Online Supplement: Should Suppliers Allow Capacity Transfers?

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A.1 Mathematical Results Used in our Proofs

Lemma A.1. *The following simplifications hold:*

$$S_i(Q_i) = Q_i - \int_{-\infty}^{Q_i} Pr(D_i < x)\,\mathrm{d}x\,; \quad T_i(Q_i,Q_j) = \int_{-\infty}^{Q_j} Pr(D_i > Q_i + Q_j - x, D_j < x)\,\mathrm{d}x\,;$$
$$\frac{\mathrm{d}T_i(Q_i,Q_j)}{\mathrm{d}Q_i} = -Pr(D_i > Q_i, D_t < Q_i + Q_j)\,; \quad \frac{\mathrm{d}T_j(Q_j,Q_i)}{\mathrm{d}Q_i} = Pr(D_i < Q_i, D_t > Q_i + Q_j)\,;$$
$$T_i(Q_i,Q_j) + T_j(Q_j,Q_i) = \int_{-\infty}^{Q_i} Pr(D_i < y)\,\mathrm{d}y + \int_{-\infty}^{Q_j} Pr(D_j < y)\,\mathrm{d}y - \int_{-\infty}^{Q_i+Q_j} Pr(D_t < y)\,\mathrm{d}y\,.$$

Proof of Lemma A.1. **First**, $S_i(Q_i) = \mathbb{E}\big[\min(Q_i, D_i)\big] = Q_i Pr(D_i > Q_i) + \int_{-\infty}^{Q_i} x f_i(x)\,\mathrm{d}x = Q_i - \int_{-\infty}^{Q_i} Pr(D_i < x)\,\mathrm{d}x$. **Second**, $T_i(Q_i,Q_j) = \mathbb{E}\big[\min(Q_j - D_j, D_i - Q_i)\mathbb{1}_{(D_i>Q_i, D_j<Q_j)}\big] = \int_{Q_i}^{\infty}\int_{Q_i+Q_j-x}^{Q_j}(Q_j - y) f(x,y)\,\mathrm{d}y\,\mathrm{d}x + \int_{Q_i}^{\infty}\int_{-\infty}^{Q_i+Q_j-x}(x - Q_i) f(x,y)\,\mathrm{d}y\,\mathrm{d}x$. Using Integration by part, we can further simplify the two integrals in the above equation as follows: $\int_{Q_i}^{\infty}\int_{Q_i+Q_j-x}^{Q_j}(Q_j - y) f(x,y)\,\mathrm{d}y\,\mathrm{d}x = (x - Q_i)Pr(D_i \geq x, D_t \leq Q_i + Q_j)|_{x=Q_i}^{\infty} - \int_{Q_i}^{\infty} Pr(D_i \geq x, D_t \leq Q_i + Q_j)\,\mathrm{d}x + \int_{-\infty}^{Q_j} Pr(D_i \geq Q_i + Q_j - x, D_j \leq x)\,\mathrm{d}x$. Similarly, $\int_{Q_i}^{\infty}\int_{-\infty}^{Q_i+Q_j-x}(x - Q_i) f(x,y)\,\mathrm{d}y\,\mathrm{d}x = \int_{Q_i}^{\infty}(x - Q_i)Pr(D_i = x, D_j \leq Q_i + Q_j - x)\,\mathrm{d}x = -(x - Q_i)Pr(D_i \geq x, D_j \leq Q_i + Q_j - x)|_{x=Q_i}^{\infty} + \int_{Q_i}^{\infty} Pr(D_i \geq x, D_j \leq Q_i + Q_j - x)\,\mathrm{d}x$. Therefore, we obtain the expression for T_i as specified in the lemma. **Third**, $\frac{\mathrm{d}T_i(Q_i,Q_j)}{\mathrm{d}Q_i} = \int_{-\infty}^{Q_j} -Pr(D_i = Q_i + Q_j - x, D_j < x)\,\mathrm{d}x = \int_{-\infty}^{Q_j} -Pr(D_i = Q_i + Q_j - x, D_t < Q_i + Q_j)\,\mathrm{d}x = Pr(D_i \leq Q_i + Q_j - x, D_t \leq Q_i + Q_j)|_{x=-\infty}^{Q_j} = -Pr(D_i \geq Q_i, D_t \leq Q_i + Q_j)$. **Fourth**, $\frac{\mathrm{d}T_i(Q_i,Q_j)}{\mathrm{d}Q_j} = Pr(D_i > Q_i, D_j < Q_j) - \int_{-\infty}^{Q_j} Pr(D_i = Q_i + Q_j - y, D_j < y)\,\mathrm{d}y = Pr(D_i > Q_i, D_j < Q_j) - Pr(D_i > Q_i, D_t < Q_i + Q_j) = Pr(D_j < Q_j, D_t > Q_i + Q_j)$. **Finally**, $T_i(Q_i,Q_j) + T_j(Q_j,Q_i) = \int_{-\infty}^{Q_j} Pr(D_i > Q_i + Q_j - x, D_j < x)\,\mathrm{d}x + \int_{-\infty}^{Q_i} Pr(D_j > Q_i + Q_j - x, D_i < x)\,\mathrm{d}x = \int_{-\infty}^{Q_j} Pr(D_j < x)\,\mathrm{d}x - \int_{-\infty}^{Q_j} Pr(D_i < Q_t - x, D_j < x)\,\mathrm{d}x + \int_{-\infty}^{Q_i} Pr(D_i < x)\,\mathrm{d}x - \int_{-\infty}^{Q_i} Pr(D_i < x, D_j < Q_t - x)\,\mathrm{d}x$. Note that, $\int_{-\infty}^{Q_j} Pr(D_i < Q_t - x, D_j < x)\,\mathrm{d}x = \int_{-\infty}^{Q_j}\int_{-\infty}^{Q_t-x}\int_{-\infty}^{x} Pr(D_i = u, D_j = v)\,\mathrm{d}v\,\mathrm{d}u\,\mathrm{d}x = \int_{-\infty}^{Q_t} Pr(D_i > y - Q_j, D_t < y)\,\mathrm{d}y$. Similarly, $\int_{-\infty}^{Q_i} Pr(D_i < x, D_j < Q_t - x)\,\mathrm{d}x = \int_{-\infty}^{Q_i}\int_{-\infty}^{x}\int_{-\infty}^{Q_t-x} Pr(D_i = u, D_t = v + u)\,\mathrm{d}v\,\mathrm{d}u\,\mathrm{d}x = \int_{-\infty}^{Q_t} Pr(D_i < y - Q_j, D_t < y)\,\mathrm{d}y$. Therefore, $T_i(Q_i,Q_j) + T_j(Q_j,Q_i) = \int_{-\infty}^{Q_i} Pr(D_i < y)\,\mathrm{d}y + \int_{-\infty}^{Q_j} Pr(D_j < y)\,\mathrm{d}y - \int_{-\infty}^{Q_i+Q_j} Pr(D_t < y)\,\mathrm{d}y = \mathbb{E}\big[\min(Q_t, D_t)\big] - \mathbb{E}\big[\min(Q_i, D_i)\big] - \mathbb{E}\big[\min(Q_j, D_j)\big]$. ■

Let $\Phi(z)$ and $\phi(z)$ be cdf and pdf of standard normal distribution, respectively. Define $\alpha \overset{def}{=}$

$\sqrt{\frac{1+\rho}{2}} < 1$. Also, define function $\Phi_\alpha(z) \stackrel{def}{=} \int_{-\infty}^{z}\int_{-\infty}^{\frac{z}{\alpha}} \frac{1}{2\pi\sqrt{1-\alpha^2}} \exp\left(-\frac{x^2-2\alpha xy+y^2}{2(1-\alpha^2)}\right) \mathrm{d}y\,\mathrm{d}x$. Note, $\Phi_\alpha(z)$ is the cdf of bivariate standard normal distribution with correlation α, calculated at $(z, \frac{z}{\alpha})$.

Lemma A.2.

1. $\frac{\Phi(\frac{z}{\alpha})}{1-\Phi(\frac{z}{\alpha})}$ *is positive and increasing in* z.
2. $\frac{\phi(z)(z+\frac{\mu}{\sigma})}{1-\Phi(\frac{z}{\alpha})}$ *is positive and increasing in* z *for* $z > -\frac{\mu}{\sigma}$.
3. $\frac{\Phi'_\alpha(z)(z+\frac{\mu}{z})+\Phi_\alpha(z)-\Phi(\frac{z}{\alpha})}{1-\Phi(\frac{z}{\alpha})}$ *is positive and increasing in* z*, for* $z > -\frac{\mu}{\sigma}$ *when* $\frac{\mu}{\sigma}$ *is large enough such that probability of having negative demand is negligible (and when* $\frac{\mu}{\sigma} \geq \frac{\sqrt{2\pi}}{2} \approx 1.26$*).*

Proof of Lemma A.2.

First, note that $\Phi(z)$ is increasing and hence $\frac{\Phi(\frac{z}{\alpha})}{1-\Phi(\frac{z}{\alpha})}$ is increasing in z. **Second**, to show item 2, it suffices to show $\frac{\phi(z)}{1-\Phi(\frac{z}{\alpha})}$ is increasing in z. For $z < 0$, we know $\phi(z)$ is increasing and thus $\frac{\phi(z)}{1-\Phi(\frac{z}{\alpha})}$ is clearly increasing. For $z > 0$, note that $\frac{\mathrm{d}\left(\frac{1-\Phi(z)}{1-\Phi(\frac{z}{\alpha})}\right)}{\mathrm{d}z} = \frac{1-\Phi(z)}{1-\Phi(\frac{z}{\alpha})}\left(-\frac{\phi(z)}{1-\Phi(z)} + \frac{\frac{1}{\alpha}\phi(\frac{z}{\alpha})}{1-\Phi(\frac{z}{\alpha})}\right) > 0$. The last inequality is true because normal distribution has increasing hazard rate, $0 < \alpha < 1$ and $z > 0$. Therefore, $\frac{\phi(z)}{1-\Phi(\frac{z}{\alpha})} = \frac{1-\Phi(z)}{1-\Phi(\frac{z}{\alpha})} \times \frac{\phi(z)}{1-\Phi(z)}$ is increasing and positive. **Third**, we focus on $\frac{\Phi'_\alpha(z)(z+\frac{\mu}{z})+\Phi_\alpha(z)-\Phi(\frac{z}{\alpha})}{1-\Phi(\frac{z}{\alpha})}$. Note $\frac{\phi(\frac{z}{\alpha})}{1-\Phi(\frac{z}{\alpha})}$ is the reciprocal of hazard rate of normal distribution; hence, it is increasing and positive. Therefore, it suffices to show $\frac{\Phi'_\alpha(z)(z+\frac{\mu}{z})+\Phi_\alpha(z)-\Phi(\frac{z}{\alpha})}{\phi(\frac{z}{\alpha})}$ is increasing and positive. In this proof, we define the followings: $\beta \stackrel{def}{=} \sqrt{1-\alpha^2}$, $R(z) \stackrel{def}{=} \frac{\Phi(\beta\frac{z}{\alpha})}{\phi(\beta\frac{z}{\alpha})}$, $R_\alpha(z) \stackrel{def}{=} \frac{\Phi(\frac{z}{\alpha})}{\phi(\frac{z}{\alpha})}$, $f_+(z) \stackrel{def}{=} \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} R(x)\frac{\phi(\frac{x}{\alpha})}{\phi(\frac{z}{\alpha})}\,\mathrm{d}x$, $f_-(z) \stackrel{def}{=} \frac{1}{2}\left(R(z) - R_\alpha(z)\right)$, $f_\phi(z) \stackrel{def}{=} \frac{\Phi'_\alpha(z)(z+\frac{\mu}{z})+\Phi_\alpha(z)-\Phi(\frac{z}{\alpha})}{\phi(\frac{z}{\alpha})}$. We can simplify $f_\phi(z) = \left(z+\frac{\mu}{\sigma}\right)\left(\frac{1}{2\alpha} + \frac{1}{\sqrt{2\pi}}R(z)\right) + f_+(z) - \frac{1}{2}R_\alpha(z)$. Also we can simplify $f_+(z) = \int_0^\infty \frac{1}{\sqrt{2\pi}} R(z-x)\frac{\phi(\frac{z-x}{\alpha})}{\phi(\frac{z}{\alpha})}\,\mathrm{d}x$. Since $R(z-x)$ is increasing and $\frac{\phi(\frac{z-x}{\alpha})}{\phi(\frac{z}{\alpha})} = e^{xz-\frac{1}{2}x^2}$ is increasing for $x > 0$, then we must have $f_+(z)$ is increasing in z and hence $f'_+(z) \geq 0$.

(1.) Consider $-\frac{\mu}{\sigma} < z < 0$. We show $f_\phi(z)$ is positive and increasing in z. Note, $f'_\phi(z) = \frac{1}{\sqrt{2\pi}}R'(z)(z+\frac{\mu}{\sigma}) - \frac{z}{2\alpha^2}R_\alpha(z) + \frac{1}{\sqrt{2\pi}}R(z) + f'_+(z)$. When $-\frac{\mu}{\sigma} < z < 0$, this expression is clearly positive. Next we show $f_\phi(z)$ is positive for $z > -\frac{\mu}{\sigma}+\epsilon$ since $\frac{\mu}{\sigma}$ is large enough for some small ϵ. Since $f_\phi(z)$ is increasing for $-\frac{\mu}{\sigma} < z < 0$, and $f_\phi(0) > 0$, for any given $\frac{\mu}{\sigma}$, there exists an $0 < \epsilon < \frac{\mu}{\sigma}$ such that $f_\phi(0) > 0$ for $-\frac{\mu}{\sigma} + \epsilon < z < 0$. Next, we argue that since $\frac{\mu}{\sigma}$ is large enough then $\epsilon \to 0$. Note $\Phi'_\alpha(z) = \phi(\frac{z}{\alpha})\left(\frac{1}{2\alpha} + \frac{1}{\sqrt{2\pi}}R(z)\right) > 0$. Also, $\frac{\mathrm{d}}{\mathrm{d}z}(\Phi_\alpha(z) - \Phi(\frac{z}{\alpha})) = \phi(\frac{z}{\alpha})\left(-\frac{1}{2\alpha} + \frac{1}{\sqrt{2\pi}}R(z)\right)$. Since $\frac{\mu}{\sigma}$ is large enough such that probability of negative demand is negligible, we have $\Phi_\alpha(-\frac{\mu}{\sigma}) - \Phi(-\frac{\mu}{\sigma\alpha})$ is negligible and it is decreasing. Hence, for a small $\epsilon > 0$, $\Phi_\alpha(-\frac{\mu}{\sigma}+\epsilon) - \Phi(\frac{-\frac{\mu}{\sigma}+\epsilon}{\alpha})$ is negligible. Also note that $\Phi'_\alpha(-\frac{\mu}{\sigma}+\epsilon)(\epsilon) > 0$. Therefore, for $z = -\frac{\mu}{\sigma}+\epsilon$, for some small $\epsilon > 0$ and large enough $\frac{\mu}{\sigma}$, $f_\phi(z) = \frac{\Phi'_\alpha(z)(z+\frac{\mu}{z})+\Phi_\alpha(z)-\Phi(\frac{z}{\alpha})}{\phi(\frac{z}{\alpha})} > 0$. In conclusion, we have shown that for $-\frac{\mu}{\sigma}+\epsilon < z < 0$ for some small $\epsilon > 0$, $f_\phi(z)$ is increasing in z and it is positive.

