The Unactualized Certainty-Actuality Correspondence

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Unactualized Certainty vs. Actuality

“Since event X just happened, its probability is 1”

True?
Not if we think of possibilities as things which have not actualized yet:
$P(\Omega)$ is supposed to be a measure over *possibilities*
X is an *Actuality*

Contrast with

"X has not happened yet, but is *certain* to occur”

Certainty is usually regarded as an epistemic state of belief, but this talk presupposes that it can be interpreted as being ontic whenever probability is interpreted as being ontic $\Longrightarrow$
“Unactualized Certainty” means ontic certainty whenever probability is ontic

Standard Axiomatic Probability vs. Axiomatic Enrichment

Standard Axiomatic Probability does **not** distinguish between possibilities and actualities.

Recently proposed an axiomatic enrichment which does[1]
Let $\Omega = \bigcup_{i=1}^{N} E_i$ be a set where N is either finite or countably infinite, $\mathcal{A} \subseteq \mathcal{P}(\Omega)$ a set of its mutually exclusive subsets E_i, and call the pair $(\Omega, \mathcal{A})$ a measurable space. Let $\Gamma = \{\gamma | \gamma = f(\omega)\}$ be a set where $f : \Omega \to \Gamma$ is a bijection and let $g : \Omega \to \Gamma$ be another map.
A real-valued function $P : \mathcal{A} \to \mathbb{R}$ satisfying

- Axiom 0: $P(g^{-1}(\gamma)) = P(\Omega)$
- Axiom 1: $P(\Omega) = 1$
- Axiom 2: $0 \leq P(E_i) \leq 1$
- Axiom 3: $P \bigcup_{i=1}^{N} E_i = \sum_{i=1}^{N} P(E_i)$

is called a *probability*.

Notice: To define a *non-probabilistic* unit measure, simply omit the red parts above; but then one obtains Kolmogorov's axioms! $\Longrightarrow$ **Original axioms do not actually define a probability!**

[1] arXiv:2003.06517

Two Correspondences

Two maps from Ω to Γ in the axiomatic enrichment

- *The Possibility-Actuality correspondence* f: Bijective, establishes same cardinality for Ω and Γ
- *The Unactualized Certainty-Actuality* $(UC - A)$ *Correspondence* g: Surjective, brings a set of elements of unit measure into correspondence with each element of Γ

The focus of this talk will be on the *concept* behind the map g

Pro-actuality

A *pro-actuality* is defined as an unactualized possibility which is a certainty.

Example: Rigged Coin with $P(H) = 1$

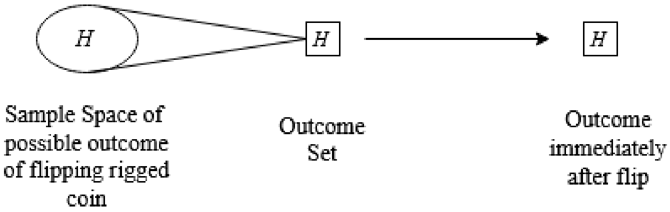


Figure: A transition from pro-actuality to actuality visualized in the axiomatic enrichment. Pro-actualities are characterized by $f = g$. The convention will be to omit the sample space in the visualization of an actual outcome.

Two types of Pro-actuality

But we could have chosen to *never flip the rigged coin!*

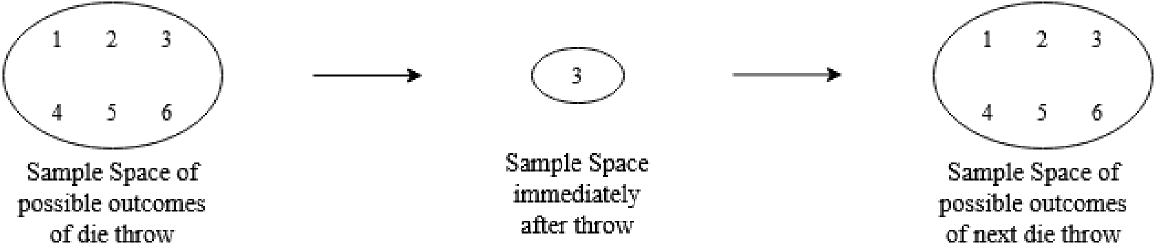
Suggests two types of pro-actuality:

- A *conditionally actualizable pro-actuality* is defined as a pro-actuality which depends on some triggering event for its actualization .
- An *unconditionally actualizable pro-actuality* is defined as a pro-actuality which does not depend upon any triggering event for its actualization.

