INTERNATIONAL TRANSACTIONS IN OPERATIONAL RESEARCH

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INTERNATIONAL TRANSACTIONS IN OPERATIONAL RESEARCH

Intl. Trans. in Op. Res. 28 (2021) 1164–1189
DOI: 10.1111/itor.12870

Optimal routing of multimodal mobility systems with ride-sharing

Xiao Yu, Huimin Miao, Armagan Bayram*, Meigui Yu and Xi Chen

Department of Industrial and Manufacturing Systems Engineering, University of Michigan-Dearborn, Dearborn, MI 48128, USA

E-mail: xiaoyuu@umich.edu [Yu]; huiminm@umich.edu [Miao]; armagan@umich.edu [Bayram]; meiguiy@umich.edu [Yu]; xichenxi@umich.edu [Chen]

Received 6 April 2019; received in revised form 25 May 2020; accepted 10 August 2020

Abstract

Multimodal transportation systems are a combination of more environmentally friendly shared transport modes including public transport, ride-sharing, shuttle-sharing, or even completely carbon-free modes such as cycling to better meet customer needs. Multimodal mobility solutions are expected to contribute in mitigating traffic congestion and carbon emissions, and to result in savings in costs. They are also expected to improve access to transportation, more specifically for those in rural or low-populated communities (i.e., difficult to serve by public transportation only). Motivated by its benefits, in this study, we consider the combination of the ride-sharing and public transportation services and formulate a mixed integer programming model for the multimodal transportation planning problem. We propose a heuristic approach (i.e., angle-based clustering [AC] algorithm) and compare its efficiency with the exact solution for different settings. We find that the AC algorithm works well in both small and large settings. We further show that the multimodal transportation system with ride-sharing can yield significant benefits on travel distances and travel times.

Keywords: mobility; multimodal transportation; ride-sharing; vehicle routing

1. Introduction

Public transportation is a form of travel provided by cities that enable affordable transportation for the residents. Public transportation systems have provided communities with a valuable means of transportation for several centuries. According to the American Public Transportation Association, there were 10.1 billion trips taken via transit in 2017 alone (Dickens, 2018). Public transportation can lead to various social, economic, and environmental benefits, for example, significant financial

*Corresponding author.

Published by John Wiley & Sons Ltd, 9600 Garsington Road, Oxford OX4 2DQ, UK and 350 Main St, Malden, MA02148, USA.

savings and more economic opportunities for passengers, higher fuel efficiency and lower emissions, and improved safety (Dickens and Neff, 2011). Despite its benefits, public transportation systems are often unavailable or unreliable for serving first- and last-mile travel, especially in rural or low-populated regions (Jennings, 2015). In fact, as much as 45% of Americans have no access to public transportation (Dickens and Neff, 2011). In recent years, private companies (e.g., Uber, Lyft) have contributed to filling the gaps by providing more flexible ride-sharing services. However, the relatively high prices of their services restrict the widespread use of ride-sharing services by most residents, especially living in rural areas (Cohen and Shaheen, 2018). To enable affordable and flexible transportation services for residents and to improve access to transportation, multimodal transportation has been introduced as a new way to provide effective and consistent transportation services.

The multimodal transportation system is the combination of various modes of transportation mechanisms such as walking, cycling, buses, trains, and ride/shuttle-sharing systems. Multimodal transportation has the capability to provide more efficient and fairer transportation compared to any single-mode transportation deployed alone (Horn, 2002; Mishra et al., 2012; Litman, 2017). Moreover, it can offer additional benefits including mitigating traffic congestions, reducing emissions, and improving customer experience (Daganzo, 2007; Yao et al., 2012). Demand for multimodal transportation is also growing. According to the study of millennials and mobility (Parker, 2017), nearly 70% of millennials use multimodal travel options several times or more per week. Similarly, people living in rural areas prefer multimodal transportation increasingly (Litman, 2018). As cities aim to improve transportation services, many cities have committed to developing multimodal transportation systems to harbor their benefits through public–private partnerships. For example, Detroit, Michigan; Summit, New Jersey; and Arlington, Texas are among the cities that partner with private companies such as Uber and Lyft in implementing the combination of public transportation systems with ride-sharing services (Boll, 2018). However, despite the increased usage and need for multimodal transportation and their observed benefits in improving mobility, the integration of different transportation modes requires effective planning and limits the large-scale adoption. In this paper, we address this important challenge by developing a model and solution algorithms for the integrated planning of a multimodal transportation system involving both public transportation system and ride-sharing service.

We study a multimodal transportation system in which the passengers are transported to their final destinations via public transportation and shared services (i.e., shuttles). Recently, many cities are looking for alternative ways to improve access to transportation, more specifically for those in rural or low-populated communities (Boll, 2018). We consider a set of passengers who go to the same or nearby locations and who can travel together (i.e., going for grocery shopping or the daily commute to work). For example, consider a setting where employees living in various regions of the city use a shuttle service to go to their work or to the public transportation station. Some of the employees may work at the same company or at the companies that are close to each other by walking distance. Hence, some employees may have common destination locations. All employees are picked via a shared vehicle (i.e., shuttle), and they have an option to transfer to a mode of public transport to reach their final destination. We consider the mixed load case where the employees traveling to a different destination but living close to each other can also share the same shared vehicle. We address the benefits of multimodal transportation with ride-sharing, and we develop answers to the following operational questions:

- Given a set of passengers at different initial locations and having different final destinations, what should be the optimal assignment, routing, and transfer decisions of passengers using multimodal transportation with ride-sharing?
- What is the value of multimodal transportation with ride-sharing in terms of vehicle travel distance and vehicle travel time?

To address these questions, we develop a mixed integer linear programming model (MILP) by considering multiple objectives to find an optimal assignment, routing, and transfer decisions. The objective of the problem is to minimize a weighted sum of the following four subgoals: (a) the sum of the distance traveled by the vehicles, (b) the maximum difference between vehicle driving time and self-driving time, (c) the average vehicle travel time, and (d) the number of transfers made by passengers. Since our problem is NP-hard, finding an optimal policy for large problem instances is difficult. Thus, we develop a heuristic (i.e., angle-based clustering [AC] algorithm) to solve the MILP model efficiently. Then, we compare the MILP model with the proposed heuristic. We find that the AC algorithm works well in both small and large settings. Finally, we analyze the benefits of the multimodal transportation system using the generated instances and show that the combination of ride-sharing and public transportation system can result in a decrease in total vehicle travel distance by 7%, and 8% in vehicle travel time.

The remainder of the paper is structured as follows. In Section 2, we review the relevant literature. In Section 3, we describe the multimodal transportation model with ride-sharing. In Section 4, we propose an AC algorithm. In Section 5, we perform numerical analyses to compare the proposed heuristic approach with the optimal policy and evaluate the benefits of multimodal transportation model with ride-sharing. Finally, our conclusions are outlined in Section 6.

2. Literature review

Multimodal transportation has been extensively studied in the context of freight transportation (Gelareh and Nickel, 2011; Ishfaq and Sox, 2011; Alumur et al., 2012; SteadieSeifi et al., 2014). Studies in the area of freight transportation focus mostly on the combination of fixed routes, and they do not consider routing decisions. Contrary to these studies, we investigate the first- and last-mile travel of passengers as well by considering the ride-sharing. In the area of urban passenger mobility, some studies investigate the multimodal transportation planning problem. However, these studies either did not take the transportation efficiency into consideration and formulated the multimodal transportation routing problem as one without time constraints (Zhang et al., 2006; Wang and Han, 2010), or did not include ridesharing and consider the combination of fixed routes (Ambrosino and Sciomachen, 2014; Sun and Lang, 2015; Zhang et al., 2015). In our study, we integrate ride-sharing with the public transportation system by considering time constraints. Different from these studies, we further evaluate the benefits of the multimodal transportation system in terms of travel time and travel distance. More relevant to our study, Maheo et al. (2017) study the combination of shared shuttles and bus routes to improve the transportation system. However, their focus is the design of the transit structure (i.e., station locations). Different from them, we investigate the planning of the first/last-mile travel of passengers using ride-sharing to and from transit stations.

