Understanding the Characteristics of Mathematical Content Knowledge for Teaching Algebra in High Schools and Community Colleges

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Abstract

In this paper we present an exploratory analysis of differences in the performance of two different samples of teachers—high school practicing teachers and community college faculty—on an instrument that assesses mathematical knowledge for teaching algebra 1. To better understand the variance in the performance within and between the two groups of teachers, we examine the relationships between the measured knowledge and teachers' educational and teaching background. Highlighting the positive effect of algebra-based teaching experience on the knowledge, we discuss the implications regarding the extent to which the instrument captures the construct it seeks to assess.

Keywords: Teacher knowledge, Experience, Algebra, High school, Community college, Mathematics teaching

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The field of mathematics education has embarked in multiple efforts to advance understanding of how programs for teacher and faculty development can be deployed to increase the quality of mathematical instruction. Standards from professional organizations advocate for the use of instructional approaches that reach all students and that present mathematics as more than memorization of facts and procedures (NCSM & NCTM, 2018). A promising line for research states that the way in which resources are deployed in the classroom makes a difference in instructional quality.

Teacher knowledge has been named as one of the most important resources, one that is tightly connected to instructional quality (Berliner, 2001). Substantive empirical research has demonstrated that there is a strong connection between teacher knowledge and quality of instruction and that high-quality instruction and high teacher knowledge result in higher student performance on standardized tests (Hill et al., 2008). This connection has been successfully established at the elementary school level, but there has been much less information of the connection at the secondary or postsecondary level. Part of the difficulty in establishing a similar connection at other levels of education stems from the scarcity of instruments available to make valid interpretations of the scores representing teachers' knowledge for teaching particular subject areas in the context of a particular school level. By being able to assess mathematical knowledge for teaching specific subject areas, for a given level of students, that is, teaching algebra for community college students, we might be in a better position to predict student performance on tests of that subject area, and possibly make a connection to quality of instruction in that subject area. Success in establishing these connections hinges on the availability of instruments that can be used to assess teacher knowledge necessary for teaching particular subject content areas. Such an instrument can also be used to assess the impact of professional development efforts that target teacher knowledge for teaching specific subject matter at a specific school level.

One of the knowledge constructs commonly assessed by an instrument measuring teachers' knowledge is content knowledge for teaching mathematics, but researchers have defined the term content knowledge in different ways. Some refer to content knowledge as pure mathematical knowledge, while others refer to mathematical knowledge specific to the work of teaching (Ball et al., 2008). More important to our study is the variety of assumptions regarding the level of mathematical content that researchers hypothesize is needed for teaching a mathematics course.

Several researchers hypothesized that a higher level (e.g., college-level) of mathematical knowledge is associated with knowledge needed for teaching lower level (e.g., secondary) mathematics courses. Some of them used teachers' subject matter preparation, such as the number of college level mathematics courses taken or the degrees attained (Begle, 1979; Monk, 1994; Rowan et al., 2002) as proxies of teachers' content knowledge predicting students' mathematics achievement. However, those proxy measures yielded inconsistent results. For example, five or fewer mathematics courses taken was positively related to student performance in mathematics (Monk, 1994), whereas more than five mathematics courses or advanced degree in mathematics had little or a negative effect on student achievement (Monk, 1994; Rowan et al., 2002). While these results have motivated researchers to further investigate the characteristics of teachers' knowledge in multifaceted ways, they have also challenged researchers to promote a consistent understanding of teachers' content knowledge. In contrast to these researchers who used indirect proxies of teachers' mathematical knowledge, McCrory et al. (2012) used assessment items measuring content knowledge of college-level mathematics to define categories of knowledge for teaching secondary school algebra.

As interest in direct measures of teachers' content knowledge for teaching has replaced those proxies, a number of instruments have been developed that purport to do that. These instruments are often developed for varied targets regarding the level of schooling, domains of knowledge, or subject specificity. For example, Hill and colleagues (Hill et al., 2004; Hill, 2007) developed different sets of items for elementary and middle school levels but did not make differences by grade levels. Some researchers have developed instruments for whole domains of mathematics (e.g., Hill et al., 2004; Saderholm et al., 2010), whereas others have developed instruments for a specific content (McCrory et al., 2012; Mohr-Schroeder et al., 2017), specific course of studies (Herbst & Kosko, 2014), or specific conceptions (Bradshaw et al., 2014). The use of existing measures in contexts other than the ones for which they were developed needs to be subject to a validity argument, that is, "the degree to which evidence and theory support the interpretations of test scores for proposed uses of tests" (AERA, APA, & NCME, 2014, p.11). Specifically, while the content of algebra in community college instruction is similar to that of algebra 1 in K-12, it is worth asking whether an instrument developed to measure the latter can be used in the former context and if so, what does it measure.

In this regard, we work with an instrument designed to measure teachers' content knowledge for teaching algebra 1 in order to understand the characteristics of content knowledge that this instrument is measuring. For example, we ask: Is it measuring content knowledge of mathematics in general? Is it measuring content knowledge associated with the work of teaching? Or is it measuring content knowledge specific to teaching a specific grade level (e.g., high school students)? To better understand the characteristics of content knowledge assessed by the instrument, we examined how the assessed knowledge is associated with teachers' educational and teaching experience. Specifically, using two groups of teachers that typically differ along their education degree and teaching experience, we sought to identify the factors (e.g., subject matter preparation or experience teaching) that accounted for the variations in their performance on the instrument. We then inferred the knowledge measured by the instrument from the associated factors.

Theoretical background

Studies on teachers' mathematical knowledge for teaching

The notion of mathematical knowledge for teaching, proposed by Ball et al. (2008), is defined as the "mathematical knowledge that teachers need to carry out their work as teachers of mathematics" (p. 4). Ball et al.'s framework (see Figure 1) extended Shulman's (1987) distinctions between teachers' content knowledge and pedagogical knowledge, by adding subcomponents that differentiated various types of knowledge based on various hypothesized activities related to teaching.