(2.) Next Consider $z \geq 0$. We show $f_\phi(z)$ is positive and increasing in z. Note since $f_\phi(0) > 0$, it suffices to show it is increasing in z for $z > 0$. Note that for $z > 0$ and $\frac{\mu}{\sigma} > \frac{\sqrt{2\pi}}{2}$,

$$\begin{aligned}
f'_\phi(z) &= \frac{1}{\sqrt{2\pi}}R'(z)(z+\frac{\mu}{\sigma}) - \frac{z}{2\alpha^2}R_\alpha(z) + \frac{1}{\sqrt{2\pi}}R(z) + f'_+(z) \\
&= \frac{1}{2}R'(z)\left(\frac{2}{\sqrt{2\pi}}(z+\frac{\mu}{\sigma})\right) + \frac{1}{2\alpha} - \frac{1}{2}R'_\alpha(z) + \frac{1}{\sqrt{2\pi}}R(z) + f'_+(z) \\
&> \frac{1}{2}R'(z) - \frac{1}{2}R'_\alpha(z) + f'_+(z) = f'_-(z) + f'_+(z)\,.
\end{aligned}$$

Therefore, it suffices to show $f'_-(z) + f'_+(z) > 0$. To show this, we show $f_+^{(k)}(0) + f_-^{(k)}(0) > 0$ for all $k \geq 0$, where $f_+^{(k)}(0)$ and $f_-^{(k)}(0)$ are the k-th derivarive of the functions $f_+(z)$ and $f_-(z)$, respectively, at zero. Thus, we could use taylor series expansion of $f'_+(z) + f'_-(z)$, around zero, to show $f'_+(z) + f'_-(z) > 0$. The rest of the proof shows that $f_+^{(k)}(0) + f_-^{(k)}(0) > 0$ for all $k \geq 0$.

(2.1.) In this step we provide expressions for the k-th derivatives of $R(z)$ and $f_-(z)$. We can verify that $R'(z) = \frac{\beta}{\alpha}\left(1+\frac{\beta}{\alpha}zR(z)\right)$ and $R^{(k)}(z) = \left(\frac{\beta}{\alpha}\right)^2\left((k-1)R^{(k-2)}(z) + z\,R^{(k-1)}(z)\right)$, for $k \geq 2$. Therefore, $R(0) = \frac{\sqrt{2\pi}}{2}$, $R'(0) = \frac{\beta}{\alpha}$, $R^{(k)}(0) = \left(\frac{\beta}{\alpha}\right)^2(k-1)R^{(k-2)}(0)$, for $k \geq 2$, $R^{(2k+1)}(0) = 2^k k!\left(\frac{\beta}{\alpha}\right)^{2k+1}$, for $k \geq 0$, $R^{(2k)}(0) = \frac{\sqrt{2\pi}}{2}\frac{(2k-1)!}{2^{k-1}(k-1)!}\left(\frac{\beta}{\alpha}\right)^{2k}$, for $k \geq 1$. Similarly, we can find derivatives of $R_\alpha(0)$. $R_\alpha^{(2k+1)}(0) = 2^k k!\left(\frac{1}{\alpha}\right)^{2k+1}$, for $k \geq 0$, $R_\alpha^{(2k)}(0) = \frac{\sqrt{2\pi}}{2}\frac{(2k-1)!}{2^{k-1}(k-1)!}\left(\frac{1}{\alpha}\right)^{2k}$, for $k \geq 1$. Therefore, for $k \geq 0$,

$$f_-^{(2k)}(0) = -\left(1-\beta^{2k}\right)\frac{(2k-1)!\sqrt{2\pi}}{2^{k+1}(k-1)!\alpha^{2k}}\,, \qquad f_-^{(2k+1)}(0) = -\left(1-\beta^{2k+1}\right)\frac{1}{2\alpha}\frac{2^k k!}{\alpha^{2k}}\,.$$

(2.2.) In this step, we provide expressions for k-th derivatives of $f_+(z)$: We can verify that $f'_+(z) = \frac{1}{\sqrt{2\pi}}R(z) + \frac{z}{\alpha^2}f_+(z)$, $f_+^{(k)}(z) = \frac{1}{\sqrt{2\pi}}R^{(k-1)}(z) + \frac{1}{\alpha^2}\left((k-1)f_+^{(k-2)}(z) + zf_+^{(k-1)}(z)\right)$, for $k \geq 2$. Therefore, $f_+(0) = \frac{1}{\sqrt{2\pi}}\arcsin(\alpha)$, $f'_+(0) = \frac{1}{2}$, $f_+^{(k)}(0) = \frac{1}{\sqrt{2\pi}}R^{(k-1)}(z) + \frac{1}{\alpha^2}(k-1)f_+^{(k-2)}(0)$, for $k \geq 2$. Using the formulas for $R^{(k)}(0)$, we can construct the following

$$f_+^{(2k+1)}(0) = \frac{2^k k!}{\alpha^{2k}}\left(\frac{1}{2} + \frac{1}{2}\sum_{j=1}^{k}\frac{(2j)!}{2^{2j}j!j!}\left(\beta^{2j}\right)\right) \qquad \text{for } k \geq 0$$

$$f_+^{(2k)}(0) = \frac{1}{\sqrt{2\pi}}\frac{(2k-1)!}{\alpha^{2k}2^{k-1}(k-1)!}\alpha\left(\frac{\arcsin(\alpha)}{\alpha} + \sum_{j=0}^{k-1}\frac{2^{2j}j!j!}{(2j+1)!}\left(\beta^{2j+1}\right)\right) \qquad \text{for } k \geq 1$$

(2.3.) Next we show that the odd derivatives of $f_+(0) + f_-(0)$ are all non-negative. Using previous steps, we have $f_+^{(2k+1)}(0) + f_-^{(2k+1)}(0) = \frac{2^{k-1}k!}{\alpha^{2k+1}}\left(\beta^{2k+1} - 1 + \alpha + \alpha\sum_{j=1}^{k}\frac{(2j)!}{2^{2j}j!j!}\beta^{2j}\right)$. Define the following two series: $S_o(k) \stackrel{def}{=} \beta^{2k+1} - 1 + \alpha + \alpha\sum_{j=1}^{k}\frac{(2j)!}{2^{2j}j!j!}\beta^{2j}$, and $L_o(k) \stackrel{def}{=} \alpha\sum_{j=k}^{\infty}\beta^{2j+1}\left(\alpha - \frac{(2(j+1))!}{2^{2(j+1)}(j+1)!(j+1)!}\beta\right)$. To show the odd derivatives of $f_+(0) + f_-(0)$ are all non-negative, it suffices to show $S_o(k)$ is non-negative. First we show that $S_o(k) = L_o(k)$ and then we show $L_o(k)$ is non-negative.

(2.3.1.) First we show that $S_o(k) = L_o(k)$. Notice that $L_o(0) = S_o(0) = \sqrt{1-\alpha^2} - (1-\alpha) \geq 0$. Also, $S_o(k+1) - S_o(k) = -\alpha\beta^{2k+1}\left(\alpha - \frac{(2(k+1))!}{2^{2(k+1)}(k+1)!(k+1)!}\beta\right) = L_o(k+1) - L_o(k)$. Therefore, we must have $S_o(k) = Lo(k)$.

(2.3.2.) Next we show $L_o(k) \geq 0$ for all k. It is easy to verify that $\frac{(2(j+1))!}{2^{2(j+1)}(j+1)!(j+1)!}$ is decreasing in j and it approaches zero as $j \to \infty$. Therefore, there exist a $\tilde{j} \geq 0$ such that $\alpha - \frac{(2(k+1))!}{2^{2(k+1)}(k+1)!(k+1)!}\beta \geq 0$ if and only if $j \geq \tilde{j}$. Therefore, we know $L_o(k) \geq 0$ for $k \geq \tilde{j}$. Consider any $k < \tilde{j}$. We know $L_o(k+1) - L_o(k) = -\alpha\beta^{2k+1}\left(\alpha - \frac{(2(k+1))!}{2^{2(k+1)}(k+1)!(k+1)!}\beta\right) \geq 0$, for $k < \tilde{j}$, and $L_o(0) = \sqrt{1-\alpha^2} - (1-\alpha) \geq 0$. Therefore, $L_o(k) \geq L_o(0) = \sqrt{1-\alpha^2} - (1-\alpha) \geq 0$ for $k < \tilde{j}$. In conclusion, $L_o(k) > 0$ for all k.

(2.4.) Next we show that the even derivatives of $f_+(0) + f_-(0)$ are all non-negative. Using previous steps, we have $f_+^{(2k)}(0) + f_-^{(2k)}(0) = \frac{1}{\sqrt{2\pi}}\frac{(2k-1)!}{\alpha^{2k}2^{k-1}(k-1)!}\left(\arcsin(\alpha) - \frac{\pi}{2}\left(1-\beta^{2k}\right) + \alpha\sum_{j=0}^{k-1}\frac{2^{2(j)}(j)!(j)!}{(2j+1)!}\beta^{2j+1}\right)$. Define the following two series: $S_e(k) \stackrel{def}{=} \arcsin(\alpha) - \frac{\pi}{2}\left(1-\beta^{2k}\right) + \alpha\sum_{j=0}^{k-1}\frac{2^{2j}j!j!}{(2j+1)!}\beta^{2j+1}$, and $L_e(k) \stackrel{def}{=} \sum_{j=k}^{\infty}\beta^{2j}\left(\left(1-\beta^2\right)\frac{\pi}{2} - \frac{2^{2j}j!j!}{(2j+1)!}\alpha\beta\right)$. To show the even derivatives of $f_+(0) + f_-(0)$ are all non-negative, it suffices to show $S_e(k)$ is non-negative. First we show that $S_e(k) = L_e(k)$ and then we show $L_e(k)$ is non-negative.

(2.4.1.) First we show that $S_e(k) = L_e(k)$. Notice that $S_e(1) = \arcsin(\alpha) = \frac{\pi}{2} - \arcsin\sqrt{1-\alpha^2} = L_e(1)$. Also, $S_e(k+1) - S_e(k) = -\beta^{2k}\left(\frac{\pi}{2}\left(1-\beta^2\right) - \frac{2^{2k}k!k!}{(2k+1)!}\alpha\beta\right) = L_e(k+1) - L_e(k)$. Therefore, we must have $S_0(k) = Lo(k)$.

(2.4.2.) Next we show $L_e(k) \geq 0$ for all k. It is easy to verify that $\frac{2^{2j}j!j!}{(2j+1)!}$ is decreasing in j and it approaches zero as $j \to \infty$. Therefore, there exist a $\tilde{j} \geq 0$ such that $(1-\beta^2) - \frac{2^{2(k)}k!k!}{(2k+1)!}\alpha\beta \geq 0$ if and only if $j \geq \tilde{j}$. Therefore, we know $L_e(k) \geq 0$ for $k \geq \tilde{j}$. Consider any $k < \tilde{j}$. We know $L_e(k+1) - L_e(k) = -\beta^{2k}\left(\frac{\pi}{2}\left(1-\beta^2\right) - \frac{2^{2(k)}k!k!}{(2k+1)!}\alpha\beta\right) \geq 0$, for $k < \tilde{j}$, and $L_e(1) = \frac{\pi}{2} - \arcsin\sqrt{1-\alpha^2} \geq 0$. Therefore, $L_e(k) \geq L_e(1) \geq 0$ for $k < \tilde{j}$. In conclusion, $L_e(k) > 0$ for all $k \geq 1$.

Steps (2.) through (2.4.2.) establishes that for $z \geq 0$, $f_\phi(z)$ is increasing in z and it is positive since $\frac{\mu}{\sigma}$ is large enough that the probability of negative demand is negligible. Also, together with step (1.), we have $f_\phi(z)$ is increasing in z for all $z > -\frac{\mu}{\sigma} + \epsilon$ and it is positive since $\frac{\mu}{\sigma}$ is large enough. ∎

Lemma A.3. *Define* $T(z) \overset{def}{=} \int_{-\infty}^{z} \Phi(x) - \Phi\left(\frac{x}{\alpha}\right)$, *and* $g(z) \overset{def}{=} \Phi(z) - \Phi_\alpha(z)$, *where* α *and* Φ *and* Φ_α *(and* ϕ*) are defined at the end of proof of Lemma A.1. Also, let* $\beta \overset{def}{=} \frac{\alpha}{\sqrt{1-\alpha^2}}$. *Then*

1. $g(\alpha\, z) = \int_0^\infty \int_0^\infty \phi(x+z)\beta\phi(\beta(x+y))\,\mathrm{d}y\,\mathrm{d}x,$
2. $\frac{1}{\alpha}T(\alpha\, z) = 2\int_0^\infty \int_0^\infty \int_0^\infty \phi(x+z-v)\beta\phi(\beta(x+y+v))\,\mathrm{d}y\,\mathrm{d}x\,\mathrm{d}v,$
3. $\frac{T(z)}{g(z)}$ *is log convex (and hence convex).*

Proof of Lemma A.3. Let $g_0(z) = g(\alpha\, z)$ and $T_0(z) = \frac{1}{\alpha}T(\alpha\, z)$. To show $\frac{T(z)}{g(z)}$ is log convex, it suffices to show $\frac{T_0(z)}{g_0(z)}$ is log convex. **First**, we simplify g_0. Notice that $g_0(z) = Pr(X_1 < \alpha z, X_t > z)$, where (X_1, X_t) are standard bivariate normal random variables with correlation α. Hence, $g_0(z) = \int_{-\infty}^{\alpha\, z}\int_z^\infty \frac{1}{2\pi\sqrt{1-\alpha^2}} \exp\left(-\frac{1}{2\sqrt{1-\alpha^2}}(x^2+y^2-2\alpha xy)\right)\mathrm{d}x\,\mathrm{d}y$. In this integral, change the variables as follows: $y_{new} = z - \frac{y}{\alpha}$ and $x_{new} = x - z$. Then, we can simplify the integral as follows: $g_0(z) = \int_0^\infty\int_0^\infty \frac{\beta}{2\pi}\exp\left(-\frac{1}{2}\left(\beta^2(x+y)^2 + (x+z)^2\right)\right)\mathrm{d}x\,\mathrm{d}y = \int_0^\infty\int_0^\infty \beta\phi(\beta(x+y))\,\phi(x+z)\;\mathrm{d}y\,\mathrm{d}x = \int_0^\infty \Phi(-\beta(x))\,\phi(x+z)\;\mathrm{d}y\,\mathrm{d}x$. **Second**, we simplify T_0. Taking $(X1, X_t)$ as defined before, we have: $T_0(z) = \frac{1}{\alpha}T(\alpha z) = \frac{1}{\alpha}\int_{-\infty}^{\alpha z}\Phi(x) - \Phi\left(\frac{x}{\alpha}\right) = \int_{-\infty}^{z}\Phi(\alpha x) - \Phi(x)\,\mathrm{d}x = \int_{-\infty}^{z}\left(Pr(X_1 < \alpha x) - Pr(X_t < x)\right)\mathrm{d}x = \int_{-\infty}^{z}\left(Pr(X_1 < \alpha x, X_t > x) - Pr(X_1 > \alpha x, X_t < x)\right)\mathrm{d}x = \int_{-\infty}^{z} g_0(u) - g_0(-u)\,\mathrm{d}u$. Using the simplified version of g_0, and by changing the variables of the integral as follows $u_{new} = z - u$, we conclude $T_0(z) = \int_0^\infty\int_0^\infty \left(\phi(x+z-u) - \phi(x-z+u)\right)\Phi(-\beta x)\,\mathrm{d}x\,\mathrm{d}u$. Change the order of integration and integrate with respect to u, we have

$T_0(z) = \int_0^\infty \left(\Phi(x+z) - \Phi(-x+z)\right)\Phi(-\beta x)\,\mathrm{d}x$. Replace $\Phi(x+z) - \Phi(-x+z) = \int_{-x}^{x}\phi(v+z)\,\mathrm{d}v$, we get $T_0(z) = \int_0^\infty\int_{-x}^{x}\phi(v+z)\Phi(-\beta x)\,\mathrm{d}v\,\mathrm{d}x$. Change the variables as follows: $\frac{x+v}{2} = x_{new}$, and $\frac{x-v}{2} = v_{new}$, we have $T_0(z) = 2\int_0^\infty\int_0^\infty \phi(x-v+z)\beta\Phi(-\beta(x+v))\,\mathrm{d}v\,\mathrm{d}x$. Expand Φ to its integral form: $T_0(z) = 2\int_0^\infty\int_0^\infty\int_0^\infty \phi(x-v+z)\beta\phi(\beta(x+y+v))\,\mathrm{d}y\,\mathrm{d}x\,\mathrm{d}v$. **Third**, we know

$$\frac{T_0(z)}{g_0(z)} = 2\int_0^\infty \frac{\int_0^\infty\int_0^\infty \phi(x-v+z)\beta\phi(\beta(x+y+v))\,\mathrm{d}y\,\mathrm{d}x}{\int_0^\infty\int_0^\infty \phi(x+z)\beta\phi(\beta(x+y))\,\mathrm{d}y\,\mathrm{d}x}\,\mathrm{d}v$$