Die Example: Standard Probability

Consider the throw of a six-sided die in three stages:

1. The die is about to be thrown.
2. The die is thrown and an outcome is obtained, say, three.
3. The die is picked up and about to be thrown again.



Die Example: Axiomatic Enrichment

Consider the throw of a six-sided die in three stages:

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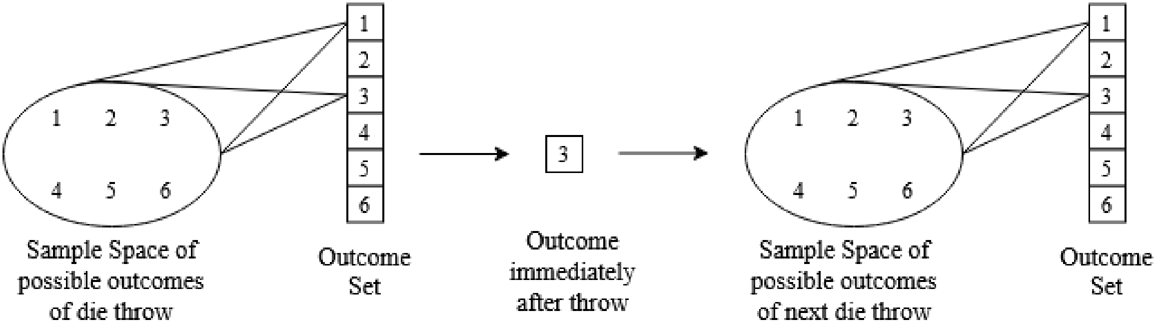
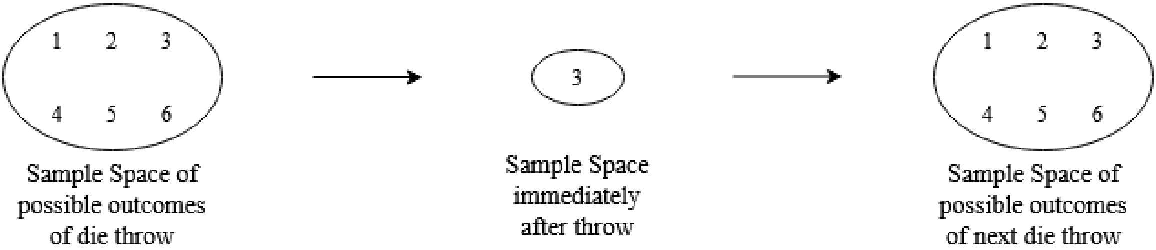


From Probability to the Quantum

Consider how we would describe the three stages if we did not know what “throwing a die” means:

1. The die is about to be thrown.
2. The die is thrown and an outcome is obtained, say, three.
3. The die is picked up and about to be thrown again.



Potentiality

Analysis by Kistler:

- Semantically: Potentiality=Dispositionality
- Pragmatically: Potentiality≠Dispositionality

  Two criteria:

  1. A potentiality ceases after it has actualized (whereas a dispositionality may not), and
  2. The probability for actualization of a potentiality is less than one (whereas for a dispositionality it can be one).

Applies this to Heisenberg's distinction between "possibilities or potentialities" and "things or facts" and finds a problem.

> *"Heisenberg's claim according to which an a quantum system undergoes, at the moment of measurement, a transition from possibility to actuality, cannot mean that, when the system is measured, it goes from a state of possible existence into a state of real existence, simply because at a time at which it only has possible existence, it has no existence at all, and a non-existent system, quantum mechanical or not, cannot enter into any interactions and cannot in particular undergo any measurement. So no measurement could bring "it" into actual existence if it had not been actual before the measurement."*

A Different Kind Unactualized Possibility

Kistler's criticism is perceptive but can be overcome: posit a different *kind* of unactualized possibility.

Suppose a quantum system emerges out of something else which in an essential way is neither that quantum system nor a "collection of its parts with a disposition to assemble into the system"

Imperfect analogy: orient a funnel over a pan containing six die molds corresponding to each outcome configuration. The "triggering event" consists of

- random alignment of the funnel with one of the molds, the pouring of a suitable fluid into it
- subsequent solidification.

