IET Smart Grid

Research Article

Fully distributed AC power flow (ACPF) algorithm for distribution systems

eISSN 2515-2947
Received on 7th April 2018
Revised 25th October 2018
Accepted on 21st November 2018
E-First on 23rd January 2019
doi: 10.1049/iet-stg.2018.0060
www.ietdl.org

Hajir Pourbabak[1], Adetokunbo Ajao[1], Tao Chen[1], Wencong Su[1,]* ✉

[1]Department of Electrical and Computer Engineering, College of Engineering & Computer Science, University of Michigan, Dearborn, USA

✉ *E-mail: wencong.su@gmail.com*

Abstract: Power flow is one of the basic tools for system operation and control. Due to its nature, which determines the complex nodal voltages, line flows, currents and losses, it enforces a large computation load on a power system. A distributed/decentralised algorithm unburdens the central controller and shares the total computation load with all agents. Therefore, such algorithms are an effective method for dealing with power flow complexity. In this study, a distributed method based on a linearised AC power system is proposed. First, the linearisation procedure of a comprehensive non-linear AC power flow (ACPF) is detailed. Second, a distributed method is presented based upon the linear ACPF equations. Three case studies are presented to evaluate the overall performance of the proposed method. In the first case study, the accuracy level of both linearised ACPF and distributed ACPF is assessed. In the second case study, the dynamic performance of distributed ACPF is investigated based on the load sudden changes. In the third case study, the scalability of the proposed distributed ACPF is evaluated by applying it to a larger power system.

1 Introduction

The power flow problem (known as load flow) is an important tool for power system monitoring, control and decision making. As a result, researchers are currently working to find an effective method for solving the power flow problem from the emergence of power systems. Naturally, the power flow imposes a heavy computation load on the power system because it determines the complex nodal voltages from which line flows, currents and losses can be derived [1]. Typical power flow solutions were first introduced about 50 years ago. Tinney and Hart [2] present Newton's method for solving the power flow problem as one of the earlier methods. Furthermore, the authors in [3, 4] suggest several different methods for solving the AC power flow (ACPF) problem. Klump and Overbye propose an approach in [5] for solving the low-voltage power problem, showing the effectiveness of their method in the speed and frequency of convergence. Matos [6] introduces an iterative process for the power flow of radial grids based on the exact power flow solution. Recently, Ghadimi [7] presents two different methods for solving the power flow based on the proportional sharing assumption and circuit laws to find the relationship between power inflows and outflows through all elements. In addition, many attempts have been made to linearise the power flow problem. Linearisation helps us to achieve a reduction in the overall computation needed for comprehensive ACPF; however, the accuracy level will be reduced as well [8–10].

In recent years, the penetration of distributed generators (DGs), including renewable energy resources and other local fossil fuel generators is increasing, exerting both the positive and negative impacts on power systems. These local generators provide energy resiliency, improve environmental benefit (carbon emissions reduction) and enhance the power quality/reliability of power systems. However, the presence of local generators and the market reconstruction result in significant challenges in power system operation, control, and protection [11–13]. Protection of the system would be very challenging because the power flow direction of the distribution grid would be changed from single to bidirectional with the integration of distributed generation. Thus, the conventional protection system cannot protect this modified power system as it would protect a traditional power system [14]. Additionally, comprehensive communication networks are required to exchange data between the central controller and the local agents. As the number of agents increases to the hundreds of thousands, the control system is confronted with some technical barriers, such as the computational complexity and a single point of failure [15, 16].

In the traditional power system, various agents (e.g. DGs and loads) are controlled by a centre. A wide range of signals and information is gathered through supervisory control and data acquisition (SCADA) from all over the system and is sent to a centre. SCADA is an advanced automation control system that centrally manages the power system by gathering data and monitoring the system's operation [17]. Then the central controller carries out a power flow or optimal power flow computations and sends control and operational commands back to the operators spread over a wide geographical area. A two-way complex communication channel for each agent is required to support all data transmission between the centre and agents [18, 19]. Consequently, this system suffers from a technical communication barrier and cannot be effective for future smart grids due to the variation in topology of both the communication and electrical network of a smart grid. Moreover, the legacy centralised approach would not be able to cope with the huge amount of data. In other words, this kind of approach is suitable for relatively small-scale systems without reconstructing the existing communication and control networks. Thus, a centralised method cannot carry out operational and control responsibilities for a large number of agents because of the high penetration of DGs, load volatility, market deregulation etc.

Power flow computation is negatively affected in systems that are integrated with numerous DGs and loads, suffering from computational complexity. Furthermore, a single point of failure is always an imminent threat to a system with a centralised controller. That is, if the central controller fails to connect to the system, the entire system will experience failure. These kinds of challenges can be completely addressed by introducing a fully distributed control approach to future power systems [20]. Therefore, an immediate and effective replacement for the centralised control approaches, which addresses the challenges raised by the launching of smart grids, is a distributed approach [21]. In this type of method, each agent makes its own operation decisions based on information exchanged with its neighbours and/or local measurements. In distributed methods, it is not required to share all information

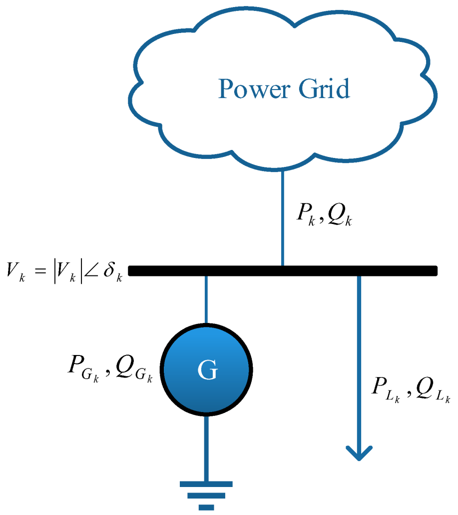

Fig. 1 *Bus configuration*

globally or send it to a central control; thus, computational load will be distributed among all agents.