How to best assign vehicles to different customers and decide on their routes is a well-studied problem in vehicle routing literature. More specifically, our problem is a special case of the pickup and delivery problem, which has been studied extensively in the operations research literature (Savelsbergh and Sol, 1995; Ropke and Pisinger, 2006; Agatz et al., 2012). However, current studies in this area lack the consideration of mixed loads and multiple modes. Moreover, the shared vehicle problem is a special case of the pickup and delivery problem that focuses on the transportation of passengers (Berbeglia et al., 2007). Thus, it is important to consider the convenience of the passengers as well. Our study differs from this literature since we measure the passenger service quality, for example, in terms of the difference between the actual drive time and direct drive time, travel time, and the number of transfers made.

Another stream of literature that is relevant to our study is on school bus routing. Generally, the school bus routing problems consider the collection of the students at their bus stops and returning to the school where the students are dropped off (Bektas and Elmastas, 2007; Riera-Ledesma and Salazar-Gonzalez, 2012; Schittekat et al., 2013). Similar to our model, some of the studies in this area allow mixed loads (i.e., the transportation of students attending different schools with the same bus) (Braca et al., 1997; Kim et al., 2012; Park et al., 2012), but these studies do not consider transfers and multiple modes. Bus routing models which model transfers of students either consider predefined transfer points (Cortes et al., 2010) or the transfers between the same modes (i.e., between buses) (Fugenschuh, 2009; Bouros et al., 2011; Bogl et al., 2015). To the best of our knowledge, none of these studies in this area explicitly considers the overall problem of shared vehicle routing, assignment, and passenger transfer decisions by considering multiple modes.

In the literature, clustering-based algorithms are widely used, and the pickup nodes have been clustered according to different features, such as vehicle information, road information, depot location, and pickup locations. The sweep algorithm is one of the first clustering-based algorithms, which was proposed by Gillett and Miller (1974). The sweep algorithm forms clusters based on the angle between the stops and the depot by considering one destination. Liu and Shen (1999) improve the angle-based sweep clustering heuristic by considering the problem with time windows, and Renaud and Boctor (2002) extend the algorithm by considering mixed size vehicles. Besides the sweep algorithm, other clustering methods are also used in vehicle routing problems. For example, some studies group the passengers into the clusters according to the main road grid system (Qu et al., 2004), some define discrete zone using a combination of spatial partitioning techniques (Ouyang, 2007), and some forms clusters according to the assigned weights of passengers (Ester et al., 1996). Different from the above literature, we extend the angle-based sweep clustering heuristic by considering different destination locations, which requires to generate angles for each destination. We also consider a mixed load in each vehicle by combining passengers having different destination locations. To ensure the mixed load, we improve the angle-based sweep clustering heuristic by adding a second stage, which combines the clusters. This step involves a simple optimization model that minimizes the distances between the combined clusters.

3. A multimodal transportation model with ride-sharing

In our study, for a given a set of vehicles, we consider assignment, routing, and transfer decisions of passengers using multimodal transportation with ride-sharing to reach their final destinations. We consider the combination of two transportation systems, for example, shared vehicles and a public transportation service with a fixed route. As a public transportation mode, we consider a more direct and faster mode than the shared vehicles (i.e., subway, train, etc.). We build on a pickup and delivery problem (Ropke and Pisinger, 2006), which considers only vehicle routing decisions, by adding the binary transferring decisions of passengers. We develop an MILP, and we aim to find the optimal routing, assignment, and transfer decisions that minimize the weighted sum of the following four subgoals: (a) sum of the distance traveled by the vehicles, (b) the maximum difference between vehicle driving time and self-driving time, (c) the average vehicle travel time, and (d) the number of passenger transfers.

We consider N number of passenger locations using the multimodal transportation system with ridesharing. Let $P = \{P_1, P_2, \ldots, P_N\}$ represent the set of passenger locations and $D = \{D_1, D_2, \ldots, D_N\}$ represent the set of passenger destinations, where D_N refers to the destination of passenger N. We further define $F = P \cup D$ to represent the set of all passenger locations and destinations. Each passenger has an individual destination, but some passengers may share the same destination. There are $|V|$ number of vehicles with capacity C, where V is the set of all shared vehicles. Each vehicle starts its tour from the initial location O (i.e., depot) and ends its tour at the end terminal location E. We define $W = \{1, 2, \ldots, |W|\}$ as the set of transfer locations and $R = F \cup W \cup O \cup E$ as the set of all nodes in the transportation system. Passengers are picked by shared vehicles and they can either transfer to the public transportation system or go to their destination via shared vehicles. We allow mixed loads where the passengers traveling to different destinations can share the same vehicle. If a passenger is transported to her/his destination via shared vehicles, we assume that there is no additional walking time. On the other hand, if a passenger chooses to transfer at the transfer station, she/he gets out of the shared vehicle and uses the public transportation system. Thus, there occurs a time delay when passengers are transferred to the public transportation system (i.e., waiting time). We assume that she/he travels to the next transfer station that is closest to her/his destination. Different from traveling via shared vehicles, the passenger also needs to walk to their destination once she/he reaches to her/his final station. To represent both the passenger walking time and the passenger waiting time due to the transfer, we define t_i^w as the delay time of passenger $i \in P$ due to the transfer. The distance and travel time between nodes $i \in R$ and $j \in R$ are defined as d_{ij} and t_{ij}, respectively. If the passenger at location $i \in P$ drives to his destination directly without stopping at any other node, it takes t_{iD_i} unit time to reach his destination. A certain time is required during the passenger pickup and drop-off. We define t_p, t_d, and t_w to denote the pickup/drop-off times needed at passengers' initial locations, at their destination locations and at the transfer stations, respectively. At each passenger pickup/drop-off location more than one passenger can be picked up/dropped off, and we define l_i to denote the number of passengers picked/dropped at passenger location $i \in F$ (i.e., l_i can take both positive and negative values). Here we have two assumptions: one is that the number of passengers at each pickup node, $l_i, i \in P$, is less than the vehicle capacity, C. Another one is that the passengers at each pickup node should be picked up at the same time by the same shared vehicle. We further define the following decision variables:

x_{ij}^v: a binary variable that equals 1 if the shared vehicle v travels from location $i \in R$ to location $j \in R$, and equals 0 otherwise.

y_{iw}^{v}: a binary variable that equals 1 if passengers at initial location $i \in P$ are transported to the transfer station $w \in W$ by vehicle $v \in V$ to use the public transportation service, and equals 0 otherwise.

s_i^v: the time when vehicle $v \in V$ arrives at location $i \in R$.

s_{Diw}: the time when passengers at location i arrive at destination location D_i by walking from transfer station w.

q_i^v: the number of passengers on vehicle $v \in V$ after serving location $i \in R$.

u : an auxiliary variable that defines the difference between the multimodal system travel time and self-driving time.

We note that we summarize all notations in the Appendix. Using the above setting, we present the mathematical model as follows:

$$\min \alpha_1 \sum_{v \in V} \sum_{(i,j) \in R \times R} d_{ij} x_{ij}^v + \alpha_2 u + \alpha_3 \frac{\sum_{v \in V} s_E^v}{|V|} + \alpha_4 \sum_{w \in W} \sum_{v \in V} \sum_{i \in P} y_{iw}^v \tag{1}$$

subject to

$$\sum_{v \in V} \sum_{j \in F \cup W} x_{ij}^v = 1 \qquad \forall i \in P \tag{2}$$

$$\sum_{w \in W} \sum_{v \in V} y_{iw}^v \leq 1 \qquad \forall i \in P \tag{3}$$

$$\sum_{w \in W} y_{iw}^v = 0 \Rightarrow \sum_{j \in R} x_{ij}^v - \sum_{j \in R} x_{jD_i}^v = 0 \qquad \forall v \in V, \forall i \in P \tag{4}$$

$$y_{iw}^v = 1 \Rightarrow \sum_{j \in R} x_{ij}^v - \sum_{j \in R} x_{jw}^v = 0 \qquad \forall v \in V, \forall i \in P, \forall w \in W \tag{5}$$

$$\sum_{v \in V} \sum_{j \in R} x_{jD_i}^v + \sum_{w \in W} \sum_{v \in V} y_{iw}^v = 1 \qquad \forall i \in P \tag{6}$$

$$\sum_{j \in P \cup E} x_{Oj}^v = 1 \qquad \forall v \in V \tag{7}$$

$$\sum_{i \in D \cup O \cup W} x_{iE}^v = 1 \qquad \forall v \in V \tag{8}$$

$$\sum_{i \in R} x_{ij}^v - \sum_{i \in R} x_{ji}^v = 0 \qquad \forall v \in V, \forall j \in F \cup W \tag{9}$$