There were no "solid die parts" before the triggering event
But solidification actualizes disposition of fluid atoms to rearrange into a solid $\implies$ this is still a potentiality

Potentiality vs. Actualizability

- A *potentiality* associated with a physical system is defined as an unactualized possibility arising out of the system's disposition to transform itself or its environment in accord with the two criteria above.
- An *actualizability* associated with a physical system is defined as an unactualized possibility arising out of a disposition for a system to come into actual existence in spacetime.

Actualizability concept forced upon us if:

1. A quantum state represents everything about a physical system there is to be represented, and
2. The physical system represented by a quantum state not under 'measurement' is a kind of ontic unactualized possibility.

The Heisenberg Interpretation of Quantum Mechanics I

What stays the same as in standard QM:

1. Quantum states are represented by vectors in Hilbert Space
2. Measurements are represented by linear Hermitian operators that are functions of the position and/or momentum operator acting on those vectors,
3. The time evolution of quantum states obeys the Schrödinger equation

The Heisenberg Interpretation of Quantum Mechanics II

What is different from standard QM:

- Postulate that Hilbert space $\mathcal{H}$ is a space of actualizabilities.
- Postulate *classical states* set $\mathcal{C}$ which contains a classical or actual counterpart to every eigenstate in the Hilbert space in every measurement basis (analog of Γ).
  - Define *classical basis sets* $\mathcal{B}_\alpha \subsetneq \mathcal{C}$ which contain the actual counterpart to each eigenstate in measurement bases α.
  - Non-unitary transformation $|\Psi\rangle \longrightarrow |\psi\rangle$ is considered an element of the converse of partial map $\mathfrak{e} : \mathcal{H} \rightharpoonup \mathcal{H}$ with range$\{|\Psi\rangle\}$ which maps its eigenstates in all possible measurement bases to it and will be called the *eigenstate map*. The converse will be called the *collapse relation*.
  - Cartesian product $\mathcal{H} \times \mathcal{C}$ makes it possible to define new relations such that we can write

$$\mathfrak{g} \circ \mathfrak{f} = \mathfrak{e} \tag{1}$$

where partial map $\mathfrak{g} : \mathcal{C} \rightharpoonup \mathcal{H}$ with range $\{|\Psi\rangle\}$ is the *classical-quantum correspondence* and partial map $f : \mathcal{H} \rightharpoonup \mathcal{C}$ is the *actualizability-actuality* correspondence.

The Heisenberg Interpretation of Quantum Mechanics III

- Postulate that the converse of the map $\mathfrak{g}$, the *actualization relation* $\mathfrak{g}^{-1}$ is subject to a probability distribution governed by the Born Rule on the elements of any subset $\mathcal{B}_\alpha$. (Inverse of $\mathfrak{f}$ will be called the *deactualization map* $\mathfrak{f}^{-1}$).
  - As in standard quantum mechanics, the value of the variable obtained upon measurement is represented by the eigenvalue obtained from operating on a quantum state with a Hermitian operator, but postulate implies it belongs to an element of $\mathcal{C}$, not $\mathcal{H}$.
- Postulate split in concept of mass for consistency:
  - *Actualizable mass m*, which characterizes physical systems the states of which are elements of $\mathcal{H}$
  - *Actual mass* $\mathfrak{f}(m)$, which characterizes physical systems the states of which are elements of $\mathcal{C}$. The notation is merely for convenience, as the actualizability-actuality correspondence characterizes states, not masses.

Frame Title

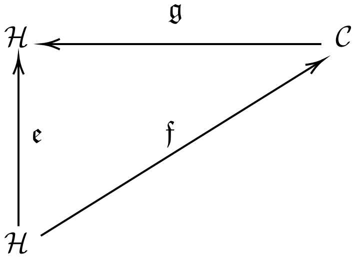


Figure: The eigenstate map $\mathfrak{e}$ as a composition of the classical-quantum correspondence $\mathfrak{g}$ after the actualizability-actuality correspondence $\mathfrak{f}$