Recently, many researchers have given their attention to the distributed methods for controlling and operating power systems. Power flow, as an important numerical analysis, could considerably benefit from the use of a distributed algorithm, playing a constructive role in determining the best operation of existing systems. A distributed power flow can provide a reliable and fast control system for power systems. A distributed power flow can be used for power system restoration, distribution system management, load shedding, microgrid control etc. [22]. In recent years, many attempts have been made to develop a distributed method for power system operation, such as distributed power flow, distributed economic dispatch and distributed optimal power flow. A distributed multi-phase power flow for the distribution grids is introduced in [23]. This method divides and separates the distribution network using several partitions based on the control capabilities of each area. A power flow of the entire grid is carried out by iteratively running centralised local power flows on each partition. This method uses distributed intelligence to share information. Nguyen and Flueck applied an agent-based distributed power flow to the unbalanced radial distribution systems considering the network's complete models. The power flow problem is solved by an iterative backward/forward sweep technique added to the distributed smart agents [24]. Warnier *et al.* in their recent paper [25] presented a new distributed computation method used for real-time monitoring to avoid cascading failures in a power system. A reduced decentralised calculation method, based on an iterative procedure, has been presented in [26] to solve the power flow problem. It helps operators have an appropriate estimation of the state of the neighbouring system. Dagdougui and Sacile proposed a decentralised control strategy of power flow to minimise the cost of energy storage and power exchanged between smart microgrids, as well as enable the system for demand support [27]. In [28], Nakayama *et al.* present a distributed approach to minimise power flow loss function of the transmission and distribution lines. Their algorithm is designed based on updating the loop variables. Erseghe introduced a fully distributed algorithm for optimal power flow based on the alternating direction multiplier method (ADMM), which does not need a central controller. ADMM helps author reach a scalable and distributed algorithm [29]. In [30], the authors presented a distributed version of DC optimal power flow (DC-OPF) for radial distribution systems based on a partial primal-dual algorithm.

In this paper, we propose a distributed ACPF based on a linearised ACPF. Contributions of this work can be considered as follows:

- AC linearised power flow benefits from a high accuracy in comparison with that of typical DC power flow (DCPF). Thus, the driven distributed ACPF obtains power flow results as accurately as ACPF. In the first case study, the simulation results show the accuracy of the distributed method. The results are compared with those of a centralised method as a benchmark.
- The proposed distributed ACPF can be applied to the distribution system because our approach does not assume the small ratio of R/X for the sake of generality.
- The proposed distributed ACPF can cope with load profile changes. The second case study shows that the distributed ACPF is readily able to follow the load changes.
- The proposed distributed ACPF is reliable to scale up. The third case study, including a 37-bus IEEE test system and a 2000-bus Texas synthetic system, confirms that the distributed method is scalable and can easily deal with a large number of agents.
- In addition, the distributed approach improves privacy because agent information is not being shared with a central controller or the whole system. Each agent is informed by its neighbours' data. Moreover, the single point of failure, which is common in centralised methods, is removed due to the nature of distributed methods.

This paper is organised as follows. Section 2 defines the power flow problem and all necessary equations. Section 3 provides the linearisation procedure of the power flow problem, including all assumptions, approximations and steps. Section 4 proposes a distributed power flow driven by the linearised power flow. Section 5 provides three case studies to demonstrate the accuracy, dynamic performance and scalability of the proposed distributed ACPF. Finally, Section 6 reviews the research opportunities and possible applications.

2 General formulation of power flow

The values of the voltage magnitude and phase angle at each bus of a power system are calculated using the power flow under steady-state conditions. In addition, the active (P) and reactive (Q) power flows of all power lines and buses are computed at the same time. Fig. 1 shows the delivered net active and reactive power to a bus, as well as its generated powers, consumed powers and other parameters [8]. At each bus, four variables, including P_k (net injected active power), Q_k (net injected reactive power), V_k (bus voltage) and δ_k (bus phase angle) are involved in the power flow computation. Two of them are considered as input information for power flow in order to calculate the other two. V_k and δ_k are known for slack bus and are 1 pu and 0°, respectively. Thus, P_k and Q_k should be determined. In PV buses, the power flow problem determines Q_k and δ_k based on known P_k and V_k. In addition, P_k and Q_k of PQ buses are known; thus, the V_k and δ_k of all PQ buses are calculated during the power flow computation.

Equation (1) prepares the nodal equations for a power system network, where $\boldsymbol{Y}_{\text{Bus}}$ is the power grid admittance matrix and $\mathscr{I}$ and $\mathscr{V}$ are $N \times 1$ vectors of buses' current and voltage. The net complex power injected into bus k is shown by (2), where I_k^* indicates the conjugate of the vector of the injected current at the kth bus

$$\mathscr{I} = \boldsymbol{Y}_{\text{Bus}} \mathscr{V} \tag{1}$$

$$P_k + jQ_k = V_k I_k^* \tag{2}$$

$$\begin{aligned} P_k &= P_{G_k} - P_{L_k} \\ Q_k &= Q_{G_k} - Q_{L_k} \end{aligned} \tag{3}$$

Equation (4) is another representation of the nodal equations based on elements of $\boldsymbol{Y}_{\text{Bus}}$

$$I_k = \sum_{n=1}^{N} Y_{kn} V_n \tag{4}$$

Substituting (2) in (4) and by taking $V_k = |V_k|\angle\delta_k$ and $Y_{kn} = |Y_{kn}|\angle\theta_{kn}$, (5) and (6) are obtained

$$1P_k + \mathrm{j}Q_k = V_k\left[\sum_{n=1}^{N} Y_{kn}V_n\right]^*, \quad k = 1, 2, 3, \ldots \tag{5}$$

$$P_k + \mathrm{j}Q_k = |V_k|\sum_{n=1}^{N} |Y_{kn}||V_n|\mathrm{e}^{\mathrm{j}(\delta_k - \delta_n - \theta_{kn})} \tag{6}$$

In (7), active and reactive power balance can be achieved as the real and imaginary parts of (6), where G_{kn} and B_{kn} are the real and imaginary parts of $\boldsymbol{Y}_{\text{Bus}}$ matrix elements, i.e. $Y_{kn} = G_{kn} + \mathrm{j}B_{kn}$.

$$\begin{aligned} P_k &= |V_k|\sum_{n=1}^{N} |V_n|[G_{kn}\cos(\delta_k - \delta_n) + B_{kn}\sin(\delta_k - \delta_n)] \\ Q_k &= |V_k|\sum_{n=1}^{N} |V_n|[G_{kn}\sin(\delta_k - \delta_n) - B_{kn}\cos(\delta_k - \delta_n)] \end{aligned} \tag{7}$$

In addition, the power flows transmitted by a line between bus k and n are calculated by (8), where g_{kn} and b_{kn} are the conductance and susceptance of the power line, respectively, both measured in Siemens.

$$\begin{aligned} P_{kn} &= |V_k|^2 g_{kn} - |V_k||V_n|g_{kn}\cos(\delta_k - \delta_n) - |V_k||V_n|b_{kn}\sin(\delta_k - \delta_n) \\ Q_{kn} &= -|V_k|^2(b_{kn} + b_k) - |V_k||V_n|g_{kn}\sin(\delta_k - \delta_n) \\ &\quad + |V_k||V_n|b_{kn}\cos(\delta_k - \delta_n) \end{aligned} \tag{8}$$

3 Linearised AC power flow

In many linearisation approaches used for power flow, such as typical DCPF [8, 9], the line resistances and active power loss are subsequently neglected. DCPF is non-iterative and, of course, a convergent method. However, the accuracy is ignored in this method because of some underlying assumptions, such as a flat voltage profile and small differences of voltage phase angles [10]. DCPF is not appropriate for a distribution system because the assumption of $R \ll X$ is no longer valid.