$$x_{ij}^v = 1 \Rightarrow s_i^v + t_{ij} + t_p \leq s_j^v \qquad \forall v \in V, \forall i \in P \cup O,\ \forall j \in R \tag{10}$$

$$x_{ij}^{v} = 1 \Rightarrow s_i^{v} + t_{ij} + t_d \leq s_j^{v} \qquad \forall v \in V, \forall i \in D, \forall j \in R \tag{11}$$

$$x_{wj}^{v} = 1 \Rightarrow s_w^{v} + t_{wj} + t_w \leq s_j^{v} \qquad \forall v \in V, \forall w \in W, \forall j \in R \tag{12}$$

$$y_{iw}^{v} = 1 \Rightarrow s_w^{v} + t_{wD_i} + t_i^{w} \leq s_{D_i w} \qquad \forall v \in V, \forall i \in P, \forall w \in W \tag{13}$$

$$\sum_{w \in W} y_{iw}^{v} = 0 \Rightarrow s_i^{v} \leq s_{D_i}^{v} \qquad \forall v \in V, \forall i \in P \tag{14}$$

$$y_{iw}^{v} = 1 \Rightarrow s_i^{v} \leq s_w^{v} \qquad \forall v \in V, \forall i \in P, \forall w \in W \tag{15}$$

$$\sum_{i \in P} y_{iw}^{v} = 0 \Rightarrow s_w^{v} = 0 \qquad \forall v \in V, \forall w \in W \tag{16}$$

$$\sum_{v \in V} y_{iw}^{v} = 0 \Rightarrow s_{D_{iw}} = 0 \qquad \forall i \in P, \forall w \in W \tag{17}$$

$$\sum_{w \in W} \sum_{v \in V} y_{iw}^{v} = 1 \Rightarrow \sum_{v \in V} s_{D_i}^{v} = 0 \qquad \forall i \in P \tag{18}$$

$$u \geq \left(s_{D_i}^{v} + \sum_{w \in W} s_{D_i w} - s_i^{v} \right) - t_{iD_i} \qquad \forall v \in V, \forall i \in P \tag{19}$$

$$x_{ij}^{v} = 1 \Rightarrow q_i^{v} + l_j = q_j^{v} \qquad \forall v \in V, \forall i \in R, \forall j \in F \tag{20}$$

$$x_{iw}^{v} = 1 \Rightarrow q_i^{v} - \sum_{j \in P} y_{jw}^{v} = q_w^{v} \qquad \forall v \in V, \forall i \in R, \forall w \in W \tag{21}$$

$$q_i^{v} \leq C \qquad \forall v \in V, \forall i \in R \tag{22}$$

$$q_O^{v} = q_E^{v} = 0 \qquad \forall v \in V \tag{23}$$

$$x_{ij}^{v} \in \{0, 1\} \qquad \forall v \in V, \forall (i, j) \in R \times R \tag{24}$$

$$y_{iw}^{v} \in \{0, 1\} \qquad \forall v \in V, \forall i \in P, \forall w \in W \tag{25}$$

$$s_i^{v} \geq 0 \qquad \forall v \in V, \forall i \in R \tag{26}$$

$$S_{D_i w} \geq 0 \qquad \forall i \in R, \forall w \in W \tag{27}$$

$$q_i^{v} \geq 0 \qquad \forall v \in V, \forall i \in R, \tag{28}$$

where α_k is the weighting factor coefficient for objective $k = \{1, \ldots, 4\}$. Our optimization model includes four types of objectives that reflect different goals involving minimization of the shared vehicle and passenger related costs. Objective 1 is a shared vehicle related objective that aims to minimize the total distance traveled by all vehicles. Objective 2 is a passenger-related objective that minimizes the maximum difference between the travel time of the multimodal transportation system with a shared vehicle and self-driving time. The objective is modeled through the variable u and the constraint (19). This objective also aims equity in travel times of all passengers as it ensures that the travel delay of all passengers is close to each other. Objective 3 is another vehicle-related objective that minimizes the average travel time of vehicles. Objective 4 is a passenger-related objective that considers the cost of transferring decisions. It minimizes the total number of transfers made by passengers.

In our model, we ensure that each passenger is picked up by one vehicle through constraint (2). Constraint (3) defines that the passengers can transfer to another transportation mode at the transfer station. Constraint (4) ensures that if the passenger $i \in P$ is picked up by vehicle $v \in V$ and if the passenger does not transfer to another transportation mode, the corresponding destination location of the passenger is visited by the shared vehicle. Constraint (5) links y_{iw}^v and x_{ij}^v variables. If the passenger uses both transportation modes, it is ensured that the shared vehicle visits the corresponding transfer station and the passenger is dropped off at the transfer station. Constraint (6) defines a condition in which the passenger reaches her/his destination location using a shared vehicle or two transportation modes (i.e., a shared vehicle and another public transportation mode). Constraints (7) and (8) ensure that every shared vehicle leaves the depot and enters the end terminal. In constraint (9), we balance the flow for each vehicle at each location. Constraints (10)–(12) are used to define s_i^v the time when vehicle $v \in V$ arrives at the specified location. Constraint (13) defines the time when passengers arrive at their destination from transfer location $w \in W$. These constraints (i.e., constraints (10)–(13)) also make subtours impossible. Constraint (14) and (15) state that the pickup time of a passenger at location $i \in P$ occurs before the passenger reaches to her/his destination. In constraints (16)–(18), we link y_{iw}^v variable with the arrival time of passengers at certain locations. More specifically, constraint (16) ensures that if no one in a shared vehicle $v \in V$ transfers at the transfer station $w \in W$, the shared vehicle does not visit the transfer station (i.e., $s_w^v = 0$). Similarly, constraint (17) states that if passenger $i \in P$ does not transfer at the transfer station $w \in W$, then (s)he will not take the public transportation service at station $w \in W$ to the destination (i.e., $s_{D_i w} = 0$). On the other hand, constraint (18) describes that if the passenger at location $i \in P$ transfers at any of the transfer station $w \in W$, then the shared vehicle does not visit the destination location of passenger $i \in P$ (i.e., $\sum_{v \in V} s_{D_i}^v = 0$).

As described above, we define variable u through constraint (19), which represents the maximum difference between the travel time of a passenger using the multimodal transportation system with ride-sharing and passenger's self-driving time. Constraints (20) and (21) define the current number of passengers in each shared vehicle at each location. We assume that the number of passengers at each pickup node, $l_i \; \forall i \in P$, is less than the vehicle capacity, C. With constraint (20), all passengers at location i will be picked up by the same vehicle at the pickup node. Similarly, with constraint (21), the number of passengers in vehicle $v \in V$ is updated at the transfer station $w \in W$ if any passenger transfers to public transportation. In constraint (22), we state that the current number of passengers at each location should be less than or equal to the capacity of the shared vehicle. Constraints (20) and (22) together ensure that if the remaining capacity of the vehicle $v \in V$ is greater than the number of passengers at pickup node $i \in P$, the vehicle will pick up all passengers at that node. If not, another vehicle satisfying that condition will pick them up. We also ensure that the vehicle is empty at the depot and the end terminal through constraint (23). Constraints (24) and (25) define integrality, and constraints (26)–(28) define nonnegativity.

In our model, there are many nonlinear constraints (i.e., constraints (4), (5), (10)–(18), (20), and (21)). These constraints can be linearized, and models (1)–(28) can be easily transformed into an MILP model. For example, we use the following two constraints to linearize constraint (4):

$$\sum_{w \in W} y_{iw}^{v} = 0 \Rightarrow \sum_{j \in R} x_{ij}^{v} - \sum_{j \in R} x_{jD_i}^{v} = 0 \qquad \forall v \in V, \forall i \in P \tag{29}$$

$$\Rightarrow \begin{cases} \sum_{j \in R} x_{ij}^{v} - \sum_{j \in R} x_{jD_i}^{v} \leq 0 + M \sum_{w \in W} y_{iw}^{v} & \forall v \in V, \forall i \in P \\ \sum_{j \in R} x_{ij}^{v} - \sum_{j \in R} x_{jD_i}^{v} \geq 0 - M \sum_{w \in W} y_{iw}^{v} & \forall v \in V, \forall i \in P \end{cases}, \tag{30}$$

where M is a large value. We determine the value of M for each nonlinear constraint separately by considering the smallest possible value for that constraint. Similarly, we convert all nonlinear constraints into linear constraints.