Shulman's teacher content knowledge (subject matter knowledge) is said to include Ball et al.'s common content knowledge, specialized content knowledge, and horizon knowledge. *Common content knowledge* refers to that "mathematical knowledge known in common with others who know and use mathematics," say bankers, nurses, engineers (Ball, et al., 2008, p. 403). *Specialized content knowledge* is defined as mathematical knowledge that teachers need to use in order to interpret, understand, and diagnose students' thinking. This knowledge is beyond the typical and common set of knowledge that people have about mathematics. *Horizon content* *knowledge* is defined as the knowledge required to understand the connections between the foundational ideas and common themes that make a discipline make sense and logically fit together. In contrast to content knowledge, Shulman's pedagogical content knowledge requires knowledge about students and teaching and in Ball's classification, it includes knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum. *Knowledge of content and students* refers to "teachers' knowledge of students' mathematical thinking" (Hill et al., 2008, p. 373). When a teacher selects a specific example or approach based on students' tendency to make a particular error, the teacher is using this type of knowledge (Ball et al., 2008, p. 404). *Knowledge of content and teaching* is defined as the intersection of knowledge of mathematical content and knowledge of teaching. Knowing which examples to select to begin a lesson, how to sequence topics, and understand the applicability of specific problem-solving processes are examples of situations in which this knowledge is deployed (Ball et al., 2008, p. 401). *Knowledge of content and curriculum* includes teachers' knowledge of mathematical concepts as presented in curricular materials used to teach a particular course, as well as alternate materials a teacher might use in addition to or instead of a prescribed curriculum (Shulman, 1986).

[Figure 1 goes here]

Regarding the relationship among the domains, subject matter knowledge tends to be considered as a necessary prerequisite for the development of pedagogical content knowledge (Friedrichsen et al., 2009). However, strong subject matter knowledge does not necessarily lead to the development of pedagogical content knowledge (Kleickmann et al., 2013; Lee et al., 2007). In relation to teacher preparation, students whose "preparation in mathematics was more demanding and rigorous" showed higher subject matter knowledge than others (Schmidt et al., 2007, p. 5).

As such, existing research has recognized multiple aspects of teachers' mathematical knowledge and conceptualized the distinctions among them.

Studies comparing teachers' mathematical knowledge across different teacher populations

To date, however, there are few studies on the comparability of subject matter knowledge or pedagogical content knowledge across different teacher populations. In other words, it remains uncertain whether the tests developed to measure mathematical knowledge for teaching a certain mathematics course allow fair comparisons of the respective groups considering that the comparisons can be meaningful only if the constructs measured by the tests are the same across the groups. The question of comparability of the test scores then arises as to whether the conceptual meaning and the structure of the knowledge based on one teacher population (e.g., elementary teachers) is applicable to another teacher population (e.g., high school teachers). As Speer et al. (2015) pointed out, a definition of content knowledge and its distinguishing features established based on analyses of elementary teachers might not be generalizable across different teacher populations. For example, what might be specialized content knowledge for an elementary school teacher (e.g., the knowledge needed to determine whether a sequence is a Fibonacci sequence) might be more a matter of common content knowledge for university professors. Among the studies measuring teachers' knowledge across different teacher populations, studies comparing the knowledge base of experienced and novice teachers (for an overview see Berliner, 2001) suggest that expert teachers not only know more than novice teachers, but that their knowledge is differently structured and may be more highly integrated. This conclusion is in line with findings from expertise research in other domains, which show that experts' knowledge bases are usually not only more extensive than those of novices, but also more connected and integrated (Chi et al., 1981; Schmidt & Boshuizen, 1992; Chase & Simon, 1973). Whether or not teachers' pedagogical content knowledge and subject matter knowledge are separable categories of knowledge may therefore be a function of different levels of expertise. Similarly, a study on teachers' knowledge at the secondary level conducted in Germany (Krauss et al, 2008) found that the "degree of cognitive connectedness between [pedagogical content knowledge] and [content knowledge] was a function of the degree of mathematical expertise" (p. 724). In the literature review on mathematical knowledge for teaching developmental courses at community colleges, Nabb and Murawska (2019) also point out that the knowledge needed to "teach mathematics in elementary levels is different than the knowledge of most mathematically-educated adults" (p. 6). Considering these findings, it seems worth questioning the applicability of the knowledge framework based on a certain teacher population to a different sample of teachers.

In our study, instead of surmising the characteristics of high school teachers' or community college faculty's knowledge based solely on the term, content knowledge, indicated by the group developing the instrument, we explored the characteristics of the knowledge by examining the variables associated with the assessed knowledge. For example, a significant relationship between the assessed knowledge and only with the number of college mathematical courses teachers had taken or academic degree would imply that the assessed knowledge has a characteristic similar to the mathematical knowledge that can be learned from college mathematics courses.

Teaching mathematics in different institutional contexts

We conjectured that the difference in subject matter preparation and pedagogical experiences of high school teachers and community college faculty during their academic studies and teaching careers is the source of the difference in the measured knowledge observed between the two groups.

In the United States, teacher certification for all K-12 grade levels is a state function, rather than a federal one. All 50 states have licensure requirements, to be completed during or after a bachelor's degree is awarded; the requirements include pedagogical content, subject matter content, general psychology courses, and practical experience in the classroom (Schmidt et al., 2011). Secondary teachers are usually required to attend professional development organized by their school districts (although the requirements vary by state and sometimes by district (see e.g., Rotermund, DeRoche, & Ottem, 2017). According to data from the Organisation for Economic Co-operation and Development (OECD, 2014), upper secondary teachers in the U.S. teach an average of 1,076 hours per year (approximately 30 hours per week).