In Lemma A.4, we show that $\partial_{z,z,v}\log(F) > 0$, where $F = \int_0^\infty\int_0^\infty \beta\phi(\beta(x+y+v))\,\phi(x-v+z)\;\mathrm{d}y\,\mathrm{d}x$ is the numerator (and also denominator with $v = 0$) of the above fraction. Hence, we must have, for every v, $\frac{\int_0^\infty\int_0^\infty \phi(x-w+z)\beta\phi(\beta(x+y+w))\,\mathrm{d}y\,\mathrm{d}x}{\int_0^\infty\int_0^\infty \phi(x+z)\beta\phi(\beta(x+y))\,\mathrm{d}y\,\mathrm{d}x}$ is log-convex; hence $\frac{T_0(z)}{g_0(z)}$ and $\frac{T(z)}{g(z)}$ are log-convex. ■

Lemma A.4. *Define* $F = \int_0^\infty\int_0^\infty \beta\phi(\beta(x+y+v))\,\phi(x-v+z)\;\mathrm{d}y\,\mathrm{d}x$, *with parameter* $\beta > 0$. *Then,* $\partial_{z,z,v}\log(F) > 0$, *where* ∂ *is the partial derivative with respect to the specified parameters.*

Proof of Lemma A.4. Define random variables (X, Y) defined for positive values with the probability distribution function $f(x,y) = \frac{1}{F}\beta\phi(\beta(x+y+v))\,\phi(x-v+z)$. Throughout this proof, we use the symbol $\mathbb{E}$ as the expected value operator with respect to distribution of (X,Y). It is easy to verify that $\partial_z \log(F) = v-z-\mu_x$ and $\partial_{z,z}\log(F) = -1+\sigma_x^2$, where, μ_x and σ_x^2 are mean and variance of random variable X and depend on parameters v and z. Specifically, $\mu_x = \frac{1}{F}\int_0^\infty\int_0^\infty x\beta\phi(\beta(x+y+v))\,\phi(x-v+z)\,\mathrm{d}y\,\mathrm{d}x$ and $\sigma_x^2 = \frac{1}{F}\int_0^\infty\int_0^\infty (x-\mu_x)^2\beta\phi(\beta(x+y+v))\,\phi(x-v+z)\,\mathrm{d}y\,\mathrm{d}x$. Define μ_t as expected value of the $X+Y$. We can evaluate μ_t as follows: $\mu_t = \frac{1}{F}\int_0^\infty\int_0^\infty (x+y)\beta\phi(\beta(x+y+v))\,\phi(x-v+z)\,\mathrm{d}y\,\mathrm{d}x = -v-\frac{1}{\beta^2 F}\int_0^\infty\int_0^\infty \beta\phi'(\beta(x+y+v))\,\phi(x-v+z)\,\mathrm{d}y\,\mathrm{d}x = -v+\frac{1}{\beta^2 F}\int_0^\infty \beta\phi(\beta(x+v))\,\phi(x-v+z)\,\mathrm{d}x$. Hence, $\beta^2(v+\mu_t) = \frac{1}{F}\int_0^\infty \beta\phi(\beta(x+v))\,\phi(x-v+z)\,\mathrm{d}x$, that we use below. Since, $\partial_{z,z}\log(F) = -1+\sigma_x^2$, from definition of σ_x^2, $\partial_{z,z,v}\log(F) = \partial_v\sigma_x^2 = \mathbb{E}\left[(X-\mu_x)^3 - \beta^2(X-\mu_x)^2(X+Y-\mu_t)\right] = \mathbb{E}\left[(X-\mu_x)^3 - \beta^2(X-\mu_x)^2(X+Y+v-v-\mu_t)\right]$. We expand parts of this expectation as follow

$$\beta^2\,\mathbb{E}\left[\beta^2(x-\mu_x)^2(v+\mu_t)\right] = \sigma_x^2\beta^2(v+\mu_t) = \sigma_x^2\frac{1}{F}\int_0^\infty \beta\phi(\beta(x+v))\,\phi(x-v+z)\,\mathrm{d}x$$

$$\mathbb{E}[\beta^2(x-\mu_x)^2(x+y+v)] = \frac{1}{F}\int_0^\infty\int_0^\infty -(x-\mu_x)^2\beta\phi'(\beta(x+y+v))\,\phi(x-v+z)\,\mathrm{d}y\,\mathrm{d}x$$

$$= \frac{1}{F}\int_0^\infty (x-\mu_x)^2\beta\phi(\beta(x+v))\,\phi(x-v+z)\,\mathrm{d}x$$

Let $f_y(y)$ be marginal distribution of Y. Notice that $f_y(0) = \frac{1}{F}\int_0^\infty \beta\phi(\beta(x+v))\,\phi(x-v+z)\,\mathrm{d}x$. Also notice that $\frac{\beta\phi(\beta(x+v))\,\phi(x-v+z)}{\int_0^\infty \beta\phi(\beta(x+v))\,\phi(x-v+z)\,\mathrm{d}x}$ is the conditional probability distribution function of X given $Y=0$. Hence, $\mathbb{E}\left[-\beta^2(X-\mu_x)^2(X+Y+v-v-\mu_t)\right] = f_y(0)\left(-\mathbb{E}[(X-\mu_x)^2|Y=0] + \mathbb{E}[(X-\mu_x)^2]\right)$ Hence, $\partial_{z,z,v}\log(F) = \mathbb{E}\left[(X-\mu_x)^3\right] + f_y(0)(-\mathbb{E}[(X-\mu_x)^2|Y=0] + \mathbb{E}[(X-\mu_x)^2])$.

Notice that (X,Y) are truncated bivariate normal distribution with a negative correlation. Hence conditional distribution of $(X|Y=0)$ second order stochastically dominates marginal distribution of X. Then since $(x-\mu_x)^2$ is convex in x, we must have $E[(x-\mu_x)^2] > E[(x-\mu_x)^2|Y=0]$. Also, since (X,Y) are truncated from below, we must have $E\left[(x-\mu_x)^3\right] > 0$. In conclusion, $\partial_{z,z,v}\log(F) > 0$.

For the sake of completeness, next, we show that conditional distribution of $(X|Y=0)$ second order stochastically dominates marginal distribution of X. Let $f_{x|y}(x)$ and $f_x(x)$ denote the conditional pdf of $(X|Y=0)$ and marginal pdf of X, respectively, we must have: $f_{x|y}(x) = \frac{1}{G}\beta\phi(\beta(x+v))\,\phi(x-v+z)$, and $f_x(x) = \frac{1}{F}\Phi(-\beta(x+v))\,\phi(x-v+z)$, where $G := \int_0^\infty \beta\phi(\beta(x+v))\,\phi(x-v+z)\,\mathrm{d}x$. To show $(X|Y=0)$ second order stochastically dominates X, we must show for any given $w > 0$, we have $\int_0^w \int_0^t (f_x(x) - f_{x|y}(x))\,\mathrm{d}x\,\mathrm{d}t > 0$. To show this, note that

$$\int_0^w \int_0^t (f_x(x) - f_{x|y}(x))\,\mathrm{d}x\,\mathrm{d}t =$$
$$\int_0^w \int_0^t \left(\frac{G}{G} f_x(x) - \frac{F}{F} f_{x|y}(x)\right) \mathrm{d}x\,\mathrm{d}t$$
$$= \int_0^w \int_0^t \int_0^\infty \left(f_{x|y}(x0) f_x(x) - f_x(x0) f_{x|y}(x)\right) \mathrm{d}x0\,\mathrm{d}x\,\mathrm{d}t$$
$$= \frac{G}{F} \int_0^w \int_0^t \int_0^\infty \left(\frac{\Phi(-\beta(x+v))}{\phi(\beta(x+v))} - \frac{\Phi(-\beta(x0+v))}{\phi(\beta(x0+v))}\right) f_{x|y}(x0) f_{x|y}(x)\,\mathrm{d}x0\,\mathrm{d}x\,\mathrm{d}t$$

Since x and $x0$ are symmetric in the integrand, we can conclude the last line is equal to

$$= \frac{G}{F} \int_0^w \int_0^t \int_t^\infty \left(\frac{\Phi(-\beta(x+v))}{\phi(\beta(x+v))} - \frac{\Phi(-\beta(x0+v))}{\phi(\beta(x0+v))}\right) f_{x|y}(x0) f_{x|y}(x)\,\mathrm{d}x0\,\mathrm{d}x\,\mathrm{d}t$$

Since, in this expression, $x < t < x_0$, and $\frac{\Phi(-\beta(x+v))}{\phi(\beta(x+v))}$ is decreasing in x, we must have the integrand is always positive in the region of integration and hence the integral must be positive. This establishes that $(X|Y = 0)$ second order stochastically dominates X. ■

Lemma A.5. *Define $f_\theta(z) \overset{def}{=} T(z) - g(z)\left(z + \frac{\mu}{\sigma}\right)$, where $T(z)$ and $g(z)$ are defined in Lemma A.3. Then*

1. *$f_\theta(z) = 0$ has a unique solution for $z > -\frac{\mu}{\sigma}$; Let z^f represent this unique solution.*
2. *$f_\theta'(z^f) > 0$.*
3. *for $z < z^f$, we have $f_\theta(z) < 0$ and for $z > z^f$, we have $f_\theta(z) > 0$.*

Proof of Lemma A.5. In Lemma A.3, we showed that $\frac{T(z)}{g(z)}$ is log-convex. This implies that $\frac{T(z)}{g(z)} - (z + \frac{\mu}{\sigma})$ is convex and has at most two solutions. Note that for, $\frac{\mu}{\sigma}$ large enough, when $z \to -\frac{\mu}{\sigma}$, we have $\left(\frac{T(z)}{g(z)} - (z + \frac{\mu}{\sigma})\right) \to 0$, and $\partial_z \left(\frac{T(z)}{g(z)} - (z + \frac{\mu}{\sigma})\right) \to -1$. Also, when $z \to \infty$, we have $\left(\frac{T(z)}{g(z)} - (z + \frac{\mu}{\sigma})\right) \to +\infty$.

In conclusion, at the lower limit, $\frac{T(z)}{g(z)} - (z + \frac{\mu}{\sigma})$ is zero and it is decreasing in z and at the upper limit it is positive. In addition, it is a convex function of z. Hence, it must have a uniques solution $z^f > -\frac{\mu}{\sigma}$. Note that $f_\theta(z) = g(z)\left(\frac{T(z)}{g(z)} - (z + \frac{\mu}{\sigma})\right)$ and $g(z) > 0$. As a result, $f_\theta(z^f) = 0$, and the sign of $f_\theta(z)$ is consistent with the sign of $\frac{T(z)}{g(z)} - (z + \frac{\mu}{\sigma})$ which is positive, for $z > z^f$ and negative for $z < z^f$. It also follows that $f_\theta'(z^f) > 0$. ■

Lemma A.6. *In case of three symmetric buyers, let the expected transfer to buyer x, when she has reserved capacity Q_x and buyer 1 and 2 reserve capacity Q each be denoted $T_{x+}(Q_x, Q)$. Also, let* *the expected transfer from buyer x to other buyers, when she has reserved capacity Q_x and buyer 1 and 2 have reserved capacity Q each be $T_{x-}(Q_x, Q)$. Then,*

$T_{x-}(Q_x, Q) = \int_{y=-\infty}^{Q_x} Pr(D_x < y, D_1 < Q, D_2 > Q_x + Q - y)\,\mathrm{d}y + \int_{y=-\infty}^{Q_x} Pr(D_x < y, D_1 < Q, D_1 + D_2 > 2Q + Q_x - y)\,\mathrm{d}y + \int_{y=-\infty}^{Q_x} Pr(D_x < y, D_1 > Q, D_2 > Q, D_1 + D_2 > Q_x + 2Q - y)\,\mathrm{d}y,$

$\frac{\mathrm{d}}{\mathrm{d}Q_x} T_{x-}(Q_x, Q) = Pr(D_x < Q_x) - Pr(D_i < Q_x, D_x + D_1 < Q_x + Q, D_x + D_1 + D_2 < Q_x + 2Q) > 0,$

$T_{x+}(Q_x, Q) = \int_{y=Q_x}^{\infty} Pr(D_x > y, D_1 > Q, D_2 < Q + Q_x - y)\,\mathrm{d}y + \int_{y=Q_x}^{\infty} Pr(D_x > y, D_1 < Q, D_1 + D_2 < 2Q + Q_x - y)\,\mathrm{d}y;$

$\frac{\mathrm{d}}{\mathrm{d}Q_x} T_{x+}(Q_x, Q) = -Pr(D_x > Q_x, D_1 > Q, D_x + D_2 < Q + Q_x) - Pr(D_x > Q_x, D_1 < Q, D_x + D_1 + D_2 < 2Q + Q_x) < 0.$

Proof of Lemma A.6. By definition of $T_{x-}(Q_x, Q)$, we have

$$
\begin{aligned}
T_{x-} = {} & \frac{1}{2}\mathbb{E}[(Q_x - D_x) \times \mathbb{I}(D_x < Q_x, D_1 < Q, D_2 > Q) \times \mathbb{I}(D_x + D_2 > Q_x + Q)] && \text{(A.1)}\\
& + \frac{1}{2}\mathbb{E}[(D_2 - Q) \times \mathbb{I}(D_x < Q_x, D_1 < Q, D_2 > Q) \times \mathbb{I}(D_x + D_2 < Q_x + Q)] && \text{(A.2)}\\
& + \frac{1}{2}\mathbb{E}[(Q_x - D_x) \times \mathbb{I}(D_x < Q_x, D_1 < Q, D_2 > Q) \times \mathbb{I}(D_x + D_1 + D_2 > Q_x + 2Q)] && \text{(A.3)}\\
& + \frac{1}{2}\mathbb{E}[(D_1 + D_2 - 2Q) \times \mathbb{I}(D_x < Q_x, D_1 < Q, D_2 > Q) \times \mathbb{I}(D_1 + D_2 > 2Q, D_x + D_1 + D_2 < Q_x + 2Q)] && \text{(A.4)}\\
& + \mathbb{E}[(Q_x - D_x) \times \mathbb{I}(D_x < Q_x, D_1 > Q, D_2 > Q) \times \mathbb{I}(D_x + D_1 + D_2 > Q_x + 2Q)] && \text{(A.5)}\\
& + \mathbb{E}[(D_1 + D_2 - 2Q) \times \mathbb{I}(D_x < Q_x, D_1 > Q, D_2 > Q) \times \mathbb{I}(D_x + D_1 + D_2 < Q_x + 2Q)] && \text{(A.6)}\\
& + \frac{1}{2}\mathbb{E}[(Q_x - D_x) \times \mathbb{I}(D_x < Q_x, D_1 > Q, D_2 < Q) \times \mathbb{I}(D_x + D_1 > Q_x + Q)] && \text{(A.7)}\\
& + \frac{1}{2}\mathbb{E}[(D_1 - Q) \times \mathbb{I}(D_x < Q_x, D_1 > Q, D_2 < Q) \times \mathbb{I}(D_x + D_1 < Q_x + Q)] && \text{(A.8)}\\
& + \frac{1}{2}\mathbb{E}[(Q_x - D_x) \times \mathbb{I}(D_x < Q_x, D_1 > Q, D_2 < Q) \times \mathbb{I}(D_x + D_1 + D_2 > Q_x + 2Q)] && \text{(A.9)}\\
& + \frac{1}{2}\mathbb{E}[(D_1 + D_2 - 2Q) \times \mathbb{I}(D_x < Q_x, D_1 > Q, D_2 < Q) \times \mathbb{I}(D_1 + D_1 > 2Q, D_x + D_1 + D_2 < Q_x + 2Q)] && \text{(A.10)}
\end{aligned}
$$

We explain in detail how we simplify the first line (A.1) and the second line (A.2) in the above sum. The details of how we simplify the remaining lines would be very similar to the first and second line and we omit the details.