In this paper, a set of assumptions and approximations are considered to have a linearised ACPF without neglecting reactive power, voltage differences, power losses and line resistances [31, 32].

As can be seen in (5) and (7) we have two different operators that make our equation non-linear. First operator is the multiplication term between voltage variable such as V_nV_k. The second operator is trigonometric, such as $\sin(\delta_k - \delta_n)$ and $\cos(\delta_k - \delta_n)$.

For most typical operating conditions, the difference angles of voltage phasors at two buses connected by a power line is 10–15°. Therefore, (7) can be written as (9)

$$\begin{aligned} P_k &= |V_k|\sum_{n=1}^{N} |V_n|[G_{kn} + B_{kn}(\delta_k - \delta_n)] \\ Q_k &= |V_k|\sum_{n=1}^{N} |V_n|[G_{kn}(\delta_k - \delta_n) - B_{kn}] \end{aligned} \tag{9}$$

After eliminating the trigonometric operations, we try to remove the multiplication operations between voltage variables in active and reactive power equations.

First, the reactive power formula mentioned in (9) can be rewritten as

$$\begin{aligned} Q_k = {} & |V_k|^2(G_{kk}(\delta_k - \delta_n) - B_{kk}) + \sum_{\substack{n=1 \\ n \neq k}}^{N} |V_k||V_n| \\ & [G_{kn}(\delta_k - \delta_n) - B_{kn}] \end{aligned} \tag{10}$$

The imaginary part of the $\boldsymbol{Y}_{\text{Bus}}$ elements can be written as: $B_{kk} = b_k + \sum_{n=1, n \neq k}^{N} b_{kn}$ and $B_{kn} = -b_{kn}$, where b_k is the shunt capacitors/reactors at bus k. In addition, we have $G_{kn} = -g_{kn}$.

Finally reactive power can be presented by

$$\begin{aligned} Q_k &= -|V_k|^2 b_k - \sum_{\substack{n=1 \\ n \neq k}}^{N} \left(|V_k|^2 b_{kn} + |V_k||V_n|[g_{kn}(\delta_k - \delta_n) - b_{kn}]\right) \\ &= -|V_k|^2 b_k - \sum_{\substack{n=1 \\ n \neq k}}^{N} \left(|V_k|^2 b_{kn} + |V_k||V_n|g_{kn}(\delta_k - \delta_n) - |V_k||V_n|b_{kn}\right) \\ &= -|V_k|^2 b_k - \sum_{\substack{n=1 \\ n \neq k}}^{N} (|V_k|b_{kn}(|V_k| - |V_n|) + |V_k||V_n|g_{kn}(\delta_k - \delta_n)) \end{aligned} \tag{11}$$

In a per-unit system, the numerical values of voltage magnitudes $|V_n|$ and $|V_k|$ are very close to 1.0. In fact, the voltage magnitudes are usually between 0.95 and 1.05. In addition, we can consider a reasonable approximation for all product terms ($|V_k||V_n|$); thus, this term is almost 1.0 pu. It is worth mentioning that this is only an approximation and it is not an assumption. Therefore, we do not consider a flat voltage profile, and voltage magnitude is not assumed to equal 1.0 pu.

However, the difference between voltage variables ($|V_k| - |V_n|$) cannot be neglected. Based on this assumption, (10) can be written as

$$Q_k = -b_k - \sum_{\substack{n=1 \\ n \neq k}}^{N} (b_{kn}(|V_k| - |V_n|) + g_{kn}(\delta_k - \delta_n)) \tag{12}$$

Now, the active power equation can be converted to a linear one based on the same approximation. The active power equation in (9) can be written as (13). Equations (14) and (15) can be obtained by substituting the equivalent of G_{kn} and B_{kn}

$$\begin{aligned} P_k = {} & |V_k|^2(G_{kk} + B_{kk}(\delta_k - \delta_k)) + \sum_{\substack{n=1 \\ n \neq k}}^{N} |V_k||V_n| \\ & [G_{kn} + B_{kn}(\delta_k - \delta_n)] \end{aligned} \tag{13}$$

$$P_k = |V_k|^2\sum_{\substack{n=1 \\ n \neq k}}^{N} g_{kn} + \sum_{\substack{n=1 \\ n \neq k}}^{N} |V_k||V_n|[-g_{kn} - b_{kn}(\delta_k - \delta_n)] \tag{14}$$

$$P_k = \sum_{\substack{n=1 \\ n \neq k}}^{N} |V_k|g_{kn}(|V_k| - |V_n|) - |V_k||V_n|b_{kn}(\delta_k - \delta_n) \tag{15}$$

Similar to reactive power, we can simplify (15) by considering $|V_k|$ and its product terms, i.e. $|V_k||V_n|$ as 1.0 pu. Then we have

$$P_k = \sum_{\substack{n=1 \\ n \neq k}}^{N} g_{kn}(|V_k| - |V_n|) - b_{kn}(\delta_k - \delta_n) \tag{16}$$

If we put both the reactive and active equations together:

$$P_k = \sum_{\substack{n=1 \\ n \neq k}}^{N} [g_{kn}(|V_k| - |V_n|) - b_{kn}(\delta_k - \delta_n)] \tag{17}$$

$$Q_k = -b_k - \sum_{\substack{n=1 \\ n \neq k}}^{N} [b_{kn}(|V_k| - |V_n|) + g_{kn}(\delta_k - \delta_n)] \tag{18}$$

δ_k and V_k can be obtained by (17) and (18), respectively.

$$\left(\sum_{\substack{n=1 \\ n \neq k}}^{N} g_{kn}\right)|V_k| - \left(\sum_{\substack{n=1 \\ n \neq k}}^{N} b_{kn}\right)\delta_k = P_k + \sum_{\substack{n=1 \\ n \neq k}}^{N} g_{kn}|V_n| - \sum_{\substack{n=1 \\ n \neq k}}^{N} b_{kn}\delta_n \tag{19}$$

$$\left(\sum_{\substack{n=1 \\ n \neq k}}^{N} b_{kn}\right)|V_k| + \left(\sum_{\substack{n=1 \\ n \neq k}}^{N} g_{kn}\right)\delta_k = -Q_k - b_k + \sum_{\substack{n=1 \\ n \neq k}}^{N} b_{kn}|V_n| + \sum_{\substack{n=1 \\ n \neq k}}^{N} g_{kn}\delta_n \tag{20}$$

