

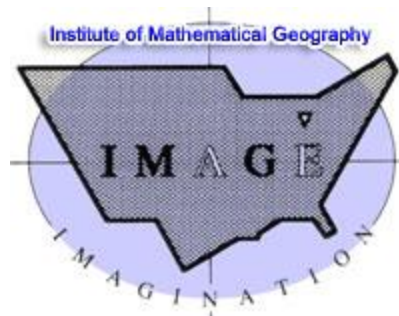
# INSTITUTE OF MATHEMATICAL GEOGRAPHY

## *MONOGRAPH SERIES*

### VOLUME 25

## **PRE-COLLEGIATE EDUCATION IN UTOPIA**

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## Chapter 1: Introduction

### A Wish-list...

#### Basic Concepts

- General strategy should be capable of implementation at a variety of geographical scales, from local to global, and across existing national and international boundaries. For example, study Latin as a root language in one set of countries while also studying simultaneously two derivative languages such as French and Spanish. In another country, study Sanskrit as a root language along with two languages derivative of Sanskrit.
- Government offers opportunity. Here we view its role as one that is additive on a cumulative basis.
- Government does not mandate in this situation; it offers guidance. Existing governmental regulation will vary from one geographical entity to another.
- Citizens are free to choose which opportunities they want for their families and themselves, independent of geographical scale. Such freedom is the hallmark of a democracy.
- Teacher, parents, and other adults need to continue to learn, and let children/students see them doing so.

#### General Concepts in Regard to Pre-Collegiate Education

- Keep improving existing structure for current public school network
- Consider supplementing or possibly replacing dysfunctional elements of the current network with tested strategies from elsewhere.
- Take advantage of current work-from-home for parents who wish educate children from home as a regular form of education.

#### Ways to Implement Concepts

##### *Ladders*

Implementation is organized as a set of 'ladders.' The ladder is chosen as an organizational metaphor because it integrates the concepts of short-range goals/planning with long-range goals/planning. To progress up the ladder is to take a sequence of short steps while keeping in mind the longer-range goal of where that sequence will lead. Too often, students see only the short-range educational project, be it a homework assignment, a daily quiz, and so forth.

Many lose track of long-range projects and convert them into short-range projects, as for example the student who has three months to complete a research paper but winds up doing it the night before it is due.

In addition, the image of the ladder suggests that one must successfully complete a step before moving on to the next higher rung. Thus, different students may move at different paces. Age does not determine location on a ladder; success at completing previous steps does. Students who start on one ladder and decide they might wish to switch can be encouraged to do so, moving swiftly past rungs on the new ladder already successfully completed on the old ladder, with booster steps where needed.

Finally, students might imagine competing at ladder climbing, both within their own group (as with intramural sports) or between ladder groupings (as in intercollegiate sports). Spelling bees, geography contests, mathematics contests, and cooking contests, are all established forms of non-sports competition.

### ***Competition***

Competition leads to success when reasonably implemented. The best competitors are likely to be those who make the fewest errors and are able to move at their top speed, bearing in mind that increase in speed is never done at the expense of accuracy and error reduction. Thus, error detection and correction are of the utmost importance.

Be sure to point out mistakes. Reward the behavior you wish to encourage; but do not gloss over mistakes. We learn from mistakes. We all make them. Adopt Strunk and White's adage: if you make a mistake, say it loudly (so it gets corrected). Do not ridicule; simply correct it, firmly but without emotion. For example, a constructive approach might go something like this: "Actually, the answer is really 36, not 37. Why is that? See if you can figure it out; if not, please tell me and I will show you why. Then, you explain why to me." There are many constructive approaches, mostly free from emotion. Emotional approaches that involve, shame, guilt, ridicule, meanness, and so forth are destructive to the entire process. So, it is important that students are corrected in a constructive manner.

The same idea applies to other activities. Encouraging students to ask questions of all sorts is a fine idea. But, some questions are 'better' than others. Hence, the

approach, “there is no such thing as a bad question’, is destructive—it suggests ‘bad’ questions, and may even generate some, as for example from the smart-alec student who starts asking ‘cute’ questions about bathroom behavior and such. Everyone knows some questions have more merit than others; reward the behavior you wish to encourage without ridiculing the less than optimal. For example, suppose a student asks, “what is the highest number?” A fine question that gives the teacher the opportunity to jump into discussions about the history of mathematics, the concept of infinity, and a host of others. The teacher might respond constructively by saying something like “that question, broadly conceptual in nature, is one that many people have asked throughout history. It appears that you have mastered conceptual thinking to the point that you would ask that question. There are many directions one can go in coming to an answer. I will offer one here (pick a candidate highest number; then add 1 to it; again and again; when does that process stop?). I will email you some links to other material you might find interesting. Please let me know which link you like best!” The teacher then asks for other questions. Now a student asks why the clock on the wall has numbers arranged from 1 to 12 in a circle when his digital watch doesn’t look anything like that. This question also is a fine academically-oriented question which is more compact in content than the first one, but still conceptual in nature, and can be answered in detail within a smaller amount of time but with links offered, once again. Notice in both cases the teacher does not evaluate the question as good or bad, but rewards it with the amount of time spent on it and a ‘gift’ of links and opportunity for extra follow-up/time. Finally, a student asks, why the teacher parts his hair on the left. The teacher simply answers this observational/practical question “because my hair falls naturally to that side of my head” and then continues on to another. Reward the behavior you wish to encourage. Time spent can be one form of reward.

## Chapter 2: Ladder Curricular Structures

These ladders are arranged in order of degree of conceptual/practical orientation: alpha, the most conceptual and least practical; gamma the least conceptual and most practical. All are worthwhile and challenging approaches to education and lead to interesting careers for a constructive citizenry.

### Alpha Ladder

*Core curriculum with heavy emphasis on conceptual orientation.*

- Long-range plan: leads to success in
  - Highest level of major university
  - Path to chosen career heading to being an international star and community leader.
- Short range steps, rungs to climb, to lead to the long-range plan:
  - Require literacy at the highest levels in core subjects. Begin such training as early as possible, but at least by age 3. Finish such training prior to onset of puberty, perhaps by age 11. By age 11, students should have completed through high school level reading, writing, and mathematics, as enumerated below. These students must be able to pass entrance requirements to a local community college at a level designed for them to succeed or some other collegiate level standardized test, such as the SAT. They should attempt such a test, on the core or subjects below, as soon as they are ready but certainly before onset of puberty.
    - Reading
      - Master the alphabet by age 3 (independent of the order in which letters are presented).
      - Master reading English (native language) at the level of a high school graduate by age 8. Use a linguistic approach.
      - Master reading at a basic level of one classical language (Latin) by age 10.
      - Master reading at a basic level of two languages (French and Spanish) derivative of the classical language by age 11.
      - Learn to discuss what has been read, analyze meaning, and debate the merits of the content read.

- Writing; develop writing skills in parallel with reading skills.
  - Write in printing and cursive at a competent level in all languages studied
  - Study calligraphy
  - Study use of digital media to communicate in writing
  - Learn to write sentences, paragraphs, essays, poems, and various other short forms of communication.
  - Learn grammar; diagram sentences. Learn full systematic approach to understanding grammar in native language and supplement it with comparison to grammars in non-native languages studied.
  - Learn to write term papers, magazine articles, research papers, book chapters, and books.
- Mathematics
  - Integers. Positive, negative, and zero. Use a number line to motivate the negative numbers. Begin at age 3.
  - Fractions, positive and negative. Begin at age 4.
  - Operations with integers. Begin at age 4.
    - Begin with addition. Use a number line as appropriate.
    - Introduce multiplications. Memorize the multiplication table through 12 x 12. Do not move on until fully mastered independent of the order tested. Consider competition, flash cards, and various devices.
    - Introduce subtraction and division as inverse operations of addition and multiplication. Use a number line as appropriate.
    - The commutative, associative, identity, and inverse properties.
    - The distributive law.
  - Operations with fractions. Begin at age 6.
    - Add fractions with like denominators. Use pie charts as appropriate.
    - Multiply fractions, independent of denominators. It is critical to learn to multiply fractions BEFORE

attempting to teach adding fractions with different denominators.