Suppose $\phi(d_x, d_1, d_2)$ is joint p.d.f. of (D_x, D_1, D_2). Focusing on the first line in the sum (A.1), note that $\int_{d_x=-\infty}^{x} \int_{d_1=-\infty}^{Q} \int_{d_2=Q_x+Q-d_x}^{\infty} \phi(d_x, d_1, d_2)\,\mathrm{d}d_2\,\mathrm{d}d_1\,\mathrm{d}d_x = Pr(D_x < x, D_1 < Q, D_x + D_2 > Q_x +$

$Q)$. Hence, $\frac{\mathrm{d}}{\mathrm{d}x} Pr(D_x < x, D_1 < Q, D_x + D_2 > Q_x + Q) = \int_{d_1=-\infty}^{Q} \int_{d_2=Q_x+Q-x}^{\infty} \phi(x, d_1, d_2)\, \mathrm{d}d_2\, \mathrm{d}d_1$.

Hence,

$$\begin{aligned}
&\mathbb{E}[(Q_x - D_x) \times \mathbb{I}(D_x < Q_x, D_1 < Q, D_2 > Q) \times \mathbb{I}(D_x + D_2 > Q_x + Q)] \\
&= \int_{x=-\infty}^{Q_x} (Q_x - x) \int_{d_1=-\infty}^{Q} \int_{d_2=Q_x+Q-x}^{\infty} \phi(d_i, d_1, d_2)\, \mathrm{d}d_2\, \mathrm{d}d_1\, \mathrm{d}x \\
&= \int_{x=-\infty}^{Q_x} (Q_x - x) \frac{\mathrm{d}}{\mathrm{d}x} Pr(D_x < x, D_1 < Q, D_x + D_2 > Q_x + Q)\, \mathrm{d}x \\
&= \int_{x=-\infty}^{Q_x} Pr(D_x < x, D_1 < Q, D_1 + D_2 > Q_x + Q)\, \mathrm{d}x
\end{aligned}$$

Similarly, focusing on the second line (A.2) of the sum:

$$\begin{aligned}
&\mathbb{E}[(D_2 - Q) \times \mathbb{I}(D_x < Q_x, D_1 < Q, D_2 > Q) \times \mathbb{I}(D_x + D_2 < Q_x + Q)] \\
&= \int_{y=Q}^{\infty} Pr(D_1 < Q, D_2 > y, D_x + D_2 < Q_x + Q)\, \mathrm{d}y \\
&= \int_{x=-\infty}^{Q_x} Pr(D_1 < Q, D_2 > Q_x + Q - x, D_x + D_2 < Q_x + Q)\, \mathrm{d}x
\end{aligned}$$

As a result, the sum of the first line (A.1) and the second line (A.2) of the sum is

$$\begin{aligned}
&\mathbb{E}[(Q_x - D_x) \times \mathbb{I}(D_x < Q_x, D_1 < Q, D_2 > Q) \times \mathbb{I}(D_x + D_2 > Q_x + Q)] \\
&+ \mathbb{E}[(D_2 - Q) \times \mathbb{I}(D_x < Q_x, D_1 < Q, D_2 > Q) \times \mathbb{I}(D_x + D_2 < Q_x + Q)] \\
&= \int_{x=-\infty}^{Q_x} Pr(D_x < x, D_1 < Q, D_x + D_2 > Q_x + Q) + Pr(D_1 < Q, D_2 > Q_x + Q - x, D_x + D_2 < Q_x + Q)\, \mathrm{d}x \\
&= \int_{x=-\infty}^{Q_x} Pr(D_x < x, D_1 < Q, D_2 > Q_x + Q - x)\, \mathrm{d}x
\end{aligned}$$

Hence, the derivative of the first and the second term of the sum is

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}Q_x} \mathbb{E}[(Q_x - D_x) \times \mathbb{I}(D_x < Q_x, D_1 < Q, D_2 > Q) \times \mathbb{I}(D_x + D_2 > Q_x + Q)] \\
&+ \frac{\mathrm{d}}{\mathrm{d}Q_x} \mathbb{E}[(D_2 - Q) \times \mathbb{I}(D_x < Q_x, D_1 < Q, D_2 > Q) \times \mathbb{I}(D_x + D_2 < Q_x + Q)] \\
&= Pr(D_x < Q_x, D_1 < Q, D_2 > Q) + \int_{x=-\infty}^{Q_x} \frac{\mathrm{d}}{\mathrm{d}Q_x} Pr(D_x < x, D_1 < Q, D_2 > Q_x + Q - x)\, \mathrm{d}x \\
&= Pr(D_x < Q_x, D_1 < Q, D_x + D_2 > Q_x + Q)
\end{aligned}$$

Sum of third line (A.3), and fourth line (A.4) is

$$\mathbb{E}[(Q_x - D_x) \times \mathbb{I}(D_x < Q_x, D_1 < Q, D_2 > Q) \times \mathbb{I}(D_x + D_1 + D_2 > Q_x + 2Q)]$$

$$+ \mathbb{E}[(D_1 + D_2 - 2Q) \times \mathbb{I}(D_x < Q_x, D_1 < Q, D_2 > Q) \times \mathbb{I}(D_1 + D_2 > 2Q, D_x + D_1 + D_2 < Q_x + 2Q)]$$

$$= \int_{x=-\infty}^{Q_x} Pr(D_x < x, D_1 < Q, D_x + D_1 + D_2 > Q_x + 2Q)$$

$$+ Pr(D_1 < Q, D_x + D_1 + D_2 < Q_x + 2Q, D_1 + D_2 > 2Q + Q_x - x)\,\mathrm{d}x$$

$$= \int_{x=-\infty}^{Q_x} Pr(D_x < x, D_1 < Q, D_1 + D_2 > 2Q + Q_x - x)\,\mathrm{d}x$$

Therefore, derivative of third line (A.3), and fourth line (A.4) is

$$\frac{\mathrm{d}}{\mathrm{d}Q_x}\mathbb{E}[(Q_x - D_x) \times \mathbb{I}(D_x < Q_x, D_1 < Q, D_2 > Q) \times \mathbb{I}(D_x + D_1 + D_2 > Q_x + 2Q)]$$

$$+ \frac{\mathrm{d}}{\mathrm{d}Q_x}\mathbb{E}[(D_1 + D_2 - 2Q) \times \mathbb{I}(D_x < Q_x, D_1 < Q, D_2 > Q) \times \mathbb{I}(D_1 + D_2 > 2Q, D_x + D_1 + D_2 < Q_x + 2Q)]$$

$$= Pr(D_x < Q_x, D_1 < Q, D_1 + D_2 > 2Q) + \int_{x=-\infty}^{Q_x} \frac{\mathrm{d}}{\mathrm{d}Q_x} Pr(D_x < x, D_1 < Q, D_1 + D_2 > 2Q + Q_x - x)\,\mathrm{d}x$$

$$= Pr(D_x < Q_x, D_1 < Q, D_x + D_1 + D_2 > 2Q + Q_x)$$

Sum of fifth line (A.5), and sixth line (A.6) is

$$\mathbb{E}[(Q_x - D_x) \times \mathbb{I}(D_x < Q_x, D_1 > Q, D_2 > Q) \times \mathbb{I}(D_x + D_1 + D_2 > Q_x + 2Q)]$$

$$+ \mathbb{E}[(D_1 + D_2 - 2Q) \times \mathbb{I}(D_x < Q_x, D_1 > Q, D_2 > Q) \times \mathbb{I}(D_x + D_1 + D_2 < Q_x + 2Q)]$$

$$= \int_{x=-\infty}^{Q_x} Pr(D_x < x, D_1 > Q, D_2 > Q, D_x + D_1 + D_2 > Q_x + 2Q)$$

$$+ Pr(D_1 > Q, D_2 > Q, D_x + D_1 + D_2 < Q_x + 2Q, D_1 + D_2 > 2Q + Q_x - x)\,\mathrm{d}x$$

$$= \int_{x=-\infty}^{Q_x} Pr(D_x < x, D_1 > Q, D_2 > Q, D_1 + D_2 > Q_x + 2Q - x)\,\mathrm{d}x$$

Derivatives of fifth line (A.5), and sixth line (A.6) is

$$\frac{\mathrm{d}}{\mathrm{d}Q_x}\mathbb{E}[(Q_x - D_x) \times \mathbb{I}(D_x < Q_x, D_1 > Q, D_2 > Q) \times \mathbb{I}(D_x + D_1 + D_2 > Q_x + 2Q)]$$

$$+ \frac{\mathrm{d}}{\mathrm{d}Q_x}\mathbb{E}[(D_1 + D_2 - 2Q) \times \mathbb{I}(D_x < Q_x, D_1 > Q, D_2 > Q) \times \mathbb{I}(D_x + D_1 + D_2 < Q_x + 2Q)]$$

$$= Pr(D_x < Q_x, D_1 > Q, D_2 > Q, D_1 + D_2 > 2Q)\,\mathrm{d}x$$

$$+ \int_{x=-\infty}^{Q_x} \frac{\mathrm{d}}{\mathrm{d}Q_x} Pr(D_x < x, D_1 > Q, D_2 > Q, D_1 + D_2 > Q_x + 2Q - x)\,\mathrm{d}x$$

$$= Pr(D_x < Q_x, D_1 > Q, D_2 > Q, D_x + D_1 + D_2 > 2Q + Q_x)$$

Because of the symmetry of the buyers, the sum of lines (A.7) to (A.10) are equal to the sum of line (A.1) to (A.4). As a results, we can summarize that

$$
\begin{aligned}
T_{x-}(Q_x, Q) = &\int_{x=-\infty}^{Q_x} Pr(D_x < x, D_1 < Q, D_2 > Q_x + Q - x)\,\mathrm{d}x \\
&+ \int_{x=-\infty}^{Q_x} Pr(D_x < x, D_1 < Q, D_1 + D_2 > 2Q + Q_x - x)\,\mathrm{d}x \\
&\int_{x=-\infty}^{Q_x} Pr(D_x < x, D_1 > Q, D_2 > Q, D_1 + D_2 > Q_x + 2Q - x)\,\mathrm{d}x
\end{aligned}
$$

Also,

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}Q_x} T_{x-}(Q_x, Q) &= Pr(D_x < Q_x, D_1 < Q, D_x + D_2 > Q_x + Q) \\
&+ Pr(D_x < Q_x, D_1 < Q, D_x + D_1 + D_2 > 2Q + Q_x) \\
&+ Pr(D_x < Q_x, D_1 > Q, D_2 > Q, D_x + D_1 + D_2 > 2Q + Q_x) \\
&= Pr(D_x < Q_x, D_1 < Q, D_x + D_2 > Q_x + Q) + Pr(D_x < Q_x, D_2 > Q, D_x + D_1 + D_2 > 2Q + Q_x \\
&= Pr(D_x < Q_x, D_x + D_1 + D_2 > 2Q + Q_x) \\
&+ Pr(D_x < Q_x, D_x + D_2 > Q + Q_x, D_x + D_1 + D_2 < 2Q + Q_x) \\
&= Pr(D_x < Q_x) - Pr(D_x < Q_x, D_x + D_1 < Q_x + Q, D_x + D_1 + D_2 < Q_x + 2Q)
\end{aligned}
$$

The details of obtaining $T_{x+}(Q_x, Q)$ and its derivative is very similar and we omit the details. Here, we present the mathematical definition $T_{x+}(Q_x, Q)$.

$$
\begin{aligned}
T_{x+} = &\mathbb{E}[(D_x - Q_x) \times \mathbb{I}(D_x > Q_x, D_1 > Q, D_2 < Q) \times \mathbb{I}(D_x + D_2 < Q_x + Q)] && \text{(A.11)} \\
&+ \mathbb{E}[(Q - D_2) \times \mathbb{I}(D_x > Q_x, D_1 > Q, D_2 < Q) \times \mathbb{I}(D_x + D_2 > Q_x + Q)] && \text{(A.12)} \\
&+ \mathbb{E}[(D_x - Q_x) \times \mathbb{I}(D_x > Q_x, D_1 > Q, D_2 < Q) \times \mathbb{I}(D_x + D_1 + D_2 < Q_x + 2Q)] && \text{(A.13)} \\
&+ \mathbb{E}[(2Q - D_1 - D_2) \times \mathbb{I}(D_x > Q_x, D_1 > Q, D_2 < Q) \times \mathbb{I}(D_1 + D_2 < 2Q, D_x + D_1 + D_2 > Q_x + 2Q)] && \text{(A.14)} \\
&+ \mathbb{E}[(D_x - Q_x) \times \mathbb{I}(D_x > Q_x, D_1 < Q, D_2 < Q) \times \mathbb{I}(D_x + D_1 + D_2 < Q_x + 2Q)] && \text{(A.15)} \\
&+ \mathbb{E}[(2Q - D_1 - D_2) \times \mathbb{I}(D_x > Q_x, D_1 < Q, D_2 < Q) \times \mathbb{I}(D_x + D_1 + D_2 > Q_x + 2Q)]\,, && \text{(A.16)}
\end{aligned}
$$

■

A.2 Proofs of Theorems and Propositions of the Paper

Proof of Proposition 1. We provide the first order condition to maximize Π_{B_i} by choosing Q_i. Recall that

$$\Pi_{B_i} = -rQ_i + (v-w)S_i(Q_i) + (1-\theta_s)\theta(v-w)T_i(Q_i,Q_j) + (1-\theta_s)(1-\theta)(v-w)T_j(Q_j,Q_i)\,,$$

Therefore, using Lemma A.1, we have

$$\begin{aligned}
\frac{1}{v-w}\frac{\mathrm{d}\Pi_{B_i}}{\mathrm{d}Q_i} &= -\frac{r}{v-w} + Pr(D_i > Q_i) + (1-\theta_s)\theta\frac{\mathrm{d}T_i(Q_i,Q_j)}{\mathrm{d}Q_i} + (1-\theta_s)(1-\theta)\frac{\mathrm{d}T_j(Q_j,Q_i)}{\mathrm{d}Q_i} \\
&= -\frac{r}{v-w} + Pr(D_i > Q_i) - (1-\theta_s)\theta Pr(D_i > Q_i, D_t < Q_i + Q_j) \\
&+ (1-\theta_s)(1-\theta)Pr(D_i < Q_i, D_t > Q_i + Q_j) \\
&= -\tilde{r} + H_i(Q_i, Q_i + Q_j)\,,
\end{aligned}$$

where $\tilde{r} = \frac{r}{v-w}$. Therefore, one can find the equilibrium order quantities by solving $H_i(Q_i,Q_t) = \tilde{r} = H_j(Q_j,Q_t)$, and $Q_i + Q_j = Q_t$. For symmetric buyers, since $Q_i = Q_j$, the equilibrium condition reduces to $H_i(Q_i, 2Q_i) = \tilde{r}$. ■

Lemma A.7. *For symmetric buyers, let Q_i be the equilibrium reservation quantity of the two buyers. That is, let Q_i satisfies the equilibrium conditions $H_i(Q_i, 2Q_i) = \tilde{r}$. Then,*

$$\begin{aligned}
\frac{\mathrm{d}Q_i}{\mathrm{d}\theta} &= -\frac{\mathrm{d}H_i}{\mathrm{d}\theta}\frac{1}{\frac{\mathrm{d}H_i}{\mathrm{d}Q_i} + 2\frac{\mathrm{d}H_i}{\mathrm{d}Q_t}} < 0 \\
\frac{\mathrm{d}Q_i}{\mathrm{d}\theta_s} &= -\frac{\mathrm{d}H_i}{\mathrm{d}\theta_s}\frac{1}{\frac{\mathrm{d}H_i}{\mathrm{d}Q_i} + 2\frac{\mathrm{d}H_i}{\mathrm{d}Q_t}} \quad \textit{and when } \theta = 0,\ \frac{\mathrm{d}Q_i}{\mathrm{d}\theta_s} < 0 \\
\frac{\mathrm{d}Q_i}{\mathrm{d}\tilde{r}} &= \frac{1}{\frac{\mathrm{d}H_i}{\mathrm{d}Q_i} + 2\frac{\mathrm{d}H_i}{\mathrm{d}Q_t}} < 0\,.
\end{aligned}$$

Proof of Lemma A.7. First, we recall the equilibrium condition in (1.). Then, we use envelop theorem and take the derivatives of the equilibrium condition with respect to θ, θ_s, and $\tilde{r}$, in (1.1.), (1.2.), and (1.3.), respectively. Hence, we would come up with expressions for $\frac{\mathrm{d}Q_i}{\mathrm{d}\theta}$, $\frac{\mathrm{d}Q_i}{\mathrm{d}\theta_s}$, and $\frac{\mathrm{d}Q_i}{\mathrm{d}\tilde{r}}$. In (2.) we show that $\frac{\mathrm{d}Q_i}{\mathrm{d}\theta}$, and $\frac{\mathrm{d}Q_i}{\mathrm{d}\tilde{r}}$ are negative, and also for $\theta = 0$, $\frac{\mathrm{d}Q_i}{\mathrm{d}\theta_s}$ is negative.