4 Distributed AC power flow

Graph theory provides us with the capability to model the various agents' relationships in the grid through undirected/directed graphs denoted by $G(\bar{\vartheta}, \psi)$. The agents are introduced by $\bar{\vartheta} = \{\vartheta_1, \vartheta_2, \ldots, \vartheta_n\}$ and their interactions are designated by a set of edges, i.e. $\psi \subseteq \vartheta \times \vartheta$. The directed edges $\vec{\omega}_{ij} = (\vartheta_i, \vartheta_j)$ and $\omega_{ij} = (\vartheta_i, \vartheta_j)$ show one-way and two-way information transmission between the two separate agents (agent i and agent j). The adjacency matrix is used to represent the communication topology of a grid. In this paper, a two-way communication channel is considered to model interaction between the two adjacent buses. Thus, the adjacency matrix, denoted by $\boldsymbol{A} = \{[a_{ij}] \big| a_{ij} \in \mathscr{R}^{\mathscr{P} \times \mathscr{P}}\}$, of an undirected graph G is symmetric. The entry a_{ij} of an adjacency matrix is a positive value if $\omega_{ij} \in \psi$ and $a_{ij} = 0$ for $\omega_{ij} \notin \psi$. Otherwise, the entry a_{ii} is assumed to be zero. The second matrix is Laplacian matrix $\boldsymbol{L} = \{[l_{ij}] \big| l_{ij} \in \mathscr{R}^{\mathscr{P} \times \mathscr{P}}\}$ in which entry $l_{ii} = \sum_j a_{ij}$ and $l_{ij} = -a_{ij}$ for $i \neq j$.

A distributed approach is applied to the power system with the help of the linearised power flow formulas. This distributed approach is based on concepts presented by [33, 34]. Some pieces of information are shared among agents in the power system network to have them reach a consensus, i.e. power balance. The admittance matrix ($\boldsymbol{Y}_{\text{Bus}}$) of a power system surprisingly corresponds to this Laplacian matrix. Thus, the elements of this matrix can be used as the elements of the Laplacian matrix to model the data transmission network.

Equation (21) is used as a distributed protocol for this paper. x_i shows the state of the ith agent in a network. Agent i can receive agent j's information or share its own information with agent j, as it has a communication channel with it, i.e. $a_{ij} \neq 0$

$$x_i(k+1) = \sum_j a_{ij} x_j(k) \tag{21}$$

All a_{ij} are defined based on the real and imaginary parts of elements of $\boldsymbol{Y}_{\text{Bus}}$ to exchange voltage magnitudes and phase angles among agents. It is important to mention that each agent shares information only with its neighbours. In other words, the communication network is defined based on the power system topology because the Laplacian matrix corresponds to the admittance matrix. Let us rewrite (19) and (20) as

$$G_k|V_k| - B_k\delta_k = P_k + V'_{G_k} - \delta'_{G_k} \tag{22}$$

$$B_k|V_k| + G_k\delta_k = -Q_k - b_k + V'_{B_k} + \delta'_{B_k} \tag{23}$$

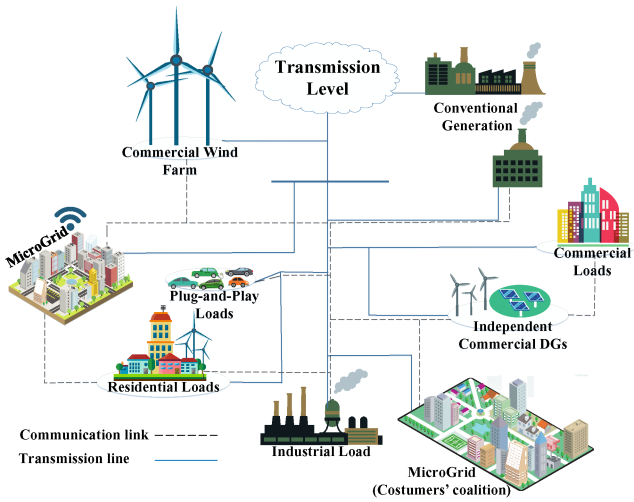

Fig. 2 *Demonstration of the communication network for the applied distributed method*

where G and B are constant values for the kth bus and are obtained by

$$G_k = \sum_{\substack{n=1 \\ n \neq k}}^{N} g_{kn}, \quad B_k = \sum_{\substack{n=1 \\ n \neq k}}^{N} b_{kn} \tag{24}$$

V'_{G_k} and V'_{B_k} are the so-called shadow voltages. They are the shadows, of adjacent buses' voltages, cast on the kth bus. V'_{G_k} and V'_{B_k} are cast by the conductance and susceptance of the power line, respectively, connected to the kth bus

$$V'_{G_k} = \sum_{\substack{n=1 \\ n \neq k}}^{N} g_{kn}|V_n|, \quad V'_{B_k} = \sum_{\substack{n=1 \\ n \neq k}}^{N} b_{kn}|V_n| \tag{25}$$

There are the same definitions for δ'_{G_k} and δ'_{B_k}, both of which are shadow phase angles of adjacent buses' phase angles on the kth bus.

Equation (26) shows the related formulae:

$$\delta'_{G_k} = \sum_{\substack{n=1 \\ n \neq k}}^{N} b_{kn}\delta_n, \quad \delta'_{B_k} = \sum_{\substack{n=1 \\ n \neq k}}^{N} g_{kn}\delta_n \tag{26}$$

Equations (22)–(26) are based on the distributed protocol previously presented by (21). They guarantee that each agent only needs the voltage and phase angle values of its neighbours. Fig. 2 clearly shows data sharing based on the provided protocol.