In contrast to K-12 teachers who have to be certified individually by their state or regional licensure agency, community college level faculty are not typically certified by a state board. The typical criterion required by the accrediting bodies for becoming a full or part time instructor at a community college is having a master's degree with at least 18 credit hours or post-baccalaureate degree preparation in the field that they teach (Higher Learning Commission, 2016). Whether faculty have the required training, is typically reviewed by a hiring committee and a regional agency that provides accreditation for the institution. In the majority of institutions, there is no single universally agreed-upon criteria for pedagogy requirements (e.g., number of years of teaching experience), but institutions are expected to have a minimum threshold of experience for faculty hiring qualifications (Higher Learning Commission, 2016). In any given semester, an instructor at a community college teaches about 15 credit-hours (between 4 and 5 courses per term). In an effort to support their faculty, many colleges offer professional development; most of the offerings are in general pedagogical strategies (e.g., using technology, or cooperative learning, see Burn et al., 2018).

There is also a difference in the context of algebra courses and the student population that high school teachers and community college instructors teach in their algebra classes. Community colleges provide students with many options to further their educational goals, some of which include remediation, transfer to university undergraduate programs, vocational training, general education, continuing education, and workforce development (Mesa, 2017). Community colleges enroll students who tend to be non-traditional (e.g., over 24 years old, working, or with family responsibilities; AACC, 2020), offer flexible schedules, and charge very low tuition compared to universities. Because of the diversity in student backgrounds, these institutions offer a broad range of mathematics courses, from developmental mathematics (designed to prepare students for collegiate level study of mathematics) to mathematics courses taught in the first two years of an undergraduate major. Mathematics courses at community colleges, especially those ostensibly designed to prepare students for college courses, have high rates of failure, ranging from 30% to 70% (Bahr, 2008, 2010; Waycaster, 2001), and tend to be taught by more adjunct or part-time instructors (Blair et al., 2018), mostly due to enrollment fluctuation.

Underlying these differences in faculty qualifications and student characteristics between high school teachers and community college faculty, we examined the differences in teachers' knowledge for teaching algebra between the two groups using the same instrument. Specifically, we ask the following questions:

1. How are high school teachers and community college faculty similar or different regarding their performance on the instrument designed to measure teachers' knowledge for teaching algebra?

2. How are high school teachers' and community college faculty's educational and teaching background associated with their knowledge measured by the instrument?
3. What do the relationships between high school teachers' and community college faculty's knowledge and their educational and teaching background imply about the characteristics of the construct measured by the instrument?

Methods

Instrument

The instrument used in this study (hereafter MKT-A) was developed as a part of the project Measure of Effective Teaching (MET; Bill and Melinda Gates Foundation, 2010). According to Phelps et al. (2014), the construct that the items intended to measure is defined as "the content knowledge used in recognizing, understanding, and responding to the content practices that teachers engage in as they teach a subject" (p. 3). As implied in their definition, the design framework of the items attended to core practices (or tasks of teaching) that teachers do in their work (e.g., evaluating student ideas evident in work, talk, actions, and interactions) rather than focusing on one dimension of content knowledge such as *common content knowledge* or *specialized content knowledge*. By grounding the design framework on the components of the work of teaching, the items aimed to capture the link between content knowledge and teaching practice. In our study, we administered the 22 MKT-A items (18 multiple choice items and four testlets with sub-questions) with 20 of the items being used for scaling participants' MKT-A. Among the 20 items, three items are testlets consisting of multiple true/false sub-questions (two testlets consisting of two sub-questions, one testlet consisting of five sub-questions) and 17 items are multiple-choice items.

Samples

The high school teachers whose knowledge was analyzed in this study came from a national sample (across 47 states) of practicing U.S. high school mathematics teachers participating in a larger project[2] from March 2015 to January 2016. Among the teachers participating in the project, 280 teachers were teaching algebra 1 at the time the test was administered and 219 teachers among them responded to at least one of the MKT-A items. Hereafter, we refer to this in-service high school teacher sample simply as the high school teachers. Of the 219 participants who responded to at least one item, 158 completed all 20 items. On average, the 219 participants had been teaching mathematics for 12.70 years (SD=8.79, min=1, max=40) and had taken an average of 13.25 college-level mathematics courses (SD=6.97, min=2, max=40). In addition, teachers had been teaching algebra 1 for an average of 8.27 years (SD=6.45, min=1, max=32).

The sample of community college faculty included 72 instructors. The instructors were recruited from eight different colleges distributed in three states, to take part in a large-scale study[3] of algebra instruction at community colleges conducted by the Authors' research group, in the Fall 2017 and Spring 2018 semesters. The recruited faculty were teaching one of two different level courses, intermediate or college algebra. As a group, these instructors taught 84 different sections of these courses. As part of the project, the faculty, which included 29 (40%) part-time instructors, responded to the MKT-A instrument and other surveys (e.g., beliefs about mathematics teaching and mathematics, teaching practices, demographics). Of the 72 participants who responded to at least one item, 69 completed all 20 items. On average, the 72 participants had been teaching mathematics for 14.33 years (SD=8.46, min=2, max=40) and had

[2] Acknowledgement to the project from which the data originates (blinded).
[3] Acknowledgement to the project from which the data originates (blinded).

been teaching various algebra courses including pre-algebra, beginning algebra, intermediate algebra, and college algebra.

Data collection

As the responses from the high school teachers and the community college faculty were collected separately by different research groups, there were differences in the procedures and the contexts in data collection. For the high school sample, the author's research group administered the MKT-A instrument through an online platform. To reduce the possibility of the effect of item location on teachers' performance and response rate on each item, the author's research group changed the order of items across different groups of teachers that were randomly assigned into 12 different groups. Participants completed background surveys asking for their educational and teaching experience (e.g., number of college mathematics courses taken, years of teaching experience, gender, age, ethnicity, etc.) prior to answering the MKT-A items.