(1.) Recall that the equilibrium conditions are $H_i(Q_i,Q_t) = \tilde{r} = H_j(Q_j,Q_t)$ and $Q_i + Q_j = Q_t$.

(1.1.) Thus, using envelop theorem, for a fixed $\tilde{r}$ and θ_s, we can take the derivatives of the above conditions with respect to θ as follows:

$$\frac{\mathrm{d}H_i}{\mathrm{d}\theta} + \frac{\mathrm{d}H_i}{\mathrm{d}Q_i}\frac{\mathrm{d}Q_i}{\mathrm{d}\theta} + \frac{\mathrm{d}H_i}{\mathrm{d}Q_t}\frac{\mathrm{d}Q_t}{\mathrm{d}\theta} = 0$$
$$\frac{\mathrm{d}H_j}{\mathrm{d}\theta} + \frac{\mathrm{d}H_j}{\mathrm{d}Q_j}\frac{\mathrm{d}Q_j}{\mathrm{d}\theta} + \frac{\mathrm{d}H_j}{\mathrm{d}Q_t}\frac{\mathrm{d}Q_t}{\mathrm{d}\theta} = 0$$
$$\frac{\mathrm{d}Q_i}{\mathrm{d}\theta} + \frac{\mathrm{d}Q_j}{\mathrm{d}\theta} = \frac{\mathrm{d}Q_t}{\mathrm{d}\theta}$$

Thus, we have three equations with three unknown variables $\frac{\mathrm{d}Q_i}{\mathrm{d}\theta}$, $\frac{\mathrm{d}Q_j}{\mathrm{d}\theta}$, and $\frac{\mathrm{d}Q_t}{\mathrm{d}\theta}$. Therefore, we can solve for $\frac{\mathrm{d}Q_i}{\mathrm{d}\theta}$ as follows

$$\left(\frac{\mathrm{d}H_i}{\mathrm{d}Q_i} + \frac{\mathrm{d}H_i}{\mathrm{d}Q_t}\frac{\mathrm{d}H_j}{\mathrm{d}Q_j}\frac{1}{\frac{\mathrm{d}H_j}{\mathrm{d}Q_j} + \frac{\mathrm{d}H_j}{\mathrm{d}Q_t}}\right)\frac{\mathrm{d}Q_i}{\mathrm{d}\theta} = -\frac{\mathrm{d}H_i}{\mathrm{d}\theta}\left(1 - \frac{\frac{\frac{\mathrm{d}H_i}{\mathrm{d}Q_t}}{\frac{\mathrm{d}H_i}{\mathrm{d}\theta}}}{\frac{\frac{\mathrm{d}H_j}{\mathrm{d}Q_j}}{\frac{\mathrm{d}H_j}{\mathrm{d}\theta}} + \frac{\frac{\mathrm{d}H_j}{\mathrm{d}Q_t}}{\frac{\mathrm{d}H_j}{\mathrm{d}\theta}}}\right)$$

If we consider symmetric buyers, we can simplify the above $\frac{\mathrm{d}Q_i}{\mathrm{d}\theta} = -\frac{\mathrm{d}H_i}{\mathrm{d}\theta}\frac{1}{\frac{\mathrm{d}H_i}{\mathrm{d}Q_i}+2\frac{\mathrm{d}H_i}{\mathrm{d}Q_t}}$.

(1.2.) Similarly, $\frac{\mathrm{d}Q_i}{\mathrm{d}\theta_s} = -\frac{\mathrm{d}H_i}{\mathrm{d}\theta_s}\frac{1}{\frac{\mathrm{d}H_i}{\mathrm{d}Q_i}+2\frac{\mathrm{d}H_i}{\mathrm{d}Q_t}}$.

(1.3.) Also, from equilibrium conditions,

$$\frac{\mathrm{d}H_i}{\mathrm{d}Q_i}\frac{\mathrm{d}Q_i}{\mathrm{d}\tilde{r}} + \frac{\mathrm{d}H_i}{\mathrm{d}Q_t}\frac{\mathrm{d}Q_t}{\mathrm{d}\tilde{r}} = 1$$
$$\frac{\mathrm{d}H_j}{\mathrm{d}Q_j}\frac{\mathrm{d}Q_j}{\mathrm{d}\tilde{r}} + \frac{\mathrm{d}H_j}{\mathrm{d}Q_t}\frac{\mathrm{d}Q_t}{\mathrm{d}\tilde{r}} = 1$$
$$\frac{\mathrm{d}Q_i}{\mathrm{d}\tilde{r}} + \frac{\mathrm{d}Q_j}{\mathrm{d}\tilde{r}} = \frac{\mathrm{d}Q_t}{\mathrm{d}\tilde{r}}$$

Therefore, $\frac{\mathrm{d}Q_i}{\mathrm{d}\tilde{r}} = \frac{1}{\frac{\mathrm{d}H_i}{\mathrm{d}Q_i}+2\frac{\mathrm{d}H_i}{\mathrm{d}Q_t}}$.

(2.) Next, we determine the signs of the derivatives we found in the previous steps. In (2.1.) we show $\frac{\mathrm{d}H_i(Q_i,Q_t)}{\mathrm{d}\theta} < 0$. In (2.2.) we show $\frac{\mathrm{d}H_i(Q_i,Q_t)}{\mathrm{d}\theta_s} < 0$ when $\theta = 0$. In (2.3), we show $H_i(Q_i, Q_t)$ is decreasing in Q_i. Finally, in (2.4.) we show $H_i(Q_i, Q_t)$ is decreasing in Q_t.

(2.1.) Note that $\frac{\mathrm{d}H_i(Q_i,Q_t)}{\mathrm{d}\theta} = -(1-\theta_s)\left(Pr(D_i < Q_i, D_t > Q_t) + Pr(D_i > Q_i, D_t < Q_t)\right) < 0$. Therefore, $H_i(Q_i, Q_t)$ is decreasing in θ.

(2.2.) When $\theta = 0$, $\frac{\mathrm{d}H_i(Q_i,Q_t)}{\mathrm{d}\theta_s} = -Pr(D_i < Q_i, D_t > Q_t) < 0$. Therefore, $H_i(Q_i, Q_t)$ is decreasing in θ_s when $\theta = 0$.

(2.3.) Rearranging $H_i(Q_i, Q_t)$, we have

$$\begin{aligned} H_i(Q_i, Q_t) =& (1-\theta_s)(1-\theta)Pr(D_t > Q_t) + (1-(1-\theta_s)(1-\theta))Pr(D_i > Q_i, D_t > Q_t) \\ &+ (1-(1-\theta_s)\theta)Pr(D_i > Q_i, D_t < Q_t), \end{aligned}$$

which is clearly decreasing in Q_i.

(2.4.) We can also rearrange $H_i(Q_i, Q_t)$ as follows:

$$\begin{aligned} H_i(Q_i, Q_t) =&(1-\theta_s)(1-\theta)Pr(D_i < Q_i, D_t > Q_t) + (1-(1-\theta_s)\theta)Pr(D_i > Q_i) \\ &+ (1-\theta_s)\theta Pr(D_i > Q_i, D_t > Q_t)\,, \end{aligned}$$

which is clearly decreasing in Q_t.

(3.) Using (1.) and (2.) together, we must have:

$$\begin{aligned} \frac{\mathrm{d}Q_i}{\mathrm{d}\theta} &= -\frac{\mathrm{d}H_i}{\mathrm{d}\theta}\frac{1}{\frac{\mathrm{d}H_i}{\mathrm{d}Q_i} + 2\frac{\mathrm{d}H_i}{\mathrm{d}Q_t}} < 0 \\ \frac{\mathrm{d}Q_i}{\mathrm{d}\theta_s} &= -\frac{\mathrm{d}H_i}{\mathrm{d}\theta_s}\frac{1}{\frac{\mathrm{d}H_i}{\mathrm{d}Q_i} + 2\frac{\mathrm{d}H_i}{\mathrm{d}Q_t}} \quad \text{and when } \theta = 0,\ \frac{\mathrm{d}Q_i}{\mathrm{d}\theta_s} < 0 \\ \frac{\mathrm{d}Q_i}{\mathrm{d}\tilde{r}} &= \frac{1}{\frac{\mathrm{d}H_i}{\mathrm{d}Q_i} + 2\frac{\mathrm{d}H_i}{\mathrm{d}Q_t}} < 0\,. \end{aligned}$$

■

Proof of Theorem 1. Fix θ_s to any number in interval $[0, 1]$ and consider optimization $\max_{(\tilde{r},\theta)}\{\Pi_s | \tilde{r} \in [0, 1],\ \theta \in [0, 1]\}$. First in (1.), we argue that in this optimization, $\tilde{r}$ is an interior solution and hence must satisfy the first order condition. Then in (2.), we show that the objective function Π_s is decreasing in θ for any $\tilde{r}$ that satisfies the first order condition, which proves our result.

(1.) We show that optimal capacity reservation fee r is an interior solution. It is easy to verify that for any given θ and θ_s, when $\tilde{r} = 0$, we have $Q_i = \infty$ and $\Pi_s = -\infty$ and for $\tilde{r} = 1$, we have $Q_i = 0$ and $\Pi_s = 0$. Therefore, we know optimal $\tilde{r}$ is an interior solution and it must satisfy the first order condition.

(2.) Next, we write the profit function of the supplier for symmetric buyers and then using Lemma A.7, we write the derivatives of the suppliers' profit function with respect to $\tilde{r}$ and θ.

(2.1.) In the symmetric case, since $Q_i = Q_j$, we can simplify the supplier's profit function as follows:

$$\begin{aligned}
\Pi_s &= (r-h)\,Q_t + (w-c)\mathbb{E}\big[\min(Q_t, D_t)\big] + \theta_s(v-w)\,(T_i(Q_i,Q_j) + T_j(Q_j,Q_i)) \\
&= 2\,(r-h)\,Q_i + 2(w-c)\mathbb{E}\big[\min(Q_i, \frac{D_t}{2})\big] + \theta_s(v-w)\left(2\int_{-\infty}^{Q_i} Pr(D_i < x)\,\mathrm{d}x - \int_{-\infty}^{2Q_i} Pr(D_t < x)\,\mathrm{d}x\right) \\
&= 2\,(r-h)\,Q_i + 2(w-c)(Q_i - \int_{-\infty}^{Q_i} Pr(D_t < 2x)\,\mathrm{d}x) \\
&+ 2\theta_s(v-w)\left(\int_{-\infty}^{Q_i} Pr(D_i < x)\,\mathrm{d}x - \int_{-\infty}^{Q_i} Pr(D_t < 2x)\,\mathrm{d}x\right)
\end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{1}{2(v-w)}\Pi_s = \left(\tilde{r} - \frac{h}{v-w}\right) Q_i + \frac{w-c}{v-w}(Q_i - \int_{-\infty}^{Q_i} Pr(D_t < 2x)\,\mathrm{d}x) \\
+ \theta_s \left(\int_{-\infty}^{Q_i} Pr(D_i < x)\,\mathrm{d}x - \int_{-\infty}^{Q_i} Pr(D_t < 2x)\,\mathrm{d}x\right).
\end{aligned}$$

(2.2.) Using Lemma A.7, we can write the derivative of Π_s with respect to $\tilde{r}$ as follows:

$$\begin{aligned}
&\frac{1}{2}\frac{1}{v-w}\frac{\mathrm{d}\Pi_s}{\mathrm{d}\tilde{r}} \equiv \\
&Q_i + \left(\left(\tilde{r} - \frac{h}{v-w}\right) + \frac{w-c}{v-w}Pr(D_t > 2Q_i) + \theta_s\left(Pr(D_t > 2Q_i) - Pr(D_i > Q_i)\right)\right)\frac{\mathrm{d}Q_i}{\mathrm{d}\tilde{r}}
\end{aligned}$$

(2.3.) Using Lemma A.7, we can write the derivative of Π_s with respect to θ as follows:

$$\begin{aligned}
\frac{1}{2}\frac{1}{v-w}\frac{\mathrm{d}\Pi_s}{\mathrm{d}\theta} &= \left(\left(\tilde{r} - \frac{h}{v-w}\right) + \frac{w-c}{v-w}Pr(D_t > 2Q_i) + \theta_s\left(Pr(D_t > 2Q_i) - Pr(D_i > Q_i)\right)\right)\frac{\mathrm{d}Q_i}{\mathrm{d}\theta} \\
&= \left(\frac{1}{2(v-w)}\frac{\mathrm{d}\Pi_s}{\mathrm{d}\tilde{r}} - Q_i\right)\frac{\frac{\mathrm{d}Q_i}{\mathrm{d}\theta}}{\frac{\mathrm{d}Q_i}{\mathrm{d}\tilde{r}}} \\
&= -\frac{\mathrm{d}H_i}{\mathrm{d}\theta}\frac{1}{2}\frac{1}{v-w}\frac{\mathrm{d}\Pi_s}{\mathrm{d}\tilde{r}} + \frac{\mathrm{d}H_i}{\mathrm{d}\theta}Q_i
\end{aligned}$$

(3.) Finally, we argue that since the optimal $\tilde{r}$ should satisfy the first order condition, Π_s is decreasing in θ. Note that in (2.3) we have shown that $\frac{1}{2}\frac{1}{v-w}\frac{\mathrm{d}\Pi_s}{\mathrm{d}\theta} = -\frac{\mathrm{d}H_i}{\mathrm{d}\theta}\frac{1}{2}\frac{1}{v-w}\frac{\mathrm{d}\Pi_s}{\mathrm{d}\tilde{r}} + \frac{\mathrm{d}H_i}{\mathrm{d}\theta}Q_i$.
Since the optimal $\tilde{r}$ is an interior solution, it must satisfy the first order condition $\frac{\mathrm{d}\Pi_s}{\mathrm{d}\tilde{r}} = 0$. Hence, for any $\tilde{r}$ that satisfies the first order condition, we have $\frac{1}{2}\frac{1}{v-w}\frac{\mathrm{d}\Pi_s}{\mathrm{d}\theta} = \frac{\mathrm{d}H_i}{\mathrm{d}\theta}Q_i < 0$.
The last inequality is true because in the proof of Lemma A.7 we showed that $\frac{\mathrm{d}H_i}{\mathrm{d}\theta} < 0$. Hence, the supplier's profit function is decreasing in θ for any $\tilde{r}$ that satisfies the first order condition. Hence, $\theta = 0$ results in highest profits for the supplier. ■

Lemma A.8. *Consider problem* $\max_{z_i}\left\{\Pi_s^0(z_i)\,|\,z_i \geq -\frac{\mu}{\sigma}\right\}$, *where*

$$\Pi_s^0(z) \overset{def}{=} s_l\left(z+\frac{\mu}{\sigma}\right) - m_r\Phi_\alpha(z)\left(z+\frac{\mu}{\sigma}\right) - (1-m_r)\int_{-\infty}^{z}\Phi\left(\frac{y}{\alpha}\right)\mathrm{d}y\,.$$

Then

1. *Since* $\frac{\mu}{\sigma}$ *is large enough such that the probability of negative demand is negligible, there is a unique solution* z^0 *to this optimization.*
2. z^0 *is decreasing in* m_r, *and increasing in* s_l.
3. z^0 *can arbitrary get close to* ∞ *when* s_l *is close enough to 1 and* m_r *is close enough to zero.*
4. z^0 *can arbitrary get close to* $-\frac{\mu}{\sigma}$ *when* s_l *is small enough.*
5. *If* $z^0 < z^f$, *then* $z_i = z^0$ *and* $\theta_s = 0$ *is the optimal solution of Problem 0.*
6. *If* $z^0 \geq z^f$, *then* $z_i = z^f$ *and* $\theta_s = [0,1]$ *is the optimal solution of Problem 0.*

Proof of Lemma A.8.