In addition, the closed-form of $|V_k|$ and δ_k for the kth bus can be shown by (27) and (28), where $C = P_k + V'_{G_k} - \delta'_{G_k}$ and $D = -Q_k - b_k + V'_{B_k} + \delta'_{B_k}$.

$$|V_k| = \frac{CG_k + DB_k}{G_k^2 + B_k^2} \tag{27}$$

$$\delta_k = \frac{DG_k - CB_k}{G_k^2 + B_k^2} \tag{28}$$

In summary, G_k, B_k, P_k, Q_k are known, constant and private information for each bus and not necessary to be shared. It is worth mentioning that G_k and B_k are from lines connected to the kth bus. The agents do not need to access the information of the entire system configuration. The only information that needs to be exchanged between the buses are V and δ of adjacent buses. In other words, if an agent connects through one line to the

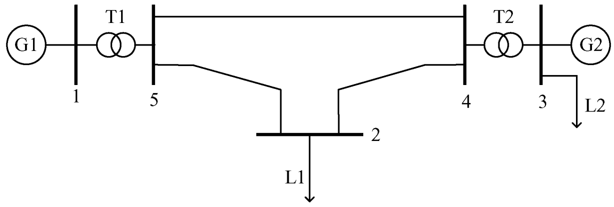

Fig. 3 *Five-bus power system*

Table 1 Injected active and reactive power of each bus

Bus	P_G, MW	P_L, MW	Q_G, MVar	Q_L, MVar
1	slack	0	slack	0
2	0	800	0	280
3	520	80	PV	40
4	0	0	0	0
5	0	0	0	0

Table 2 Comparison of ACPF, L-ACPF, D-ACPF

Bus	i	1	2	3	4	5
ACPF	V, pu	1.000	0.834	1.050	1.019	0.974
	δ, deg	0.00	−22.41	−0.60	−2.83	−4.55
L-ACPF	V, pu	1.00	0.815	1.050	1.004	0.963
	δ, deg	0.00	−19.0686	0.2978	−2.1682	−4.2144
DCPF	V, pu	1.00	1.00	1.00	1.00	1.00
	δ, deg	0.00	−18.6948	0.5238	−1.9972	−4.1253
D-ACPF	V, pu	1.000	0.816	1.050	1.004	0.963
	δ, deg	0.00	−19.0812	0.3384	−2.1654	−4.2340

distribution system it will access information through a point of common coupling.

5 Performance evaluation

In this section, three case studies are presented to assess the performance of the proposed distributed ACPF. All software simulations are conducted in the MATLAB 2017a environment on an ordinary desktop PC with an Intel Core(TM)i7 CPU @ 2.13 GHz, 8-GB RAM memory.

In the first case study, we provide a numerical example to evaluate the algorithm performance from accuracy prospective in a relatively small-scale system (five buses). The numerical results are compared with typical ACPF, linearised ACPF and DCPF as the benchmark results.

In the second case study, we tested the changes in load level to see how distributed ACPF copes with real-time changes in load.

In the third case study, a larger system (Texas 2000-buses) is considered to demonstrate the scalability of proposed the ACPF.

5.1 Accuracy analysis

In this section, a numerical proof for the accuracy analysis of both the proposed linearised power flow and distributed protocol are provided. A sample power system is selected from the power flow example of the PowerWorld software [8]. Fig. 3 shows a five-buses system, including two generators, transformers and loads. The elements of $\boldsymbol{Y}_{\text{Bus}}$ are: $Y(1,1) = 13.73 - \text{j}49.72$, $Y(1,5) = -3.73 + \text{j}49.72$, $Y(2,2) = 2.68 - \text{j}28.46$, $Y(2,4) = -0.89 + \text{j}9.92$, $Y(2,5) = -1.79 + \text{j}19.84$, $Y(3,3) = 7.46 - \text{j}99.44$, $Y(3,4) = -7.46 + \text{j}99.44$, $Y(4,4) = 11.92 - \text{j}147.96$, $Y(4,5) = -3.57 + \text{j}39.68$, $Y(5,5) = 9.09 - \text{j}108.58$.

In addition, Table 1 provides injected active and reactive powers at each bus of five-buses system. Buses 1 and 3 are the slack and PV buses of the system, respectively. The desired voltage for the slack bus and PV bus are 1 pu and 1.05 pu, respectively.

Table 2 shows a comprehensive comparison between ACPF, linearised ACPF (L-ACPF), DCPF and the proposed distributed ACPF (D-ACPF). As can clearly be seen, the results of the proposed linearised ACPF are much more precise than those of the typical DCPF. The voltage profile of L-ACPF is no longer flat as it is in DCPF because both the active and reactive powers' equations are incorporated into the power flow formulas. However, voltages of L-ACPF are slightly different from those of conventional ACPF due to some approximations used for the linearisation of non-linear ACPF.

In addition, phase angles of L-ACPF are closer to that of ACPF. Therefore, L-ACPF can reach more accurate results with lighter mathematical calculation. Furthermore, the proposed D-ACPF has the appropriate results that are more precise than those of DCPF and are very close to those of ACPF and L-ACPF. The solution mismatch between the D-ACPF and centralised ACPF (ACPF) methods is 0.95 and 1.95% of the average for voltages and phase angles, respectively.

5.2 Dynamic performance test

As mentioned earlier, ACPF imposes a heavy computational load on the system. Continuous monitoring of a large system is one of the technical issues raised by ACPF. D-ACPF, however, shares computational load among various agents; therefore, continuous running of ACPF is theoretically possible meaning the results of D-ACPF can follow loads variations.

In the second case study, the dynamic performance of D-ACPF is studied by intentional changes in the level of demand in PQ buses. The active demand (P_L) and reactive demand (Q_L) are suddenly changed from 800 MW and 280 MVAR to 280 MW and 120 MVAR, respectively. The goal of this test is to indicate that D-ACPF easily follows the change of load and calculates ACPF based on the new demand level. As can be seen in Fig. 4, D-ACPF can carry out real-time power flow based on real demand profile. Thus, D-ACPF can be a very effective tool for system operation and control because it can easily provide operators and agents with real-time ACPF results.