For the community college faculty sample, the authors' research group distributed a list of links for three surveys that they were asked to complete online via Qualtrics prior to the beginning of the semester in which data were collected. The first survey asked for characteristics of the course they were teaching (e.g., cooperative learning, inquiry-based learning, lecture, mastery learning, emphasis on communication skills, project-based learning) and collected personal characteristics (e.g., years of teaching experience, degree, amount of professional development, gender, age, ethnicity, etc.); the second survey asked for their beliefs about mathematics and the teaching of mathematics; the third survey was the MKT-A. Both groups of participants were allowed to take the test at their own convenience (regarding time and place) but were asked to answer the questions by themselves, without assistance.

Analysis

Before scaling participants' MKT-A using all of the administered items, we evaluated the properties of items to identify ones that were problematic regarding the difficulty level (too easy or too difficult[4]) or the correlation between each item and the sum of the rest of the item scores. The evaluations were conducted with participants' responses scored as 1 for correct and 0 for incorrect responses. In the evaluation of correct response rates calculated within each group, three items were commonly identified as being too easy in both groups; there were no too difficult items identified. We decided against retaining three too easy items for two reasons: 1) to avoid the issue of empty cells in the bivariate tables that will be used in a later latent factor model and 2) because there were other items measuring similar contents with appropriate difficulty levels. Thus, we excluded the items that did not contribute much to the construct but could cause an issue and increase the number of parameters. In the evaluation of the item correlations conducted within a whole sample, we identified one multiple-choice item that was not correlated with other items (less than 0.2 item-rest correlation). We also excluded this item for the subsequent analyses. With the remaining set of 20 items, we examined whether the items coherently measured one latent construct (MKT-A). As acceptable inter-item correlations do not necessarily mean that there is a single latent construct influencing item responses, we further conducted a confirmatory item factor analysis to ensure that there was a unidimensional construct underlying a set of item responses. In the analysis, we conducted structural equation modeling (SEM)-based unidimensional item factor analysis in which all 20 items are loaded into one latent construct, that is, MKT-A, using WLSMV[5] estimator. After confirming that the

[4] The thresholds used for "too easy" and "too difficult" were 0.95 and 0.05 correct response rate respectively.

[5] WLSMV stands for weighted least squares means and variance.

unidimensional model fits the data from two groups of teachers, we proceeded to the analysis comparing MKT-A scores between high school teachers and community college faculty. In all SEM analyses conducted in this study, we set latent factor variances and means to 1 and 0, respectively, and all item factor loadings were freely estimated.

After establishing the measurement model, we examined the latent factor (MKT-A) mean difference between the two groups of participants. Specifically, we tested a Multiple Indicators, Multiple Causes (MIMIC) model in which the participants' MKT-A was regressed on the indicator representing the participant group (0: high school teachers; 1: community college faculty)[6]. In a MIMIC model, the mean comparisons with covariate variables are made in the context of a latent variable measurement model, so the measurement errors and correlated residuals are adjusted in the analyses (Brown, 2006, p. 267). In other words, as the MIMIC model allows conducting regression of MKT-A scores on a covariate (here, group membership) and item factor analysis simultaneously, we could examine the factor mean difference more accurately than conducting regression analysis with an exported MKT-A score treated as an observed variable. In the MIMIC model, we also examined differential item functioning (DIF) to identify items that might function differently between two groups of instructors. We used Mplus version 7.4 (Muthén & Muthén, 1998 – 2015) for all the item factor analysis and MIMIC analyses conducted in this study.

Next, the effects of participants' educational and teaching experience on the participants' MKT-A were examined through multiple regression models in which the MKT-A score is regressed by each of the covariates representing an aspect of the participants' educational or

[6] We are aware of the disadvantage of MIMIC models over multiple-group comparison. However, considering the small number of community college faculty (N=72), we decided to use MIMIC because of its less restrictive sample size requirement.

teaching experience. This method (two-step approach), where the regression analysis is conducted separately from the measurement model, was chosen as an alternative to a unified approach, where an effect of a covariate is estimated with a measurement model simultaneously, because of an empty cell in a table that associates a covariate and an item score. This is discussed in more detail in the section Results.

As the survey questions asking for educational and teaching experience were different for high school teachers and college faculty, the analyses were conducted for each group separately. In the analysis, the MKT-A score exported from the unidimensional model was used as an outcome variable and each of the background variables was used as an independent variable predicting the participants' MKT-A. The regression analyses were conducted using STATA 15.1 (StataCorp, 2017). To examine the effect of high school teachers' educational experience on their MKT-A scores, we used the following variables: the number of college-level algebra courses taken, the number of college-level geometry courses taken, and the number of college-level mathematics courses taken. To examine the effect of community college faculty' educational experience, we used the following variables: whether or not a participant had a bachelor's, master's, or doctoral degree in mathematics or mathematics education. The participants' teaching experience was represented by the number of years teaching mathematics and all the variables indicating their experience in teaching a specific course: whether or not a participant had experience teaching algebra 2, calculus, trigonometry or pre-calculus, geometry, statistics (for high school teachers); whether they had experience teaching pre-algebra, trigonometry, combined course of college algebra and trigonometry, linear algebra, precalculus, calculus 1, calculus 2, calculus 3, probability and statistics. For community college faculty, we also used a variable representing the frequency of using Inquiry-based Learning practices in their classroom. The specific questions used for these variables are presented in Table 2 and Table 3 for high school teachers and community college faculty, respectively.

Results

This section consists of two main results from the analysis comparing the knowledge measured by the MKT-A instrument between high school teachers and community college instructors. The differences and similarities in knowledge were examined in terms of the participants' performance on MKT-A items and the relationships between their performance and their educational and teaching background. The comparison results were then used to infer the characteristics of the knowledge construct measured by the instrument.