- Multiplication by the number 1, in different forms.
- Addition of fractions with different denominators. Emphasis on the use of multiplication of fractions and on the use of multiplication by 1 in different forms.
- Subtraction and division of fractions, again, as inverse processes.
- Efficiency in performing operations with fractions: cancellation prior to manipulation.
- Commutative, associative, identity, and inverse properties.
- The distributive law.
- Competitions with arithmetic manipulations in one's head; illustrate the use of the distributive law. Complete full competence with arithmetic by age 8.
- Algebra; complete high school algebra by age 9, emphasizing that algebra is simply arithmetic that is rearranged (somewhat similar to derivative language idea). Require being able rigorously prove the quadratic formula.
- Plane geometry; complete by age 10. Study Euclidean geometry with emphasis on proving theorems rigorously.
- Solid geometry; recognize and name shapes. Draw the three-dimensional solids on a sheet of paper; emphasize the idea of perspective.
- Trigonometry; functional approach. Complete by age 11.
- Now, at age 11, take the standardized test that covers elements of these core subjects. Some students may wish to consider enrolling in a local community college at this point; or elsewhere; or work from home.

This alpha ladder, through the core subjects, is for parents who want something entirely different, perhaps with both conventional and home-based components, or entirely home-based components. Include contemporary technology readily available in the home. Nothing is too 'hard' in this group. There is risk, but also great reward.



Children who are young are capable of immense amounts of rapid learning. Begin schooling as soon as a child can walk, understand language, and use the toilet reliably. Continue schooling under this plan until puberty is reached and hormonal influences and imbalances set in. That time slot might be from ages 3 to 11, which is why the ladder is based on those ages. During that time, develop a firm foundation for all future learning, as measured by successful completion of high school mastery of the core subjects. Learning is a serious business. But it needs to be compelling, fun, and something one wants to do. The goal is for children to succeed.

### ***Fleshing Out Alpha Ladder Steps***

Listed below are a few ideas of how to flesh out the ladder steps above.

- Language:
  - Speaking: encourage question asking, comments, thought-sharing, story-telling. Secure the child in his native language from a talking standpoint.
  - Reading: Use a modified Bloomfield/linguistic approach. Words and nonsense words. No pictures. Then as fun, Dr. Seuss books, built on this approach. Read along with children. If a parent holds a young child on his lap, and reads a book with him, the child feels the love of the parent and learns to transfer that to a love of reading.
  - Writing:
    - Nonsense words emphasize patterns in English and are suggestive of different roles in communication for different words—grammar. Play with patterns of English.
    - Create a new language; try pig-Latin; try ‘ab’ or ‘op’ (insert ‘ab’ before each vowel; thus, “SANDY” becomes “SabANDabY”) or whatever.
    - As an activity, show them how to use a keyboard on an electronic device (although they may already know). As another activity, teach them to write/print using a pen or pencil (as small hands and coordination permit) or drafting equipment in support of calligraphy.
  - Other languages:

- Master classical Latin at a basic level. Dead languages are the best way to learn grammatical structure. Their grammar is fixed. Living languages are dynamic and rules change, causing difficulty.
- Shortly after starting Latin, also start two derivative languages, like French and Spanish. Show them how the living languages derive from the dead language.
- Mathematics
  - Teach all elements of arithmetic, shapes/geometry. Teach them how to do arithmetic quickly in their heads. Have contests involving acquisition of information to have in their heads (e.g. multiplication table). Have contests on multiplying large numbers in their heads. There are many available materials, most of which are not used in a traditional school.
  - Teach algebra; it is nothing more than arithmetic with a few letters thrown in:  $2+2=?$  Is arithmetic.  $2+x=4$  is algebra. Go all the way through fractions. Teach quadratic equations and creative use of the distributive law. Proof of the quadratic formula is a first proof.
  - Build on the first proof to teach elements of plane geometry.
  - Introduce solid geometry.
  - Introduce analytic geometry.
  - Do two-week course in trigonometry based on functions.
  - Elements of symbolic logic; draw together math and language.
- Lab work or learning adventures: take children out in the real world each day.
  - Walk around the neighborhood. Read street signs and any other words. Teach them to read all written text as a natural thing to do; street addresses, posters, everything. Teach them to be observant not only of patterns in words and numbers but also in nature. Teach them to ask questions. Why is the sky blue? Why are plants green? Are all plants green? What kinds of animals do they see. What do they all have in common (they all drink water, for example). How are animals different from plants (animals can move by themselves, plants are generally fixed in the ground). And so on.

- Take them grocery shopping with you. Make it a learning adventure, too. Read labels with them. Add prices. Keep a running tally of prices in your head (ballpark figures at least). Make a contest out of who can get the closest to the actual checkout figure. Winner gets a prize. Reward success. And so on.
  - Go to museums when it's rainy. Or to the mall. Or to concerts. Or to a library story hour.
  - The list is endless of things in any community, all of which will contain elements of what is being learned more formally.
  - All of these learning adventures will feed into more formal training in science, history, social science, etc. later.
  - The pre-puberty learning time is the time of establishing a rock-solid foundation on which to base all other learning.
- Success is measured by achievement levels. Kids move quickly or slowly according to interest and ability.
  - Abilities and talents; we all have different ones. It is good to get exposed to a wide variety of content but success and rapid advancement generally comes by doing what one likes best which is often what one is best at. So, have them play simple games on their smartphones and track their choices from a list provided. For example, a child who excels and often chooses to play a game involving tying and untying knots, at the 2D and 3D levels, may enjoy moving forward to studying mathematics leading to the study of knot theory and topology. Another who often chooses Sudoku may like subjects dealing with the manipulation of numbers. With advice about abilities and interests in hand, it will become possible to tailor an exciting and constructive learning experience.
  - Long-range plan: leads to success in a wide-range of conceptually-oriented professions. Students might score near the top on standardized tests of achievement and ability and might enjoy taking such tests. Encourage early practice, perhaps at age 11, on entrance requirements to a local community college. Once achieved, move on to the more integrative subjects, with eventual mastering of SAT tests at high levels. All this leads to entering any college or university, subsequent graduate/professional schooling, and success at a high level in a chosen career. This level focuses heavily toward the conceptual; students with a heavy practical bent, leading perhaps to a career that is more mechanical in nature, might benefit more from Ladder Beta or Gamma.

