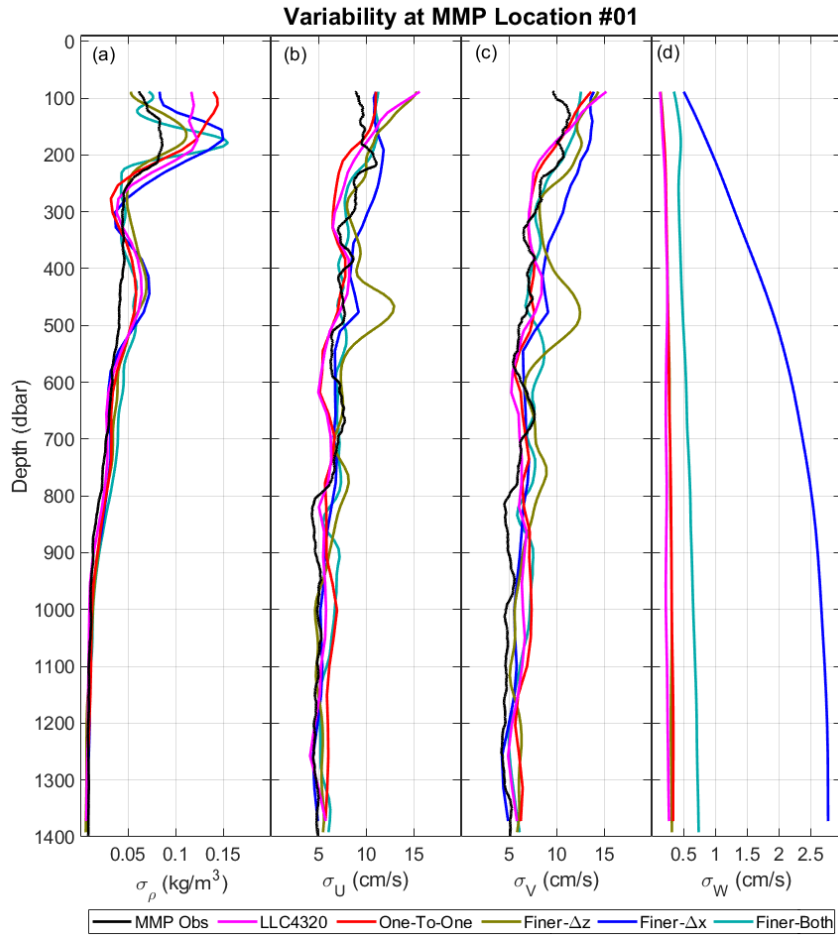
321 are not WKBJ-scaled. These results, along with the results in the following subsections,  
 322 are similar at the other MMP locations. Thus, for the sake of brevity, we will only show  
 323 results at the southern-most MMP location in this paper.



**Figure 2.** The time mean over the observational period of the variables of interest at the southern-most MMP location from all data sources. These variables include (a) temperature, (b) salinity, (c) buoyancy frequency (computed as the root-mean-square), (d) zonal velocity, and (e) meridional velocity. These results are not WKBJ-scaled.

324 Figure 2 indicates that the thermodynamic variables display reasonable agreement  
 325 between the observations and simulations, which both exhibit the strongest vertical gra-  
 326 dients of temperature and salinity between 100m and 300m depth. The observed and  
 327 simulated mean velocities are opposite in sign, with the simulations exhibiting a stronger  
 328 mean velocity in the zonal direction and a weaker mean velocity in the meridional di-  
 329 rection relative to the observations. However, as will be shown, the standard deviation  
 330 of the velocities are larger than their means at nearly all depths, so this result is not sta-

331 tistically significant. Additionally, the record length is short relative to the timescales  
 332 of intrinsic variability in this region due to structures such as mesoscale eddies. Since  
 333 the model is not data-assimilative, there is no reason to expect these larger, slower struc-  
 334 tures to match between the model and the real world over this short time period, so this  
 335 sign reversal in the mean flow is not surprising.



**Figure 3.** The temporal standard deviations of the (a) potential density, (b) zonal velocity, (c) meridional velocity, and (d) vertical velocity at the southern-most MMP location from all data sources. Velocities are not WKBJ-scaled.

336 The temporal standard deviations of the non-WKBJ-scaled potential density and  
 337 velocities are shown in Figure 3. As explained previously, vertical velocities from the MMPs  
 338 are disregarded in this study. Below 300m, the variabilities of the velocities are fairly con-

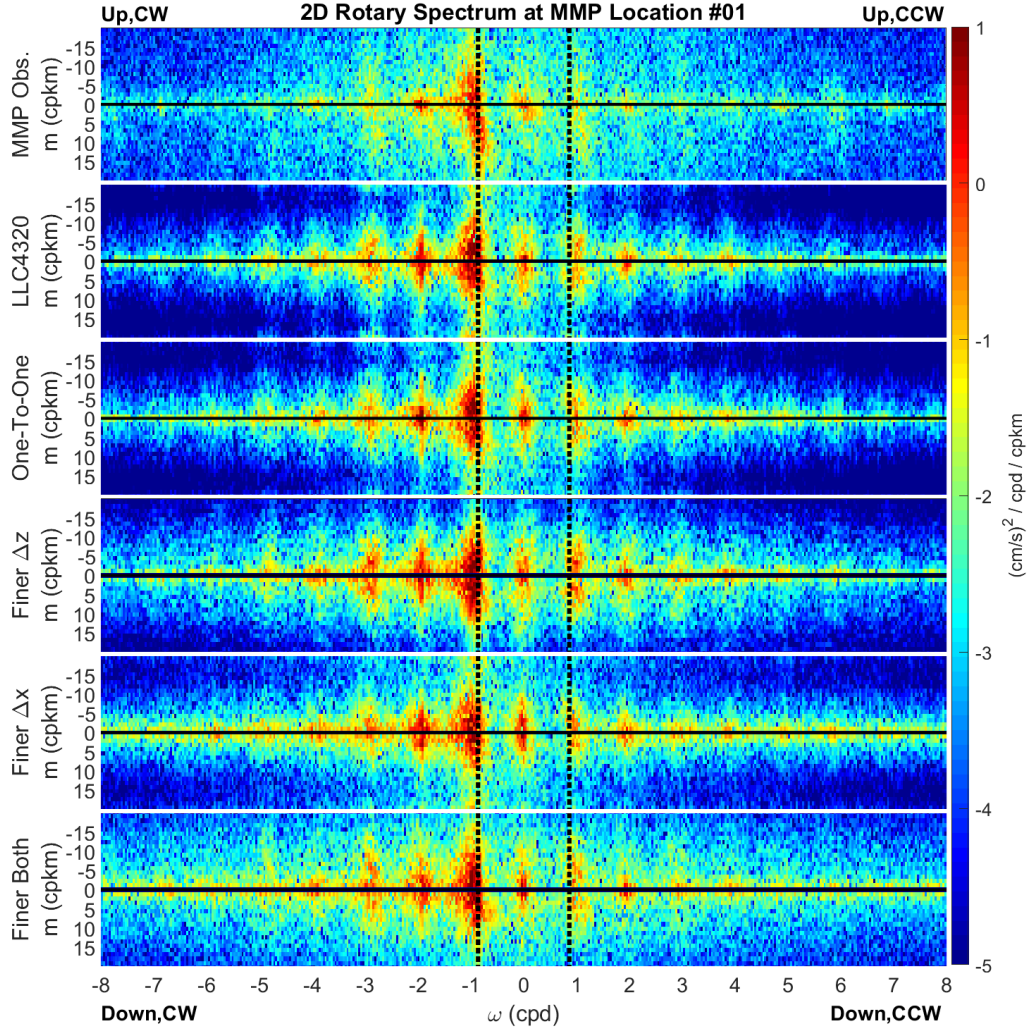
339 stant in depth with magnitudes that are comparable between the observations and all  
340 simulations, with the simulations showing a slightly stronger variance overall. In con-  
341 trast, the variability in the vertical velocity varies significantly depending on the model  
342 grid spacings. For the Finer-Both simulation, the variability in the vertical velocity is  
343 about twice as large as the vertical velocity variability in the LLC4320, One-To-One, and  
344 Finer- $\Delta z$  simulations at all depths. The vertical velocity in the Finer- $\Delta x$  simulations is  
345 significantly more variable relative to the other simulations. This striking increase in the  
346 vertical velocity when only the horizontal grid spacing is decreased may or not be due  
347 to a mismatch between the horizontal and vertical model grid spacings as discussed in  
348 Section 3.4.

## 349 4.2 Frequency - Vertical Wavenumber Rotary Spectra

350 Figure 4 displays the frequency-vertical wavenumber spectra of the observations,  
351 LLC4320, and the four MITgcm runs performed on the Niagara supercomputer. All sim-  
352 ulations show a large amount of variance at the near-inertial, diurnal, and semidiurnal  
353 frequencies. The peaks at these frequencies are indicative of forcing from the winds and  
354 the astronomical tidal potential. There is a strong asymmetry between clockwise (CW)  
355 and counterclockwise (CCW) rotating velocities, with both the observations and sim-  
356 ulations showing more energy at the CW forcing frequencies rather than CCW. This asym-  
357 metry is as expected in the Northern Hemisphere, and it is important that, before any-  
358 thing else, one verifies that the simulations capture this CW/CCW asymmetry.

359 In the One-To-One simulation, the spectrum has a majority of the variance resid-  
360 ing within peaks near the forcing frequencies and their harmonics, compared to the ob-  
361 servations which have variance spread out across the spectral continuum. This “peak-  
362 iness” may be indicative of a lack of nonlinear interactions in this simulation necessary  
363 to transfer energy from these low-frequency bulk motions into the high-frequency IWs.

364 When the vertical grid spacing is decreased, the spectra remain similar to that of  
365 the One-To-One simulation, although more variance is present at higher vertical wavenum-  
366 bers. When the horizontal grid spacing is decreased, there is a similar effect, except that  
367 more variance is present at higher frequencies and in between the forcing bands and har-  
368 monics, while the spectrum still exhibits the “peakiness” of the One-To-One spectrum.  
369 When both horizontal and vertical grid spacings are decreased, the spectrum becomes



**Figure 4.** The 2D WBKJ-scaled frequency-vertical wavenumber spectrum of horizontal kinetic energy at the southern-most MMP location for the observations (top 1) and the MITgcm simulations (bottom 5). Dashed lines indicate the local inertial frequency. Positive/negative frequencies indicate CCW/CW rotation in time. Positive/negative vertical wavenumbers indicate CCW/CW rotation in (WBKJ-stretched) depth. The peaks are typically present at inertial and tidal frequencies and their higher-order harmonics.



370 much more comparable to that seen in the observations, with much less “peakiness” at  
371 the forcing frequencies and much more energy spread out into the continuum, especially  
372 at higher vertical wavenumbers.