MIMIC model and DIF

Before comparing the mean scores of MKT-A between high school teachers and community college instructors, we conducted the confirmatory item factor analysis to ensure that the unidimensional model of MKT-A fits the full sample data (N=291) well. The model fit indices of the unidimensional model with all 20 items loaded onto one factor MKT-A indicated that it is reasonable to estimate the participants' MKT-A in terms of a single score (RMSEA=0.017; CFI=0.983; TLI=0.981). Table 1 presents the standardized factor loadings and its significance level of each item used in estimating the unidimensional MKT-A. As shown in the table, all of the items contributed to the latent factor with factor loadings greater than 0.3 standardized factor loadings, except for one item (Q3). The internal consistency of the single score estimated in terms of Cronbach's alpha was 0.77, indicating that the score was adequately reliable.

[Table 1 goes here]

After establishing this unidimensional model, we added a covariate—group indicator—to the model to compare MKT-A between the two groups.

To test the equality of the latent mean (mean of MKT-A) between the two groups, we used a MIMIC model that regresses the latent factor on the variable indicating whether a participant is a high school teacher or a community college faculty. The model suggested that there is a significant direct effect of the group membership (0: high school teachers; 1: community college faculty) on the latent factor, MKT-A (standardized estimate $\beta=0.79$, unstandardized estimate $B=0.83$, $SE=0.17$, $p < 0.001$). Specifically, the group of community college faculty is 0.79 standardized scores higher than that of the group of high school teachers on the MKT-A and this effect size is large according to the criterion suggested by Cohen (1988) (Cohen's d= 0.20, 0.50, 0.80, for small, medium, and large, respectively).

The equality of item intercepts was also examined for each item while controlling for a latent factor MKT-A. This was to evaluate differential item functioning (DIF) that identifies any biased items that function differently for high school teachers and community college faculty. The result suggested that, when both groups' overall MKT-A level is the same, community college faculty are likely to have higher scores on items Q9 and Q23 than high school teachers, whereas high school teachers are likely to have higher scores on items Q3 and Q8 than community college instructors. To understand influences that may have caused differences in the item scores between the two groups, we analyzed the contents of these items shown to have DIF.

One characteristic of the items community college instructors showed higher performance than high school teachers when controlling for MKT-A is that the items ask participants to choose an option that best characterizes a given student's approach based on the student's written work. The responses from the two groups of participants suggested that a higher proportion of high school teachers than community college faculty tended to choose an option describing a procedural step (e.g., “the student should have first divided by 3”) rather than an option describing the student’s reasoning as an appropriate evaluation of the student’s work. On the other hand, the DIF items that the community college instructors showed lower performance than high school teachers when controlling for MKT-A suggested that community college faculty may consider omitting some steps in solving a problem as an indication of the lack of students’ mathematical reasoning. Thus, community college faculty might conclude that the student work does not provide evidence of correct student reasoning, even though the option is designed to present correct student work. This might be because there is a perception that remediation is much needed in the community college context, so community college faculty, in particular, the faculty who teach pre-college courses may tend to evaluate student work as incomplete when it does not include every step needed to reach a solution, even though the students’ work is mathematically correct. In contrast, high school teachers may think that not presenting the steps and doing the work correctly indicates that students are becoming more proficient in using mathematical procedures. As our DIF analysis implies, some factors other than teacher’ mathematical knowledge such as teachers’ expectations on the level of details or the use of certain methods may influence how teachers evaluate mathematical completeness of students’ work. Freeing parameters for these DIF items in the MIMIC model (RMSEA=0.011; CFI=0.992; TLI=0.991) yielded better fit than the initial unidimensional model (RMSEA=0.017; CFI=0.983; TLI=0.981) reported above. Considering the interpretable source of the DIF and the improved model fit, we decided to free the parameters for these items. The modified model showed that the significance and the strength of the factor loadings were improved while the significance of the effect of the group membership on MKT-A did not significantly change with the controls for DIF.

The consistent result showing higher MKT-A of community college faculty than high school teachers raised intriguing questions regarding the nature of knowledge measured by MKT-A. Specifically, we questioned whether teachers' MKT-A measured by the instrument is associated with their advanced mathematics degree or experience teaching higher level mathematics courses. To examine this question, we conducted a series of multiple regressions examining the effects of teachers' educational and teaching experience on their MKT-A.

Difference in the relationships with educational and teaching experience

As the data of high school teachers and community college faculty were collected from two different projects, the specific questions asking for the participants' background were different between the groups. Thus, we used these background variables as covariates of teachers' MKT-A only within each group. Table 2 and Table 3 present descriptive statistics for the participating high school teachers' and community college faculty's educational background and their teaching experience along with the descriptions of the used questions, separately for each group.

[Table 2 goes here]

[Table 3 goes here]

Although the specific questions used for the participants' educational and teaching background were different between the groups, the questions share common themes as the questions ask about participants' experience related to learning subject matter (the number of courses taken for high school teachers, degree for community college faculty) or their experience in teaching (number of total years teaching, experience teaching a course).

A series of multiple regressions, where the MKT-A factor score is regressed on one of the educational or teaching background variables, was conducted to better understand the characteristics of the construct measured by MKT-A items and the differences in MKT-A between the two groups. From Table 4 to Table 7 we present unstandardized effect (B), standardized effect (β), standard error, and associated *p*-value for each of the regressions. We acknowledge the potential increase in type 1 error when conducting multiple tests on the same samples. However, we have decided to report the results in terms of effect sizes and unadjusted conventional significance levels (0.05, 0.01, 0.001) considering that there is still a controversy regarding the need for multiplicity control (Cribbie, 2017). Thus, the *p*-values reported in the tables need to be interpreted cautiously.

Regarding the effect of the participants' educational experience, the number of college level mathematics courses taken was not significantly associated with high school teachers' MKT-A (Table 4). However, community college instructors who had a Bachelor's or Master's degree in mathematics had significantly higher MKT-A scores than the instructors who did not have the degrees (Table 5). The results may imply that fine-grained differences in subject matter preparation such as the number of specific courses taken do not make significant differences in the level of MKT-A, but that having a degree in mathematics, which requires a longer period of preparation than a series of single courses, make a significant difference in instructors' MKT-A. In contrast to degrees in mathematics, degrees in mathematics education had no significant effect on MKT-A. Considering these results, we suggest that MKT-A measured in this study might be more closely related to a type of subject matter knowledge than the knowledge specific to mathematics education (e.g., curriculum, teaching methods).