### ***Beyond the Core Subject Matter***

- Beyond the pre-pubescent period. Students can now enter any formal school or continue along the same path. They are ready to learn more integrative content that draws on much of the established foundation: biology, botany, zoology, astronomy, physics, chemistry, history, geography, philosophy, and more now become available on a more systematic basis. In this period, they still acquire new knowledge well but become more 'sensitive' about their person and such.
  - In a formal school setting, all wear uniforms, even something simple such as a smock, to remove obvious 'class' distinctions; there is a long history of this, dating back as far as 1870 in French Boys' schools.
  - Keeping simple mantras in mind may help to reinforce the otherwise eroding pre-pubescent confidence:
    - It is ok to disagree; it is not ok to do so disagreeably
    - The only failure is the failure to try.
    - Perfection is not a human trait; to strive for perfection is to set yourself up for failure. Strive, instead for excellence.
- Education here is integrated as a systematic approach with the full range of core math and language skills, in subjects such as:
  - Mathematical systems
    - Continuous mathematics: Calculus (with analytic geometry) and more mathematical analysis based on calculus, differential equations (arithmetic/algebra with calculus objects)
    - Discrete mathematics, graph theory, set theory, topology.
    - Geometry: advanced Euclidean and non-Euclidean.
    - Computer science and digital systems
  - Scientific systems
    - Biological and Environmental Studies
    - Chemistry and Physics
    - Geology and Astronomy
  - Wide range of others.
  - Cultural systems
    - History and Geography, integrated as a system, based on primary references and factual evidence

- Local
- State
- National
- Global
- Philosophy, Religion
- Music
- Art
- Sports
  - Physical
    - Golf, tennis
    - Swimming
  - Mental
    - Checkers
    - Chinese checkers
    - Chess
    - Bridge
    - Go
    - Internet games; knot tying, word games, etc.
    - Debate
- Social systems useful to becoming constructive citizens and community leaders
  - Basic personal care of self and others
  - Basic culinary arts
  - Basic interior and exterior environmental care
  - Basic sewing and use of simple carpentry tools (screwdriver, hammer, etc.)
  - Basic use of the computer and Internet to acquire goods and services
  - Basic good manners in writing (the thank you note; letter salutations and closings), sports, speech, and elsewhere.
  - Community service and understanding of local governmental process



## ALPHA: Conceptual Practical

## **Beta Ladder: Curriculum with moderate or mild emphasis on conceptual orientation.**

- Long-range plan: leads to success in a wide range of professions with a mix of conceptual and practical orientations (engineering or sales/marketing might be straightforward examples). Established school systems will have provisions for collegiate guidance and standardized testing to lead to a wide variety of choices in College and in careers.
- Short range steps, rungs to climb, to lead to the long-range plan:
  - Require a good level of literacy sufficient to live independently in the adult world and to help guide dependents. Students must be able to pass entrance requirements to a college of their choice at a level designed for them to succeed.
    - Reading
    - Writing
    - Mathematics

The traditional public or private school curriculum will take a series of short steps to lead to this long range goal. What is suggested here is supplementing that with materials from the alpha ladder, boosts, or extensions, as desired.

- Cultural systems
  - History and Geography, integrated as a system, based on primary references and factual evidence. Supplement traditional social studies.
    - Local
    - State
    - National
    - Global
  - Music; supplement traditional opportunity
  - Art; supplement traditional opportunity
  - Sports
    - Physical
      - Land sports (e.g. baseball, football, basketball, golf)
      - Street games (e.g. hopscotch, jump rope)
      - Water sports
    - Mental
      - Checkers

- Chinese checkers
- Chess
- Bridge
- Go
- Internet games; knot tying, word games, etc.
- Debate
- Social systems useful to live in the world and succeed in moving forward.
  - Basic personal care of self and others
  - Basic cooking and grocery shopping
  - Basic housekeeping/lawn care
  - Basic sewing and use of simple carpentry tools (screwdriver, hammer, etc.)
  - Basic use of the computer and Internet to acquire goods and services
  - Basic good manners in sports, speech, and elsewhere.

Consider developing a parallel public network, similar to the existing public school network, and perhaps incorporating elements of existing home-schooling networks or elements from ladder alpha. Learn from successful models elsewhere such as (but not limited to):

- [Drew Charter School](#), East Lake Village Georgia place;
- [Friedman Foundation for Educational Choice](#);
- [Khan Academy](#)





**BETA: Conceptual Practical**

## **Gamma Ladder: Curriculum with heavy emphasis on practical orientation.**

- Long-range plan: leads to success in
  - Community College
  - Apprenticeship in a trade and mastery of a trade, mechanics, culinary arts, and so forth.
- Short range steps, rungs to climb, to lead to the long-range plan:
  - Require literacy sufficient to live independently in the adult world and to help guide dependents. Students must be able to pass entrance requirements to a local community college at a level designed for them to succeed. They should attempt such a test, on the 'core' subjects below, as soon as they are ready.
    - Reading
    - Writing
    - Mathematics

Once they pass the test--repeat until it is passed--they move on to other topics such as those below, all of which have relation in some degree to these core topics.

- Offer extensive practical training of various sorts, integrated as a systematic approach with appropriate basic math and language skills, in subjects such as:
  - Scientific systems: practical approaches to
    - Biological and Environmental Studies
    - Chemistry and Physics
  - Computer and electronics systems--hands-on experience
  - Mechanical systems, automotive and so forth
  - Construction systems
  - Culinary systems
  - Wide range of others.
- Cultural systems
  - History and Geography, integrated as a system, based on primary references and factual evidence
    - Local
    - State
    - National
    - Global

- Music
- Art
- Sports
  - Physical
    - Land sports (e.g. baseball, football, basketball, golf)
    - Street games (e.g. hopscotch, jump rope)
    - Water sports
  - Mental
    - Checkers
    - Chinese checkers
    - Chess
    - Bridge
    - Go
    - Internet games; knot tying, word games, etc.
    - Debate
- Social systems useful to live in the world and succeed in moving forward.
  - Basic personal care of self and others
  - Basic cooking and grocery shopping
  - Basic housekeeping/lawn care
  - Basic sewing and use of simple carpentry tools (screwdriver, hammer, etc.)
  - Basic use of the computer and Internet to acquire goods and services
  - Basic good manners in sports, speech, and elsewhere.



## **GAMMA: Conceptual Practical**

## Chapter 3: Ladder Boosts and Extensions

‘Boosts’ are designed to assist students in overcoming traditional sticking points in core subjects to create a rock-solid foundation on which to move forward. ‘Extensions’ offer a way to go beyond the core subjects.

### Boosts

Students on any ladder, at any point on that ladder, may become stuck and have trouble progressing. Parents and experienced teachers can help to ‘boost’ students back up the ladder toward success. Parents will know what types of boosts will help their child; experienced teachers will know, from a general perspective, what sorts of boosts often work. In order to get the student successfully re-launched, obviously it works best when parent and experienced teacher coordinate as a team. We offer, here, a set of boosts, which from our perspectives as university-level teachers, professional tutors, and students have been helpful in filling gaps we have observed over a combined experience of more than a century.

#### ***Boosts in Foundational, Core Subjects***

##### *Reading, Foundations*

- Master recognizing and saying the letters of the alphabet. Not just in order. Use flash cards with one letter per card. Make a game of it. Get down on the floor with the child; enter his space. Talk to him in a friendly and straightforward manner with sugar-coating thoughts or language; no ‘baby-talk’ or cutesy substitutions for correct names of objects, people, and ideas.
- In a formal learning setting at home (or elsewhere), try a linguistic approach to reading—with no pictures as clues. Learn to read the words with no distractions. One that we have used successfully in two extended independent one-on-one trials, and one observed trial, is: Leonard Bloomfield and Clarence Barnhart, *Let’s Read: A Linguistic Approach*.
  - From the [Internet](#):  
Review

The system is totally orderly, totally logical, and virtually foolproof. It is strictly a reading system. Reading is means, process, and end. (Charles C. Walcutt *Council for Basic Education (CBE) Bulletin*)

Let's Read may well be the most important book on beginning reading in our time; the important point is that here is a new methodology soundly based on psychology and linguistics . . . The lessons are ingeniously constructed to induce smooth, rapid progress. Let's Read is not just another reading text; it is a new system. (Russell Cosper *Journal of Reading Development*)

#### Book Description

Let's Read was published for the first time by Wayne State University Press in 1961. It has been reprinted many times, and is a recognized classic in the field of reading instruction.