4. Heuristic approach

Our problem is a variant of the vehicle routing problem, and it is an NP-hard problem. NP-hard problems are usually difficult to solve for large instances due to the curse of dimensionality. In our model, as the number of passengers, transfer locations, and vehicle capacity increase, it becomes intractable to compute the optimal objective function and find the optimal assignment, routing, and transfer decisions. In this section, to address computational and practical challenges, we propose an AC algorithm.

4.1. Angle-based clustering algorithm

In this section, we propose a three-stage AC algorithm that splits the problem into several clusters and reduces the size of the optimization model. We assume that some passengers may have the same destination location or may have nearby destinations (i.e., by walking distance). Let $D' = \{D'_1, \ldots, D_z'\}$ represent the set of different destination locations where $D' \subseteq D$ and $P_z = \{P_{11}, \ldots, P_{iz}\}$ represent the set of passengers' initial locations traveling to destination D'_z, where $P_z \subseteq P$. P_{iz} denotes passenger i traveling to destination D'_z. In the first stage of the algorithm, we aim to create a set of clusters of passengers who have the same destination or having close destinations. We assume that cluster $m'_z \in \{1, 2, \ldots, m_z\}$ is formed for destination location D'_z, where m_z is the number of clusters formed for that destination location. We use $h(m'_z)$ to represent the number of passengers assigned to each cluster m'_z. We determine clusters using angles. Let $\angle A_z$ represent the largest angle formed by $P_{iz} D'_z P_{jz}, \forall D'_z \in D'$ and $\forall P_{iz}, P_{jz} \in P_z$, and $\angle B_z = \frac{\angle A_z}{m_z}$ represent equal angles for all different destination locations D'_z. We describe the first stage of the AC algorithm with details as follows.

AC algorithm—stage 1

Step 1: Calculate the largest angle $\angle A_z$ formed by $P_{iz}D'_zP_{jz}$, $\forall D'_z \in D'$, and $\forall P_{iz}, P_{jz} \in P_z$ so that passengers fall within the angle.
Step 2: Calculate the number of clusters m_z for each different destination location D'_z using

$$m_z = \frac{\text{Numbers of passengers traveling to destination } D'_z}{\text{Shared vehicle capacity}} \times |D'|$$

Step 3: Calculate $\angle B_z$ for each different destination location D'_z using $\angle B_z = \frac{\angle A_z}{m_z}$.
Step 4: Assign passengers to clusters as follows:
for $z \rightarrow 1$ **to** $|D'|$ **do**
  for $j \rightarrow 1$ **to** $|P_z|$ **do**
    for $m'_z \rightarrow 1$ to m_z **do**
      if $(\angle P_{1z}D'_zP_{jz} \leq m'_z\angle B_z)$ **then**
        P_{jz} is assigned to cluster m'_z
        $h(m'_z) \leftarrow h(m'_z) + 1$; **break**
Step 5: Rearrange clusters by considering the shared vehicle capacity.
  for $z \rightarrow 1$ **to** $|D'|$ **do**
    if $h(m'_z) > C$ **then**
      Step 5.1: Split the cluster m'_z equally, until each new subcluster does not exceed the vehicle capacity as follows:
      for $\varrho'_z \leftarrow 1$ **to** ϱ_z **do**
        if $(\angle P_{1z}D'_zP_{jz} \leq (m'_z - 1) \cdot \angle B_z + \varrho'_z \cdot \angle B'_z)$ **then**
          Step 5.1.2: P_{jz} is assigned to cluster $m'_z = m_z + (\varrho'_z - 1)$
          $h(m'_z) \leftarrow h(m'_z) + 1$
          Step 5.1.3: Update the number of cluster as $m_z = m_z + (\varrho_z - 1)$
      If $h(m'_z) > C$ where $m'_z = m_z + (\varrho'_z - 1)\forall \varrho'_z \in \{1, \ldots, \varrho_z\}$ **then**
        Update ϱ_z as $\varrho_z \leftarrow \varrho_z + 1$ and go to step 5.1

In stage 1, we form the clusters of passengers sharing the same destination location by considering the initial location of each passenger. In the AC algorithm, if the size of the any of the formed clusters exceeds the vehicle capacity, we split those clusters into equal-angled subclusters. To split the clusters that exceed capacity, we use the same approach that we used to determine the initial clusters. To this end, we define ϱ_z to represent the number of subclusters that are obtained after the split in cluster z. We split the clusters with equal angles and ensure that each subcluster has the same angle. Hence, we define $\angle B'_z$ to represent the angle of each subcluster.

To allow the mixed load, in stage 2, we combine the clusters formed in stage 1 by considering the distances between each cluster and assign one shared vehicle to each cluster. Let $Cent_{m'_z}$ and $Cent_{m''_z}$ represent the centroids of clusters m'_z, $m''_z \in \{1, \ldots, m_z\}$, respectively. We use $d_{m'_zm''_z}$ to denote the distance between clusters m'_z and m''_z, where $z \in \{1, \ldots, |D'|\}$. We use an optimization model to combine the clusters as a group. We further define a binary decision variable $k_{m'_zm''_z}$, where it equals to 1 if clusters m'_z and m''_z are grouped together, and equals to 0 otherwise. We describe the second stage of the algorithm as follows:

AC algorithm—stage 2

Step 1: Calculate the $Cent_{m'_z}$ where $m'_z \in \{1, \ldots, m_z\}$ for all clusters of all destination locations (i.e.,the centroid of each cluster formed in stage 1).
Step 2: Solve the following optimization model:

$$\min \sum_{z=1}^{|D'|} \sum_{m'_z=1}^{m_z} \sum_{m''_z=1, m''_z \neq m'_z}^{m_z} d_{m'_z m''_z} k_{m'_z m''_z} \tag{31}$$

subject to

$$\sum_{z=1}^{|D'|} \sum_{m''_z=1, m''_z \neq m'_z}^{m_z} k_{m'_z m''_z} \geq |D'| \qquad \forall m'_z \in \{1, \ldots, m_z\} \tag{32}$$

$$k_{m'_z m''_z} = k_{m''_z m'_z} \qquad \forall m'_z, m''_z \in \{1, \ldots, m_z\} \tag{33}$$

$$k_{m'_z m''_z} \in \{0, 1\} \qquad \forall m'_z, m''_z \in \{1, \ldots, m_z\} \tag{34}$$

Equation (31) is the objective function that minimizes the sum of the distances between clusters that are grouped together. Constraint (32) states that at least $|D'|$ clusters should be grouped together since there are at least $m_z = \frac{\text{Numbers of passengers traveling to destination } D'_z}{\text{Shared vehicle capacity}} \times |D'|$ clusters for each different destination D'_z. In constraint 33, we define that variable $k_{m'_z m''_z}$ is symmetric (i.e., if cluster 1 is grouped with cluster 2, it means that cluster 2 is grouped with cluster 1). Finally, we define integrity conditions through constraint (34). We assign one shared vehicle for each cluster formed in stage 2. In stage 3, we fix the vehicle assignment decisions and solve each cluster simultaneously using the MILP model to find optimal shared vehicle routing and passenger transfer decisions for given shared vehicle assignment decisions. Next, we describe a simple example to illustrate the AC algorithm.

Example 1. Suppose there are 10 passengers and 2 different destination locations (i.e., D'_1 and D'_2). Let shared vehicle capacity be five passengers. As shown in Fig. 1, we assume that passengers from 1 to 5 are traveling to D'_1, and passengers from 6 to 10 are traveling to D'_2. Using stage 1 of the proposed heuristic approach, we define two clusters for each destination (i.e., $m_1 = m_2 = \frac{5}{5} \times 2 = 2$). We show obtained clusters in Fig. 1a. More specifically, passengers 1–3 are assigned to cluster 1 of D'_1, passengers 4 and 5 are assigned to cluster 2 of D'_1, passengers 6, 9, and 10 are assigned to cluster 1 of D'_2, and passengers 7 and 8 are assigned to cluster 2 of D'_2. No cluster contains more than five pickup nodes, so we move on to stage 2.