[Table 4 goes here]

[Table 5 goes here]

Table 6 and Table 7 present the results of regression models in which MKT-A is regressed on each of the variables reflecting the participants' experience teaching mathematics. The results suggest that high school teachers who have taught algebra 2 (including those who were currently teaching at the time of the test) had significantly higher MKT-A scores than high school teachers who have not taught the course. Similarly, high school teachers who have taught calculus had significantly higher MKT-A scores than the high school teachers who have not taught calculus. Having experience in teaching trigonometry or precalculus also showed a significant effect on teachers' MKT-A (Table 6). However, the number of total years of teaching mathematics in general or having experience teaching mathematics courses that were not closely related to algebra 1 (geometry) did not have a significant effect on MKT-A. Taken together, these results suggest that experience in teaching advanced mathematics courses related to algebra 1 had significant and positive effects on teachers' MKT-A scores.

[Table 6 goes here]

Similar to the results of high school teachers, community college instructors' total years of teaching in general was not significantly associated with their MKT-A score (Table 7). Also, experience in teaching mathematics courses whose content is related to algebra but more advanced than algebra 1 (Linear algebra, Calculus_1, Caculus_2) had a significantly (marginally significant for Calculus_2) positive influence on the instructors' MKT-A. Moreover, community college instructors who self-reported that they frequently use inquiry-based learning (IBL) practices in their classrooms had higher MKT-A scores than others who reported never or rarely using IBL practices.

[Table 7 goes here]

To check the consistency of the result, we also conducted the unified model approach where each background variable is added as a covariate predicting the MKT-A in the unidimensional model. Although we countered a case of an empty cell in the bivariate table when incorporating the variable Teaching_Prealgebra, other background variables showed the consistent results. In other words, only algebra-related coursework showed significant associations with participants' MKT-A. Also, among the education related variables, only the variables representing having a bachelor's and master's in mathematics showed significant effect on MKT-A.

In summary, the results suggest that the construct MKT-A measured in this study is a type of knowledge associated with teaching practices in that experience teaching algebra related courses has a significant effect on MKT-A for both groups of teachers. Importantly, the significant effect is not applicable to general mathematics courses, but to courses specifically related to advanced level of algebra.

Limitations

The generalizability of these results is subject to limitations. For instance, unlike the national sample of high school teachers from 47 states, the sample of the community college faculty is limited to three states. Because of the nature of the project, we could only work with participants who were willing to agree to a semester-long process of data collection which included student work and multiple video-taped sessions. In addition, the intrusive form of data collection may have limited the volunteer pool to faculty who might be somewhat more comfortable in their teaching and more open to discuss instruction. We might have naturally selected a more homogeneous set of community college instructors. The sample characteristics (57% female instructors, 60% of full-time, all indicated receiving some sort of professional development with 68% saying that it was math-specific) differ slightly from the characteristics of the national population of community college mathematics faculty (52% female, 33% full-time, and 82% of institutions requiring continued professional development; Blair et al., 2018).

The small number of community college faculty responses may have affected statistical power in detecting effects of instructors' educational background and teaching experience on their MKT-A. Thus, to develop a full picture of how subject matter preparation and teaching experience affect instructors' MKT-A, additional studies with a large number of samples having diverse backgrounds will be needed. At the same time, this investigation with this particular sample provides important insights regarding community college faculty that has not been done before and provides important considerations for future research with this population.

The small number of the faculty also limited the scope of analysis used in this study. In other words, we used MIMIC instead of multiple-groups CFA considering that the MIMIC model estimates fewer parameters. Even though MIMIC allowed us to compare the mean of MKT-A within the established measurement model, it examines a latent factor mean difference under the assumption on the measurement invariance (e.g., equality of factor loadings, residual variances) that we didn't test. A further study with a larger number of samples could assess multiple aspects of measurement invariance between high school teachers and community college faculty and establish a greater degree of accuracy on the comparison of MKT-A between the groups.

Discussion and Conclusions

The present study examined the differences and similarities between high school teachers and community college faculty in the knowledge measured by the instrument designed to measure teachers' mathematical knowledge for teaching algebra 1. The findings of this study have a number of implications.

First, we found that community college faculty scored higher than high school teachers in the MKT-A instrument. There are two likely explanations associated with the differences in the performance between the groups. One is that the knowledge measured by the instrument is algebra content knowledge; in this case, we could say that community college faculty scored higher than high school teachers because they have better understanding of algebra as a mathematical discipline than high school teachers. This explanation can be supported by one of our results showing that in this sample of community college faculty those with a graduate degree in mathematics perform better than those who do not. The effect of a graduate degree could not be examined across groups or within high school teachers, because the degree information was not collected for high school teachers. However, considering that a graduate degree in mathematics is a typical requirement for community college faculty, whereas only 56% high school teachers have graduate degrees in mathematics (National Center for Education Statistics, 2011-2012), it is reasonable to assume that more community college faculty have graduate degrees than high school teachers, and that a graduate degree can contribute to community college faculty's higher MKT-A.

Another possible explanation for this finding is that the knowledge measured by the instrument is mathematical knowledge specific to the work of teaching; in this case, we could say that community college faculty scored higher because they have experience teaching more diverse algebra-based mathematics courses (e.g., intermediate and college algebra, trigonometry, pre-calculus, and calculus) than high school teachers whose teaching might be mainly confined to geometry, algebra 1, and algebra 2. We conjecture that this is the case because the wider range of algebra-based courses that community college faculty teach exposes them to many more teaching experiences related to the content providing the opportunity to develop a broader foundation in algebra as measured by the instrument. This may contribute to their higher performance on the instrument. It is possible, therefore, that a combination of both having the subject matter preparation through graduate programs and being exposed to many algebra-based courses with diverse students who have different needs, which is not typically what would happen in HS, may lead community college faculty to have higher MKT-A scores than high school teachers.