#### About the Author

Leonard Bloomfield served on the staffs of the universities of Wisconsin, Illinois, Ohio State, and Chicago. He was Sterling Professor of Linguistics at Yale at the time of his death in 1949. Among his many significant writings, three stand out as basic to modern linguistic science, "A Set of Postulates for the Science of Language" (*Language*, II.3, 1926); *Language* (New York, 1933); and "Linguistic Aspects of Science" (*International Encyclopedia of Unified Science*, I.4, Chicago, 1939).

Clarence Barnhart is best known as the co-editor of the Thordike-Barnhart Dictionary Series (Chicago, 1952-1971) and of the World Book Dictionary (New York, 1936, 1976) and The Dictionary of New English since 1963 (New York, 1973).

- Remember that children are quick to memorize things. Do not assume that because a child can recite words on a page that he/she has seen before, that he/she is actually reading them. Use flash cards or digital schemes to mix up the order. Here is content from *Let's Read*, first lesson.
  - Vocabulary: can, Dan, fan, man, Nan, pan, ran, tan, an ban, van.
  - Phrases and sentences using the vocabulary: a can; a fan; a pan; a man; a van; a tan van; a tan fan; Dan ran; Nan ran; Van ran; A man ran; Nan can fan Dan; Can Dan fan Nan?; Dan can fan Nan; Dan, fan Nan; Dan ran a van; Dan ran a tan van; A man ran a tan fan.Subsequent lessons build new vocabulary and patterns, integrate it with old vocabulary and patterns, and review. The content from the second lesson is shown below.
  - Vocabulary: bat, cat, fat, hat, mat, Nat, pat, rat, sat, at, tat, vat.
  - Phrases and sentences using the accumulated vocabulary: a bat; a cat; a rat; a hat; a mat; At bat; A fat cat; A fat rat; A fat cat ran; A fat man ran; Can Pat fan Dan? Pat can fan Dan; Nan can pat a cat; A fat rat ran; A fat cat ran at a fat rat; Can Nan fan a fat man? Can a fat man pat a cat? Nat at bat! Dan at bat! Can Pat bat?

The building process is gradual and is, itself a ladder, as children add new content, from short lessons with only words (no pictures) after mastering a previous step. The content from lesson 25 (out of 245 lessons) is shown below:

- Vocabulary: bet, get, jet, met, net, pet, set, wet, yet, vet, sunset.
  - Phrases and sentences using the accumulated vocabulary: Let Dan bat; Did Al get wet? Nat had a pet cat; Get up, Tad! Let us in, Sis! Sis, let us in! Let Sid pet a pup; Jim let Pam tag him; Jim let Pam win at tag; Nan met Dad at sunset; Nan met Dad at a bus; Did Sam bat yet? Let Sam bat; Hit it, Sam! Get a hit, Sam! Sam hit it; Sam can get a run.
- Children may quickly read out loud as a pattern-recognition response. It may take longer for them to realize that the words they say in response to patterns translate to mean the same as words they select from their own internal vocabulary. It may also take them longer to learn to read silently.
  - In a bedtime story setting, for very young children, hold the child on your lap while reading; the child feels your love while reading to him/her and associates it with reading; thus transfers it to a love of reading. For older children, an arm on the shoulder, or some appropriate way to have them feel direct parental love or love from a sibling, once again can transfer to love for reading. Make sure that the experience is constructive—do not transfer negative feelings (tension, aggression) or other negative associations (strong, offensive aromas, etc.)!

### *Writing, Foundations*

Children, and others, may find it difficult to commit to placing their thoughts and words on paper—they think it is ‘final’ or ‘set in stone’. Digital capability helps greatly to overcome such hesitancy.

Short-writing projects are ones that students often do well on. As long as they can see the goal, they tend to do well (assuming they are motivated to try to do well). Where they have problems is with long-range written projects: term papers, research papers, magazine articles, books, and so forth. We offer a strategy to help overcome this difficulty; it should be implemented flexibly bearing in mind that different students have different interests and capabilities.

- Read Strunk and White, [The Elements of Style](#), noting in particular their encouragement to speak up when you don't know...get your mistakes and lack of knowledge adjusted so you can move forward!
- Become competent with various digital formats that permit sharing of documents: the Internet, cloud-based technology, whatever works for you.
- Write a term/research paper over the course of three months
  - Do NOT try to write an abstract or summary or introduction to your material at the beginning. Those items should be the last things written; so that they actually introduce or summarize the final material that is written about.
  - Read about your topic. Write down, at the top of a digital page, the citation to the material, and then write down the thoughts that were triggered about how the material relates to your topic.
  - If you find images that are useful, capture those digitally, noting the source, and write about them—why they are of interest and how they relate to your topic. Note needs for copyright issues.
  - Arrange words and images in a digital document. Look it over. See what extra thoughts your arrangement triggers. Write them down and insert them in appropriate places. Begin to format your document.
  - Share your document with a teacher. Keep sharing it so that he/she can look over your shoulder as you work on it and locations where there are errors (with accompanying suggestions for correction) as well as constructive ideas or links for you to consider.
  - Do some work on your document every day (5 days a week) even if it is just for half an hour or other small amount of time. Save your work so it is in shared space. Make changes in association with feedback as it comes in, rather than letting it pile up.
  - When you adopt a general strategy such as this, with continuing feedback and response, when you are done, you will succeed in creating a fine piece of work.



*Arithmetic, Foundations***Fractions**

Problem with adding fractions of unlike denominators before learning to multiply them

Distributive law; the power of 1.

Distributive law; rapid calculation

**Root Concepts**

**Concept: The Distributive Law.**

$$a*(b+c) = a*b + a*c$$

**Remember: Order of Operations...Consider:**

$$2*(3 + 4)$$

Using Order of Operations, the steps are:  $2*(3 + 4) = 2*7 = 14$ .

That doesn't expose the Distributive Law (DL)...of course, using DL for steps should give the same answer:  $2*(3 + 4) = 2*3 + 2*4 = 6 + 8 = 14$ .

Simple problems like this don't expose any power to DL...so, why bother with it? Let's see...

**Book Pricing Example:**

One book costs \$7.99. How much do 8 books cost? Use DL:

$$8*(800 - 1) = 8*800 - 8*1 = 6400 - 8 = 6392 \text{ or } \$63.92$$

Here, using the DL enables solving the problem quickly in your head; it might be hard to solve  $8*799$  in your head otherwise.

Notice that DL applies when there is a minus sign because  $(800 - 1)$  is the same as  $(800 + (-1))$ .

**Rapid Multiplication: Difference of Two Squares:**

Find the product:  $(x - y)*(x + y)$

a in the DL is  $(x-y)$ ;  $(b+c)$  is  $(x+y)$ .

So, using DL:

$$(x-y)(x+y) = (x-y)x + (x-y)y \quad \text{One use of DL}$$

$$= x^2 - y^2 \quad \text{Another use of DL}$$

$= x^2 + 0 - y^2$  Notice that  $y^2$  is the same as  $x^2$  and that adding zero causes no change

$$= x^2 - y^2$$

Rapid calculation using the Difference of Two Squares:

$$29 * 31 = (30 - 1)(30 + 1) = 30 * 30 - 1 * 1 = 900 - 1 = 899.$$

$$67 * 73 = (70 - 3)(70 + 3) = 70 * 70 - 3 * 3 = 4900 - 9 = 4891.$$

### ***Squaring a number ending in 5***

Try  $25 * 25$ . Use the DL (more than once):

$$\begin{aligned} (20 + 5)(20 + 5) &= (20+5) * 20 + (20+5) * 5 = 20 * 20 + 5 * 20 + 20 * 5 + 5 * 5 = 20(20+5+5) + 25 \\ &= 20 * 30 + 25 = 2 * 10 * 3 * 10 + 25 = 2 * 3 * 100 + 25 = 625. \end{aligned}$$

Try  $75 * 75$ .

$$\begin{aligned} (70+5)(70+5) &= (70+5) * 70 + (70+5) * 5 = 70 * 70 + 5 * 70 + 70 * 5 + 5 * 5 = 70(70+5+5) + 25 = 70 * 80 + 25 \\ &= 7 * 8 * 100 + 25 = 5625 \end{aligned}$$

Generally:

$$\begin{aligned} (10x+5)(10x+5) &= (10x+5) * 10x + (10x+5) * 5 = 10x * 10x + 5 * 10x + 10x * 5 + 5 * 5 = 10x(10x+5+5) + 25 \\ &= 10x(10x+10) + 25 = 10 * 10x(x+1) + 25 = x(x+1) * 100 + 25 \end{aligned}$$

So, a quick way to express it:  $45 * 45$  ends in 25 and the front part is  $4 * 5$ , so 2025.