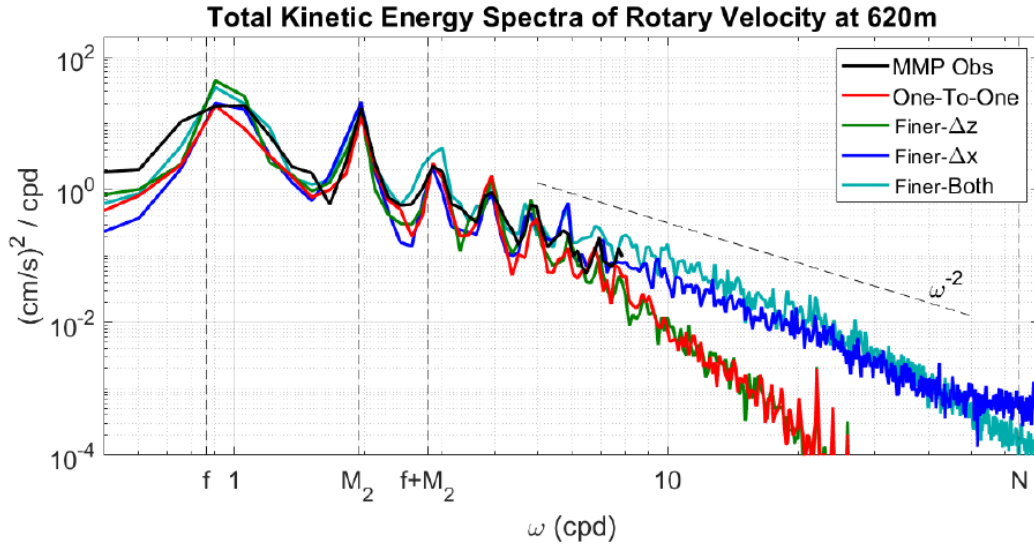
373 One troubling feature is the noticeable increase in variance with vertical wavenum-  
374 ber at high vertical wavenumbers. This increasing variance at high vertical wavenum-  
375 bers only occurs at wavelengths shorter than the coarsest vertical spacing, so this increase  
376 in variance is attributed to either aliasing from lower wavenumbers or from model “speck-  
377 ling”, an instability where grid points have alternating positive and negative values that  
378 flip sign and increase in amplitude with each time step. We do not know whether this  
379 is due to failure of the hydrostatic assumption or some other aspect of the model.

### 380 4.3 Frequency Power Spectra

381 The total (CW plus CCW) horizontal kinetic energy frequency spectra at 620m are  
382 shown in Figure 5. The separate frequency spectra of CCW and CW horizontal kinetic  
383 energy and vertical kinetic energy are shown in Figure 6. In Figure 7, the theoretical and  
384 measured consistency relations in Equations 1 and 2 are displayed. The 90% confidence  
385 interval (5% to 95% confidence limits) of the consistency relations are computed using  
386 a million-member bootstrap based on the confidence intervals of the associated spectra  
387 and are denoted by the shaded regions. WBKJ scaling assumes  $\omega \ll N$ , but this isn’t  
388 always true for the simulation output which has an integration frequency exceeding the  
389 time mean of  $N$  at some depths. Thus, only the frequency spectra of the non-WBKJ-  
390 scaled data are explored here. The spectra are also smoothed in log-log space to aid in  
391 visually separating the curves.

392 From Figure 5 and Figure 6 (a) and (b), it is apparent that the variance of hor-  
393 izontal velocity, and its partitions into CW and CCW components, is similar between  
394 the observations and all of the simulations up to frequencies at about 5 cpd. The obser-  
395 vations are different in that there is less variance at the forcing peaks and more variance  
396 in the continuum. This “peakiness” is consistent with a lack of nonlinear small-scale dy-  
397 namics in the simulations as previously discussed.

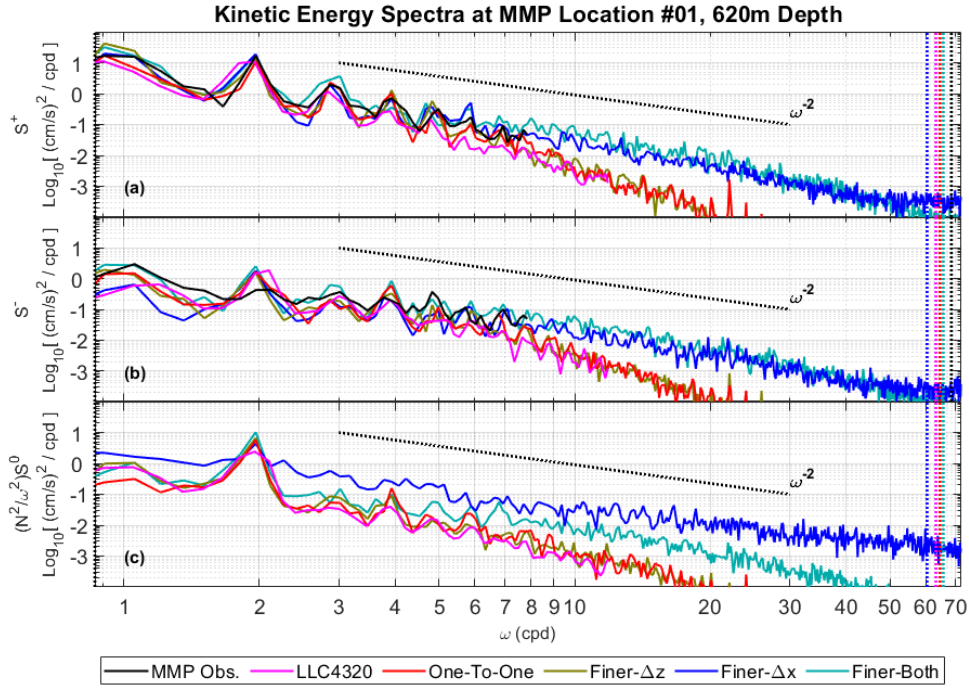
398 At frequencies higher than about 5 cpd, the spectra of the simulations with the coarser  
399 horizontal grid spacing begin to decrease relative to the observations, the finer horizon-  
400 tal grid spacing simulations, and the GM76 model. There are some noticeable peaks near



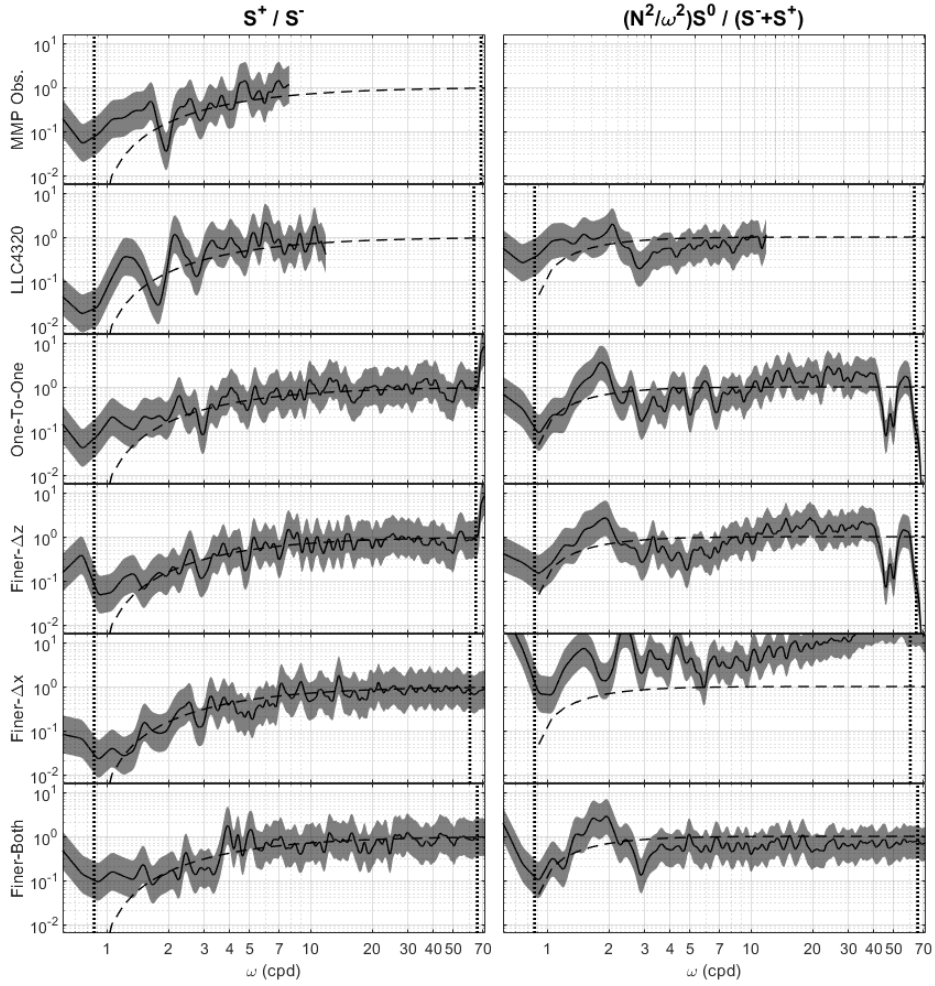
**Figure 5.** The total (CW plus CCW) horizontal frequency spectra at 620m depth at the southern-most MMP location. These spectra are not WKBJ-scaled. There are marks on the x-axis for the inertial frequency  $f$ , the semidiurnal lunar tide  $M_2$ , their sum  $f + M_2$ , and the buoyancy frequency from the observations  $N$  (computed as the root of the time mean of  $N^2$ ). The dashed curve above the spectra represents the spectral slope theorized by the GM76 model.

401 24 and 48 cpd, likely due to the hourly forcing at the simulation boundaries, and near  
 402 the Nyquist frequency of 72 cpd due to model speckling, but even with these peaks the  
 403 spectra still do not rise to the variance of the Finer- $\Delta x$  and Finer-Both spectra, and are  
 404 hence not visible in Figure 5 and Figure 6 (a) and (b). Comparatively, the Finer- $\Delta x$  sim-  
 405 ulation retains a GM76 model-like spectral slope of approximately  $\omega^{-2}$  out to about 50  
 406 cpd, after which it begin to flatten. The Finer-Both simulation maintains a slope of  $\omega^{-2}$   
 407 out to the “Nyquist” frequency associated with the model output frequency; 72 cpd.

408 In the vertical kinetic energy spectra of Figure 6 (c), once again the spectra from  
 409 the simulations with coarser horizontal grid spacings begin to decrease relative to the  
 410 simulations with finer horizontal grid spacing after about 5 cpd. However, the Finer- $\Delta x$   
 411 simulation exhibits a higher vertical velocity variance and a spectral slope shallower than  
 412  $\omega^{-2}$  at all frequencies. It is unknown whether the differences in the Finer- $\Delta x$  simu-  
 413 lation’s vertical velocity spectrum relative to the other simulations is the result of a pos-  
 414 sible artifact as described previously or some other mechanism.