Frame Title

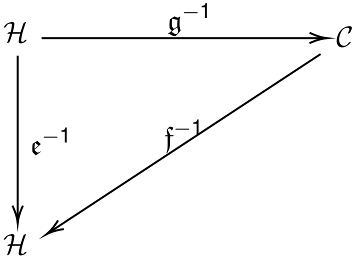
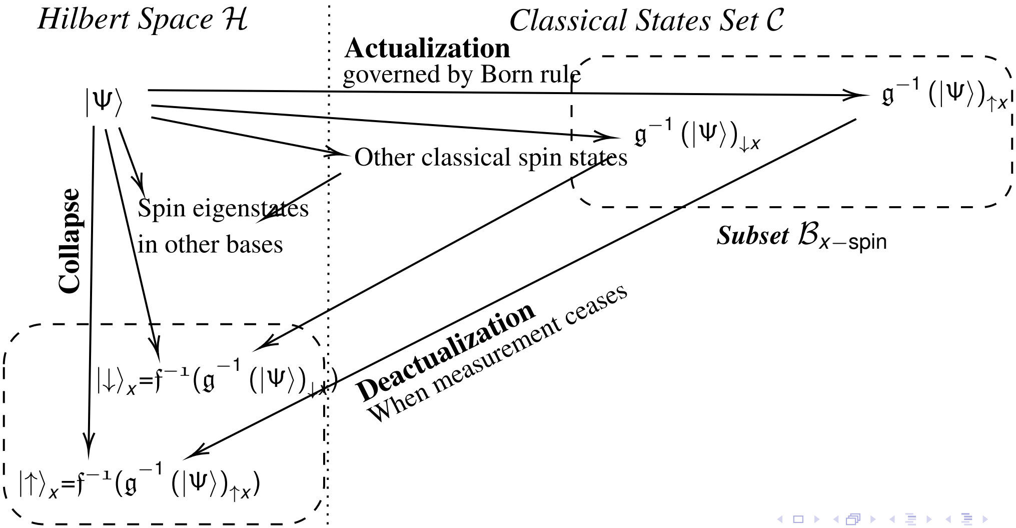


Figure: The collapse relation $\mathfrak{e}^{-1}$ as a composition of the deactualization map $\mathfrak{f}^{-1}$ after the actualization relation $\mathfrak{g}^{-1}$

An Example: $x-$Spin $1/2$



Quantum Entanglement: A Difficulty

Consider a singlet state of spin $1/2$ particles A and B and suppose we carry out a measurement on A and obtain spin up. We might be tempted to write:

$$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \longrightarrow |\uparrow\rangle_A |\downarrow\rangle_B \tag{2}$$

but this can't be quite right:

- fails to account for the fact that one particle has been measured while the other has not
- behaves differently than an ordinary product state

In an ordinary product state, both particles will undergo time evolution, whereas in the partially measured singlet state only the measured particle will. The other one will stay "stuck" in its state until measured $\Longrightarrow$ difference from ordinary product state is not just "philosophical" but has observable consequences!

A Permanent Proactuality

According to HI, measuring A turns B into a time-independent conditionally actualizable pro-actuality. (in that measurement basis):

$$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \longrightarrow \mathfrak{g}^{-1}_{\uparrow A}(|\psi\rangle) + |\downarrow\rangle^{p}_{B} \tag{3}$$

Here, $\mathfrak{g}^{-1}_{\uparrow A}(|\psi\rangle)$ is that image in $\mathcal{C}$ of $|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\uparrow\downarrow\rangle)$ under this relation which is the classical counterpart to the spin-up state of A in that measurement basis. By the above compositions

$$\mathfrak{g}^{-1}_{\uparrow A}(|\psi\rangle) = \mathfrak{f}(|\uparrow\rangle_{A}) \tag{4}$$

Probability can only apply to the unactualized part, so

$$\langle\downarrow\,|\,\downarrow\rangle^{p}_{B} \longrightarrow P(\mathfrak{f}(|\downarrow\rangle_{B})|\,|\downarrow\rangle^{p}_{B}) = 1 \tag{5}$$

Time-independence of this equation indicates that it is a permanent pro-actuality. The subscript on $|\downarrow\rangle^{p}_{B}$ indicates this symbolically.

No Non-local Influence

Consider the same scenario and suppose A and B are spacelike separated.

Observer frame 1: A measured before B

$$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \longrightarrow \mathfrak{f}(|\uparrow\rangle_A) + |\downarrow\rangle_B^p \tag{6}$$

Oberserver frame 2: B measured before A

$$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \longrightarrow |\uparrow\rangle_A^p + \mathfrak{f}(|\downarrow\rangle_B) \tag{7}$$

In either frame, no physical influence of any kind is transmitted or could have been transmitted because the supposed “recipient” did not even exist until it was “measured”!