The derivative of the objective function $\Pi_s^0(z_i)$ with respect to z_i is $\frac{\mathrm{d}\Pi_s^0(z_i)}{\mathrm{d}z_i} = -(1-\Phi(\frac{z}{\alpha}))H^0\left(z_i\right)$, where $H^0(z) \overset{def}{=} m_r\left(\frac{\Phi'_\alpha(z)(z+\frac{\mu}{\sigma})}{1-\Phi(\frac{z}{\alpha})} + \frac{\Phi_\alpha(z)-\Phi(\frac{z}{\alpha})}{1-\Phi(\frac{z}{\alpha})}\right) + (1-s_l)\frac{1}{1-\Phi(\frac{z}{\alpha})} - 1$.

By Lemma A.2, (i) $\left(\frac{\Phi'_\alpha(z)(z+\frac{\mu}{\sigma})}{1-\Phi(\frac{z}{\alpha})} + \frac{\Phi_\alpha(z)-\Phi(\frac{z}{\alpha})}{1-\Phi(\frac{z}{\alpha})}\right)$ is positive and increasing in z; (ii) $\frac{1}{1-\Phi(\frac{z}{\alpha})}$ is positive and increasing in z. Hence $H^0(z)$ is increasing in z. Also note that since $\frac{\mu}{\sigma}$ is large enough such that the probability of negative demand is negligible, we have $\lim_{z\to-\frac{\mu}{\sigma}} H^0(z) = -s_l$. Also we can verify $\lim_{z\to\infty} H^0(z) = \infty$. Hence $H^0(z) = 0$ has a unique solution z^0. Also since both $\left(\frac{\Phi'_\alpha(z)(z+\frac{\mu}{\sigma})}{1-\Phi(\frac{z}{\alpha})} + \frac{\Phi_\alpha(z)-\Phi(\frac{z}{\alpha})}{1-\Phi(\frac{z}{\alpha})}\right)$ and $\frac{1}{1-\Phi(\frac{z}{\alpha})}$ are increasing in z, the solution z^0 is decreasing in m_r and increasing in s_l. Also, z^0 can arbitrary get close to ∞ when s_l is close enough to 1 and m_r is close enough to 0 and z^0 can arbitrary get close to $-\frac{\mu}{\sigma}$ when either s_l is small enough.

Next, we show if $z^0 < z^f$, then $z_i = z^0$ and $\theta_s = 0$ is the optimal solution of Problem 0.

Suppose $z^0 < z^f$. Note that for any $z_i \leq z^f$,

$$\Pi_s^0(z_i) \leq \max_{z_i}\left\{\Pi_s^0(z_i)\,|\,z_i \leq z^f\right\} = \Pi_s^0(z^0) \tag{A.17}$$

Notice that the left hand side expression $\Pi_s^0(z_i)$ is the objective function of Problem 0. Since by Lemma A.5, $f_\theta(z^0) < 0$, we know that ($z_i = z^0$ and $\theta_s = 0$) is a feasible solution of Problem 1 and by this feasible solution, the objective function achieves its upper bound which is the right hand side expression $\Pi_s^0(z^0)$. Hence, ($z_i = z^0$ and $\theta_s = 0$) is optimal solution of Problem 0. Also since for any $z_i < z^f$, we have $\theta \in (0,1]$ are not feasible solutions of Problem 0, and for $z_i = z^f$,

$\Pi_s^0(z^f) < \Pi_s^0(z^0)$, we know $\theta \in (0,1]$ is not optimal.

Finally, we show if $z^0 \geq z^f$, then ($z_i = z^f$ and $\theta_s = [0,1]$) is the optimal solution of Problem 0. Suppose $z^0 \geq z^f$. Since the derivative of $\Pi_s^0(z_i)$ with respect to z_i is a negative number times an increasing function $H^0(z_i)$ (as shown earlier), and z^0 is the solution to $H^0(z^0) = 0$, we know that the objective function $\Pi_s^0(z_i)$ is increasing in z_i for any $z_i \leq z^0$. Hence, the solution to optimization $\max_{z_i} \left\{\Pi_s^0(z_i) \,|\, z_i \leq z^f\right\}$ is $z_i = z^f$. Note that for any $z_i \geq z^f$,

$$\Pi_s^0(z_i) \leq \max_{z_i} \left\{\Pi_s^0(z_i) \,|\, z_i \leq z^f\right\} = \Pi_s^0(z^f)$$

Notice that the left hand side expression $\Pi_s^0(z_i)$ is the objective function of Problem 0. Since by Lemma A.5, $f_\theta(z^f) = 0$, we know that ($z_i = z^f$ and $\theta_s = [0,1]$) are feasible solutions of Problem 0 and by these feasible solutions, the objective function achieves its upper bound which is the right hand side expression $\Pi_s^0(z^f)$. Hence, ($z_i = z^f$ and $\theta_s = [0,1]$) are optimal solutions of Problem 0. ■

Lemma A.9. *Consider optimization* $\max_{z_i} \left\{\Pi_s^0(z_i) + m_r f_\theta(z_i) \,|\, z_i \geq -\frac{\mu}{\sigma}\right\}$. *Then,*

1. *Since $\frac{\mu}{\sigma}$ is large enough such that the probability of negative demand is negligible, there is a unique solution z^1 to this optimization.*
2. *z^1 is decreasing in m_r and increasing in s_l.*
3. *z^1 can arbitrary get close to ∞ when m_r is close enough to zero and s_l is close enough to 1.*
4. *z^1 can arbitrary get close to $-\frac{\mu}{\sigma}$ when s_l is small enough.*
5. *If $z^1 > z^f$, then $z_i = z^1$ and $\theta_s = 1$ is the optimal solution of Problem 1.*
6. *If $z^1 \leq z^f$, then $z_i = z^f$ and $\theta_s = [0,1]$ is the optimal solution of Problem 1.*
7. *It is not possible to have $z^1 \leq z^f \leq z^0$.*

Proof of Lemma A.9. The derivative of the objective function with respect to z_i is $\frac{\mathrm{d}\left(\Pi_s^0(z_i) + m_r f_\theta(z_i)\right)}{\mathrm{d}z_i} = -(1-\Phi(\frac{z}{\alpha}))H^1(z_i)$, where, $H^1(z) \overset{def}{=} m_r \frac{\phi(z)(z+\frac{\mu}{\sigma})}{1-\Phi(\frac{z}{\alpha})} + (1-s_l)\frac{1}{1-\Phi(\frac{z}{\alpha})} - 1$. By Lemma A.2, we know (i) $\frac{\phi(z)(z+\frac{\mu}{\sigma})}{1-\Phi(\frac{z}{\alpha})}$ is increasing in z; (ii) $\frac{1}{1-\Phi(\frac{z}{\alpha})}$ is increasing in z. Hence $H^1(z)$ is increasing in z. Also, since $\frac{\mu}{\sigma}$ is large enough such that the probability of negative demand is negligible, $\lim_{z \to -\frac{\mu}{\sigma}} H^1(z) = -s_l$ and $\lim_{z\to\infty} H^1(z) = \infty$. Hence, $H^1(z) = 0$ has a unique solution z^1. Also, since both $\frac{\phi(z)(z+\frac{\mu}{\sigma})}{1-\Phi(\frac{z}{\alpha})}$ and $\frac{1}{1-\Phi(\frac{z}{\alpha})}$ are increasing and positive, z^1 should be decreasing in m_r and increasing in s_l. Also, z^1 can arbitrary get close to $-\frac{\mu}{\sigma}$ when m_r is small enough and s_l is large enough and z^1 can arbitrary get close to ∞ when either m_r is large enough or s_l is small enough.

Next, we show if $z^1 > z^f$, then $z_i = z^1$ and $\theta_s = 1$ is the optimal solution of Problem 1.

Suppose $z^1 > z^f$. Note that for any given $\theta_s \in [0,1]$, and for any $z_i \geq z^f$,

$$\Pi_s^0(z_i) + m_r\theta_s f_\theta(z_i) \leq \Pi_s^0(z_i) + m_r f_\theta(z_i) \leq \max_{z_i}\left\{\Pi_s^0(z_i) + m_r f_\theta(z_i)\,|\, z_i \geq z^f\right\} = \Pi_s^0(z^1) + m_r f_\theta(z^1) \tag{A.18}$$

Notice that the left hand side expression $\Pi_s^0(z_i) + m_r\theta_s f_\theta(z_i)$ is the objective function of Problem 1. Since by Lemma A.5, $f_\theta(z^1) > 0$, we know that ($z_i = z^1$ and $\theta_s = 1$) is a feasible solution of Problem 1 and by this feasible solution, the objective function achieves its upper bound which is the right hand side expression $\Pi_s^0(z^1) + m_r f_\theta(z^1)$. Hence, ($z_i = z^1$ and $\theta_s = 1$) is optimal solution of Problem 1. Also since for $\theta \in [0,1)$ and $z_i > z^f$, the first inequality in (A.18) is strict, $\theta < 1$ is not optimal.

Finally, we show if $z^1 \leq z^f$, then ($z_i = z^f$ and $\theta_s = [0,1]$) is the optimal solution of Problem 1. Suppose $z^1 \leq z^f$. Since the derivative of $\Pi_s^0(z_i)+m_r f_\theta(z_i)$ is a negative number times an increasing function $H^1(z)$ (as shown earlier), and z^1 is the solution to $H^1(z^1) = 0$, we know that the objective function $\Pi_s^0(z_i) + m_r f_\theta(z_i)$ is decreasing in z_i for any $z_i \geq z^1$. Hence, the solution to optimization $\max_{z_i}\left\{\Pi_s^0(z_i) + m_r f_\theta(z_i)\,|\, z_i \geq z^f\right\}$ is $z_i = z^f$. Note that for any given $\theta_s \in [0,1]$, and for any $z_i \geq z^f$,

$$\Pi_s^0(z_i) + m_r\theta_s f_\theta(z_i) \leq \Pi_s^0(z_i) + m_r f_\theta(z_i) \leq \max_{z_i}\left\{\Pi_s^0(z_i) + m_r f_\theta(z_i)\,|\, z_i \geq z^f\right\} = \Pi_s^0(z^f)$$

Notice that the left hand side expression $\Pi_s^0(z_i) + m_r\theta_s f_\theta(z_i)$ is the objective function of Problem 1. Since by Lemma A.5, $f_\theta(z^f) = 0$, we know that ($z_i = z^f$ and $\theta_s = [0,1]$) are feasible solutions of Problem 1 and by these feasible solutions, the objective function achieves its upper bound which is the right hand side expression $\Pi_s^0(z^f)$. Hence, ($z_i = z^f$ and $\theta_s = [0,1]$) are optimal solutions of Problem 1.

Next, we show that it is not possible to have $z^1 \leq z^f \leq z^0$. Suppose to the contrary we have $z^1 \leq z^f \leq z^0$. Also note that $H^0(z^f) - H^1(z^f) = m_r\frac{f'_\theta(z^f)}{1-\Phi(\frac{z^f}{\alpha})}$. Therefore, since both $H^1(z)$ and $H^0(z)$ are increasing in z and by Lemma A.5, $f'_\theta(z^f) > 0$ we must have $0 = H^0(z^0) \geq H^0(z^f) > H^1(z^f) \geq H^1(z^1) = 0$, which is a contradiction. ■

Proof of Theorem 2 and Proposition 2. First, we introduce the optimization problem of the supplier and then transform that to an equivalent optimization problem. Then we explain how we solve the equivalent optimization.

Supplier's Optimization Problem

From Theorem 1, we know optimal $\theta = 0$. Hence, we set $\theta = 0$ and maximize the supplier's profit function by choosing θ_s and $\tilde{r}$. That is the supplier's optimization problem is $\max_{(\tilde{r},\theta_s)}\{\Pi_s|\tilde{r} \in [0,1],\, \theta_s \in [0,1]\}$, assuming that $\theta = 0$ and that Q_i satisfies the equilibrium condition.

Knowing optimal $\theta = 0$ and for symmetric buyers, the equilibrium condition simplifies to

$$\tilde{r} = Pr(D_i > Q_i) + (1-\theta_s)Pr(D_i < Q_i, D_t > 2Q_i)\,.$$

Since the right hand side is decreasing in Q_i, there is a one to one correspondence between $\tilde{r}$ and Q_i for any given θ_s. Hence, instead of optimizing Π_s for θ_s and $\tilde{r}$, we can optimize for θ_s and Q_i or equivalently for θ_s and $z_i \stackrel{def}{=} \frac{Q_i-\mu}{\sigma}$. In the profit function of the supplier, we set $\tilde{r} = Pr(D_i > Q_i) + (1-\theta_s)Pr(D_i < Q_i, D_t > 2Q_i)$, and $z_i = \frac{Q_i-\mu}{\sigma}$. Assuming bivariate normal demand distribution, the supplier profit function reduces to $\Pi_s = 2\sigma(v-c)\left(\Pi_s^0(z_i) + \theta_s m_r f_\theta(z_i)\right)$. Hence, the supplier optimization problem is equivalent to

Problem S: $\max_{(z_i,\theta_s)}\{\Pi_s^0(z_i) + m_r\theta_s f_\theta(z_i)|z_i \geq -\frac{\mu}{\sigma},\, \theta_s \in [0,1]\}$.

Next, we explain how we solve Problem S.

Solution of Problem S

We divide the feasible region of Problem S into two regions: (i) $f_\theta(z_i) \geq 0$ and (ii) $f_\theta(z_i) \leq 0$. Hence instead of solving Problem S, we solve two optimizations:

Problem 0: $\max_{z_i,\theta_s}\left\{\Pi_s^0(z_i) + m_r\theta_s f_\theta(z_i)\,|\,f_\theta(z_i) \leq 0\,, z_i \geq -\frac{\mu}{\sigma},\, \theta_s \in [0,1]\right\}$,

Problem 1: $\max_{z_i,\theta_s}\left\{\Pi_s^0(z_i) + m_r\theta_s f_\theta(z_i)\,|\,f_\theta(z_i) \geq 0\,, z_i \geq -\frac{\mu}{\sigma}\,, \theta_s \in [0,1]\right\}$.

The optimal solution of Problem S, can be obtained by comparing the optimal solutions of Problem 0 and Problem 1 together and choosing the better one. In the following we outline the remainder of the proof.