**Figure 6.** The total (CW plus CCW) horizontal kinetic energy spectra at 620m depth at the southern-most MMP location, including (a) the CCW kinetic energy spectra, (b) the CW kinetic energy spectra, and (c) the vertical kinetic energy spectra. The dotted curves in (a)-(c) represent the spectral slope theorized by the GM76 model. The root-mean-squares of  $N$  from each data source are plotted as vertical dotted lines colored as given in the legend and marked as  $N$  on the x-axis. These spectra are not WKBJ-scaled.



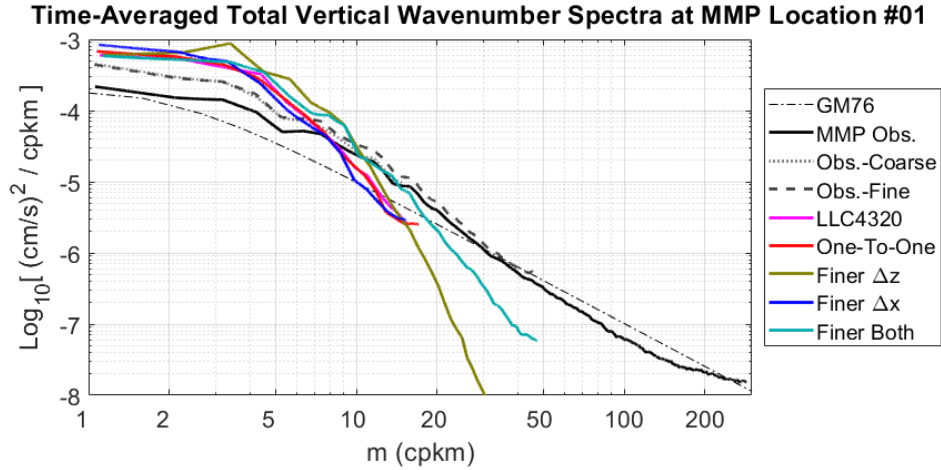
**Figure 7.** Consistency relations at 620m depth at the southern-most MMP location, including (left column) the ratio of CCW to CW kinetic energy spectra, and (right column) the ratio of the vertical kinetic energy spectra to the total horizontal kinetic energy spectra. Shading denotes the 90% confidence interval (5% to 95% confidence bounds).

415 The two consistency relations in Figure 7 are followed closely by the observations  
416 and most of the simulations except near the forcing peaks and their harmonics, but there  
417 are some notable deviations from theory. In the left column of Figure 7, which plot the  
418 ratios of the CCW to CW horizontal kinetic energy spectra, all data sources are statis-  
419 tically indistinguishable from theory except at frequencies approaching the Coriolis fre-  
420 quencies. We speculate that this may be due to the presence of other submesoscale mo-  
421 tions such as eddies which begin to manifest at these lower frequencies, but further work  
422 is needed to verify this speculation. In addition, the observations and LLC4320 also de-  
423 viate from theory at about 1 and 2 cpd due to tidal forcing, but these deviations do not  
424 show up in the regional simulations. This deviation could be due to the regional sim-  
425 ulations beginning on the first of March rather than the beginning of the year as in LLC4320,  
426 highlighting the importance of model spin-up in the regional domain even in the pres-  
427 ence of remote forcings.

428 In the right column of Figure 7, which plot the ratio of the vertical to horizontal  
429 kinetic energy spectra, the One-To-One and Finer- $\Delta z$  simulations deviate significantly  
430 from theory after about 40 cpd, and the Finer- $\Delta x$  simulation exhibits a ratio statisti-  
431 cally greater than 1 at nearly all frequencies, and increases as frequency increases start-  
432 ing at about 10 cpd. In both plots, all data sources also exhibit a discrepancy near 2 cpd,  
433 likely due to the same reasons as discussed for the previous consistency relation. This  
434 discrepancy is consistent with results from other works (e.g. Müller et al., 1978). Note  
435 that in the Finer- $\Delta x$  simulation, the consistency relation fails to hold at nearly all fre-  
436 quencies, once again highlighting the importance of matching the horizontal and verti-  
437 cal grid spacings as previously discussed. Altogether, these two consistency relations pro-  
438 vide statistical confidence that the model does indeed simulate IWs, and that these IWs  
439 are better simulated when both horizontal and vertical grid spacings are decreased.

#### 440 4.4 Vertical Wavenumber Power Spectra

441 The vertical wavenumber spectra computed by integrating the rotary spectra in  
442 Figure 4 across all frequencies at the southern-most MMP location are shown in Figure  
443 8. For convenience, the CW and CCW spectra are summed together so as to reduce the  
444 number of curves on the plot. Obs.-Coarse and Obs.-Fine are the observations processed  
445 as defined in Section 3.1, and then analyzed in the same manner as the model outputs.  
446 The difference in the three observation-based spectra at low wavenumbers stems solely



**Figure 8.** The record-length-averaged total (CCW+CW) vertical wavenumber spectra of horizontal velocity at the southern-most MMP location. Observations averaged within the 90-layer and 270-layer model layer configurations are labeled with dotted and dashed curves respectively.

447 from the WKB scaling and stretching; when WKB is not applied, the three curves lay  
 448 on top of each other (not shown). The vertical wavenumber spectrum from GM76 is  
 449 included as well, and was generated using The Garrett and Munk internal wave spectra  
 450 MATLAB toolbox (<http://jkllymak.github.io/GarrettMunkMatlab/>).

451 The MMP vertical wavenumber spectra have more variance than GM76 at wavenum-  
 452 bers below about 30 cpkm and less variance than GM76 as higher wavenumbers. This  
 453 decrease continues until the spectrum levels out beginning at about 100 cpkm. The roll-  
 454 off at high  $m$  is consistent with the presence of a noise floor as found in previous work  
 455 (Alford, 2010), albeit in WKB-stretched coordinates. There is also a roll-off at low  $m$ ,  
 456 but the presence of a spectral peak near 4 cpkm makes it difficult to estimate the limit  
 457 at which this roll-off occurs. The theoretical value of  $m$  at which this low- $m$  roll-off oc-  
 458 curs in GM76 is given by (Polzin et al., 1995) as  $(3\pi/Z)(N/N_0)$ , where  $Z$  is the bathymetry  
 459 at this location (1300km) and  $N/N_0 \approx 1$ , yielding a value of about  $7 \times 10^{-3}$  rad/m or  
 460 about 1 cpkm.

461 In comparison to the spectra from the MMP observations, the spectra from the sim-  
 462 ulations contain more variance at low  $m$ , and less variance at high  $m$ . This pattern is  
 463 strongest for the simulations with finer vertical grid spacing, and there is little improve-  
 464 ment between the simulations with low and high horizontal grid spacing, although the

465 Finer-Both simulation does slightly better. This result is in agreement with the discus-  
466 sions at the end of Section 4.1. The simulated spectra also have their low- $m$  roll-off be-  
467 ginning near 5 cpkm, and possess less distinctive spectral peaks than the observed spec-  
468 trum.

469 In terms of agreement with observations, the Finer-Both simulation displays the  
470 greatest degree of agreement between the MMP observations and GM76 predictions, main-  
471 taining a  $m^{-2}$  slope out to about 15 cpkm, and lying closer to observations at the high-  
472 est resolved vertical wavenumbers than any of the other simulations.

## 473 5 Conclusions

474 The primary conclusion of this work is that decreasing the model grid spacing in  
475 regional models, up to levels that are not feasible in today’s global models, and using  
476 a boundary forcing which carries in remotely generated IWs, does indeed lead to a greater  
477 “filling out” of the IW spectrum and thus bringing models closer to observations and em-  
478 pirical models such as GM76. Along with the inclusion of remotely generated IWs, de-  
479 creasing only the horizontal grid spacing significantly fills out the frequency and verti-  
480 cal wavenumber spectra, while decreasing only the vertical grid spacing resulted in lit-  
481 tle to no change in these spectra. Decreasing both grid spacings provides the best out-  
482 come, filling out the frequency spectrum out to the frequency of the model output (72  
483 cpd).

484 The vertical wavenumber spectrum becomes arguably more deficient as the ver-  
485 tical grid spacing decreases without a corresponding decrease in horizontal grid spacing,  
486 with more variance observed at low wavenumbers, less variance at higher wavenumbers,  
487 and a steeper slope in between, relative to the observations. Decreasing both horizon-  
488 tal and vertical grid spacings does provide the best improvement in the spectra at all ver-  
489 tical wavenumbers, but the improvement is less substantial than the improvement seen  
490 in the frequency spectra. As hinted at by Stewart et al. (2017), the less dramatic improve-  
491 ment in the vertical wavenumber spectra may be due to a “mismatching” of the hori-  
492 zontal and vertical grid spacings, but this has not been verified.

493 Additionally, consistency relations for IWs derived from linear internal wave the-  
494 ory are applied to provide statistical confidence that the improvement seen in the IW  
495 spectrum with remote IW forcing at the boundaries and decreased grid spacing is indeed

496 due to IWs being generated at higher frequencies and vertical wavenumbers. However,  
497 a more in-depth investigation using, for example, theoretical frameworks for the forma-  
498 tion of a power-law spectrum in a fluid, may provide even stronger evidence.

499 Models of IWs may help us to better understand the spatial geography of mixing  
500 in the ocean, and are playing an increasingly important role in planning for satellite mis-  
501 sions (e.g., Wang et al., 2018). Improving the representation of IWs in models will ben-  
502 efit these research endeavors, and the results discussed here represent another step in this  
503 continual progression of improvement. In future work we plan to apply the same fine-  
504 scale parameterization techniques used to extrapolate dissipation rates from observations  
505 to IW models (e.g. Polzin et al., 1995), at least in modest-sized regions.