Or,  $65 * 65$  ends in 25 and the front part is  $6 * 7$  so 4225.

Use the quick way; but make sure you understand WHY it works, that is, PROVE it...the answer for all of these is the Distributive Law. When you know WHY things work, you remember them and can be creative with them.

### ***Putting things together...***

What is  $24 * 26$ ?

$$(25-1)(25+1)=25*25-1*1=625-1=624$$

What is  $32 * 38$ ?

$$(35 - 3)(35 + 3)=35*35-3*3=1225-9=1216$$

**KEEP LOOKING FOR THE DISTRIBUTIVE LAW...SEE IF YOU CAN MAKE UP OTHER INTERESTING USES OF IT...THE DISTRIBUTIVE LAW WILL COME UP IN ARITHMETIC, ALGEBRA, GEOMETRY, TRIGONOMETRY, CALCULUS AND MANY OTHER PLACES...IT IS TRULY A ROOT CONCEPT! ARE THERE OTHER ROOT CONCEPTS EMBEDDED IN THIS DOCUMENT? STAY TUNED...**

### Root Concepts

**Concept: The Distributive Law.**

$$a*(b+c) = a*b + a*c$$

**Concept: Multiplicative Identity.**

$$a * 1 = 1 * a = a$$

**Concept: Additive Identity.**

$$a + 0 = 0 + a = a$$

**Concept: Associative.**

$$a + (b + c) = (a + b) + c$$

$$a*(b*c) = (a*b)*c$$

**Concept: Commutative.**

$$a*b = b*a$$

$$a+b = b+a$$

**Concept: Symmetry.**

$$a = b \text{ is the same as } b = a$$

**Terminology:**

The parts of an expression linked by + signs are called 'terms'.

The parts of an expression linked by \* signs are called ‘factors’.

In a fraction, the top part is the “numerator” and the lower part, under, is called the denominator.

1. Where have we already assumed we are using any of these (look at the material on the Distributive Law).
2. Multiply fractions:

$$a/b * c/d = (ac)/(bd)$$

$$\text{Samples: } 2/3 * 4/5 = (2*4)/(3*5)$$

$$6/15 * 45/48 = (6*45)/(15*48), \text{ or more efficiently, use the concept that } a*1=a:$$

$$6/15 = 3*2/5*3 = 2/5 * 3/3 = 2/5 * 1 = 2/5. \text{ Cancel first!}$$

$$45/48 = 15*3/16*3 = 15/16 * 3/3 = 15/16 * 1 = 15/16. \text{ Cancel first!}$$

So, now the problem has become:

$$2/5 * 15/16$$

Which is  $2/5 * (3*5)/16$ ; look to cancel again...cancel the 5s.

So, now the problem is:

$$2/1 * 3/16 = 2/1 * 3/(2*8) \text{ cancel the 2s.}$$

So now it is:

$$1/1 * (3/8) = (3/8).$$

You can do this problem in your head when you use cancellation to make things more efficient! Look to cancel first before performing the multiplication. Could you have done this problem even more efficiently? Answer is YES...figure it out! See why it is good to be quick with the multiplication table, both in multiplying two numbers to get an answer and in doing it the other way around, expressing a single number as a product of factors.

3. Adding fractions.

$$\text{Like denominators: } a/c + b/c = (a+b)/c$$

$$\text{Different denominators: } a/b + c/d = (ad + bc)/bd$$

Why does this work? If we can reduce the case with different denominators to the first case with like denominators, then we are done. To do so, use  $a \cdot 1 = a$ , or written equivalently,  $a/a = 1$ .

So,  $a/b + c/d = a/b \cdot d/d + c/d \cdot b/b$  (multiplying by 1, and choosing values to yield the common denominator that we want)

$$= ad/bd + cb/db \text{ (multiplying fractions...see why you learn to do that first!)}$$

$$= (ad + bc)/bd \text{ (using simple adding of fractions and commutative).}$$

When teaching, it is CRITICAL therefore that students first understand well how to multiply fractions, then how to add simple fractions with the same denominator, and finally learn to add fractions with different denominators.

$$\text{Samples: } 1/5 + 2/3 = (1 \cdot 3 + 2 \cdot 5)/3 \cdot 5 = (3+10)/15 = 13/15.$$

$$6/48 + 10/15 =$$

Look for cancellation possibilities...

**GENERAL IDEA: THINK ABOUT WHAT YOU ARE DOING BEFORE YOU APPLY SOME MEMORIZED CUTE RULE...BE EFFICIENT AND DO AS MUCH AS POSSIBLE IN YOUR HEAD TO MAKE LIFE EASY! MATH IS FUN...NOT TEDIOUS!**

## Root Concepts

**Concept: The Distributive Law.**

$$a*(b+c) = a*b + a*c$$

**Concept: Multiplicative Identity.**

$$a * 1 = 1 * a = a$$

**Concept: Additive Identity.**

$$a + 0 = 0 + a = a$$

**Concept: Associative.**

$$a + (b + c) = (a + b) + c$$

$$a*(b*c) = (a*b)*c$$

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$$a = b \text{ is the same as } b = a$$

**Terminology:**

The parts of an expression linked by + signs are called ‘terms’.

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In a fraction, the top part is the “numerator” and the lower part, under, is called the denominator.

4. Multiply fractions:

$$a/b * c/d = (ac)/(bd)$$

5. Adding fractions.

$$\text{Like denominators: } a/c + b/c = (a+b)/c$$

Different denominators:  $a/b + c/d = (ad + bc)/bd$

6. Division of Fractions
7. Negative Numbers
8. Absolute Value
9. Inequalities

### Root Concepts

**Concept: The Distributive Law.**

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**Concept: Commutative.**

$$a*b = b*a$$

$$a+b = b+a$$

**Concept: Symmetry.**

$$a = b \text{ is the same as } b = a$$

In previous work, we assumed we were dealing with non-negative numbers and dealt only with addition and multiplication of them. Now let's look at subtraction and division.

### SUBTRACTION

Here, it is logical to think in terms of both non-negative and negative numbers. Why??

Because:

$$a - b = a + (-b)$$

That is subtraction is really just addition but with a negative number involved. So, all the concepts and processes that you learned that involved addition are also the same for subtraction. But, the answer in the case of subtraction may be either positive or negative (or zero of course).

Create and practice solving problems involving subtraction viewed as addition.

### **DIVISION**

Here, notice that:

$$a/b = a * 1/b.$$

So, division reduces to multiplication (given that one knows about  $1/b$ ).

Create and practice solving problems.

### **DIVISION OF FRACTIONS**

Although the observation above is sufficient to do division of fractions, an observation of a couple of sample problems might be helpful.

a.  $(1/2)/(2/3)$ . This is the same as  $(1/2)*(1/(2/3))$  or  $(1/2)*(3/2)$ . So, that's just  $3/4$ . Notice that this displays where the process of "invert the denominator" comes in. Often students just try to memorize that and then later, when not using it for a while, forget where to apply it. Again, it's good to understand the logic.

b.  $(1/2)/(3/2)$ . This is the same as  $(1/2)*(1/(3/2))$  or  $(1/2)*(2/3)$  or  $1/3$  (did you remember to cancel the 2s before performing the multiplication?).