1. Since $\Pi_s^0(z_i)$ and $f_\theta(z_i)$ are not functions of θ_s, in Problem 0, the optimal solutions θ_s and z_i should satisfy: either ($\theta_s = 0$ and $f_\theta(z_i) < 0$) or ($\theta_s = [0,1]$ and $f_\theta(z_i) = 0$). In other words, for Problem 0, $f_\theta(z_i)\theta_s = 0$.
2. Since $\Pi_s^0(z_i)$ and $f_\theta(z_i)$ are not functions of θ_s, in Problem 1, the optimal solutions θ_s and z_i satisfy ($\theta_s = 1$ and $f_\theta(z_i) > 0$) or ($\theta_s = [0,1]$ and $f_\theta(z_i) = 0$).
3. We establish in Lemma A.5 that $f_\theta(z_i) = 0$ has a unique solution $z^f > -\frac{\mu}{\sigma}$ and $f_\theta(z) < 0$ if and only if $z < z^f$. Hence, we can write Problem 0 and Problem 1 as follows:

Problem 0: $\max_{z_i,\theta_s} \left\{\Pi_s^0(z_i) \,|\, -\frac{\mu}{\sigma} \leq z_i \leq z^f,\, \theta_s \in [0,1],\, \theta_s f_\theta(z_i) = 0\right\}$,

Problem 1: $\max_{z_i,\theta_s} \left\{\Pi_s^0(z_i) + m_r \theta_s f_\theta(z_i) \,|\, z_i \geq z^f,\, \theta_s \in [0,1]\right\}$.

4. We establish in Lemma A.8 that the relaxed constrained optimization $\max_{z_i} \left\{\Pi_s^0(z_i) \,|\, z_i \geq -\frac{\mu}{\sigma}\right\}$ has a unique solution, which we call z^0. In that lemma we show if $z^0 < z^f$, then $z_i = z^0$ and $\theta_s = 0$ is the optimal solution of Problem 0 and if $z^0 \geq z^f$, then ($z_i = z^f$ and $\theta_s \in [0,1]$) are the optimal solutions of Problem 0
5. We establish in Lemma A.9 that the relaxed constrained optimization $\max_{z_i} \left\{\Pi_s^0(z_i) + m_r f_\theta(z_i) \,|\, z_i \geq -\frac{\mu}{\sigma}\right\}$ has a unique solution, which we call z^1. In that lemma, we show if $z^1 > z^f$, then $z_i = z^1$, and $\theta_s = 1$ is the optimal solution of Problem 1 and if $z^1 < z^f$, then $z_i = z_f$ and $\theta_s \in [0,1]$ is the optimal solution of Problem 1.
6. Next we characterize the optimal solution of Problem S, using Lemma A.8, and Lemma A.9. There are four possibilities:
   (a) If $z^0 < z^f$ and $z^1 \leq z^f$, then $z_i = z^0$ and $\theta_s = 0$ is the optimal solution of Problem S. This is because, in this case, (i) optimal solution of Problem 0 is $z_i = z^0$ and $\theta_s = 0$, (ii) optimal solutions of Problem 1 are ($z_i = z^f$ and $\theta_s = [0,1]$) which are feasible solutions of Problem 0.
   (b) If $z^0 \geq z^f$ and $z^1 > z^f$, then $z_i = z^1$ and $\theta_s = 1$ is the optimal solution of Problem S. This is because, in this case, (i) optimal solutions of Problem 0 are $z_i = z^f$ and $\theta_s = [0,1]$ which are feasible solutions of Problem 1, (ii) optimal solution of Problem 1 is ($z_i = z^1$ and $\theta_s = 1$).
   (c) If $z^0 < z^f$ and $z^1 > z^f$, then either ($z_i = z^1$ and $\theta_s = 1$) or ($z_i = z^0$ and $\theta_s = 0$) is the optimal solution of Problem S. This is because, in this case, (i) optimal solution of Problem 0 is $z_i = z^0$ and $\theta_s = 0$, (ii) optimal solution of Problem 1 is ($z_i = z^1$ and $\theta_s = 1$). Therefore, $\theta_s = 0$ or $\theta_s = 1$ should be optimal solution of Problem S.
   (d) We establish in Lemma A.9 that it is not possible to have ($z^0 \geq z^f$ and $z^1 \leq z^f$).
7. When s_l is small enough, using Lemma A.8, and Lemma A.9, z^1 and z^0 both become smaller than z^f (which does not depend on s_l or m_r). Hence $\theta_s = 0$ is optimal solution of Problem S as discussed in the previous step.
8. Similarly, when m_r is small enough and s_l is large enough, using Lemma A.8, and Lemma A.9, both z^1 and z^0 become larger than z^f (which does not depend on $\tilde{h}$ or m_r). Hence $\theta_s = 1$

is optimal solution of Problem S as discussed earlier.

This established the proof of the Theorem. ■

Proof of Theorem 3.

If optimal $\tilde{r} = 0$, then for any θ and θ_s, we have $Q_i \to \infty$ and $Q_j \to \infty$, which results in negative profit for the supplier if $h > 0$. Therefore, optimal $\tilde{r} > 0$.

If optimal $\tilde{r} = 1$, then for any θ and θ_s, we have $Q_i \to 0$ and $Q_j \to 0$, which results in zero profit for the supplier. However, with $\tilde{r} = 1 - \epsilon > 0$, for small enough ϵ, the profit of the supplier is positive. Hence, optimal $\tilde{r} < 1$. In conclusion, optimal $\tilde{r} \in (0, 1)$.

Suppose $\tilde{r}^* \in (0, 1)$, θ^*, and θ_s^* are optimal solutions that maximize the supplier's profit.

First we show that either $\theta \in \{0, 1\}$ or $\theta_s \in \{0, 1\}$. That is, at least one of θ or θ_s is at the boundary. Suppose to the contrary that $\theta_s^* \in (0, 1)$ and $\theta^* \in (0, 1)$. Assume Q_i and Q_j and Q_t are the corresponding equilibrium quantities. Also let Φ_i, Φ_j, Φ_t, Φ_{it}, and Φ_{jt} be the corresponding probabilities as defined before. First suppose $\Phi_i = \Phi_j$. Then using the equilibrium conditions, we must have $\Phi_{it} = \Phi_{jt}$. Take an arbitrary small $\delta > 0$ and define $\theta_2 = \theta^* - \delta$, $\theta_{s2} = \theta_s^*$, and $\tilde{r}_2 = \tilde{r}_1^* + \delta(1-\theta_s^*)(\Phi_i + \Phi_t - 2\Phi_{it}) > \tilde{r}_1^*$. Since $\tilde{r}^*$, θ^*, and θ_s^* are all interior solutions by assumption, when δ is small enough, we must have $\tilde{r}_2 \in (0, 1)$, $\theta_2 \in (0, 1)$, and $\theta_{s2} \in (0, 1)$. Also, for such δ, the same Q_i and Q_j satisfies the equilibrium conditions under $\tilde{r} = \tilde{r}_2$, $\theta = \theta_2$, and $\theta_s = \theta_{s2}$. Then $\Pi_s(\tilde{r}_2, \theta_2, \theta_{s2}) - \Pi_s(\tilde{r}^*, \theta^*, \theta_s^*) = \delta(1 - \theta_s^*)(\Phi_i + \Phi_t - 2\Phi_{it})Q_t > 0$ which is a contradiction.

Next suppose $\Phi_i \neq \Phi_j$. Define $y = \frac{\Phi_{it} - \Phi_{jt}}{2(\Phi_{it} - \Phi_{jt}) - (\Phi_i - \Phi_j)}$. For a given multiplier δ, define $\tilde{r}_2 = \tilde{r}^* + \delta\left((1 - y)(\Phi_t - \Phi_{it}) - y(\Phi_i - \Phi_{it})\right)$, $\theta_2 = \theta^* + \delta\frac{2y-1}{(1-\theta_s^*)(1-\theta_s^*-\delta)}$, and $\theta_{s2} = \theta_s^* + \delta$. Since $\tilde{r}^*$, θ^*, and θ_s^* are all interior solutions by assumption, when $|\delta|$ is small enough, we must have $\tilde{r}_2 \in (0, 1)$, $\theta_2 \in (0, 1)$, and $\theta_{s2} \in (0, 1)$. Also, for such δ, the same Q_i and Q_j satisfies the equilibrium conditions under $\tilde{r} = \tilde{r}_2$, $\theta = \theta_2$, and $\theta_s = \theta_{s2}$.

Define $M_{\theta_s} = T_i(Q_i, Q_j) + T_j(Q_j, Q_i) + (1 - y)(\Phi_t - \Phi_{it})Q_t - y(\Phi_i - \Phi_{it})Q_t$.

If $M_{\theta_s} > 0$, take a small enough $\delta > 0$ such that $\tilde{r}_2 \in (0, 1)$, $\theta_2 \in (0, 1)$, and $\theta_{s2} \in (0, 1)$. Then $\Pi_s(\tilde{r}_2, \theta_2, \theta_{s2}) - \Pi_s(\tilde{r}^*, \theta^*, \theta_s^*) = \delta M_{\theta_s} > 0$, which is a contradiction.

If $M_{\theta_s} < 0$, take $\delta < 0$, where $|\delta|$ small enough such that $\tilde{r}_2 \in (0, 1)$, $\theta_2 \in (0, 1)$, and $\theta_{s2} \in (0, 1)$. Then $\Pi_s(\tilde{r}_2, \theta_2, \theta_{s2}) - \Pi_s(\tilde{r}^*, \theta^*, \theta_s^*) = -\delta M_{\theta_s} > 0$, which is a contradiction.

If $M_{\theta_s} = 0$, take $\delta > 0$ large enough such that $\tilde{r}_2 \in [0, 1]$, $\theta_2 \in [0, 1]$, and $\theta_{s2} \in [0, 1]$ and at least one of them is at the boundary. Then $\Pi_s(\tilde{r}_2, \theta_2, \theta_{s2}) - \Pi_s(\tilde{r}^*, \theta^*, \theta_s^*) = \delta M_{\theta_s} = 0$. That is, $\tilde{r}_2$, θ_2, and θ_{s2} are as good as $\tilde{r}^*$, θ^*, and θ_s^*. If $\tilde{r}_2$ is at the boundary, then we have a contradiction, since, at any optimal solution we must have $\tilde{r} \in (0,1)$. Hence, either θ_2, or θ_{s2} must be at the boundary. Therefore, in any case, we can find an optimal solution such that either optimal $\theta \in \{0,1\}$ or optimal $\theta_s \in \{0,1\}$.

Next, we show that either $\theta_s^* = 1$ or $\theta^* \in \{0,1\}$.

Previously, we showed that either optimal $\theta^* \in \{0,1\}$ or optimal $\theta_s^* \in \{0,1\}$. Therefore we have one the four cases for optimal θ_s^* and the optimal θ^*: (i) $\theta_s^* = 1$, $\theta^* \in [0,1]$; (ii) $\theta_s^* = 0$, and $\theta^* \in [0,1]$; (iii) $\theta_s^* \in [0,1]$ and $\theta^* = 0$; or (iv) $\theta_s^* \in [0,1]$, and $\theta^* = 1$. Note that when $\theta_s^* = 1$, the value of θ is irrelevant. Hence, to show that either $\theta_s^* = 1$ or $\theta^* \in \{0,1\}$, it is enough to show that it is impossible to have $\theta_s^* = 0$, and $\theta^* \in (0,1)$.

Suppose to the contrary that $\theta_s^* = 0$ and $\theta^* \in (0,1)$ and $\tilde{r}^* \in (0,1)$ are optimal. Suppose the corresponding equilibrium quantities are Q_i^*, Q_j^* and $Q_t^* = Q_i^* + Q_j^*$. Let $\Phi_i^* = Pr(D_i < Q_i^*)$, $\Phi_t^* = Pr(D_t < Q_t)$, $\Phi_{it}^* = Pr(D_i < Q_i^*, D_t < Q_t^*)$.

Choose $\tilde{\theta} = 0$. Also let $\tilde{H}_i(Q_i) = H_i(Q_i, Q_t^*; \theta = \tilde{\theta})$. Note that $\tilde{H}_i(Q_i^*) - \frac{r^*}{v-w} = (\theta - \tilde{\theta})(\Phi_i^* + \Phi_t^* - 2\Phi_{it}^*) > 0$, for $i \in \{1,2\}$. Also note $\frac{\mathrm{d}\tilde{H}(Q_i)}{\mathrm{d}Q_i} = -\tilde{\theta}\frac{\mathrm{d}\Phi_i}{\mathrm{d}Q_i} + (2\tilde{\theta} - 1)\frac{\mathrm{d}\Phi_{it}}{\mathrm{d}Q_i} < 0$, for $i \in \{1,2\}$.

Without loss of generality, suppose $\tilde{H}_1(Q_1^*) > \tilde{H}_2(Q_2^*)$ (in case of asymmetric buyers, they cannot be equal). Since $\tilde{H}_i(Q_i)$ is decreasing in Q_i, there exists a $\tilde{Q}_2 < Q_2^*$ such that $\tilde{H}_1(Q_t^* - \tilde{Q}_2) = \tilde{H}_2(\tilde{Q}_2) > \tilde{H}_i(Q_i^*) > \tilde{r}^*$.

Let $\tilde{r} = \tilde{H}_2(\tilde{Q}_2)$. Consider the supplier's profit by choosing $r = \tilde{r}$, $\theta = 0$, and $\theta_s = 0$. With these decision variables, the equilibrium quantities are $Q_1 = Q_t^* - \tilde{Q}_2$, $Q_2 = \tilde{Q}_2$, $Q_t = Q_t^*$.

Then we have $\Pi_s(r = \tilde{r}, \theta_s = 0, \theta = \tilde{\theta}) - \Pi_s(r = r^*, \theta_s = 0, \theta = \theta^*) = (\tilde{H}_i(Q_i^*) - \tilde{r}^*)(Q_t^*)(v - w) > 0$. That is the profit improves. Hence, if $\theta_s^* = 0$, θ^* cannot be greater than zero and it must be $\theta = 0$.

As a result, the optimal pair (θ, θ_s) can only be the following options:

1. if $\theta_s = 0$, we must have $\theta = 0$;
2. if $\theta_s = 1$, θ is irrelevant;
3. if $\theta_s \in [0,1]$, we must have $\theta \in \{0,1\}$.

Therefore, either $\theta_s = 1$ or $\theta \in \{0,1\}$. ■

Proof of Proposition 3. Consider a quantity Q that satisfies $\frac{r}{v-w} = Pr(D_x > Q) + (1 -$

$\theta_s)\theta dT_{x+}(Q)+(1-\theta_s)(1-\theta)dT_{x-}(Q)$. We claim that Buyer i, Buyer 1, and Buyer 2, each ordering Q is the equilibrium. To show that each buyer ordering Q is equilibrium, due to symmetry of buyers, we only need to show that if Buyers 1 and 2 reserve capacity Q, then Buyer i cannot unilaterally improve her profit by deviating from reserving capacity Q and instead reserve a capacity Q_i. Suppose Buyer 1 and Buyer 2 reserve capacity Q. We will show that the quantity that maximizes the profit of buyer i is $Q_i = Q$.

Optimal Q_i must satisfy the first order condition of the buyer i's profit function. Note that

$$\frac{1}{v-w}\frac{\mathrm{d}}{\mathrm{d}Q_i}\Pi_{B_i} = -\frac{r}{v-w} + Pr(D_x > Q_i) + (1-\theta_s)\theta\frac{\mathrm{d}}{\mathrm{d}Q_i}T_{x+}(Q_i,Q) + (1-\theta_s)(1-\theta)\frac{\mathrm{d}}{\mathrm{d}Q_i}T_{x-}(Q_i,Q)$$

Hence, optimal Q_i must satisfy $\frac{r}{v-w} = Pr(D_x > Q_i) + (1-\theta_s)\theta\frac{\mathrm{d}}{\mathrm{d}Q_i}T_{x+}(Q_i,Q) + (1-\theta_s)(1-\theta)\frac{\mathrm{d}}{\mathrm{d}Q_i}T_{x-}(Q_i,Q)$. Note that, by definition of Q, Q satisfies this first order condition. That is, if Buyer 1 and Buyer 2 reserve quantity Q that satisfies $\frac{r}{v-w} = Pr(D_x > Q)+(1-\theta_s)\theta dT_{x+}(Q)+(1-\theta_s)(1-\theta)dT_{x-}(Q)$, buyer i, would do the same. Hence, we have the statement of the proposition.