## 506 **Acronyms**

507 **IW** Internal (gravity) wave

508 **MITgcm** Massachusetts Institute of Technology global circulation model

509 **ECMWF** European Center for Medium-Range Weather Forecasts

510 **MMP** McLane Moored Profiler

511 **WKBJ** Wentzel-Kramers-Brillouin–Jeffreys

512 **CCW** Counterclockwise

513 **CW** Clockwise

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523 [deepblue.lib.umich.edu/data/concern/data\\_sets/9s1616296](https://deepblue.lib.umich.edu/data/concern/data_sets/9s1616296). The MMP observa-  
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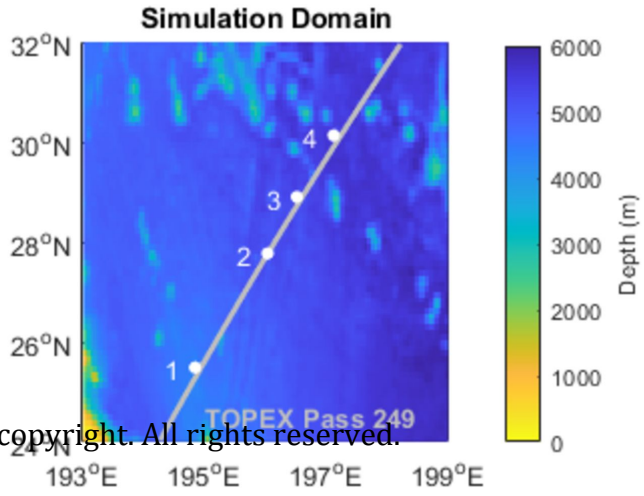
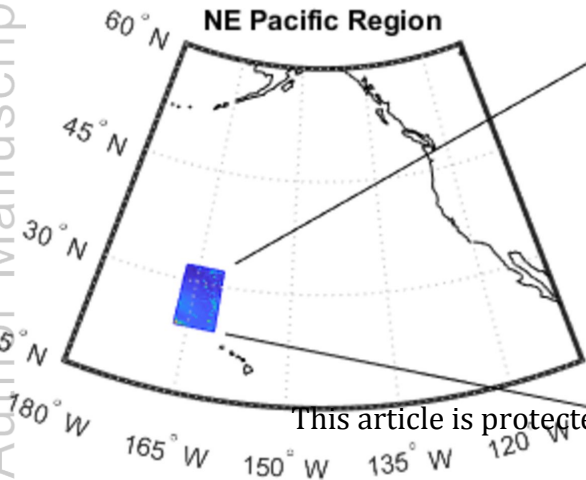
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Figure 1.

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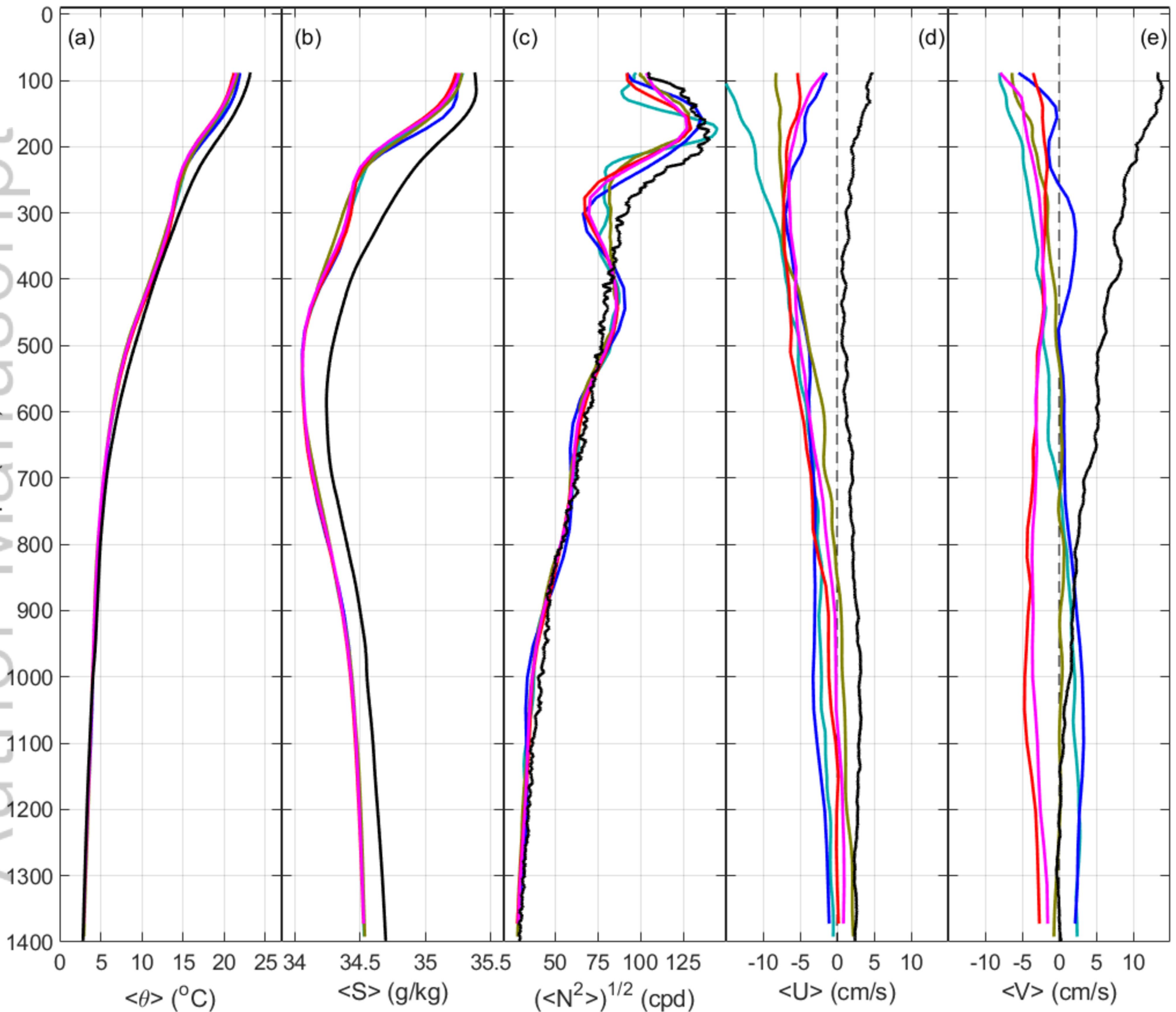
Figure 2.

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# Statistics at MMP Location #01

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— MMP Obs — LLC4320 — One-To-One — Finer- $\Delta z$  — Finer- $\Delta x$  — Finer-Both

Figure 3.

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# Variability at MMP Location #01

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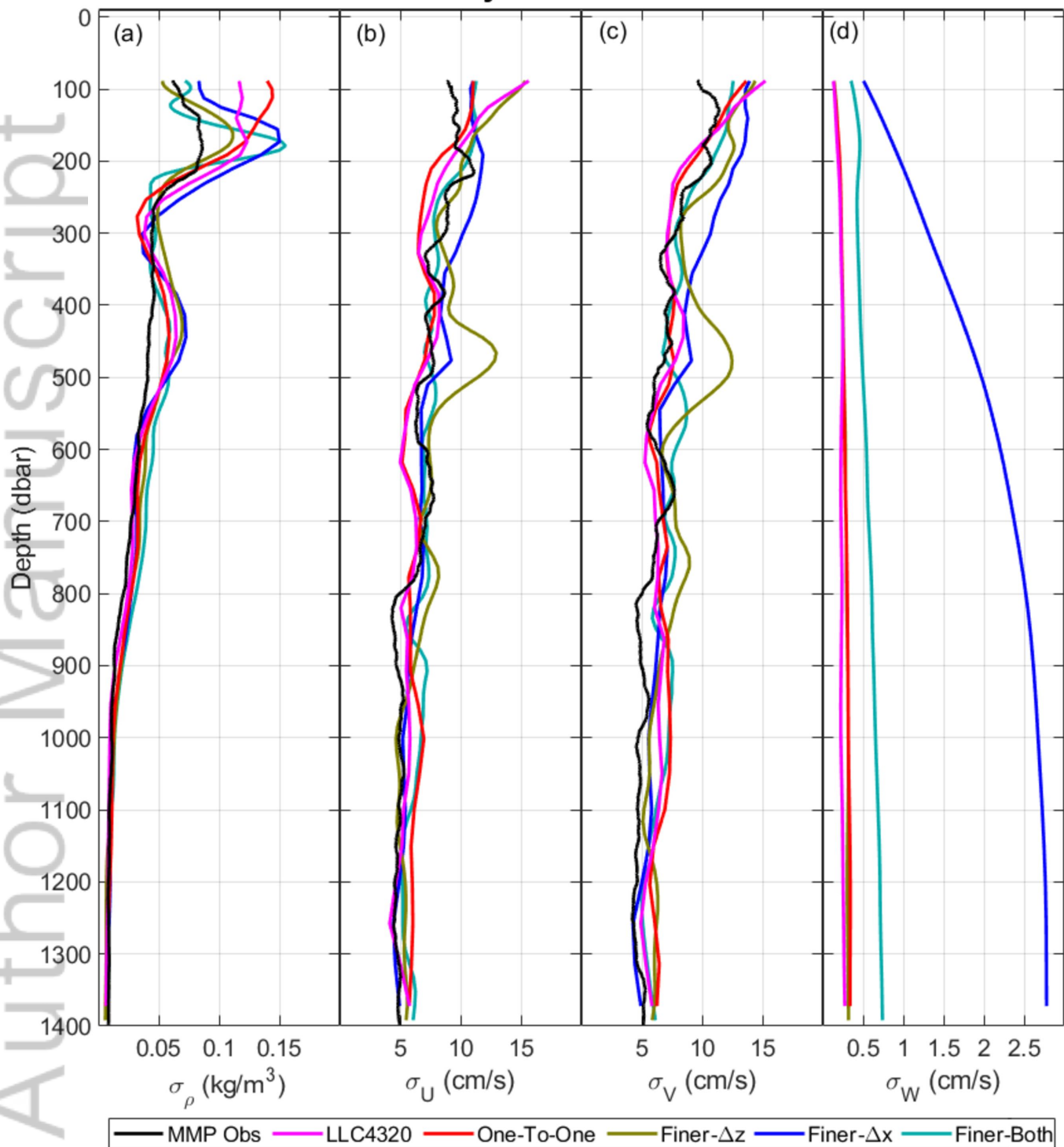


