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MLMC method to estimate propagation of uncertainties in electromagnetic fields scattered from objects of uncertain shapes

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We estimate the propagation of uncertainties in electromagnetic wave scattering problems. The computational domain is a dielectric object with uncertain shape. Since classical Monte Carlo (MC) method is too expensive, we suggest to use a modified multilevel Monte Carlo (MLMC) method. This method uses a hierarchy of spatial meshes and optimally balances the statistical and discretisation errors. MLMC performs most of the simulations using low-fidelity models and only a few simulations using high-fidelity models. As a result, the final computational cost is becoming significantly smaller.

More details about this research can be found in [1].
Problem: electromagnetic wave scattering from a dielectric object of uncertain shape, see schema in Fig. 1, left.
Input uncertainties: shape, geometry.
Output uncertainties: electromagnetic fields, radar and scattering cross sections (RCS and SCS).
Goal: estimate output uncertainties.
Methods: the continuation multilevel Monte Carlo (CMLMC) method [2].
During the last 10 years the multilevel MC (MLMC) methods have shown their efficiency, robustness, and simplicity [3].
The exact shape of the computational domain could be unknown. We suggest to model and parameterise this shape by random variables. Random sampling could be done by the traditional MC method with the error convergence rate $\mathcal{O}(N^{-1/2})$, where N is the number of samples. The quasi-MC method requires more smoothness and may have the convergence rate $\mathcal{O}(N^{-1})$. Sparse grids were applied in [4,5]. Another class of methods is the surrogate schemes [6–8].

Let $\boldsymbol{\xi}$ be the vector of random variables. The quantity of interest (QoI) be $g(\boldsymbol{\xi})$. The goal of the MLMC method is to approximate the expected value, $\mathrm{E}[g]$, to a guaranteed tolerance TOL with predefined probability, and minimal computational cost. To achieve this, the MLMC method constructs a hierarchy of meshes (see Fig. 2, left) and performs most of the simulations using low-fidelity models (the problem is discretised and solved on a coarse mesh) and only a few simulations using high-fidelity models (a fine mesh is used). Figure 1(center) shows the averaged computing time vs. tolerance TOL. As TOL

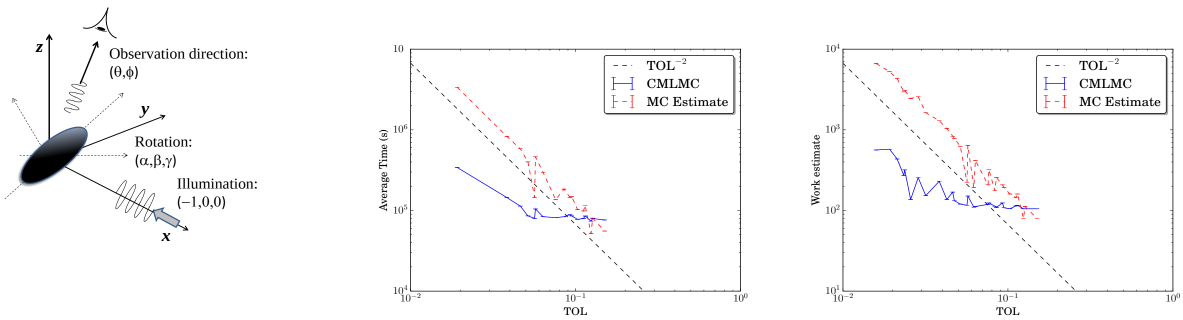

Fig. 1: (left) Illustration of the scattering problem; (center) Computation times required by the CMLMC and MC methods vs. TOL; (right) Computational work estimate for the CMLMC and MC methods vs. TOL.

gets smaller, the CMLMC algorithm becomes more efficient than the MC method. For values of TOL close to 0.02, the CMLMC algorithm is roughly 10 times faster than MC. Figure 1(right) shows the estimated work vs. TOL. The job done by CMLMC is again much smaller than the job done by MC.
Deterministic solver. We use the Poggio-Miller-Chan-Harrington-Wu-Tsai surface integral equation (PMCHWT-SIE) [1]. The PMCHWT-SIE is discretized using the method of moments (MoM) and the iterative solution of the resulting matrix system is accelerated using a (parallelized) fast multipole method (FMM) - fast Fourier transform (FFT) scheme (FMM-FFT) [9].

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Even with these modern numerical techniques the computational time for real-life problems may vary from a few hours to a few days.