In the first stage of the heuristic, four clusters are formed. Assigning one vehicle for each cluster with a capacity of five would be costly. Thus, in the second stage of the heuristic, we group clusters to make the vehicle assignment decisions by allowing the mixed load. We use the optimization model and ensure that passengers who are close to each other are traveling together. Since we have two different destinations (i.e., $D' = 2$), we assign two clusters for each group. According to the

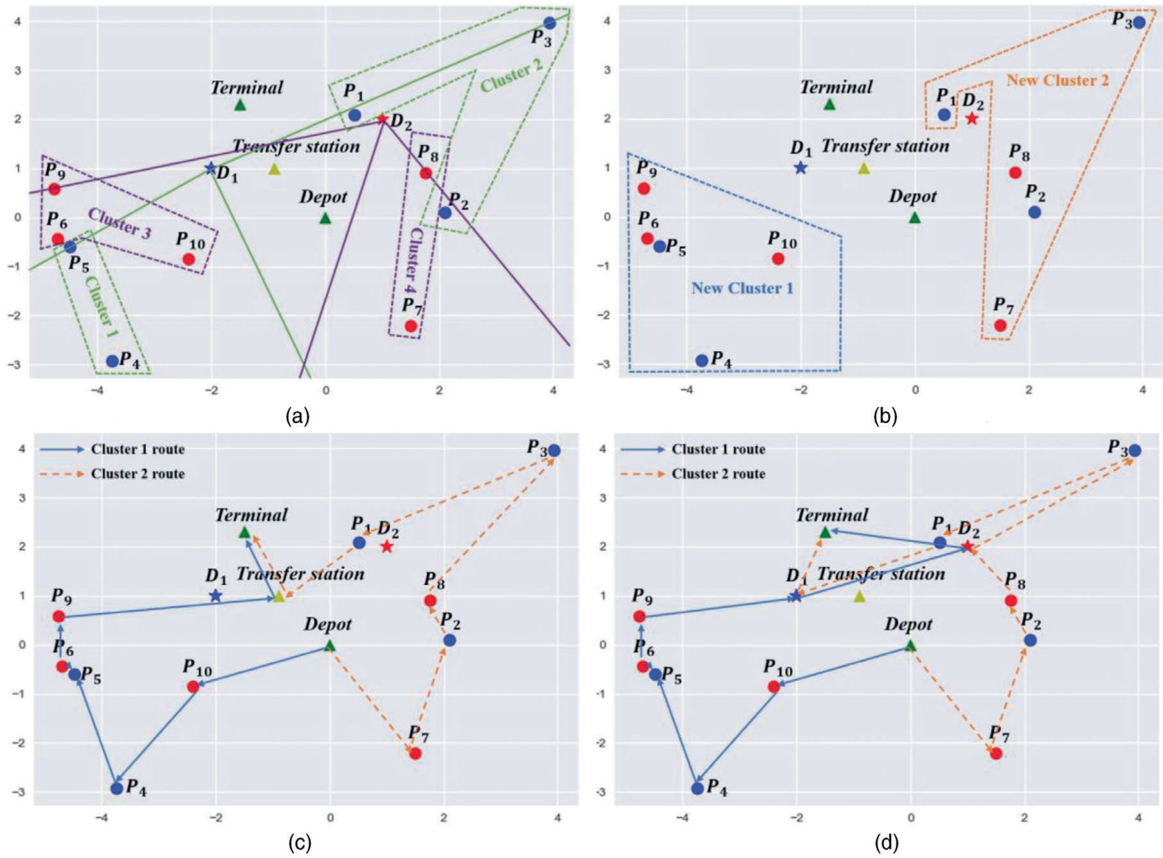

Fig. 1. Illustration of the angle-based clustering algorithm. (a) Example of the AC algorithm—stage 1, (b) example of the AC algorithm—stage 2, (c) example of the AC algorithm—stage 3, and (d) example of the AC algorithm—stage 3.

optimization model, passengers 1, 2, 3, 7, and 8 are assigned to one group, and passengers 4, 5, 6, 9, and 10 are assigned to another group.

We illustrate our results in Fig. 1b. Finally, in the third stage, we fix the shared vehicle assignment decisions, and we use the original MILP model to find optimal routing and transfer decisions for two new clusters formed in stage 2. We show two feasible routes where the transfer station is visited and the transfer station is not visited in Fig. 1c and d, respectively.

5. Numerical experiments

This section comprises three main parts. First, we describe the instance generation process that we use in our numerical experiments in Section 5.1. Second, we compare the performance of the heuristic with that of the MILP solution in Section 5.2. Third, in Section 5.3, we evaluate the benefits of the multimodal transportation system in terms of travel distance and travel time by comparing the multimodal transportation system with the single-mode system (i.e., shared vehicles only).

Table 1
Summary of parameter settings of generated instances

Number of passengers	Vehicle capacity	Number of vehicles	Total number of instances
10	5	2 Vehicles	12
15	5	3 Vehicles	12
20	5	4 Vehicles	12
	10	2 Vehicles	12
30	5	6 Vehicles	12
	10	3 Vehicles	12
50	5	10 Vehicles	12
	10	5 Vehicles	12

5.1. Instance generation

In our numerical experiments, we use a realistically generated data set by considering Detroit's transportation system. We consider a shared-shuttle service that can take employees to and from the nearest high-frequency public transportation stop or directly to their work. In all instances, we assume that the passengers' locations are distributed within a 10-mile radius around their destination, and we generate these pickup locations randomly. More specifically, we generate the distance between locations by sampling from a uniform distribution and by considering a Manhattan distance structure. In our numerical experiments, we consider that there is one public transfer station, one depot, one end terminal, and two destination locations and we vary the number of pickup locations, the number of vehicles, and the capacity of vehicles. We summarize the settings and the number of instances in each setting in Table 1.

As shown in Table 1, in our numerical experiments, we consider cases where there are 10, 15, 20, 30, and 50 passengers. When there are 10 and 15 passengers, we use vehicles with a capacity of 5, and when there are more than 15 passengers we use vehicles with a capacity of both 5 and 10. We consider a varying number of vehicles as well. For example, when there are 20 passengers, and when the capacity of the vehicles is 5, we consider that there are 4 vehicles. Similarly, when there are 20 passengers, and when the capacity of the vehicles is 10, we consider that there are 2 vehicles. In each setting, we ensure that total vehicle capacity can meet the total number of passengers. We also vary the weights of the subobjectives (i.e., $\alpha_i \ \forall \ i \in \{1, \ldots, 4\}$). More specifically, we consider two different weighted objective settings: (a) $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.25$ and (b) $\alpha_1 = \alpha_3 = 0.5$, $\alpha_2 = \alpha_4 = 0$. For each defined scenario setting, we consider six instances. For example, there are a total of 6 instances for the scenario setting where there are 10 passengers and 2 vehicles with a capacity of 5 and when the objective function weights are $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.25$. Since we consider two different weighted objective settings for this scenario, we analyze $6 \times 2 = 12$ instances in total when there are 10 passengers. The number of total instances can be calculated with a similar approach for the remaining settings as illustrated in Table 1.

We further take into account the differences between the speeds of different modes, since we consider two modes of transportation. The average driving speed of several U.S. cities without traffic is stated as 29 MPH (Trigg, 2015). We use 24 MPH for the speed of the shared vehicle by considering the traffic congestion during the rush hours. As stated in our model assumption, we consider a more direct and faster mode than the shared vehicles as a second transportation mode (i.e., subway, train, etc.). The subway/train speed is stated as around 30 MPH (Johnson, 2010), and we use this value in our numerical experiments. The speed of the different modes impacts the time it takes to travel between locations.

In our model, we include the time it takes to get on and get off the vehicle. We consider that it takes around one minute (i.e., $t_p = 1$ minute) for a passenger to get on a shared vehicle. Considering that there might be more than one passenger getting off the shared vehicle or subway/train, we use two minutes (i.e., $t_d = t_w = 2$ minutes) for getting off at the destination and transfer locations. For passenger waiting time at the transfer stations, we review subway schedules (i.e., NYC MTA) and train schedules (i.e., Detroit) during the rush hours where a subway/train is scheduled around every four to six minutes. Thus, we use a uniformly distributed waiting time at the transfer stations with an average waiting time of 2.5 minutes. We also take into account a walking time from the transfer station to the passenger's destination for passengers using the second mode (i.e., train, subway). In our instances, we use uniformly distributed walking times changing from 1 to 10 minutes. We provide an example to illustrate one of the instances in Section A2 in the Appendix.

5.2. Comparison of solution algorithms

In this section, we compare the MILP solution with the proposed methodology (i.e., the AC algorithm) and use it as a benchmark value for the proposed algorithm. We solve the problem for all instances, which are defined in Section 5.1, and all instances are solved using the CPLEX solver. We note that we also provide a comparison of algorithms for small-sized settings (i.e., when $N = 5$) in Section A2 in the Appendix. As described before, we define six instances for each scenario setting, and thus, we present our results for each scenario setting in each table. Moreover, since four subobjectives have different units and magnitudes, we normalize the value of each subobjective before calculating the weighted sum of four subobjectives. Hence, we present the normalized objective function values in the tables.