### **NEGATIVE NUMBERS**

It's helpful when beginning with these to view them on a number line. Draw some and illustrate some addition problems.

You have all the root concepts to do the arithmetic.



Please practice.

### **ABSOLUTE VALUE**

Here is a link that might be helpful. Also, look around elsewhere on this site (there is lots of good stuff out there on the internet):

<https://www.mathsisfun.com/numbers/absolute-value.html>

### **INEQUALITIES**

For now, just make sure you know what the symbols mean:  $>$  and so forth. What is next to the wide, or big, part of the symbol is the big value; what is next to the point, or small, part of the symbol is the small value, no matter how the symbol is oriented. It may also have part of an equals sign associated with it.

**Next:** we move on to algebra, which is not really very different from all this. Just a matter of some rearrangement. So, if you are really comfortable with arithmetic, algebra will not be hard.

### ***Boosts in Other Subjects***

Language: these are items we found in online searches; materials have not been tested by the authors. There may be a fee for materials; there may be free samples online.

- Latin: [search results](#) on 'Latin for children'. Also, the [Henle](#) series of materials.
- French: [search results](#) on 'French for children'.
- Spanish: [search results](#) on 'Spanish for children'.

Beyond 'boosts' there are also 'extensions'. The boundary between the two terms is not well-defined. There may well be overlap and that is a good thing in terms of providing comfort in transition from one set of materials to another. For the most part, 'boosts' apply to the critical materials that we all need to learn. Sometimes, we just need a bit of help to overcome something that is difficult for us that might not be for others. There is no shame in having such difficulties; we all need a boost from time to time. But, what one person needs (and when she needs it) may not be what another needs which is why it is so important to keep open lines of communication, across varying generations and educational levels, that are free from emotion and value judgment.

Boosts in subjects we choose to study, tend to become more like 'extensions'. A difficult concept might be explained by offering lab work puzzles that demonstrate the concept, as an extension of an existing lab. Or, ease with using a computer, and mouse or fingering skills, might be facilitated by playing simple logic games on an app. The thoughtful parent or teacher will be able to design suitable extensions in response to student questions and needs as a bit of extra work that often pays enormous dividends.

## Extensions

With a solid foundation, with no evident gaps, students can progress very quickly in all manner of subject at various levels. Some subjects are linear in nature, moving from one course, as a pre-requisite, to a related higher level course. Other disciplines permit simultaneous samplings of a variety of topics within disciplinary boundaries. Still others are inter-disciplinary in nature and possess a certain element of risk, in terms of success, but may also offer the greatest reward when success is achieved simultaneously in multiple disciplines.

### *Fine Arts*

We believe children should be exposed to the fine arts along with the core curriculum. The fine arts are what reveal our unique humanity. They separate individual humans from each other and from a robotic, one-size-fits-all, developmental pattern; they encourage constructive creativity. Hence it is important to have an opportunity to see some of the finest examples in the world. We offer a sample below, based on personal preferences.

### *Music.*

- Play music for the baby in the womb and for the very young baby and toddler (for that matter, people of any age might enjoy these). Try straightforward classical music, especially Mozart; an approach that may, or may not, have advantage. However, it seems harmless.

Some of our favorites are found on the following links (all You Tubes); see what you find, too:

- Mozart: [Eine Kleine Nachtmusik](#).
- Mozart: 12 Variations on, [Ah! Vous Dirai-je, Maman](#). (Familiar music; variations on Twinkle, Twinkle, Little Star).
- Mozart: [Alleluja](#) from Exsultate, Jubilate, sung by Leontyne Price.
- Mozart: Marriage of Figaro, "[Opera Song](#)" as characterized in the movie the Shawshank Redemption.
- Mozart: [Piano Concerto #21](#). Selection that appeared in the movie, Elvira Madigan
- Mozart: The Magic Flute, [Die Strahlen der Sonne vertreiben die Nacht](#).

There are many articles on the topic online. Related articles:

- [Babies Are Stimulated by Classical Music in the Womb.](#)
- Rauscher, F., Shaw, G. & Ky, C. Music and spatial task performance. *Nature* **365**, 611 (1993). <https://doi.org/10.1038/365611a0>
- General extensions into the realm of classical music, for all ages. Children will readily find their own music. Offer them some selections that have endured over time that are generally regarded as ‘great’. Links below offer a few of our favorites, both as linked performances and as links to orchestras (with derivative links to performances) we have enjoyed for many years. Beyond our enjoyment, however, there may be deeper meaning; songs sung in Latin, by Italians, may offer language support while students learn Latin. Songs sung in French offer similar support. Songs reflective of particular periods of time may show that language use, and content, considered beautiful and appropriate within an historical context from the past, may no longer be viewed as suited in the world of 2020. Were these earlier songs, appropriate within the context of their times, ‘wrong’? Or just reflective of different times? Students might ask themselves how will what we think is beautiful, dignified, and appropriate today be viewed by future generations?
  - Traditional Classical Music
    - Mozart, Symphonies, [39](#), [40](#), [41](#). Von Karajan. Operas: The [Marriage of Figaro](#) von Karajan, [The Magic Flute](#), Bohm. [Sampler](#).
    - Beethoven, Symphonies (Berliner Philharmoniker, Herbert von Karajan). Selections from [#3](#) (Eroica), [#5](#), [#6](#) (Pastoral), [#9](#).
    - Schubert, [Unfinished, Symphony #8](#). Herbert von Karajan.
    - Haydn, [Surprise Symphony](#), von Karajan
    - Tchaikovsky, [Swan Lake Ballet](#); [Symphony #5](#).
    - [Mendelssohn](#) sampler.
    - Verdi. Aida. [Triumphal March](#). Metropolitan Opera House, New York
  - Young People Emphasis
    - Prokofiev, [Peter and the Wolf](#), Leonard Bernstein, New York Philharmonic
    - Bernstein, [Young People’s Concerts](#). Links to individual concerts—search the Internet.
  - Language

- Latin
  - Pavarotti, [Adeste Fidelis](#). Christmas Special in Montreal, 1978.  
Luciano Pavarotti, [Natale a Notre-Dame, Montreal](#).
  - Pavarotti, [Petits Chanteurs du Mont-Royal](#)
  - [Gaudeamus Igitur](#)
  - [Panis Angelicus](#)
- French
  - [Cantique de Noel](#) (O Holy Night).
  - [Au Clair de la Lune](#)
- English
  - [Amazing Grace](#), Jessye Norman, Kennedy Center, New York
  - J. Kern and O. Hammerstein II. [Ol' Man River](#), Paul Robeson (Showboat, 1936).
  - Johnny Cash, [I Walk the Line](#).
  - Willie Nelson, [On the Road Again](#).
  - Elvis Presley, [Love Me Tender](#) (sung in 1956 on The Ed Sullivan Show)
- Patriotic/Historical Connection
  - Dvorak, [Largo, from the New World Symphony](#)
  - Sibelius, [Finlandia](#) von Karajan
  - [Battle Hymn of the Republic](#) as sung by Andy Williams at the funeral of Robert Kennedy. 1968.
- National Anthems
  - [United States of America](#)
  - Canada: English ([O Canada](#); [God Save the Queen](#)), [French](#)
  - [Mexico](#)
  - [France](#)
- Orchestras, listed with our famous music directors from the past
  - [New York Philharmonic](#), Leonard Bernstein et al.
  - Boston Symphony Orchestra, [Erich Leinsdorf](#)
  - Philadelphia Orchestra, [Eugene Ormandy](#)
  - Chicago Symphony Orchestra, [Fritz Reiner](#)
  - Cleveland Orchestra, [George Szell](#)
  - NBC Symphony Orchestra, New York, [Arturo Toscanini](#), [Bruno Walter](#)