This conjecture is in accord with the argument in McCrory et al. (2012) that demonstrated the importance for teachers to draw both advanced mathematical knowledge beyond the course level (i.e., algebra 2 or calculus for teaching algebra 1) and teaching knowledge as they engage in appropriate teaching practices that require mathematical knowledge specific to teaching. Although it is difficult to detect any direct evidence for the reason of this finding within the present study, our second finding, described below, provides some support for the second conjecture—the difference in the performance might be partly due to differences in how instruction is enacted in each context rather than mere differences in the level of disciplinary knowledge.

Our second finding is that faculty and high school teachers who have taught more advanced mathematics courses (e.g., linear algebra for community college faculty; algebra 2 and calculus for high-school sample) obtain higher scores on the MKT-A instrument. We conjecture that this is because, by teaching calculus or more advanced mathematics courses, faculty and high school teachers could have a first-hand experience of how difficulties with algebraic ideas interfere with their understanding in high-level classes (e.g., calculus). This experience thus increases their awareness (and therefore build up their knowledge) to better recognize difficulties with algebra (as assessed with the MKT-A items) than instructors or high school teachers who only see the content in algebra 1. This example can be illustrated by one MKT-A item that requires participants to determine whether a potential connection between slope (a concept taught in algebra 1) and derivative (a concept taught in calculus) is expressed in students' discussion on the slope of a function. To respond to a correct answer, participants need to have mathematical knowledge in both concepts and understand which of the conceptions the students are referring to in their statements.

More importantly, we found that the experience effect is specific to the course. In other words, community college faculty and high school teachers who have experience teaching advanced algebra courses showed higher MKT-A than others, whereas the faculty and teachers who have experience teaching non-algebraic courses such as geometry did not show significantly higher MKT-A than others who do not have the experience. This finding is consistent with that of Herbst and Kosko (2014) who showed that teachers' mathematical knowledge for teaching geometry is specifically associated with years of experience teaching geometry. Our results provide support for the hypothesis that the course-specific effect of teaching experience on teachers' mathematical knowledge can be applied to other mathematics courses than geometry and also applied to other mathematics teacher populations than high school teachers. Furthermore, considering that the course-specific effect is shown only with teaching experience, but not with college coursework, we conjecture that this effect might be due to the work associated with teaching the different courses rather than differences between the mathematical disciplines (e.g., between algebra and geometry). By presenting each item of the instrument in a context that addresses the need of using mathematical knowledge for teaching a specific course (e.g., understanding students' work/statements on the concept of function), the items appear able to tap the knowledge used in teaching rather than pure mathematical knowledge. Taken together, our findings, while preliminary, suggest that the MKT-A instrument measures mathematical content knowledge specific to teaching algebra courses and that teachers' experience teaching advanced algebra-based courses has positive effects on their mathematical knowledge for teaching algebra.

Methodologically, our finding of course-specific effects of teaching experience suggest that the need of careful consideration of using the number of total years teaching for the proxy of the knowledge or skills gained from work experience. To use it as a predictor of differentiated performance in teaching mathematics, the experience measure may need to be specified according to the kinds of experience as well as on the kinds of courses (e.g., algebra, geometry). In particular, considering that there is greater participation of teachers in professional development in their 4^{th} to 9^{th} year of experience than early career teachers (Lewis et al., 1999), the identified effect of years of experience might indicate the effectiveness of professional development activities rather than years of teaching experience. In future investigations, it might be possible to use not only further specified indicators of experience teaching including multiple types of professional activities (e.g., learning new methods of teaching or educational technology) but also specified measures of teachers' knowledge. For example, the multiple measures could be developed according to different tasks of teaching presented by items. An additional study with these specified knowledge measures and predictors would allow us to diagnose aspects of knowledge associated with different kinds of experience and contribute to the development of a full picture of the nature of the knowledge needed for teaching a specific mathematics course.

Disclosure statement

This study was funded by X. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of X. The authors declare that they have no potential conflict of interest.

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Table 1

Factor loadings for unidimensional model (N=291)

	Std. estimate	S.E.	P-Value
MKT-A BY			
Q1	0.528	0.098	0.000
Q2	0.756	0.059	0.000
Q3	0.234	0.089	0.009
Q4	0.769	0.072	0.000
Q6	0.474	0.070	0.000
Q7	0.414	0.079	0.000
Q8	0.453	0.076	0.000
Q9	0.396	0.081	0.000
Q10	0.563	0.072	0.000
Q12	0.419	0.079	0.000
Q13	0.307	0.084	0.000
Q14	0.723	0.065	0.000
Q15	0.631	0.052	0.000
Q16	0.580	0.068	0.000
Q17	0.417	0.080	0.000
Q18	0.600	0.077	0.000
Q19	0.393	0.080	0.000
Q20	0.418	0.101	0.000
Q22	0.420	0.082	0.000
Q23	0.507	0.081	0.000

Table 2

Descriptive statistics for high school teachers' background variables (N=219)

Educational background				
	Mean	S.D.	Min	Max[7]
CollegeAlgCourses: How many college-level mathematics courses focusing on Algebra topics have you taken?	3.46	2.55	0	20
CollegeGeoCourses: How many college-level mathematics courses focusing on Geometry topics have you taken?	1.75	1.33	0	8
CollegeMathCourses: Please indicate the total number of college-level mathematics courses you have taken?	13.25	6.97	2	40
Teaching experience				
Total Years Teaching (Including the most recently completed year, how many total years have you been teaching?)	12.70	8.79	1	40
Including up to the most recently completed school year, please indicate all the mathematics courses you have taught at the secondary level.				
	Number of participants who have taught the course			
Algebra 2	162 (74%)			
Calculus	76 (35%)			
Trigonometry or Precalculus	113 (52%)			
Geometry	173 (79%)			
Statistics	43 (20%)			