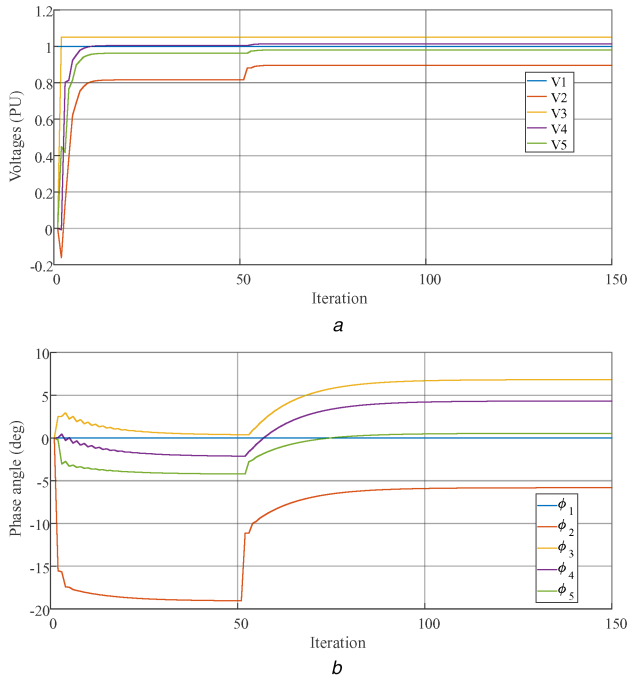

Fig. 4 *Dynamic performance test for voltage phase angle (five buses)*

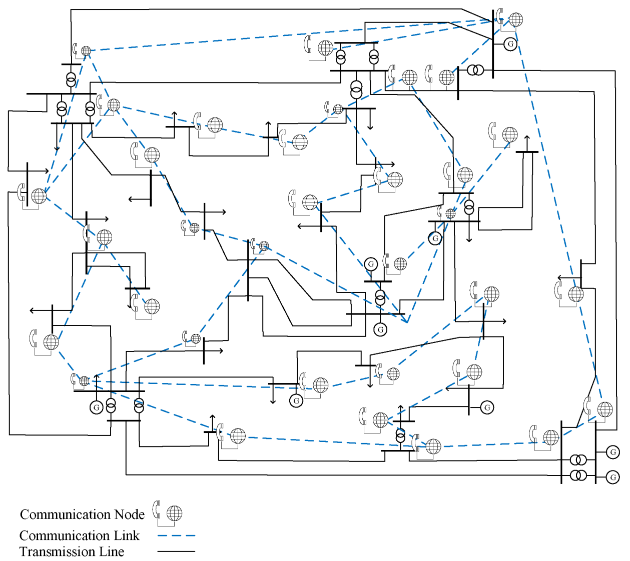

Fig. 5 *37-bus power system: the configuration of the physical and communication network*

5.3 Scalability test (37 and 2000-bus system)

Scalability testing evaluates the performance of an algorithm in the case of a higher number of agents to measure how much it can be scaled up. Sometimes, new algorithms work very well with a relatively small-scale system. However, they may not be able to cope with a larger system because of the heavier computations, accumulated error and more uncertainties.

In this paper, the investigation of scalability is carried out by a 37-bus IEEE test system (shown in Fig. 5) and a 2000-bus Texas synthetic system (shown in Fig. 6). IEEE test system includes 9 generators, 26 loads, 8 shunt capacitors, 43 power lines and 14 transformers. Detailed information can be found in Chapter 6 of [8]. For this system, we also examine accuracy. Fig. 7 shows the voltage and phase angle profile of this system. As can be seen, D-ACPF can meet the benchmark criteria based on ACPF because its calculated voltages and phase angles are very close to those of ACPF. The solution mismatch of the power flow results between the D-ACPF and centralised ACPF methods is 0.86% and 2.66% of the average for the voltage and phase angle profile, respectively. Fig. 8 shows the evolution of voltage amplitude and voltage angle

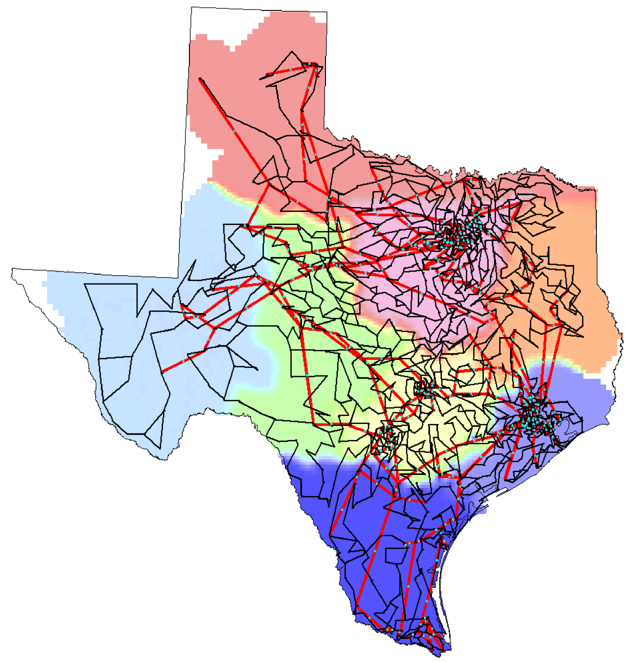

Fig. 6 *2000-bus power system: the configuration of the physical system built from public information and a statistical analysis of real power systems (University of Illinois at Urbana-Champaign, Coordinated Science Laboratory)*

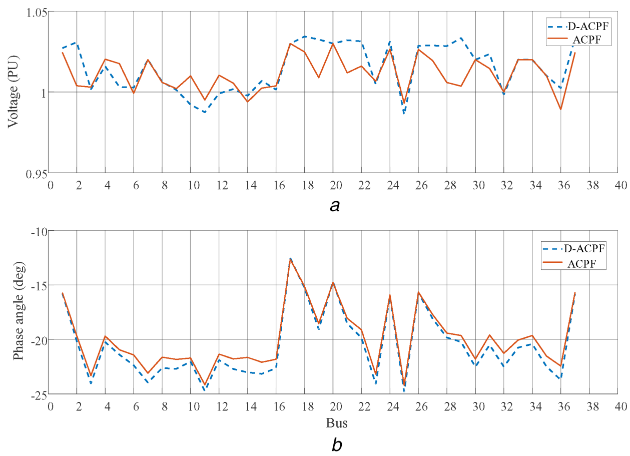

Fig. 7 *37-bus power system*

of 37-bus system which successfully converged to solution. Texas synthetic system, which is built from public information and a statistical analysis of real-power systems, includes 2007 buses, 282 generators, 1417 loads, 41 shunt capacitors, 2481 lines and 562 transformers. Fig. 9 shows the convergence of the results for this huge system. As can be seen, both voltage amplitude and voltage angle successfully converged.

The simulation results of this test and the small mismatch between results of D-ACPF and those of typical ACPF demonstrate that D-ACPF can be an effective replacement for a typical ACPF in the case of scalability.