Domain of locality principle is over actualities and related properties (e.g. setting up a field). Actualizabilities are outside its domain because there is no physical system that could be subject to non-local influences.

Correlated Actualizations

- Actualizability concept denotes a complete negation of “realism” applied to physical systems
- Heisenberg interpretation gives up realism in favor of locality full stop.
- Potentiality concept could not have been used because it still maintains a measure of “realism”

Within this interpretation, the spacelike separated correlations are to be understood in terms of *correlated actualizations*. The mechanism for their enforcement lies not at the level of the spacetime objects but whatever it is that they emerge from.

Where is the $UC - A$ correspondence in Quantum Mechanics?

To locate the $UC - A$ correspondence in QM, **locate its domain and co-domain**.

Co-domain:
According to the Born rule, probability enters the theory through inner products, but now given slightly different interpretation:

$$|\langle\phi|\psi\rangle|^2 \longrightarrow P(\mathfrak{f}(|\phi\rangle)) \text{ given the state } |\psi\rangle \tag{8}$$

- $|\phi\rangle \longrightarrow \mathfrak{f}(|\phi\rangle)$ means range to which probability distribution applies is $\mathcal{B}_\alpha$.
- The superset of all $\mathcal{B}_\alpha$ is $\mathcal{C} \implies \mathcal{C}$ is the Co-domain!

Domain:
To find the domain, notice that $\langle\psi|\psi\rangle$ describes a proactuality. In Probability, the domain of g can be built up by collecting elements of the domain of f as proactualities of its singleton subsets $\{\omega\} \subset dom(f) = \Omega$:

$$\{\omega : \omega \in dom(f)\} = \Omega \tag{9}$$

But if we try an analogous approach here, we run into a problem:

$$\{\langle\psi|\psi\rangle : \langle\psi| \in \overline{\mathcal{H}}, |\psi\rangle \in \mathcal{H}\} = \{1\} \tag{10}$$

The Quantum State as a Label

By the Born rule, $\{\langle\psi|\psi\rangle : \langle\psi| \in \overline{\mathcal{H}}, |\psi\rangle \in \mathcal{H}\}$ *should* have been the domain of the $UC - A$ correspondence

A way around this difficulty: Label each unactualized certainty!

$|\psi\rangle$ is the most natural label, so **reinterpret**

$$|\psi\rangle = \langle\psi|\psi\rangle\,|\psi\rangle \quad (11)$$

$|\Psi\rangle$ is a mere label for the true objects of quantum mechanics: *Unactualized Certainties.*

Hilbert space functions as a "substitute" for the domain of the $UC - A$ correspondence: its basis elements $|\psi_i\rangle$ for any measurement basis α are really just labels for $\langle\psi_i|\psi_i\rangle$ labelling their inner concept or *intension*

$\implies \mathfrak{g}^{-}1$ is quantum analog of the map g, *provided* it is governed by the Born Rule.

Measurement Contexts

What does it mean to say that "the objects of quantum mechanics are unactualized certainties labeled by quantum states"?

If a system is not being measured, it is not "there". Instead, there is an unactualized certainty which manifests itself as follows:

Distinctions labeled by the basis representation of a quantum state in basisα are in terms of the conditions necessary so that *if* a "measurement" is carried out under those conditions, one will **with certainty** measure the property of a physical system the state of which is an element of $\mathcal{B}_\alpha$.

- A *Measurement context* M_α is a set of experimental or natural conditions or configurations such that if a quantum measurement occurs in connection with a given quantum state under those conditions, an observable which represents the property of a system with a classical state which is the image of one of the eigenstates of that quantum state in the measurement basis α under $\mathfrak{f}$ will be found with certainty.

A measurement context M_α, when connected to an unactualized certainty, permits it to be indirectly represented as a quantum state in a particular measurement basis α.

Indirect and Direct Representations of Unactualized Certainties

A quantum state, considered abstractly, is a label for an unactualized certainty. But when represented in terms of componenents, it can also represent an unactualized certainty.