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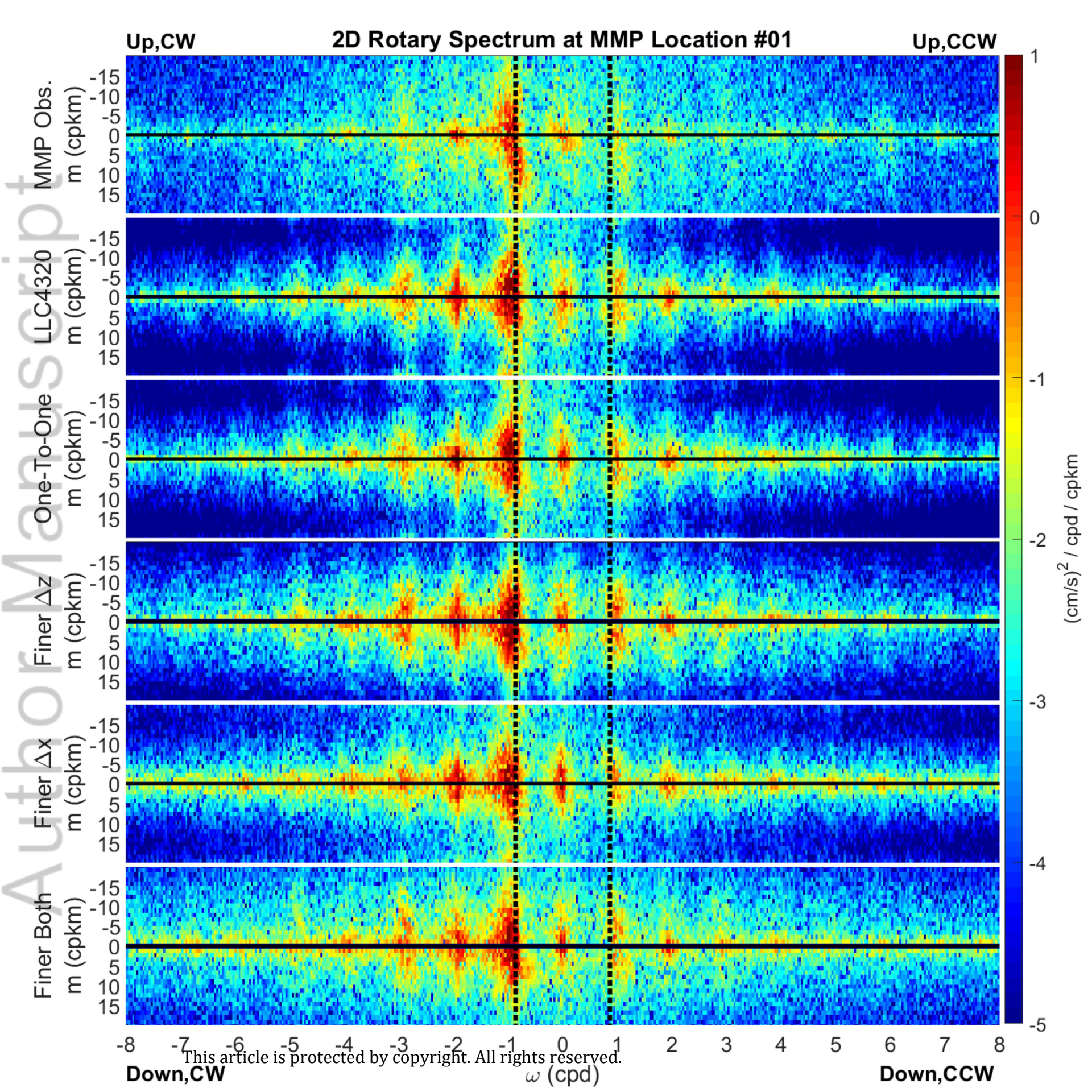


Figure 5.

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# Total Kinetic Energy Spectra of Rotary Velocity at 620m

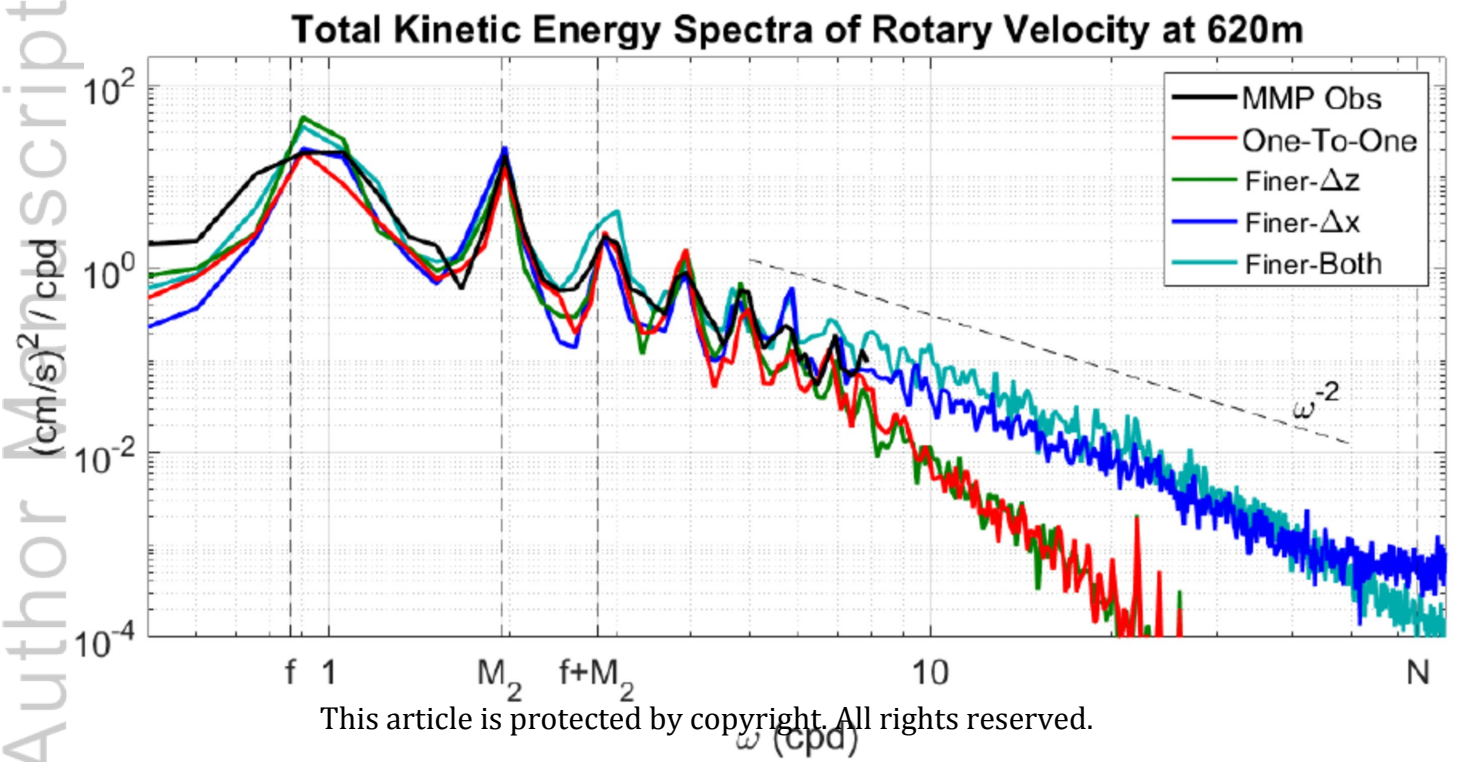
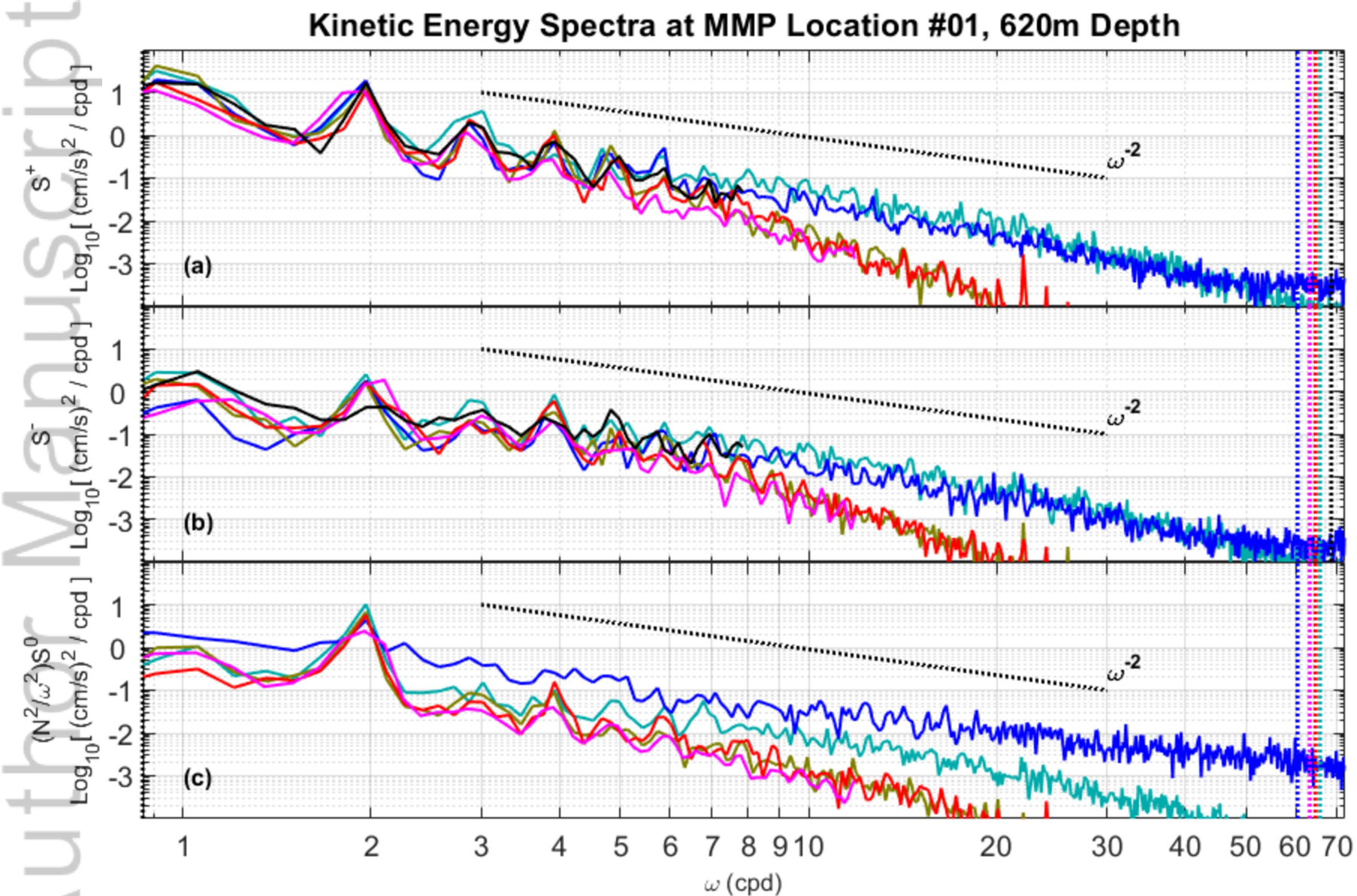


Figure 6.

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# Kinetic Energy Spectra at MMP Location #01, 620m Depth



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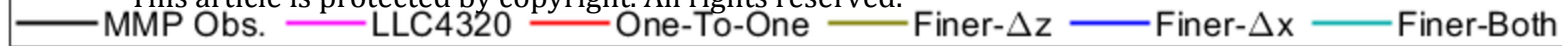


Figure 7.