## Art

- Famous paintings and sculpture
  - [Mona Lisa](#), Leonardo da Vinci
  - [The Last Supper](#), Leonardo da Vinci
  - [Sistine Chapel](#) ceiling, Michelangelo
  - [The Creation of Adam](#), Michelangelo
  - [David](#), Michelangelo
  - [The Pieta](#), Michelangelo
  - [Starry Night](#), Vincent van Gogh
  - [The Birth of Venus](#), Botticelli
  - [Guernica](#), Picasso
  - [American Gothic](#), Grant Wood
  - [The Persistence of Memory](#), Salvador Dali
  - [The Night Watch](#), Rembrandt
  - [A Sunday Afternoon on the Island of La Grande Jatte](#), Georges Seurat
  - [Bal du Moulin de la Galette](#), Renoir
  - [Sunrise](#), Monet
  - [The Garden of Earthly Delights](#), Bosch
  - [The Thinker](#), Auguste Rodin
  - [Venus de Milo](#), Alexandros of Antioch
  - Statue of Liberty National Monument, Bartholdi
- Famous museums, a few are listed below. Look them up online to see their dazzling array of famous paintings. We list a few and we also list some smaller museums, and extras, we have enjoyed.
  - Louvre, Paris, France. Leonardo DaVinci, Mona Lisa.
  - Vatican Museums, Vatican City, Rome, Italy. Michelangelo's Sistine Chapel ceiling.
  - The Metropolitan Museum of Art, New York. Claude Monet, The Water Lily Pond.
  - State Hermitage Museum, St. Petersburg, Russia. World's largest collection of paintings.
  - The Art Institute of Chicago. Grant Wood, American Gothic.
  - Museo Nacional del Prado, Madrid, Spain. Hieronymous Bosch, The Garden of Earthly Delight (triptych).
  - Musee d'Orsay. Paris, France. Paul Cezanne, The Cardplayers.

- The Uffizi Gallery, Florence, Italy. Botticelli's The Birth of Venus.
- National Gallery of Art, Washington, D.C. American, and other, artists.
- Detroit Institute of Arts, Detroit, Michigan. Diego Rivera murals.
- Toledo Art Museum, Glass Pavilion.
- Smithsonian Museums, Washington D.C.
- Museum of Science and Industry, Chicago, Illinois
- Exploratorium, San Francisco, California

### *Architecture*

- Famous buildings
  - [Empire State Building](#), New York
  - [Eiffel Tower](#), Paris, France
  - [Leaning Tower of Pisa](#), Pisa, Italy
  - [Sydney Opera House](#), Sydney, Australia
  - [Taj Mahal](#), Agra, India
  - [Colosseum](#), Rome, Italy
  - [Hagia Sophia](#), Istanbul, Turkey
  - [The White House](#), Washington, D.C.
  - [Parthenon](#), Athens, Greece
  - [St. Basil's Cathedral](#), Moscow, Russia
  - [Space Needle](#), Seattle, Washington
  - [Big Ben](#), London, England
  - [One World Trade Center](#), New York
  - [Pantheon](#), Rome, Italy
  - [Willis Tower](#), Chicago, Illinois
  - [The Gateway Arch](#), St. Louis, Missouri
  - [St. Paul's Cathedral](#), London, England
  - [The Great Pyramid of Giza](#) Egypt
  - [Buckingham Palace](#), London, England
  - [Westminster Abbey](#), London, England
  - [Cathedrale Notre Dame](#), Paris, France
  - [St. Peter's Basilica](#), Rome, Vatican City, Italy
  - [United States Capitol](#), Washington, D. C.
  - [CN Tower](#), Toronto, Ontario, Canada
  - [Washington Monument](#), Washington, D. C.
  - [Baha'I Lotus Temple](#), Delhi, India

- [The Blue Mosque](#), Istanbul, Turkey
- [Palace of Versailles](#), Versailles, France
- [Cologne Cathedral](#), Cologne, Germany
- [Dome of the Rock](#), Jerusalem
- [Great Wall of China](#)
- Famous skylines
  - New York City
  - Chicago, Illinois
  - Hong Kong, China
  - Dubai, United Arab Emirates
  - Sydney, Australia
  - Toronto, Canada
  - Paris, France
  - Rio de Janeiro, Brazil
  - Vancouver, Canada

### ***History and Geography; Philosophy***

History and geography can work hand in hand to enhance the study of each other. Often, however, that does not appear to happen in formal education. In each case, we would like to see formal study proceed as follows: from the local, to the regional, to the national, to the international, to the global.

So, if your child is studying the geography of Mississippi, make sure he also learns about its history, at the same time. The other way around, when studying only its history, accompany that with maps of how it has changed over time. Use appropriate geography with appropriate history, and vice-versa.

At the national, international, or global level, when one is studying the Civil War in the United States, ask what was going on at the same time in other parts of the world. We do not live in a vacuum. There are various ways to integrate history and geography.

Travel with your child; experience first-hand a Civil War battlefield. Invoke the imagination; conjure up images of soldiers on a field. Or take a page out of intellectual history and imagine physicists and chemists under the West Stands of Stagg Field at the University of Chicago as they created the first self-sustaining



nuclear reaction. Where is Stagg Field today? Where are the West Stands today? How do we know they existed? What is there now? What other scientists were alive at the time, in other parts of the world, and did they all know each other? Travel, even local travel to a museum, may create exciting experiences for the child who has been taught to read everything and to think about the world around him, all the time. The luster of an adrenaline-pumping fast roller coaster ride may become a cheap, transient thrill in the memory back of a consistently thoughtful child.

In all cases, stick to facts. Use primary sources, when possible, rather than secondary sources. For example, what was the cause of the Civil War? Look it up. Find various statements; there will be ones that conflict with each other. Then go back to the primary documents of the time. Notice that some reports become laced with emotion; point out that that is never appropriate. Stick to facts: what is not offensive (indeed, possible meant as respectful) at one point in time within one culture may not subsequently be viewed that way. One is not 'right' or 'wrong': just different. The cultural context within which history is viewed is critical and it varies with both time and place.

Introduce associated philosophy for opinion. For example, on the notion of not seeing what is respectful or offensive within one's own context, because one is embedded in it rather than looking at it with perspective, consider reading *Flatland* by Edwin A. Abbot.

### ***Natural Sciences***

Sciences such as Biology, Chemistry, and Physics, typically have a required Laboratory component, often best left to formal institutional organizations that have the necessary, expensive, infrastructure already in place. There are, however, things one can do at home in support of, and supplementary to, the school-based endeavors. We look online for a few suggestions and offer those with which we have had experience (albeit limited).

- Physics.
  - Follow along as your child learns physics. Look around your world and see where concepts from physics might play out in that world. For example, when Newton's Second Law,  $F=MA$  (Force equals Mass times Acceleration) is studied, think about pointing out to your student how

you slow down before going over a bump in the road. The mass of the car is constant; higher acceleration thus leads to higher force.

Demonstrate with closing a door slowly or more swiftly. Or with using the fist of your right hand hitting the palm of your left hand at varying accelerations. There will be any number of real-world applications that are straightforward and require nothing more than flexible and creative thought processes.