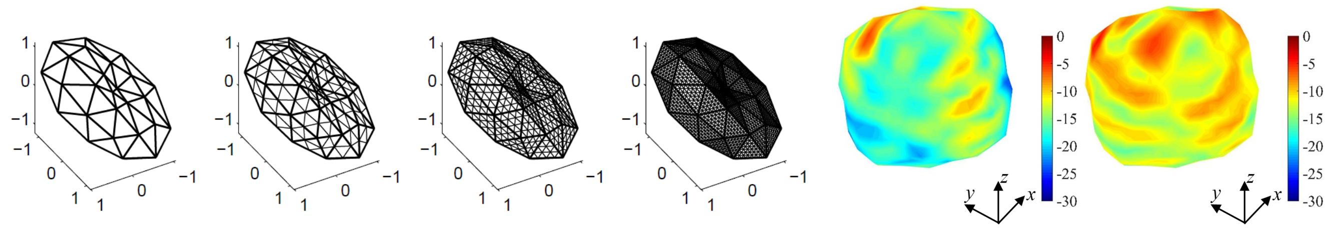

Fig. 2: (left) An example of four nested meshes of a perturbed sphere with N, $4N$, $16N$ and $64N$ triangular elements; Amplitudes of magnetic (center) and electric (right) surface current densities induced on the perturbed domain under excitation by an x-polarized plane wave propagating in z direction at 300 MHz (dB scale).

CMLMC method constructs a sequence of (nested) meshes $\{P_\ell\}_{\ell=0}^{L}$. In our case, all meshes consist of $\{N, 4N, \ldots, 4^L N\}$ triangular elements (see an example in Fig. 2, left). Perturbations are generated in the following way by perturbing the nodes of the initial mesh P_0 with N elements:

$$v(\vartheta_m, \varphi_m) \approx \tilde{v}(\vartheta_m, \varphi_m) + \sum_{k=1}^{K} a_k \kappa_k(\vartheta_m, \varphi_m). \tag{1}$$

Here ϑ_m and φ_m denote angular coordinates of mth node, $v(\vartheta_m, \varphi_m)$ is its (perturbed) radial coordinate, and $\tilde{v}(\vartheta_m, \varphi_m) = 1$ is its (unperturbed) radial coordinate on the unit sphere. Here, $\kappa_k(\vartheta, \varphi)$ are obtained from spherical harmonics by re-scaling their arguments and a_k are uncorrelated random variables. In numerical tests we used $K = 2$, $\kappa_1(\vartheta, \varphi) = \cos(\alpha_1 \vartheta)$ and $\kappa_2(\vartheta, \varphi) = \sin(\alpha_2 \vartheta)\sin(\alpha_3 \varphi)$, $\alpha_1 = 2$, $\alpha_2 = 3$, and $\alpha_3 = 2$. See also [10] to learn how to generate random fields. Note, that no uncertainties are added on meshes P_ℓ, $\ell > 0$; the uncertainty is introduced only at level $\ell = 0$.

The random variables used in generating random perturbations in P_0 are the weights a_k, $k = 1, \ldots, K$, the rotation angles φ_x, φ_y, and φ_z, and the scaling factors l_x, l_y, and l_z, making the dimension of the stochastic space $K + 6$, i.e., random parameter vector

$$\boldsymbol{\xi} = \{a_1, \ldots, a_K, \varphi_x, \varphi_y, \varphi_z, l_x, l_y, l_z\}. \tag{2}$$

In the example shown in Fig. 1, the CMLMC algorithm is executed for uniform random variables a_1, $a_2 \sim U[-0.14, 0.14]$ m, φ_x, φ_y, $\varphi_z \sim U[0.2, 3]$ rad, and l_x, l_y, $l_z \sim U[0.9, 1.1]$. The CMLMC algorithm is run for TOL, decreasing from 0.2 to 0.008. At the lowest value of TOL, the CMLMC algorithm requires $L = 5$ mesh levels. The QoI is the scattering cross-section (SCS) computed over the cone $\Omega = [1/6, 11/36]\pi\,\mathrm{rad} \times [5/12, 19/36]\pi\,\mathrm{rad}$. The scatterer resides in free space (vacuum) with $\mu_0 = 4\pi \times 10^{-7}$ H/m and the frequency $f = 300$ MHz.

Conclusion. We successfully applied CMLMC method to efficiently and accurately characterize EM wave scattering from dielectric objects with uncertain shapes. For some settings we observed that the CMLMC algorithm is roughly 10 times faster than MC.

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