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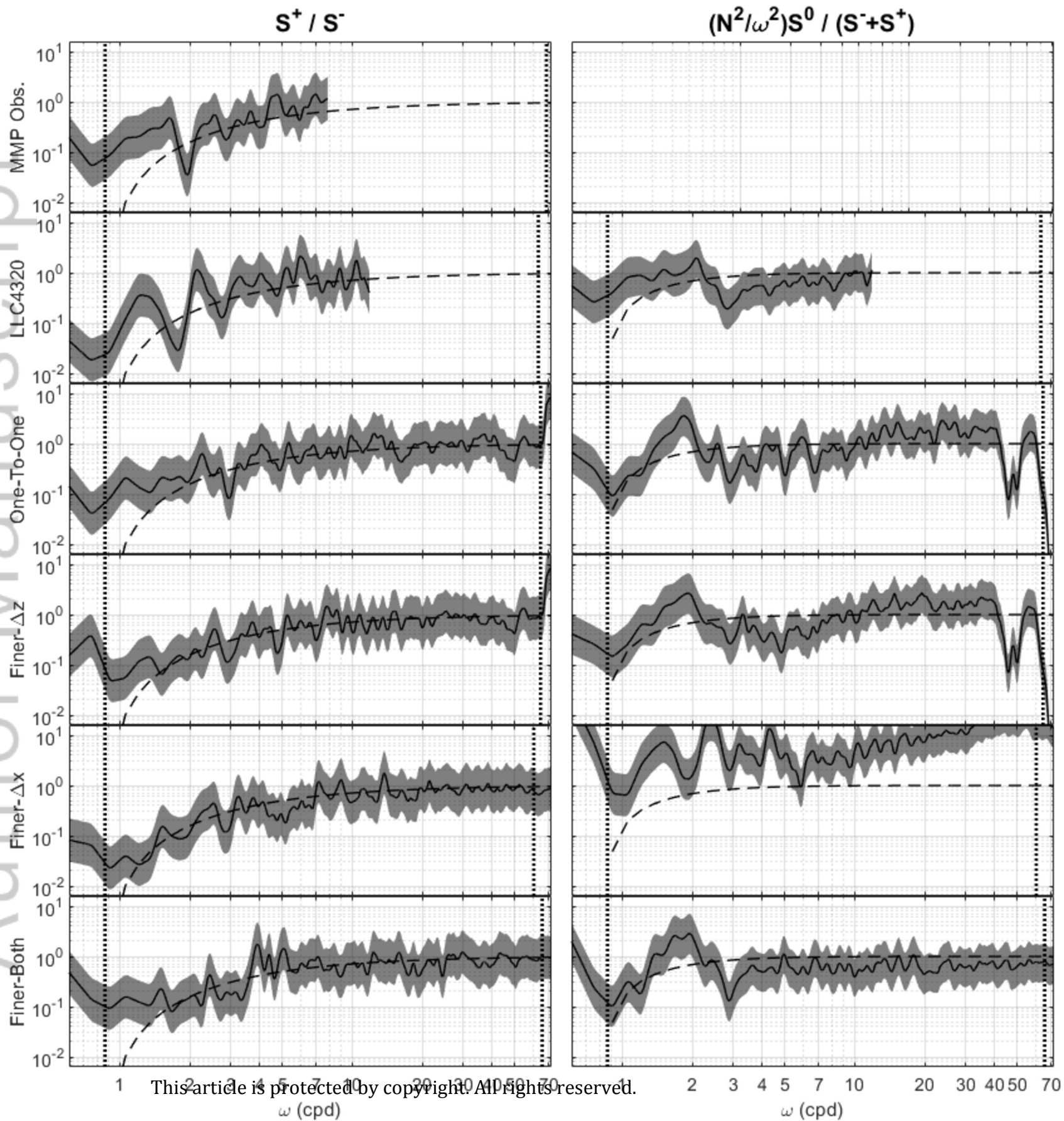
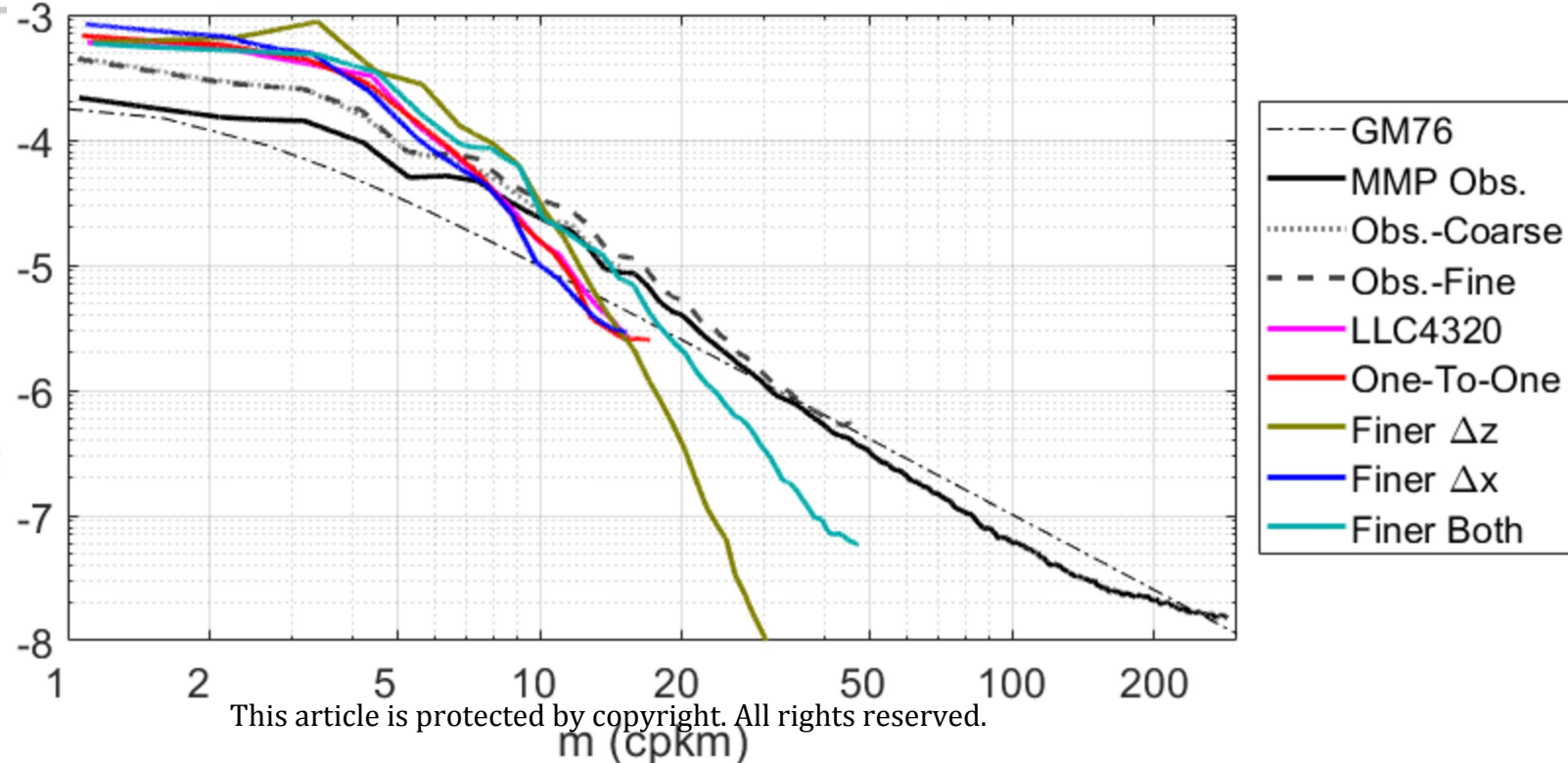


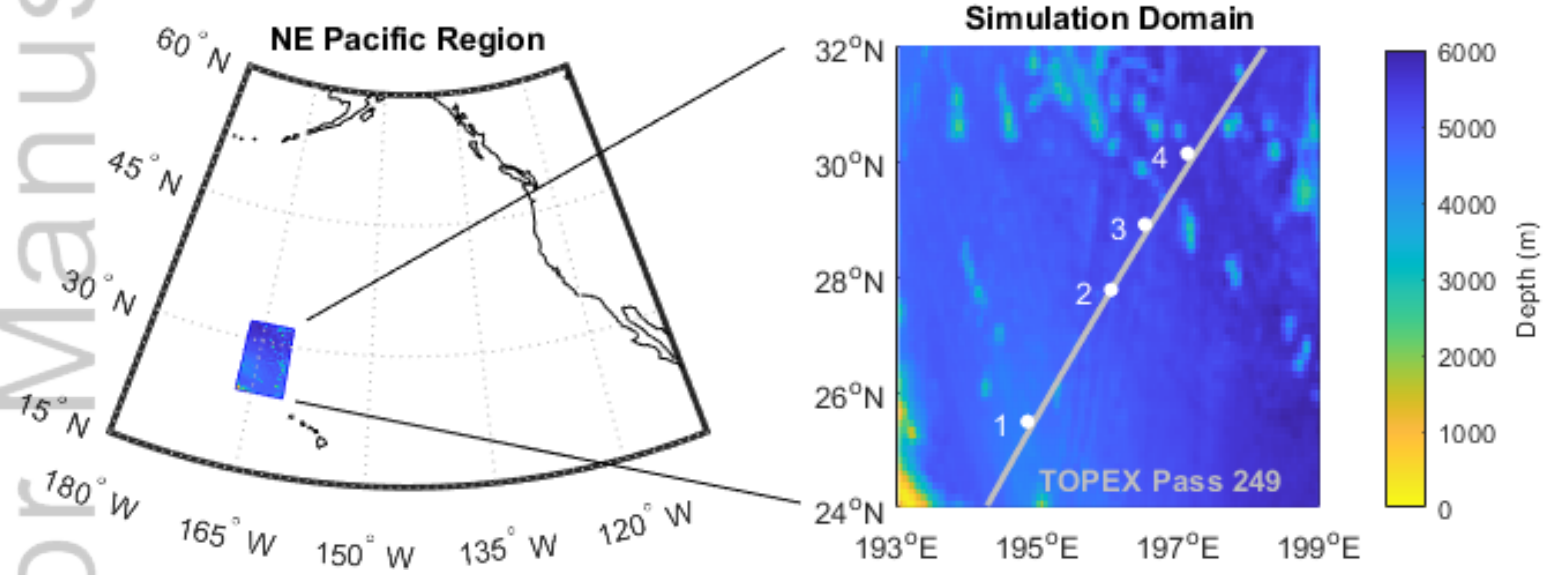
Figure 8.