- Read materials found online about great physicists and the history of physics; when using Newton's Second Law, for example, get the child accustomed to naturally reading about Newton (or any other scientist whose name appears in association with a concept).
- One classical text/lab approach is through PSSC Physics, originally developed in the late 1950s as a response to ramping up pre-collegiate training in science in the wake of the Russian launch of Sputnik. One review of it on [Amazon](#) speaks to various issues associated with teaching physics—the takeaway?—teaching laboratory science has difficulties that other subjects do not but there are many fine materials available online:

The text that this lab manual comes from is my favorite Physics text of any I have seen. The lab manual follows this excellence.

It is clear, easy to follow, and is not a "cookbook". It requires the student to think about what data will be collected and how to record it, and then guides the student in interpretation of the labs.

The bad part is the expense. The later half of this book requires some very expensive equipment. My school cannot afford this equipment. I am in favor of doing labs, even with quantum mechanics and the more advanced topics, but I think that sometimes the authors of these books need to look at reality.

A few wealthy city schools may be able to afford this equipment, but that is all. Rural or less wealthy schools, like mine, will never be able to afford this equipment. I can write my own labs for those sections using "cheaper" equipment, but I also have a Physics degree. Those who lack my training or experience will be more likely to pick a different text.

- Chemistry
  - Follow along as your child learns chemistry. Look around for examples of chemistry in your world. Try making [rock candy](#) at home.
  - Read, with your child, about famous chemists. Discuss their discoveries.

- Read in the world around you. Look at advertising slogans: “[Better Things through Better Living with Chemistry](#)” is one that has been around for a long time. Is it true? Discuss pros and cons.
- Biology and Environmental Sciences
  - Follow along as your child learns biology. Look around your yard or nearby park, for interesting plants, animals, and local environments. Build elements of a local weather station. Why is the weather important in considering local environments? What is ‘climate’? How are ‘weather’ and ‘climate’ related? Read widely on the Internet, with your child. Once again, as he sees your love for doing so, he too will learn to love asking, and satisfying, scientific questioning.
  - In the grocery store, ask your child how food gets from the farm to the table. Do carrots grow in plastic bags? Do baby carrots grow that way? Are all parts of a radish edible? What does hamburger look like before it is put in plastic wrap? Tie back to philosophy, too. Illustrate interdisciplinary questions. Do the cows and carrots that we harvest to eat know they are going to be harvested? Indeed, does a carrot, which we pull totally from the ground and kill the whole plant (rather than harvesting berries but leaving the plant to live), have a rational soul? Oddball speculation, of an academic sort, is interesting to engage in to foster creative thinking; often, however, it is better left at home than taken into the formally structured classroom.
  - Check out local and Internet sites that promote ‘green tables’ and related topics. One such organization is [Les Dames d’Escoffier](#).
  - In addition to reading about famous scientists, encourage your child to talk to local scientists; talk to his pediatrician about biology. It is likely that many scientists will enjoy talking to a genuinely curious child.
- Other sciences, such as geology (under the Earth’s surface), oceanography (oceans), physical geography (the Earth’s surface), climatology (just above the Earth’s surface), and astronomy (well beyond the Earth’s surface), will respond equally well to the sort strategy suggested above. Look to integrate sciences across disciplinary boundaries when possible. Train the child to look for the big picture of how things are interrelated, in addition to the internal workings and detail of each science.

## Help Beyond the Usual

### *Government*

Young children may learn to embrace the idea of seizing an opportunity when one is offered. For example, a child may enjoy being involved in the process of making decisions about dinner: “we are having a vegetable with our protein tonight—which one of peas, carrots, or corn would you like?” The child chooses peas, and happily eats them because he was involved in decision making. Carpe diem! Later, the parent might say, “we are having milk for lunch; would you like plain milk, chocolate milk, or malted milk?” The child chooses chocolate milk and again recognizes an opportunity and takes advantage of it. He continues to be able to identify opportunity and embrace it. After a while, he may learn to create opportunity. Now he comes back to the parent and says, “instead of milk for lunch, let’s have beer!” The parent of course says, “no, ask again in 15 years; care to try for something else right now?” The child asks for lemonade and it’s provided. Learning to identify, create, and take advantage of opportunity is a big part of the child’s path toward success. It is, as well, for parents and teachers.

Not everyone has access to the same opportunities. Some have a full wi-fi network in their homes with high speed access and state-of-the-art digital equipment in every room and in every pocket. Others have very little. It’s not so much what you do or do not have; it is more about learning to identify opportunities that are available to you at a particular time and place and make the most of those. Then, you move forward.

To create the most opportunity for yourself and your family, get to know all the resources available to you. Clearly, knowing your child’s teachers and friends is critical to building a constructive team. In addition, get to know your local members of government, independent of political party. Government (as distinct from politics) may have a number of opportunities available to you. But you need to ask to find out about them. Consider getting to know you local City Council person, Mayor, State Representative, State Senator, or Governor. There may be programs they have that fit your needs for enhanced Internet access or opportunity for access to digital equipment. Or other may be tuition grants for attending online education programs or scholarships in other forms. There may be grants to purchase books or art supplies. Check out local libraries, too.

For example, we know of one instance where the local municipal government worked with the local state university to offer free training in opera singing through the university's school of music. It was an unusual opportunity that a few took advantage of.

Perhaps your local government has some sort of educational opportunity office funded directly by the Mayor and staffed by retired college professors. Or perhaps you can work to create such opportunity. Just remember, though, that such a request may turn out to be like the six year old's request of beer for lunch. But if it does, remember that he did not give up and restructured his request, which was subsequently granted. So, in this case, backtrack to ask instead for the Mayor's office to form some sort of 'educational opportunity zone' in association with the local Community College. Ask for what you would like most but be prepared to respond creatively to the answer. If you don't ask you won't get: the only failure is the failure to try.

### ***Universities***

Schools will have extensive guidance counseling to help guide your student toward higher education. As with anything else though, you know your student best. Involve yourself as part of a team process. Big decisions are going to be made that involve the entire family, at least indirectly. Read online about colleges and universities. What appears below is simply the tip of a very large iceberg.

- Public
  - Four-year colleges and universities: top set, U.S. News and World Report, 2021, Best Public University [Ranking](#): 1. UCLA. 2. Univ. of California, Berkeley. 3. Univ. of Michigan, Ann Arbor. 4. Univ. of Virginia. 5. Univ. of North Carolina, Chapel Hill. 6 (tie). Univ. of Florida and Univ. of California, Santa Barbara.
    - Not originally a land grant university. The name is often a clue as to status in this regard. In states that have both University of X, and X State University, the former is not land grant and the latter is.
    - Land grant university, often with colleges of agriculture.
  - Two-year colleges, often named X Community College. One [ranking](#) (there are many different rankings) appears on College Choice, October 20, 2020. 1. Santa Barbara City College (Santa Barbara, CA). 2. Broward

- College (Ft. Lauderdale, FL). 3. Meridian Community College (Meridian, MS). 4. Santa Fe College (Gainesville, FL). 5. Flint Hills Technical College (Emporia, KS). 6. Los Medanos College (Pittsburg, CA).
- Private: top set, U.S. News and World Report, 2021, Best National University [Ranking](#). There are many different ways these have been sorted into subsets. Some of the more common names associated with such subsets are: Ivy League, Seven Sisters; Jesuit; online; etc. According to the linked ranking: 1. Princeton Univ. 2. Harvard Univ. 3. Columbia Univ. 4 (tie). M.I.T. and Yale Univ. 6 (tie). Univ. of Chicago and Stanford Univ.

### References to other Material by these Authors.

- [Institute of Mathematical Geography](#) (persistent link), general active [link](#).
- [Spatial Synthesis](#)
  - [Appendices](#)
    - [Eratosthenes's Measurement of the Earth](#)
    - [Latitude and Longitude](#)
    - [Earth-Sun Relations](#)
    - [Trigonometry](#)
    - [Thiessen Polygons](#)
  - [Bibliography](#)