6 Conclusions

In this paper, we proposed a distributed approach (D-ACPF) based on a linearised ACPF. The distributed method can easily be applied to a power system based on the distributed protocol and can benefit from the accuracy of L-ACPF. The proposed L-ACPF provides us with a simple power flow formulation that is much more precise than that of DCPF. The simulation results show that regular ACPF can be replaced by the proposed distributed ACPF. Three different simulation tests are provided to confirm accuracy, dynamic performance and scalability of the proposed distributed ACPF. The

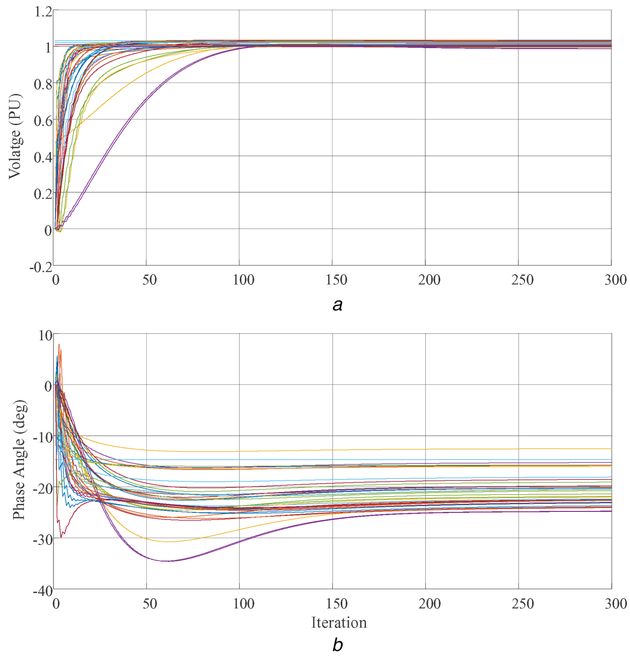

Fig. 8 *Buses voltage and phase angle convergence of 37-bus power system*

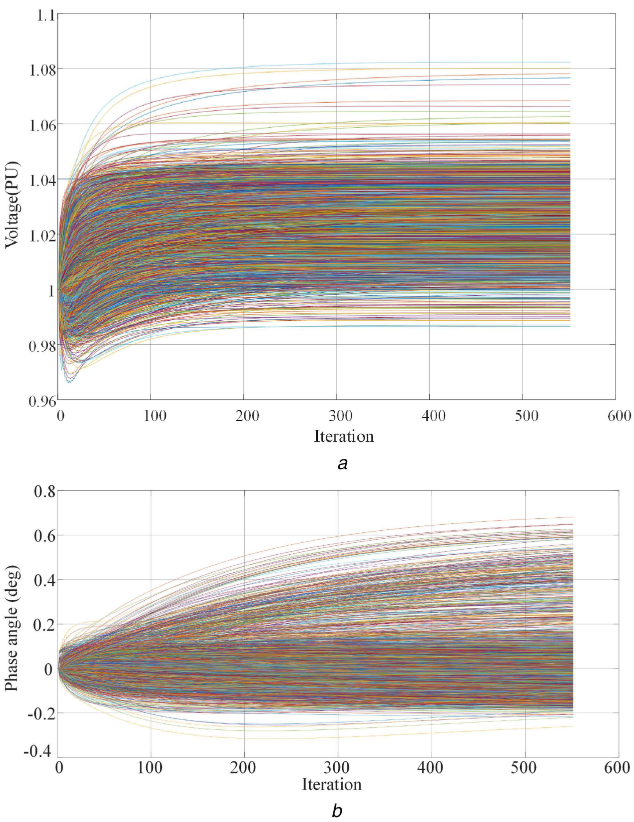

Fig. 9 *Buses voltage and phase angle convergence of 2000-bus power system*

proposed D-ACPF can be applied to other studies of power systems, as well, such as load shedding and demand response.

7 Acknowledgment

The authors would like to thank the anonymous reviewers for their valuable comments and suggestions to improve the quality of this paper.

8 References

[1] Glavitsch, H., Bacher, R.: 'Optimal power flow algorithms', *Anal. Control Syst. Tech. Electr. Power Syst.*, 1991, **41**, pp. 135–205. Available from: http://linkinghub.elsevier.com/retrieve/pii/B9780120127412500087

[2] Tinney, W., Hart, C.: 'Power flow solution by Newton's method', *IEEE Trans. Power Appar. Syst.*, 1967, **PAS-86**, (11), pp. 1449–1460. Available from: http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=4073219

[3] da Costa, V.M., Martins, N., Pereira, J.L.R.: 'Developments in the Newton Raphson power flow formulation based on current injections', *IEEE Trans. Power Syst.*, 1999, **14**, (4), pp. 1320–1326. Available from: http://ieeexplore.ieee.org/document/801891/

[4] Ward, J.B., Hale, H.W.: 'Digital computer solution of power-flow problems [includes discussion]', *Trans. Am. Inst. Electr. Eng. Part III: Power Appar. Syst.*, 1956, **75**, (3), pp. 398–404. Available from: http://ieeexplore.ieee.org/document/4499318/

[5] Klump, R.P., Overbye, T.J.: 'A new method for finding low-voltage power flow solutions'. In 2000 Power Engineering Society Summer Meeting (Cat. No.00CH37134), Seattle, USA, 2000, vol. 1, pp. 593–597. Available from: http://ieeexplore.ieee.org/document/867653/

[6] Matos, M.A.: 'A new power flow method for radial networks'. In 2003 IEEE Bologna Power Tech Conf. Proc., Bologna, Italy, 2003, vol. **2**, pp. 359–363. Available from: http://ieeexplore.ieee.org/document/1304335/

[7] Ghadimi, N.: 'Two new methods for power flow tracing using bus power balance equations', *J. Central South Univ.*, 2014, **21**, (7), pp. 2712–2718. Available from: http://link.springer.com/10.1007/s11771-014-2233-8

[8] Glover, J.D., Overbye, T.J., Sarma, M.S.: '*Power system analysis & design*' (Boston, MA, 2017, 6th edn.)

[9] Wood, A.J., Wollenberg, B.F., Sheble, G.B.: '*Power generation, operation, and control*' (John Wiley & Sons, Inc., Hoboken, 2014). Available from: https://www.worldcat.org/title/power-generation-operation-and-control/oclc/886509477{&}referer=brief{_}results

[10] Stott, B., Jardim, J., Alsac, O.: 'DC power flow revisited', *IEEE Trans. Power Syst.*, 2009, **24**, (3), pp. 1290–1300

[11] Hatziargyriou, N., Asano, H., Iravani, R., *et al.*: 'Microgrids: an overview of ongoing research, development, and demonstration projects', *IEEE Power Energy Mag.*, 2007, **5**, (July 2007), pp. 78–94. Available from: https://building-microgrid.lbl.gov/sites/all/files/journal-lbnl-62937

[12] Hamidi, R.J., Livani, H., Hosseinian, S.H., *et al.*: 'Distributed cooperative control system for smart microgrids', *Electr. Power Syst. Res.*, 2016, **130**, pp. 241–250. Available from: http://dx.doi.org/10.1016/j.epsr.2015.09.012