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# Time-Averaged Total Vertical Wavenumber Spectra at MMP Location #01

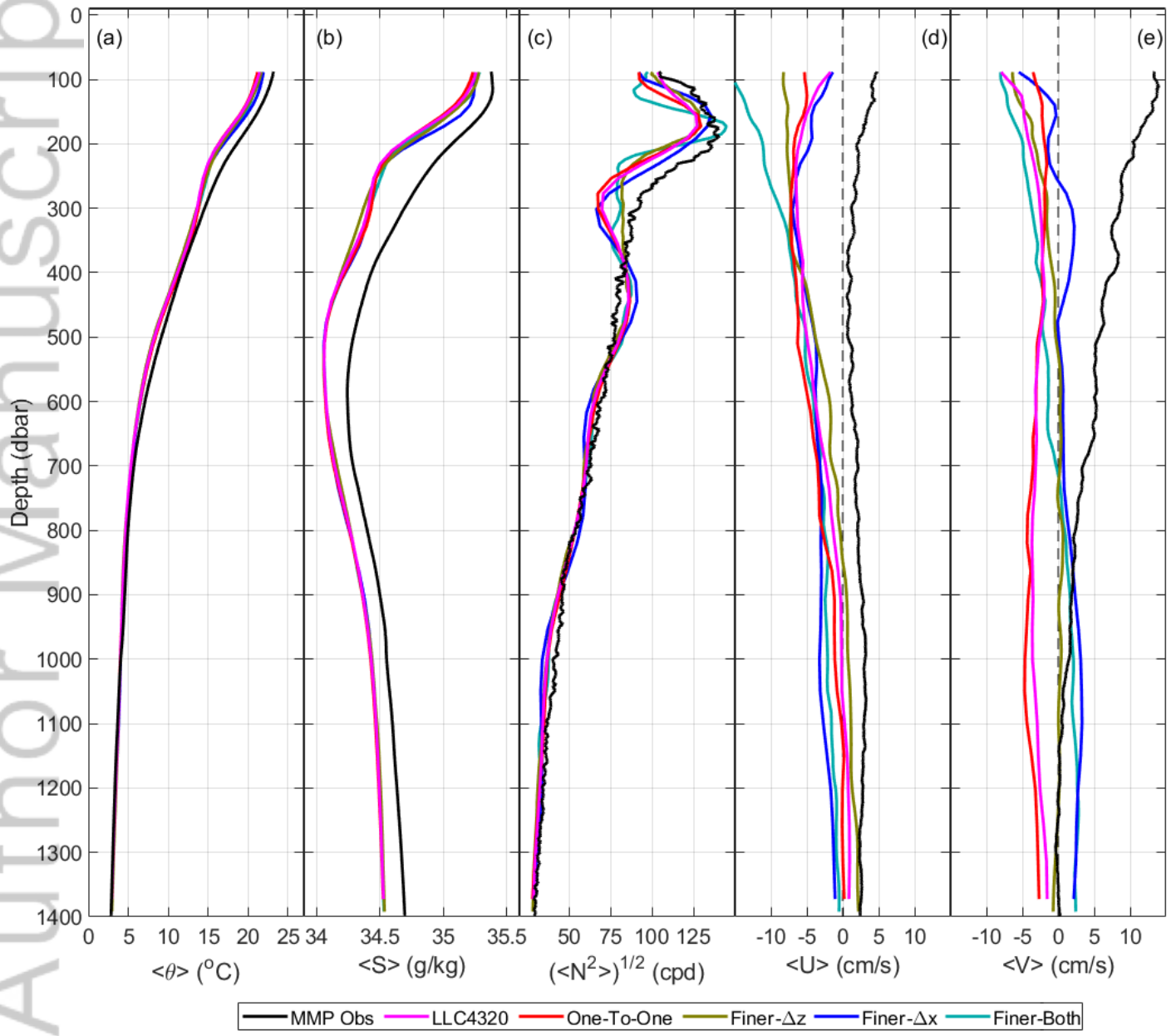


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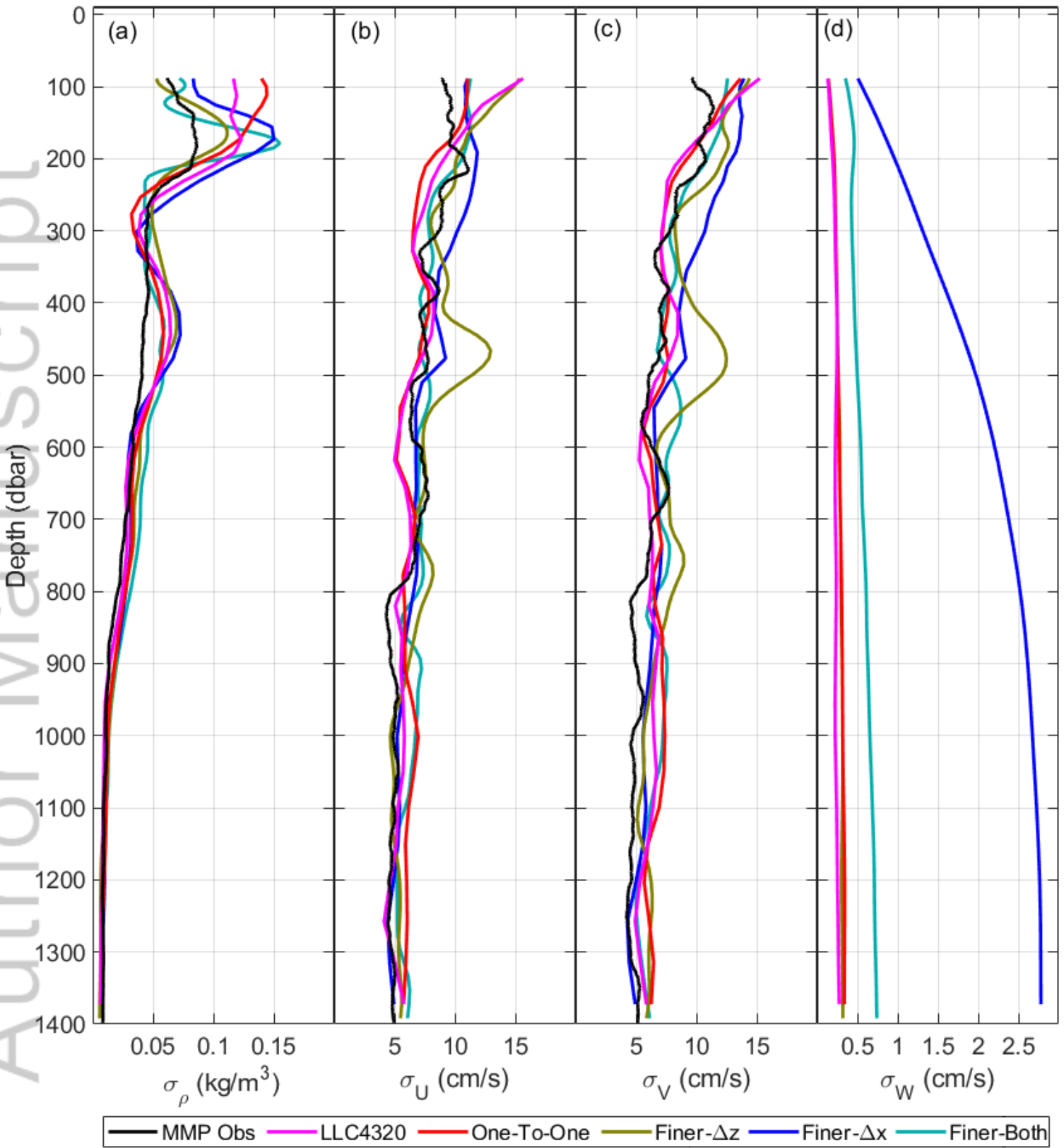
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### Statistics at MMP Location #01