In each table, we present the numerical results of the exact MILP and the AC algorithm for each instance. For the MILP result, we present the objective value of the MILP (i.e., Obj.), the optimality gap obtained at the end of the running time by the CPLEX solver (i.e., Gap%), and the computation time in seconds (i.e., Run time (seconds)). Similarly, for the proposed algorithm, we present the corresponding objective value (i.e., Obj.), percentage difference with the MILP solution at the end of the running time (i.e., %Gap with MILP), and the computation time in seconds (i.e., Run time (seconds)). We calculate the percentage difference with the MILP solution using the following formula:

$$\%\,\text{Gap with MILP} = \frac{\text{Obj. of heuristic algorithm} - \text{Obj. of MILP}}{\text{Obj. of MILP}}. \tag{35}$$

In Table 2, we summarize the comparison results for six instances. We present the run results for the MILP and AC algorithm when the vehicle capacity is 5 (i.e., $C = 5$), when the subobjectives are equally weighted (i.e. , $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.25$), and when there are 10 passengers

Table 2
Comparison of algorithms for setting: $N = 10,\ C = 5,\ \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.25$

	MILP			AC algorithm		
Instances	Obj.	Gap%	Run time (seconds)	Obj.	%Gap with MILP	Run time (seconds)
1	0.3235	55.72	3600	0.3235	0.00	180
2	0.4637	60.31	3600	0.4408	−4.93	180
3	0.3070	59.20	3600	0.3142	2.37	180
4	0.3465	58.30	3600	0.3465	0.00	180
5	0.3899	59.18	3600	0.400	2.74	180
6	0.3618	53.93	3600	0.3686	1.88	180
Average		57.77			0.34	

Table 3
Comparison of algorithms for setting: $N = 15,\ C = 5,\ \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.25$

	MILP			AC algorithm		
Instances	Obj.	Gap%	Run time (seconds)	Obj.	%Gap with MILP	Run time (seconds)
1	0.3652	57.95	3600	0.3700	1.32	180
2	0.3277	58.71	3600	0.3199	−2.36	180
3	0.3487	61.50	3600	0.3430	−1.63	180
4	0.4128	54.45	3600	0.4068	−1.44	180
5	0.3102	52.34	3600	0.3116	0.45	180
6	0.4006	53.23	3600	0.4115	2.73	180
Average		57.77			−0.15	

(i.e., $N = 10$). As shown in Table 2, the average "%Gap with MILP" for the AC algorithm is 0.34%, which means that the AC algorithm yields slightly worse objective function value than that of the MILP solution on average for this scenario setting. However, the run time of MILP is one hour, while that of the AC algorithm is three minutes (i.e., 180 seconds).[1] Hence, the AC algorithm is more time efficient than the MILP for this setting.

Similarly, in Table 3, we present the run results for the MILP and AC algorithm when the vehicle capacity is 5 (i.e., $C = 5$), when the subobjectives are equally weighted (i.e., $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.25$) and when there are 15 passengers (i.e., $N = 15$). As shown in Table 2, the average "%Gap with MILP" for the AC algorithm is −0.15%. Different from the results of Table 2, the AC algorithm outperforms the MILP in terms of both objective function value and run time. On average, the AC algorithm reaches a better objective function value within three minutes.

For a setting where the vehicle capacity is 5, we also consider the case with 50 passengers. In this case, due to a large number of passengers, MILP cannot obtain any feasible solution within one hour, whereas the AC algorithm obtains feasible solutions within five minutes. Since there is no MILP solution to compare, the "%Gap with MILP" column cannot be calculated. Thus, it is

[1]We limit the computation time of the MILP to one hour and the run time of the AC algorithm to three minutes.

Table 4
Comparison of algorithms for setting: $N = 20,\ C = 10,\ \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.25$

	MILP			AC algorithm		
Instances	Obj.	Gap%	Run time (seconds)	Obj.	%Gap with MILP	Run time (seconds)
1	–	–	3600	0.4908	–	300
2	0.4494	65.00	3600	0.4234	−5.78	300
3	0.2297	57.40	3600	0.2195	−4.43	300
4	–	–	3600	0.2204	–	300
5	0.2203	54.91	3600	0.2170	−1.48	300
6	0.2679	65.20	3600	0.2529	−5.58	300
Average		60.62			−4.31	

clear that the AC algorithm outperforms the MILP in this scenario setting as well. We present our comparison results in Table A5.

We further compare the results of the settings when the vehicle capacity is 10. In Tables 4 and in A6, we present the results where there are 20 and 30 passengers, respectively. For both tables, the vehicle capacity is 10 and the subobjectives are equally weighted. As shown in tables, the MILP cannot obtain solutions for some settings, specifically, for the settings where the number of passengers is large (i.e., $N = 30$). For the instances where there is no MILP solution, the percentage gap with MILP cannot be calculated. When there are 20 passengers, it is shown that the AC algorithm obtains a better objective function value in five minutes than the MILP can obtain within one hour. We note that the results for the remaining settings are similar to our findings. We present the tables for the remaining instances in Section A2 in the Appendix.

Overall, given the complexity of the problem, the exact MILP model cannot find a feasible solution within one hour when the number of passengers is greater than 20. On the other hand, the AC algorithm reduces both the number of decision variables in the MILP model and the size of the problem by splitting the model into clusters that can be solved by the MILP simultaneously. Thus, the AC algorithm can find a feasible solution for all instances within five minutes. Moreover, the AC algorithm is more efficient (i.e., can solve within five minutes) and more straightforward compared to the MILP. Given that our analysis focuses on ride-sharing and point-to-point pickup and delivery service, we can conclude that high-quality solutions can be obtained within five minutes using the AC algorithm.

5.3. Value of the multimodal transportation system

In recent years, transportation planning has expanded to include more emphasis on nonautomobile modes, to maximize traffic speeds, minimize congestion, reduce pollution emissions, and to minimize the cost of traveling. A multimodal transportation system is an alternative option to the single mode (i.e., private vehicles or ride-sharing mode only), and it is expected to minimize the use of vehicles. In our study, we calculate the benefits of the multimodal transportation system on the vehicle-driven distance and vehicle travel time. Hence, we consider two settings: (a) single mode,

Table 5
Comparison of multimode and single-mode transportation systems in terms of shared vehicle travel distance (i.e., $\alpha_1 = 1$) and travel time (i.e., $\alpha_3 = 1$)

			Objective function when $\alpha_1 = 1$			Objective function when $\alpha_3 = 1$		
Instances	Vehicle capacity	Number of passengers	Single mode	Multimode	Percentage change	Single mode	Multimode	Percentage change
1	5	10	0.5515	0.4907	−11.03%	0.6405	0.5528	−13.69%
2	5	10	0.7730	0.7385	−4.46%	0.9533	0.8467	−11.18%
3	5	10	0.6326	0.6260	−1.03%	0.7605	0.7367	−3.13%
4	5	15	0.5943	0.5468	−8.00%	0.9740	0.8538	−12.34%
5	5	15	0.5323	0.4993	−6.19%	0.8053	0.7844	−2.59%
6	5	15	0.5685	0.5166	−9.13%	0.9242	0.8599	−6.95%
7	10	20	0.3639	0.3320	−8.77%	–	–	–
8	10	20	0.3292	0.2870	−12.80%	–	–	–
	Average				−7.68%			−8.31%

where the use of public transportation is not allowed and (ii) multimode, where the use of public transportation is allowed. We run the same instances by considering both single- and multimode options and calculate the percentage difference between the objective function values of these two options. The calculated difference gives us the obtained benefits in terms of vehicle-driven distance and vehicle travel time when the multimode system is used instead of the single-mode system. We use the exact MILP model for comparison. We analyze the change in the vehicle travel distance (i.e., $\alpha_1 = 1$) and vehicle travel time (i.e., $\alpha_3 = 1$) when multi- and single-mode transportation systems are compared. We use the same instances that are introduced in Section 5.2. Since the exact MILP solution does not provide a feasible solution in some cases, we illustrate our results for the cases where a solution can be obtained within one hour (Table 5).