- **Indirect Representation:**

$$|\psi\rangle = \langle\psi|\psi\rangle\,|\psi\rangle = \langle\psi|\psi\rangle \sum c_i\,|\psi_i\rangle = \sum c_i\,\langle\psi_i|\psi_i\rangle\,|\psi_i\rangle \tag{12}$$

- **Direct Representation:**

$$\langle\psi|\psi\rangle = \sum |c_i|^2\,\langle\psi_i|\psi_i\rangle \tag{13}$$

Building up the Context for Hilbert Space

Let

$$\mathcal{S} = \{\mathcal{B}_\alpha : \mathcal{B}_\alpha \subsetneq \mathcal{C}\} \tag{14}$$

Let

$$\bigsqcup_\alpha \mathcal{H}_\alpha = \bigsqcup \mathbb{H} \tag{15}$$

Define the *measurement context set* $\mathcal{M}$ as

$$\mathcal{M} = \{M_\alpha : \forall(\mathcal{H}_\alpha, \alpha) \in \mathbb{H}, \varphi((\mathcal{H}_\alpha, \alpha)) = \eta(M_\alpha)\} \tag{16}$$

where

- bijection $\varphi : \mathbb{H} \longrightarrow \mathcal{S}$ will be called the *fate map*
- bijection $\eta : \mathcal{M} \longrightarrow \mathcal{S}$ will be called the *emergence map*

$\mathcal{S}$ is related to two other sets via bijections, which permits us to relate them to each other:

- bijection $\kappa : \mathcal{M} \longrightarrow \mathbb{H}$ will be called the *context representation map*

The Unactualized Certainty Diagram for the HI

The bijections commute

$$\eta = \varphi \circ \kappa \tag{17}$$

which can be visualized by a **unactualized certainty diagram**:

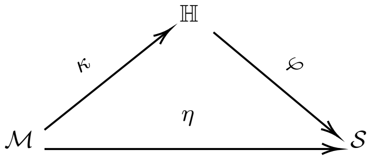


Figure: The emergence map η as a composition of the fate map φ after the context representation map κ.

Unactualized Certainty Diagram for a Hidden Variable Variant

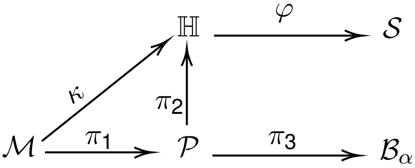


Figure: The unactualized certainty diagram of a hypothetical “hidden variable variant” of the Heisenberg interpretation. Here, whereas φ merely specifies with certainty a set $\mathcal{B}_\alpha$, π_3 specifies with certainty one of its elements because $\mathcal{P}$ supplies the specifications needed (i.e. hidden variables) to continue any deterministic causal chain from before to after a measurement.

Unamendable Probability

In order to have a deterministic physical theory, two ingredients are needed:

1. A set of deterministic laws, *complete* in the sense that they apply to all those properties of all physical systems in the theory which under those laws dispose systems to behave deterministically.
2. A set of initial conditions, *complete* in the sense that they specify definite values for all properties to which those laws apply

- Classical physics has both ingredients
- Under the HI, Quantum mechanics cannot have a complete set of initial conditions
  - Absence of System before a Measurement $\Longrightarrow$ List of specifications of initial conditions incomplete $\Longrightarrow$ breaks causal deterministic chains from before measurement to after

Under the Heisenberg Interpretation, Probablity in Quantum Mechanics is unamendable

Summary

1. The $UC - A$ correspondence is embodied by the map g in the axiomatic enrichment of probability, and by the actualization relation $\mathfrak{g}^{-1}$ in the Heisenberg Interpretation.
2. When there is only one unactualized possibility available $\implies$ pro-actuality
   - highlights conflation of pro-actuality with actuality in standard probability
   - helps conceptualize partially measured entangled states

   .
3. Unactualized possibilities $\implies$ potentialities in probability
   Unactualized possibilities $\implies$ actualizabilities in quantum mechanics under the HI.
   Actualizability concept enforces locality, anti-realism and contextuality in that interpretation and provides reason for unamendable probability in QM.
4. **The objects of quantum mechanics under the Heisenberg Interpretation are unactualized certainties.**
   - Quantum states are *labels* for unactualized certainties which connect measurement contexts to classical states, such that the state which obtains upon measurement will *with certainty* belong to $\mathcal{B}_\alpha$.

Conclusion

The paper on which this talk is based is available at:

http://dx.doi.org/10.7302/2334

Thank you!