[13] Kazemi, A., Pourbabak, H.: 'Islanding detection method based on a new approach to voltage phase angle of constant power inverters', *IET Gener. Transm. Distrib.*, 2016, **10**, (5), pp. 1190–1198. Available from: http://digital-library.theiet.org/content/journals/10.1049/iet-gtd.2015.0776

[14] Zou, G., Gao, H., Tong, B., *et al.*: 'Directional pilot protection method for distribution grid with DG'. 12th IET Int. Conf. on Developments in Power System Protection (DPSP 2014), Copenhagen, Denmark, 2014, (1), pp. 1–5. Available from: http://digital-library.theiet.org/content/conferences/10.1049/cp.2014.0089

[15] Su, W., Wang, J., Ton, D.: 'Smart grid impact on operation and planning of electric energy systems', in '*Handbook of clean energy systems*' (John Wiley & Sons, Ltd, Chichester, UK, 2015), pp. 1–13. Available from: http://doi.wiley.com/10.1002/9781118991978.hces030

[16] Pourbabak, H., Chen, T., Su, W.: 'Consensus-based distributed control for economic operation of distribution grid with multiple consumers and prosumers'. In 2016 IEEE Power and Energy Society General Meeting (PESGM), Boston, MA, 2016, vol. **2016-Novem.**, pp. 1–5. Available from: http://ieeexplore.ieee.org/document/7741083/

[17] Boyer, S.A.: '*Scada: supervisory control and data acquisition*' (International Society of Automation, USA, 2009, 4th edn.). Available from: https://www.isa.org/store/scada-supervisory-control-and-data-acquisition,-fourth-edition/44478161

[18] Higgs, M.A.: 'Electrical SCADA systems from the operator's perspective'. In IEE Seminar Condition Monitoring for Rail Transport Systems, London, UK, 1998, pp. 3/1–3/4. Available from: http://digital-library.theiet.org/content/conferences/10.1049/ic{_}19980976

[19] Pourbabak, H., Luo, J., Chen, T., *et al.*: 'A novel consensus-based distributed algorithm for economic dispatch based on local estimation of power mismatch', *IEEE Trans. Smart Grid*, 2018, **9**, (6), pp. 5930–5942. Available from: https://ieeexplore.ieee.org/document/7913707

[20] Pourbabak, H., Chen, T., Zhang, B., *et al.*: 'Control and energy management system in microgrids', in '*Clean energy microgrids*' (Institution of Engineering and Technology, Stevenage, UK, 2017), pp. 109–133. Available from: http://digital-library.theiet.org/content/books/10.1049/pbpo090e{_}ch3

[21] Xu, S., Pourbabak, H., Su, W.: 'Distributed cooperative control for economic operation of multiple plug-in electric vehicle parking decks', *Int. Trans. Electr. Energy Syst.*, 2017, p. **27**, e2348. Available from: http://doi.wiley.com/10.1002/etep.2348

[22] McArthur, S.D.J., Davidson, E.M., Catterson, V.M., *et al.*: 'Multi-agent systems for power engineering applications – part I: concepts, approaches, and technical challenges', *IEEE Trans. Power Syst.*, 2007, **22**, (4), pp. 1743–1752. Available from: http://ieeexplore.ieee.org/document/4349106/

[23] Kleinberg, M., Miu, K., Nwankpa, C.: 'Distributed multi-phase distribution power flow: modeling, solution algorithm and simulation results'. *Simulation-Transactions of the Society for Modeling and Simulation Int.*, 2008, **84**, (8–9), pp. 403–412. Available from: http://journals.sagepub.com/doi/pdf/10.1177/0037549708098121

[24] Nguyen, C.P., Flueck, A.J.: 'A novel agent-based distributed power flow solver for smart grids', *IEEE Trans. Smart Grid*, 2015, **6**, (3), pp. 1261–1270. Available from: http://ieeexplore.ieee.org/

ielx7/5165411/7086423/06987287.pdf?tp={&}arnumber=6987287{&}isnumber=7086423

[25] Warnier, M., Dulman, S., Koç, Y., *et al.*: 'Distributed monitoring for the prevention of cascading failures in operational power grids', *Int. J. Crit. Infrastruct. Prot.*, 2017, **17**, pp. 15–27. Available from: http://www.sciencedirect.com/science/article/pii/S1874548216300427

[26] Iggland, E., Andersson, G.: 'On using reduced networks for distributed DC power flow'. In 2012 IEEE Power and Energy Society General Meeting, San Diego, USA, 2012, pp. 1–6. Available from: http://ieeexplore.ieee.org/document/6345302/

[27] Dagdougui, H., Sacile, R.: 'Decentralized control of the power flows in a network of smart microgrids modeled as a team of cooperative agents', *IEEE Trans. Control Syst. Technol.*, 2014, **22**, (2), pp. 510–519

[28] Nakayama, K., Zhao, C., Bic, L.F., *et al.*: 'Distributed power flow loss minimization control for future grid', *Int. J. Circuit Theory Appl.*, 2015, **43**, (9), pp. 1209–1225. Available from: http://doi.wiley.com/10.1002/cta.1999

[29] Erseghe, T.: 'Distributed optimal power flow using ADMM', *IEEE Trans. Power Syst.*, 2014, **29**, (5), pp. 2370–2380

[30] Disfani, V.R., Fan, L., Miao, Z.: 'Distributed DC optimal power flow for radial networks through partial primal dual algorithm'. In 2015 IEEE Power & Energy Society General Meeting, Denver, USA, 2015, pp. 1–5. Available from: http://ieeexplore.ieee.org/document/7286528/

[31] Nikoobakht, A., Mardaneh, M., Aghaei, J., *et al.*: 'Flexible power system operation accommodating uncertain wind power generation using transmission topology control: an improved linearised AC SCUC model', *IET Gener. Transm. Distrib.*, 2017, **11**, (1), pp. 142–153

[32] Nikoobakht, A., Aghaei, J., Niknam, T., *et al.*: 'Towards robust OPF solution strategy for the future ac/dc grids: case of VSCHVDC-connected offshore wind farms', *IET Renew. Power Gener.*, 2018, **12**, (6), pp. 691–701

[33] Olfati-Saber, R., Fax, J.A., Murray, R.M.: 'Consensus and cooperation in networked multi-agent systems', *Proc. IEEE*, 2007, **95**, (1), pp. 215–233. Available from: http://ieeexplore.ieee.org/document/4118472/

[34] Saber, R.O., Murray, R.M.: 'Consensus protocols for networks of dynamic agents'. Proc. of the 2003 American Control Conf., Denver, USA, 2003, vol. **2**, pp. 951–956. Available from: http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=1239709