From left to the right, Table 5 presents the instance number, the capacity of the shared vehicle, the number of passengers, objective function value of the single-mode system when $\alpha_1 = 1$, objective function value of the multimode system when $\alpha_1 = 1$, percentage difference between different modes when $\alpha_1 = 1$, objective function value of the single-mode system when $\alpha_3 = 1$, objective function value of the multimode system when $\alpha_3 = 1$, and the percentage difference between different modes when $\alpha_3 = 1$. We calculate the percentage difference between different modes using the following formula:

$$\begin{array}{c}\text{Percentage difference between}\\ \text{different modes}\end{array} = \frac{\text{Obj. value of multimode} - \text{Obj. value of single mode}}{\text{Obj. value of single mode}}. \tag{36}$$

As shown in Table 5, when $\alpha_1 = 1$ (i.e., when the objective function is to minimize the total travel distance solely), the result indicates that the average total travel distance over all instances decreases by 7.68% with the existence of multiple modes. When $\alpha_3 = 1$ (i.e., when the objective function is to minimize the average travel time solely), the result shows that the average travel time decreases by 8.31% with the existence of multiple modes. When multimode is preferred, the shared vehicle does not need to travel to the passenger's destination all the time, which results in savings in both travel distance and travel time. It is also expected that the decreased use of shared vehicles will lead to reduced traffic density and traffic delay. Overall, the existence of multiple modes can reduce the travel distance and the travel time of the shared vehicle to a large extent, which saves cost, energy, and time.

6. Conclusions

Multimodal mobility systems are essential in day-to-day transportation of commuters, such as employees who work in big cities or people who live in low-populated neighborhoods. Multimodal transportation systems ensure more flexible, less costly (i.e., regarding traffic density, travel time, and travel distance) transportation compared to the single-mode systems. Multimodal transportation system also contributes to improving the accessibility of the residents to the resources that they need. Although its implementation with shared vehicles is relatively new, the demand is increasing, and cities are investigating solutions to implement an efficient integration of multiple modes. Thus, it is important to analyze efficient ways to implement multiple modes with shared vehicles and evaluate its value.

In this study, we build an MILP model for multimodal transportation systems with shared vehicles. Since the investigated problem is NP-hard, it is computationally intractable to obtain optimal (and even feasible) solutions for large problem instances. Thus, we propose an AC algorithm and we compare its performance with the MILP solution. We show that the AC algorithm is a more practical procedure that outperforms the MILP. Moreover, the AC algorithm requires less computational effort and can solve relatively large scale multimodal transportation system problem, since it splits the problem into clusters and solves all of them simultaneously. We further show that the AC algorithm generates a feasible solution for real-world instances in a reasonable time.

As part of future research, first, variants of the proposed multimodal mobility model can be considered to allow even more flexibility to the pickup and delivery problems. For example, configurations where both delivery and pickup operations are performed within specific time windows, a mixed fleet of vehicles with different capacities can be considered. Second, more than two different modes can be considered, which can provide more flexibility and options to passengers. We build a deterministic model to analyze the multimodal transportation system with ride-sharing. As the third extension, a stochastic programming model can be introduced to model the uncertainty in waiting and travel times, which would be a practically relevant variation to the problem.

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Appendix A

A1. Summary of the notation

Table A1
Summary of the notation

Notation	Definition
Model setup	
N	The number of passengers using multimodal transportation system with ride-sharing
P	The set of passenger locations
P_i	The passenger location i
D	The set of passenger destinations
D_i	The destination of passengers from passenger location i
$F = P \cup D$	The set of all passenger locations and destinations
V	The set of all shared vehicles
$\|V\|$	The number of shared vehicles
C	The capacity of shared vehicle
O	The initial location (i.e., depot)
E	The end terminal location
W	The set of transfer locations
w	The transfer location w
$R = F \cup W \cup O \cup E$	The set of all nodes the transportation system
t_i^w	The delay time of passenger $i \in P$ due to the transfer at transfer location $w \in W$
d_{ij}	The distance between location $i \in R$ and location $j \in R$
t_{ij}	The travel time between location $i \in R$ and location $j \in R$
t_{iD_i}	The time that passenger $i \in P$ drives to his destination directly
t_p	The pickup time needed at passengers' initial locations
t_d	The drop-off time needed at passengers' destination locations
t_w	The drop-off time needed at the transfer stations
l_i	The number of passengers picked at location $i \in F$
l_w	The number of passengers can be dropped off at transfer location $w \in W$
Variables for MILP	
x_{ij}^v	A binary variable that equals to 1 if the shared vehicle v travels from location $i \in R$ to location $j \in R$, and equals to 0 otherwise
y_{iw}^v	A binary variable that equals to 1 if a passenger at initial location $i \in P$ is transported to the transfer station $w \in W$ by vehicle $v \in V$ to use the public transportation service, and equals to 0 otherwise
s_i^v	The time when vehicle $v \in V$ arrives at location $i \in R$
s_{Diw}	The time when a passenger at location i arrives at destination location D_i by walking from transfer station w
q_i^v	The number of passengers on vehicle $v \in V$ after serving location $i \in R$
u	An auxiliary variable that defines the difference between shared vehicle driving time and self-driving time
α_k	The weighting factor coefficient for objective $k = 1, \dots, 4$
M	A large value used for linearizing nonlinear constraints

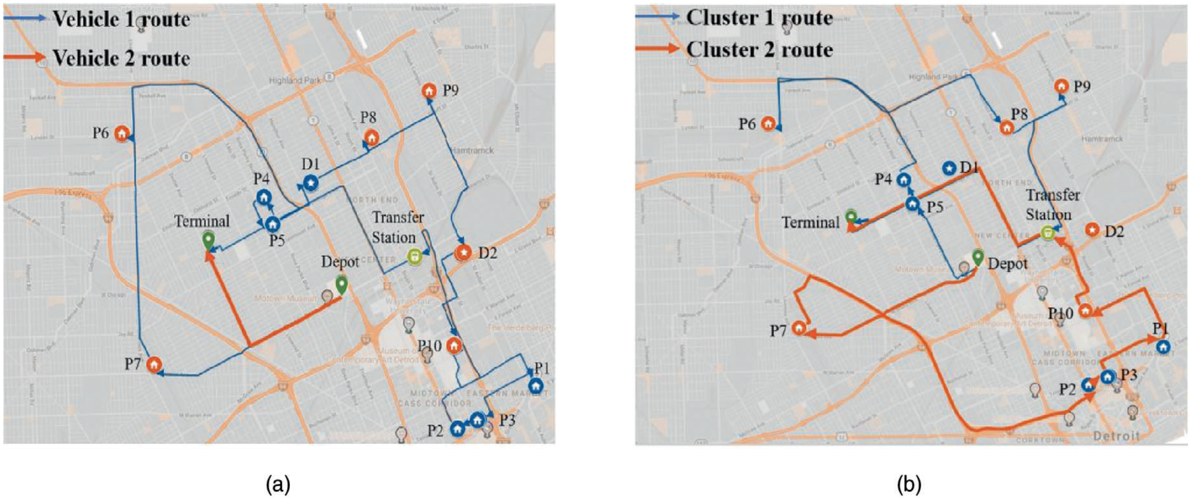

Fig. A1. Feasible MILP route and clustering route for $\alpha_1 = 0.5,\ \alpha_2 = 0,\ \alpha_3 = 0.5,$ and $\alpha_4 = 0$. (a) Feasible MILP route and (b) feasible AC algorithm route.