2019JC015974-f02-z-.png

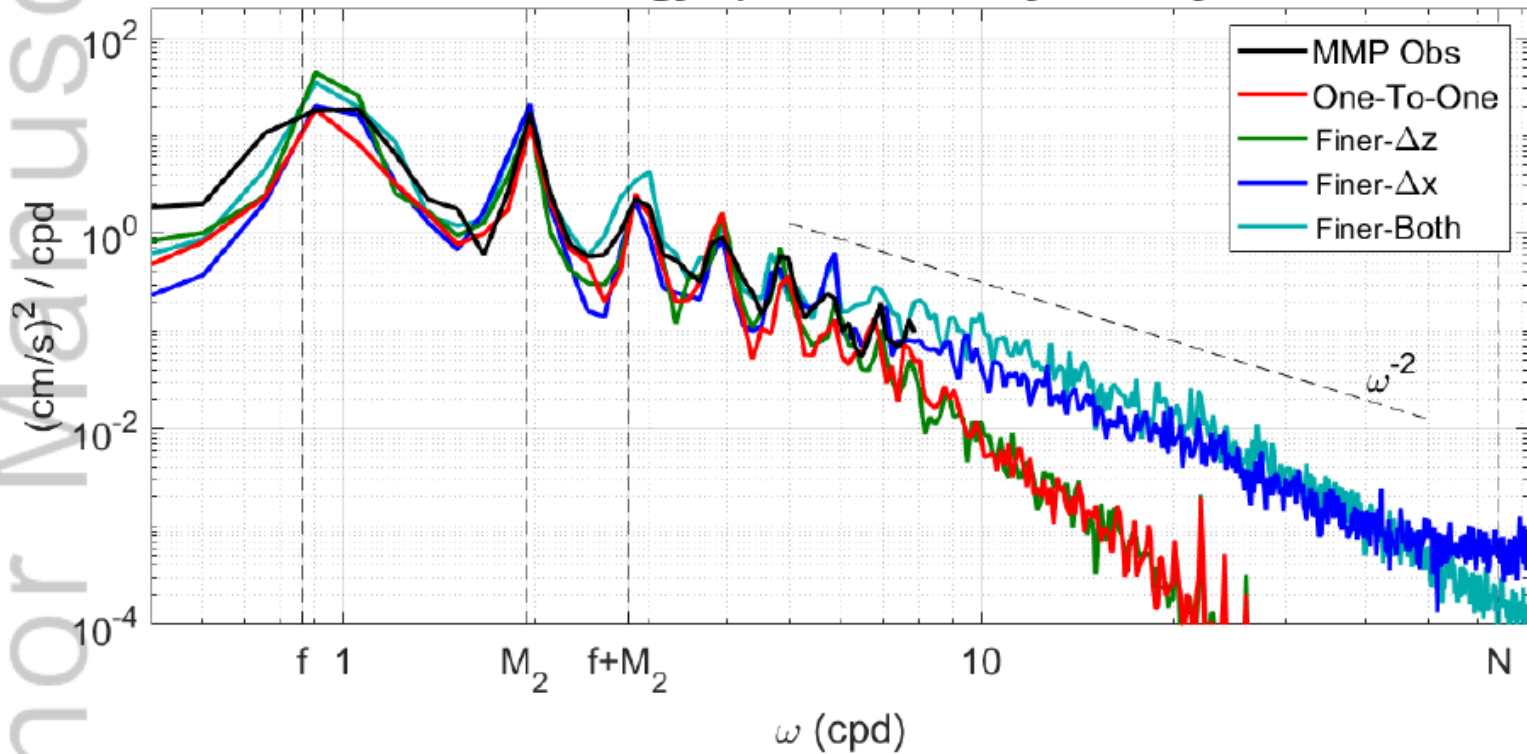
### Variability at MMP Location #01





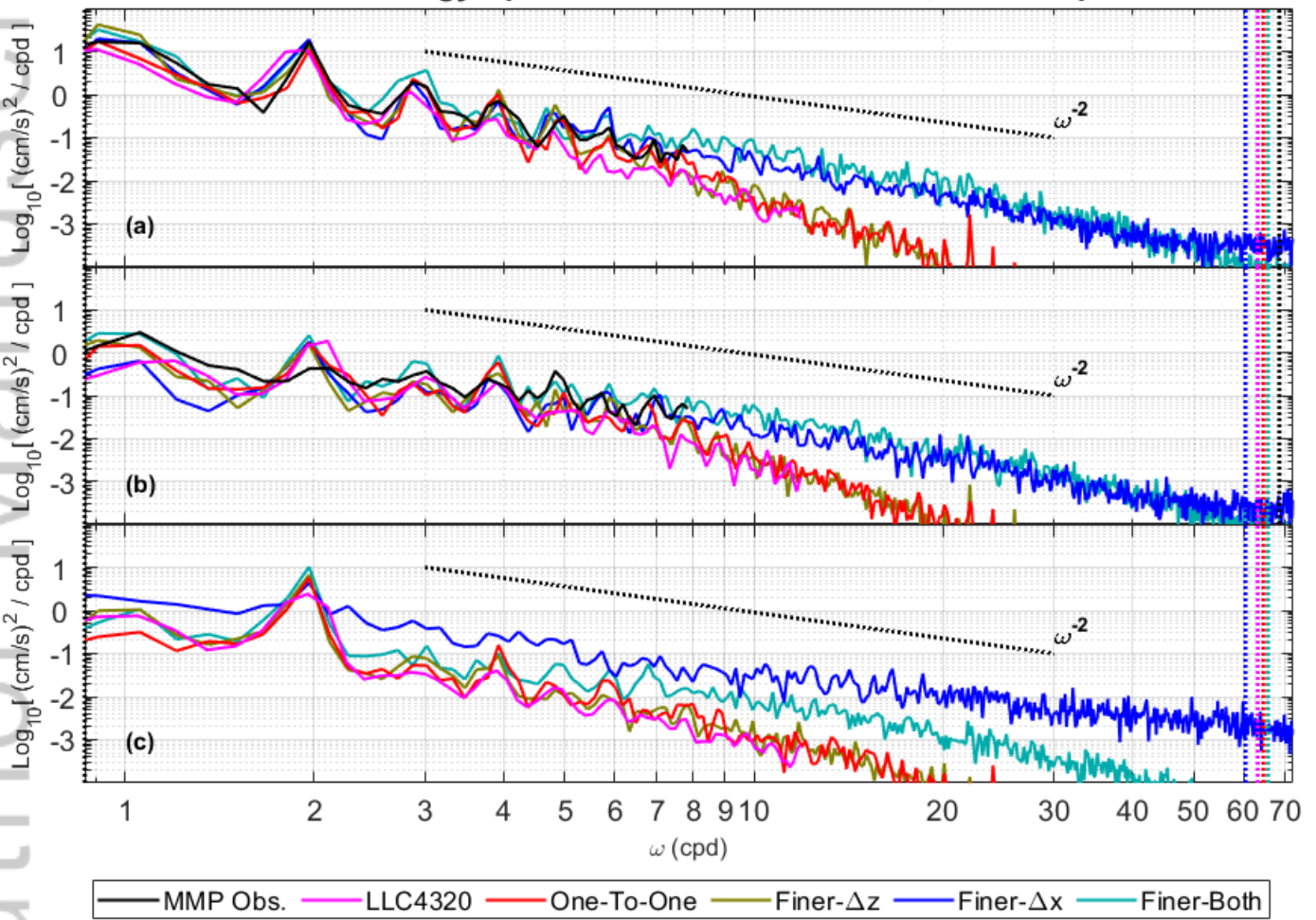


**Total Kinetic Energy Spectra of Rotary Velocity at 620m**

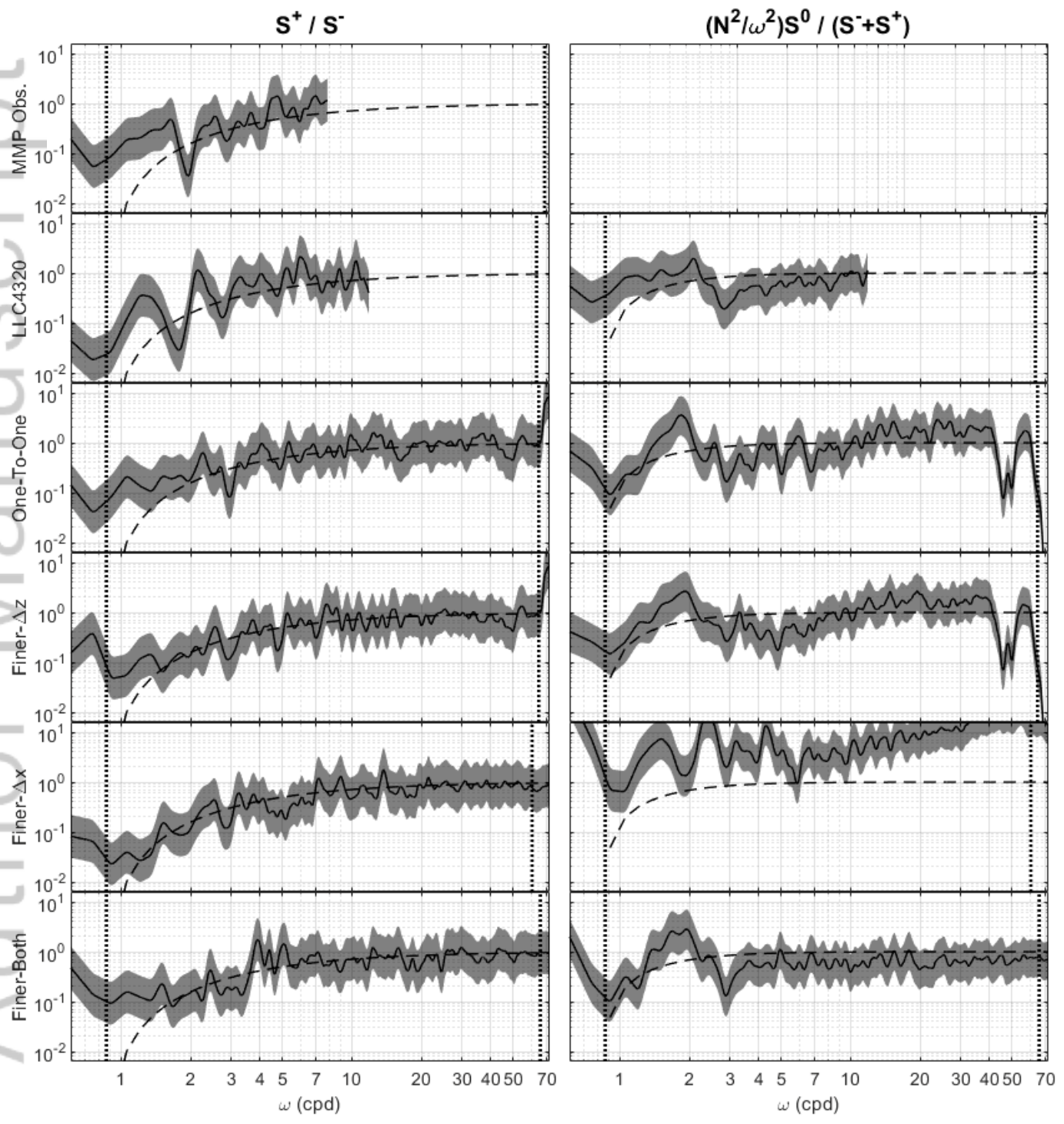


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### Kinetic Energy Spectra at MMP Location #01, 620m Depth

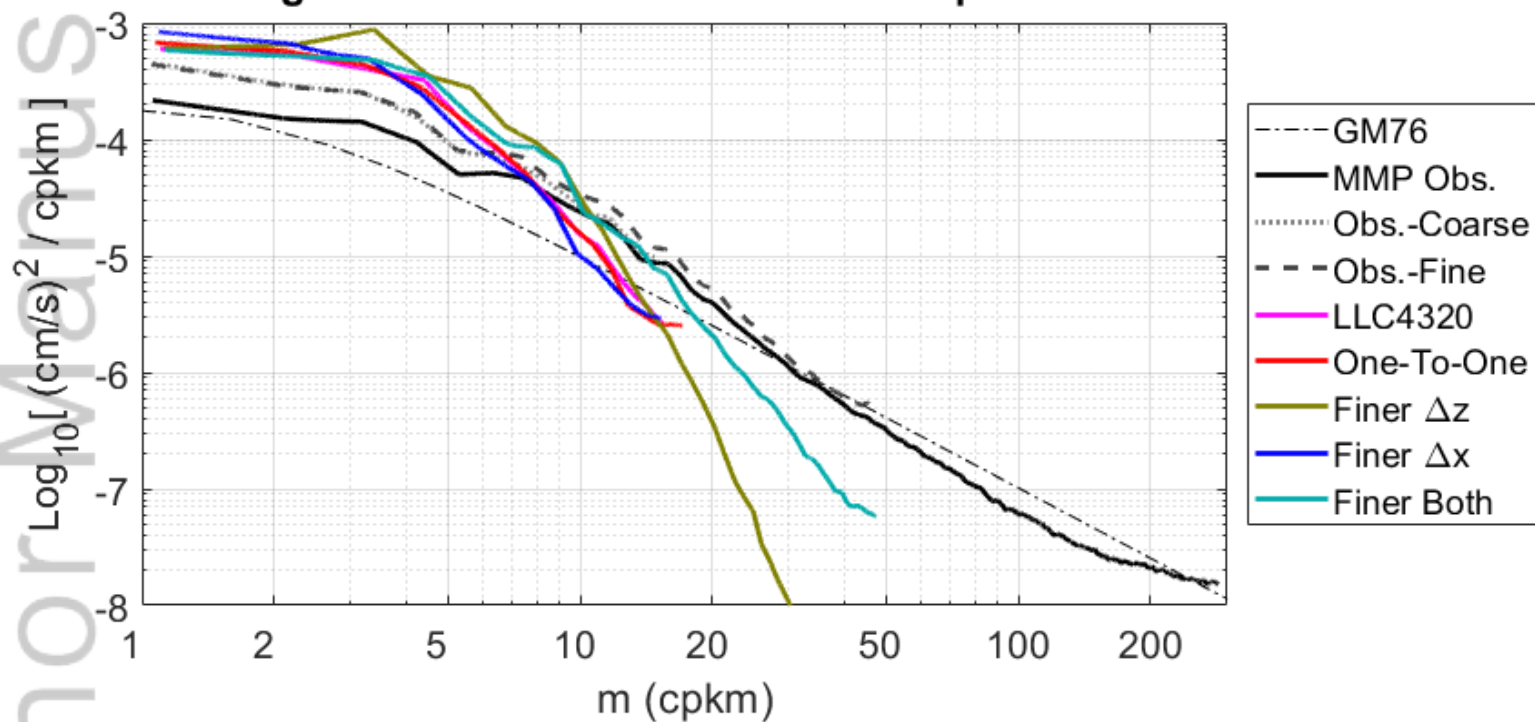


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### Time-Averaged Total Vertical Wavenumber Spectra at MMP Location #01



2019JC015974-f08-z-.png