7 The participants were asked to choose one of the options ranging from 0 to 40.

Table 3

Descriptive statistics for community college faculty's background variables (N=72[8])

Educational background	
What degrees have you completed?	Number of participants who have completed the degree
Bachelor_MathEducation (0: Not applicable; 1: Yes)	11(15%)
Bachelor_Mathematics (0: Not applicable; 1: Yes)	42(58%)
Master_MathEducation (0: Not applicable; 1: Yes)	17(24%)
Master_Mathematics (0: Not applicable; 1: Yes)	47(65%)
PhD_MathEducation (0: Not applicable; 1: Yes)	1(1%)
PhD_Mathematics (0: Not applicable; 1: Yes)	3(4%)

Teaching experience	Mean	S.D.	Min	Max
YearsTeaching_fulltime_math: How many total years of full-time-equivalent teaching experience mathematics do you have?	14.33	8.46	2	40
Freq_IBL: During the last year, how frequently in the semester did you use Inquiry-based Learning practices in the classroom portion of your course? (1: Never; 2: Once or twice a month; 3: Once or twice a week; 4: Once or twice in a class; 5: Multiple times in a class)	1.75	1.07	1	5

Which of the following math courses have you taught before?	Number of participants who have taught the course
Prealgebra	57 (79%)
Trigonometry	53 (74%)
College Algebra and Trigonometry	39 (54%)
Linear Algebra	13 (18%)
Precalculus	52 (72%)
Calculus_1	48 (67%)
Calculus_2	36 (50%)
Calculus_3	14 (19%)
Probability and Statistics	31 (43%)

[8] 71 participants responded to the question for "Freq_IBL."

Table 4

Summary of effects of high school teachers' mathematics coursework level on MKT-A (N=219)

Educational experience (coursework)	B	β	SE	p-value
College Algebra Courses	-0.005	-0.016	0.020	0.818
College Geometry Courses	0.014	0.024	0.038	0.721
College Math Courses	0.013	0.122	0.007	0.072

(*$p < 0.05$, **$p < 0.01$, ***$p < 0.001$)

Table 5

Summary of effects of community college faculty's degree level on MKT-A (N=72)

Educational experience (degree)	B	β	SE	p-value
Bachelor_MathEducation	-0.220	-0.094	0.277	0.431
Bachelor_Mathematics	0.516**	0.303**	0.194	0.010
Master_MathEducation	-0.176	-0.089	0.235	0.457
Master_Mathematics	0.536**	0.304**	0.201	0.009
PhD_MathEducation	-0.409	-0.057	0.855	0.634
PhD_Mathematics	-0.366	-0.087	0.500	0.467

(*$p < 0.05$, **$p < 0.01$, ***$p < 0.001$)

Table 6

Summary of effects of high school teachers' teaching experience on MKT-A (N=219)

Teaching experience	B	β	SE	p-value
Total years teaching	0.006	0.065	0.006	0.335
Taught_Algebra2	0.271*	0.159*	0.114	0.019
Taught_Calculus	0.211*	0.134*	0.106	0.047
Taught_TrigPreCalc	0.251*	0.167*	0.100	0.013
Taught_Geometry	-0.088	-0.048	0.125	0.483
Taught_Stats	0.024	0.013	0.128	0.854

(*$p < 0.05$, **$p < 0.01$, ***$p < 0.001$)

Table 7

Summary of effects of community college instructors' teaching experience on MKT-A (N=72)

Teaching experience	B	β	SE	p-value
Total Years Teaching	0.009	0.087	0.012	0.465
Frequency of using IBL teaching	0.268**	0.335**	0.091	0.004
Taught_Prealgebra	-0.064	-0.031	0.247	0.797
Taught_Trigonometry	0.302	0.159	0.224	0.182
Taught_CollegeAlgebraTrig	0.253	0.151	0.199	0.207
Taught_LinearAlgebra	0.622*	0.285*	0.250	0.015
Taught_Precalculus	0.301	0.161	0.221	0.178
Taught_Calculus_1	0.436*	0.245*	0.206	0.038
Taught_Calculus_2	0.378	0.226	0.195	0.057
Taught_Calculus_3	0.073	0.034	0.253	0.774
Taught_Probability_Stats	0.205	0.121	0.201	0.312

(*$p < 0.05$, **$p < 0.01$, ***$p < 0.001$)

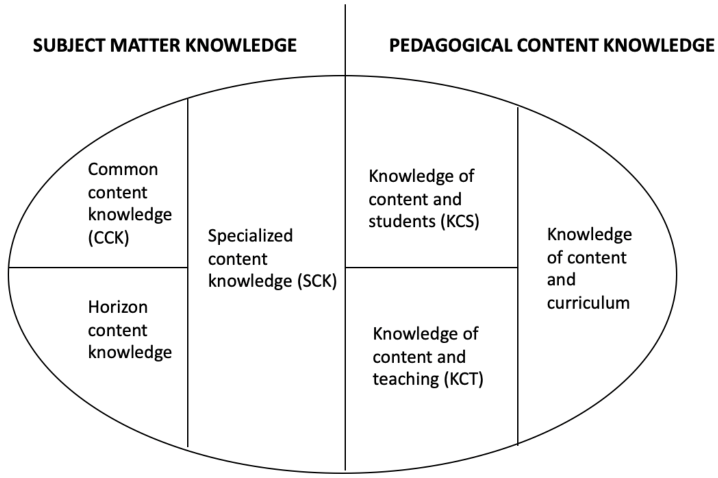

Figure 1. Domains of mathematical knowledge (Reprinted from Ball, Thames, & Phelps, 2008).