A2. Supplemental numerical experiments

A2.1. An illustrative example

Example 2. In Fig. A1, we present an illustrative example on the map of Detroit to visualize the described problem and solutions of two approaches (i.e., MILP and AC algorithm). In this example, we consider that there are 10 passengers, one terminal, one depot, one transfer station, and two shared vehicles with a capacity of five. There is one passenger at each location. Passengers from 1 to 5 are traveling to destination 1, and passengers from 6 to 10 are traveling to destination 2. We use the following setting for the weights of subobjectives: $\alpha_1 = \alpha_3 = 0.5$, $\alpha_2 = \alpha_4 = 0$, where the total vehicle travel distance and vehicle travel time are minimized. Figure A1a presents feasible routes of vehicles that are obtained by the exact MILP, while Fig. A1b presents feasible routes of vehicles that are obtained by the AC algorithm. As illustrated, in the MILP solution only one vehicle is used to pick up all 10 passengers. Shared vehicle 1 starts the route at the depot, picks up the passengers 7, 6, 8, and 9 in order, and drops them at their destination (namely at D_2). Then, shared vehicle 1 picks up passengers 10, 1, 2, and 3 in order, and drops passenger 10 at the transfer station. The vehicle, then, travels to destination 1 to drop the remaining passengers. Finally, it picks up passengers 4 and 5 and completes its route after dropping them at destination 1. The second shared vehicle travels from the depot to the terminal directly indicating that this vehicle is not used to pick up any passenger. On the other hand, in the solution of the AC algorithm, both vehicles are used and passengers 4, 5, 6, 8, 9 are picked up by vehicle 1, and passengers 1, 2, 3, 7, 10 are picked up by vehicle 2. All passengers are dropped at the transfer station to travel their final destinations. We further compare the objective functions and run times of both MILP and AC algorithm for the above setting. MILP finds the described route within one hour, while the AC algorithm finds the described route in five minutes. Further, the objective function value obtained by the AC algorithm is better than the objective function

Table A2
Comparison of algorithms for setting: $N = 5,\ C = 5,\ \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.25$

	MILP			AC algorithm		
Instances	Obj.	Gap%	Run time (seconds)	Obj.	%Gap with MILP	Run time (seconds)
1	0.1617	0.00	180	0.1617	0.00	180
2	0.1763	0.00	258	0.1763	0.00	258
3	0.1273	0.00	345	0.1273	0.00	345
4	0.0961	0.00	255	0.0961	0.00	255
5	0.1292	0.00	169	0.1292	0.00	169
6	0.1170	0.00	108	0.1170	0.00	108

value of the MILP, where the percentage difference is -1.6% (i.e., $\frac{\text{AC algorithm solution} - \text{MILP solution}}{\text{MILP solution}} = \% - 1.6$). We note that the above example describes just one of the settings and the solution for this setting. In the next section, we compare the MILP with the AC algorithm for all generated instances.

A2.2. Comparison of algorithms for small-sized instances

In this section, we generate small cases where we can actually solve the problem optimally. We consider that there are five passengers to pick up and one vehicle with a capacity of five. We further consider that the subobjectives are equally weighted (i.e., $\alpha_1 = 0.25, \alpha_2 = 0.25, \alpha_3 = 0.25, \alpha_4 = 0.25$). We present the comparison of MILP and AC algorithm for the described setting in Table A2.

Similar to the previous tables, in Table A2, we present the numerical results of the exact MILP and AC algorithm. For the MILP result, we present the objective value of the MILP (i.e., Obj.), the optimality gap obtained at the end of the running time by the CPLEX solver (i.e., Gap%), and the computation time in seconds (i.e., Run time (seconds)). Similarly, for the proposed algorithm, we present the corresponding objective value (i.e., Obj.), the percentage difference with the MILP solution at the end of the running time (i.e., %Gap with MILP), and the computation time in seconds (i.e., Run time (seconds)). As illustrated in Table A2, the "Gap%" column for the MILP and the "%Gap with MILP" column for the heuristics is 0 for all instances. This indicates that the obtained solutions are optimal for all instances. Moreover, since the number of passengers is only five, we do not create any subclusters for the AC algorithm. Hence, the MILP and AC algorithm solve the same problem and their run times are same.

A2.3. Comparison of algorithms for the remaining settings

In this section, we present the table results for the remaining instances that are presented in the main paper for MILP and AC algorithm.

Table A3
Comparison of algorithms for setting: $N = 20,\ C = 5,\ \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.25$

	MILP			Clustering		
Ins.	Obj.	Gap%	Run time (seconds)	Obj.	%Gap with MILP	Run time (seconds)
1	–	–	3600	0.3664	–	300
2	–	–	3600	0.3160	–	300
3	–	–	3600	0.3485	–	300
4	–	–	3600	0.3223	–	300
5	–	–	3600	0.3293	–	300
6	–	–	3600	0.2993	–	300

Table A4
Comparison of algorithms for setting: $N = 30,\ C = 5,\ \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.25$

	MILP			AC algorithm		
Ins.	Obj.	Gap%	Run time (seconds)	Obj.	%Gap with MILP	Run time (seconds)
1	–	–	3600	0.3963	–	300
2	–	–	3600	0.3035	–	300
3	–	–	3600	0.2995	–	300
4	–	–	3600	0.3220	–	300
5	–	–	3600	0.3005	–	300
6	–	–	3600	0.2918	–	300

Table A5
Comparison of algorithms for setting: $N = 50,\ C = 5,\ \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.25$

	MILP			AC algorithm		
Ins.	Obj.	Gap%	Run time (seconds)	Obj.	%Gap with MILP	Run time (seconds)
1	–	–	3600	0.3010	–	300
2	–	–	3600	0.2933	–	300
3	–	–	3600	0.2774	–	300
4	–	–	3600	0.2978	–	300
5	–	–	3600	0.3466	–	300
6	–	–	3600	0.1790	–	300

Table A6
Comparison of algorithms for setting: $N = 30,\ C = 10,\ \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.25$

	MILP			AC algorithm		
Ins.	Obj.	Gap%	Run time (seconds)	Obj.	%Gap with MILP	Run time (seconds)
1	–	–	3600	0.4534	–	300
2	–	–	3600	0.5855	–	300
3	–	–	3600	0.3486	–	300
4	–	–	3600	0.2123	–	300
5	–	–	3600	0.1879	–	300
6	–	–	3600	0.2151	–	300

Table A7
Comparison of algorithms for setting: $N = 50,\ C = 10,\ \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.25$

	MILP			AC algorithm		
Ins.	Obj.	Gap%	Run time (seconds)	Obj.	%Gap with MILP	Run time (seconds)
1	–	–	3600	0.1689	–	300
2	–	–	3600	0.1835	–	300
3	–	–	3600	0.1814	–	300
4	–	–	3600	0.1866	–	300
5	–	–	3600	0.2098	–	300
6	–	–	3600	0.1782	–	300

Table A8
Comparison of algorithms for setting: $N = 10,\ C = 5,\ \alpha_1 = 0.5,\ \alpha_2 = 0,\ \alpha_3 = 0.5,\ \alpha_4 = 0$

	MILP			AC algorithm		
Ins.	Obj.	Gap%	Run time (seconds)	Obj.	%Gap with MILP	Run time (seconds)
1	0.5396	48.50	3600	0.5396	0.00	300
2	0.7671	51.42	3600	0.7690	0.26	300
3	0.5258	51.51	3600	0.5306	0.91	300
4	0.5244	43.81	3600	0.5164	–1.52	300
5	0.6144	46.47	3600	0.6258	1.86	300
6	0.6805	48.23	3600	0.6641	–2.41	300

Table A9
Comparison of algorithms for setting: $N = 15,\ C = 5,\ \alpha_1 = 0.5,\ \alpha_2 = 0,\ \alpha_3 = 0.5,\ \alpha_4 = 0$

	MILP			AC algorithm		
Ins.	Obj.	Gap%	Run time (seconds)	Obj.	%Gap with MILP	Run time (seconds)
1	0.5947	49.05	3600	0.6306	6.04	300
2	0.5349	49.39	3600	0.5585	4.42	300
3	0.5832	54.40	3600	0.5679	–2.63	300
4	0.6953	46.23	3600	0.6723	–3.31	300
5	0.5286	48.23	3600	0.5236	–0.94	300
6	0.7496	49.05	3600	0.6897	–8.00	300

Table A10
Comparison of algorithms for setting: $N = 20,\ C = 5,\ \alpha_1 = 0.5,\ \alpha_2 = 0,\ \alpha_3 = 0.5,\ \alpha_4 = 0$

	MILP			AC algorithm		
Ins.	Obj.	Gap%	Run time (seconds)	Obj.	%Gap with MILP	Run time (seconds)
1	–	–	3600	0.6281	–	300
2	–	–	3600	0.5462	–	300
3	–	–	3600	0.5874	–	300
4	–	–	3600	0.5091	–	300
5	–	–	3600	0.5527	–	300
6	–	–	3600	0.5191	–	300

Table A11
Comparison of algorithms for setting: $N = 30,\ C = 5,\ \alpha_1 = 0.5,\ \alpha_2 = 0,\ \alpha_3 = 0.5,\ \alpha_4 = 0$

	MILP			AC algorithm		
Ins.	Obj.	Gap%	Run time (seconds)	Obj.	%Gap with MILP	Run time (seconds)
1	–	–	3600	0.5220	–	300
2	–	–	3600	0.4972	–	300
3	–	–	3600	0.5116	–	300
4	–	–	3600	0.5589	–	300
5	–	–	3600	0.4726	–	300
6	–	–	3600	0.4671	–	300