■

Proof of Theorem 4. Let $dT_{x+}(Q) = \frac{\mathrm{d}}{\mathrm{d}Q_i}T_{x+}(Q_i,Q)|_{Q_i=Q}$ and $dT_{x-}(Q) = \frac{\mathrm{d}}{\mathrm{d}Q_i}T_{x-}(Q_i,Q)|_{Q_i=Q}$. Note that based on Lemma A.6, $dT_{x+}(Q) < 0$, and $dT_{x-}(Q) > 0$.

Replacing the equilibrium quantities in the expression of the supplier's profit function and simplifying the expression, we have

$$\begin{aligned}\frac{1}{3}\frac{1}{v-c}\Pi_s &= s_l Q + m_r(-Pr(D_x < Q) + (1-\theta_s)\theta dT_{x+}(Q) + (1-\theta_s)(1-\theta)dT_{x-}(Q))Q \\ &- (1-m_r)\int_{-\infty}^{Q} Pr(\frac{D_1+D_2+D_x}{3} < x)\,\mathrm{d}x \\ &+ \theta_s m_r\left(\int_{-\infty}^{Q} Pr(D_x < x)\,\mathrm{d}x - \int_{-\infty}^{Q} Pr(\frac{D_1+D_2+D_x}{3} < x)\,\mathrm{d}x\right)\end{aligned}$$

Since there is a one to one correspondence between (r,θ,θ_s) and (Q,θ,θ_s), we can let the supplier choose (Q,θ,θ_s). Note that, for any given Q, and $\theta_s < 1$,

$$\frac{1}{3}\frac{1}{v-w}\frac{\mathrm{d}}{\mathrm{d}\theta}\Pi_s = (1-\theta_s)(dT_{x+}(Q) - dT_{x-}(Q)) < 0.$$

Hence optimal $\theta = 0$ when its value is relevant, i.e., when $\theta_s < 1$. The supplier's profit function at

$\theta = 0$ reduces to:

$$\frac{1}{3}\frac{1}{v-c}\Pi_s = s_l Q + m_r(-Pr(D_x < Q) + dT_{x-}(Q))Q - (1-m_r)\int_{-\infty}^{Q} Pr(\frac{D_1+D_2+D_x}{3} < x)\,\mathrm{d}x$$
$$+\theta_s m_r\left(\int_{-\infty}^{Q} Pr(D_x < x)\,\mathrm{d}x - \int_{-\infty}^{Q} Pr(\frac{D_1+D_2+D_x}{3} < x)\,\mathrm{d}x - dT_{x-}(Q)Q\right)$$

Notice that, for any given Q, the profit function Π_s is linear in θ_s. Hence, the boundaries $\theta_s = 0$ or $\theta_s = 1$ is included in the optimal solution for any given Q. That is, optimal $\theta_s \in \{0, 1\}$. As a result, we have the statement of the Theorem. ■

Proof of Theorem 5. Please note that the capacity building cost only appears in the supplier's profit function. As a result, from the buyer's perspective, capacity building cost is irrelevant in their decision about how much capacity to reserve. Hence, similar to our analysis of the main model, each buyer reserve quantity Q of capacity, where Q satisfies:

$$\frac{r}{v-w} = Pr(D_i > Q) - (1-\theta_s)\theta Pr(D_i > Q, D_t < 2Q) + (1-\theta_s)(1-\theta)Pr(D_i < Q, D_t > 2Q)\,.$$

Since the equilibrium equation is a one to one correspondence between (r, θ, θ_s) and (Q, θ, θ_s), the supplier can decide the second triple and deduce the optimal r.

Replacing the equilibrium conditions, the supplier's profit would be

$$\frac{1}{2(v-c)}\Pi_s = Q - \frac{h(2Q)}{2(v-c)} - (1-m_r)\left(\int_{-\infty}^{Q} Pr(D_t < \frac{y}{2})\,\mathrm{d}y\right)$$
$$+ m_r\left(-Pr(D_i < Q) - (1-\theta_s)\theta Pr(D_i > Q, D_t < 2Q) + (1-\theta_s)(1-\theta)Pr(D_i < Q, D_t > 2Q)\right)Q$$
$$+\theta_s m_r\left(\int_{-\infty}^{Q} Pr(D_i < x)\,\mathrm{d}x - \int_{-\infty}^{Q} Pr(D_t < \frac{y}{2})\,\mathrm{d}y\right)$$

For any given value of (Q, θ_s), the supplier's profit function is decreasing in θ. Consequently, the optimal $\theta = 0$.

We define $z = \frac{Q-\mu}{\sigma}$. The supplier's problem is to find the optimal pair (Q, θ_s) or equivalently, the optimal pair (z, θ_s). Define, $\Pi_{0i}(z) = (z + \frac{\mu}{\sigma}) - \frac{h_i}{2\sigma(v-c)} - (1-m_r)\left(\int_{-\infty}^{z} \Phi(\frac{y}{\alpha})\,\mathrm{d}y\right) - m_r\Phi_\alpha(z)(z+\frac{\mu}{\sigma})$, and $\Pi_i(z) = \Pi_{0i}(z) + \theta_s m_r f_\theta(z)$, where $f_\theta(z)$ is defined in Lemma A.5.

With the assumption that the demand is normally distributed, and the fact that optimal $\theta = 0$, we can verify that the supplier's profit function Π_s in the interval $(q_{i-1}, q_i]$ is $2\sigma(v-c)\Pi_i(z)$.

To find the optimal (Q, θ_s) (or equivalently (z, θ_s)), for each piece i of the step function $h(Q)$, the supplier solves optimization i defined as $\max_{(z,\theta_s)}\{\Pi_i : s.t.\, q_{i-1} \le \sigma z + \mu \le q_i\}$. Then the global optimal (z, θ_s) is determined based on the best of all optimal objective function of all such optimizations. We need to show that in any optimization i optimal $\theta_s \in \{0, 1\}$.

Recall that, in Lemma A.5, we show that $f_\theta(z)$ has a unique solution z^f and it is increasing at its solution, i.e. $f'_\theta(z^f) > 0$, furthermore, $f_\theta(z) < 0$ for $z < z^f$ and $f_\theta(z) > 0$ for $z > z^f$. As a result, (1) if $q_i < \sigma z^f + \mu$, then optimization i is decreasing in θ_s for any feasible z, and consequently, for optimization i, optimal $\theta_s = 0$; (2) if $q_{i-1} > \sigma z^f + \mu$, then optimization i is increasing in θ_s for any feasible z, and consequently, for optimization i, optimal $\theta_s = 1$; (3) Next, we focus on the case where $q_{i-1} \leq \sigma z^f + \mu \leq q_i$.

In this case, the supplier's optimization problem is

$$\max \left\{ \max_{(z,\theta_s)} \{\Pi_i(z) : s.t. \, \frac{q_{i-1} - \mu}{\sigma} \leq z \leq z^f\}, \max_{(z,\theta_s)} \{\Pi_i(z) : s.t. \, z^f \leq z \leq \frac{q_i - \mu}{\sigma}\} \right\}.$$

In this case, for any $z < z^f$, the supplier's profit function is decreasing in θ_s and hence for any $z < z^f$, optimal $\theta_s = 0$. Similarly, for any $z > z^f$, the supplier's profit function is increasing in θ_s and hence for any $z > z^f$, optimal $\theta_s = 1$. That is, for any $z \neq z^f$, optimal θ_s is at the boundary. Next, we explain that z^f cannot be optimal solution. We need to show, either $\frac{\mathrm{d}(\Pi_{0i}(z^f) + m_r f_\theta(z^f))}{\mathrm{d}z} > 0$ or $\frac{\mathrm{d}\tilde{\Pi}_{0i}(z^f)}{\mathrm{d}z} < 0$ (or both), which means deviating from z^f improves the objective function. Suppose to the contrary, $\frac{\mathrm{d}\tilde{\Pi}^1_s(z^f)}{\mathrm{d}z} \leq 0$ and $\frac{\mathrm{d}\tilde{\Pi}^0_s(z^f)}{\mathrm{d}z} \geq 0$. Then, $m_r f'_\theta(z) = \frac{\mathrm{d}\tilde{\Pi}^1_s(z^f)}{\mathrm{d}z} - \frac{\mathrm{d}\tilde{\Pi}^0_s(z^f)}{\mathrm{d}z} \leq 0$ which is contradiction to the fact that $f'_\theta(z^f) > 0$. Therefore, optimal $z \neq z^f$.

Therefore, we can conclude that at optimal z, optimal θ_s is either zero or one. ■

Proof of Theorem 6. The buyer i's profit function when she reserves capacity Q_i and the other buyer reserves capacity Q_j is

$$\Pi_{B_i} = -rR(Q_i) + (v - w)S_i(Q_i) + (v - w)(1 - \theta_s)(\theta T_i(Q_i, Q_j) + (1 - \theta)T_j(Q_j, Q_i)).$$

The optimal decision of the buyer regarding her reserved quantity satisfies the first order condition. That is, the optimal solution Q_i satisfies

$$\frac{\mathrm{d}\Pi_{B_i}}{\mathrm{d}Q_i} = -rR'(Q_i) + (v - w)H_i(Q_i, Q_i + Q_j) = 0,$$

where H_i is the same as what is defined before and we know, for any given Q_i and Q_j, it is decreasing in θ. With symmetric buyers, we must have that the equilibrium capacity Q reserved by each buyer should satisfy $-rR'(Q) + (v-w)H_i(Q, 2Q) = 0$ or equivalently, $r = (v-w)\frac{H_i(Q,2Q)}{R'(Q)}$. Next, we solve the supplier's problem. The supplier's profit function, when each buyer has reserved capacity Q is

$$\Pi_s = 2rR(Q) - \beta h(2Q) + (w-c)S_t(2Q) + 2(v-w)\theta_s(T_i(Q,Q)).$$

The supplier maximizes his profit by choosing r, θ, and θ_s and with the constraint that at equilibrium, we must have $r = (v-w)\frac{H_i(Q,2Q)}{R'(Q)}$. Note that, since $R(Q)$ is increasing in Q and convex, $R'(Q)$ is positive and increasing in Q. Also, as shown in proof of Lemma A.7, $H_i(Q,2Q)$ is decreasing in Q for any given θ and θ_s. Hence, for any given θ and θ_s, $\frac{H_i(Q,2Q)}{R'(Q)}$ is decreasing in Q. Therefore, for any given θ and θ_s, there is a one-to-one correspondence between r and Q. Hence, the manufacturer can choose Q instead and based on the equilibrium condition, deduce the corresponding r and announce it to buyers. Hence, the supplier's optimization problem is

$$\begin{aligned}
\max_{Q,\theta,\theta_s} \quad & \Pi_s = 2(v-w)\frac{R(Q)}{R'(Q)}H_i(Q,2Q) - \beta h(2Q) + (w-c)S_t(2Q) + 2(v-w)\theta_s(T_i(Q,Q)) \\
s.t. \quad & r = (v-w)\frac{H_i(Q,2Q)}{R'(Q)}
\end{aligned}$$

Note, using Lemma A.7, $\frac{\mathrm{d}\Pi_s}{\mathrm{d}\theta_s} = 2(v-w)\frac{R(Q)}{R'(Q)}\frac{\mathrm{d}H_i(Q,2Q)}{\mathrm{d}\theta} < 0$. Therefore, Π_s is decreasing in θ and hence the optimal $\theta = 0$. We can simplify the profit function of the supplier as follows:

$$\begin{aligned}
\Pi_s =& 2(v-w)\frac{R(Q)}{R'(Q)}\left(Pr(D_i > Q, D_t < 2Q) + Pr(D_t > 2Q)\right) - \beta h(2Q) + (w-c)S_t(2Q) \\
&+ 2(v-w)\theta_s\left(T_i(Q,Q) - \frac{R(Q)}{R'(Q)}Pr(D_i < Q, D_t > 2Q)\right)
\end{aligned}$$

This function is linear in θ_s and hence, the optimal θ_s is at the boundary. ■

Proof of Theorem 7.

For any given θ, we maximize the supplier's profit function by choosing θ_s and $\tilde{r}$. That is the supplier's optimization problem is $\max_{(\tilde{r},\theta_s)}\{\Pi_s | \tilde{r} \in [0,1],\, \theta_s \in [0,1]\}$, assuming Q_i satisfies the equilibrium condition.

For symmetric buyers, the equilibrium condition simplifies to

$$\tilde{r} = 1 - \Phi(z) - (1-\theta_s)\theta(\Phi(\frac{z}{\alpha}) - \Phi_\alpha(z)) + (1-\theta_s)(1-\theta)(\Phi(z) - \Phi_\alpha(z)),$$

where $z = \frac{Q_i - \mu}{\sigma}$. Since the right hand side is decreasing in Q_i, there is a one to one correspondence between $\tilde{r}$ and Q_i for any given θ_s. Hence, instead of optimizing Π_s for θ_s and $\tilde{r}$, we can optimize for θ_s and Q_i or equivalently for θ_s and z. We can simplify the profit function of the supplier, using the equilibrium condition and definition of z: $\frac{1}{2\sigma(v-c)}\Pi_s = f_2(z,\theta) + m_r\theta_s f_3(z,\theta)$, where

$$f_2(z,\theta) = s_l\left(z+\frac{\mu}{\sigma}\right) - (1-m_r)\int_{-\infty}^{z}\Phi(\frac{x}{\alpha})\,\mathrm{d}x + m_r\left(-\theta\Phi(z) - \theta\Phi(\frac{z}{\alpha}) + (2\theta-1)\Phi_\alpha(z)\right)\left(z+\frac{\mu}{\sigma}\right),$$

$$f_3(z,\theta) = \int_{-\infty}^{z}\left(\Phi(x) - \Phi\left(\frac{x}{\alpha}\right)\right)\,\mathrm{d}x - \left(-\theta\Phi(\frac{z}{\alpha}) + (2\theta-1)\Phi_\alpha(z) + (1-\theta)\Phi(z)\right)\left(z+\frac{\mu}{\sigma}\right)$$

Note $\Phi_\alpha(z)$ is c.d.f of bivariate standard normal distribution with correlation α, calculated at z and $\frac{z}{\alpha}$ as specified before.

Notice that the profit function Π_s is linear in θ_s. Hence, the boundaries $\theta_s = 0$ or $\theta_s = 1$ is included in the optimal solution for any given θ and any given z. ■

Proof of Proposition 4. We use backward induction. At the last stage, the buyers decide the order quantity Q. The buyer's profit function is:

$$\Pi_{B_i} = -rQ_i + (v-w)S_i(Q_i) + (1-\theta_s)\theta(v-w)T_i(Q_i,Q_j) + (1-\theta_s)(1-\theta)(v-w)T_j(Q_j,Q_i)\,.$$

Given θ, θ_s, and r, the problem of the buyers to solve for the optimal quantity is the same as our original model. Hence the equilibrium quantity of the buyers satisfies the same condition $H_i(Q,2Q) = \frac{r}{v-w}$, where

$$H_i(Q_i,Q_t) = Pr(D_i > Q_i) - (1-\theta_s)\theta Pr(D_i > Q_i, D_t < Q_t) + (1-\theta_s)(1-\theta)Pr(D_i < Q_i, D_t > Q_t)\,.$$

Since at equilibrium $T_i(Q,Q) = T_j(Q,Q)$, the Buyer i's profit function reduces to:

$$\Pi_{B_i} = -rQ + (v-w)(Q - \int_{-\infty}^{Q} Pr(D_i < x)\,\mathrm{d}x) + (1-\theta_s)(v-w)\int_{-\infty}^{Q} Pr(D_i > 2Q - x, D_j < x)\,\mathrm{d}x\,.$$

Hence

$$\frac{\mathrm{d}\Pi_{B_i}}{\mathrm{d}\theta} = (v-w)\frac{\mathrm{d}Q}{\mathrm{d}\theta}\left(-\frac{r}{v-w} + 1 - \theta_s Pr(D_i < Q) - (1-\theta_s)Pr(D_t < 2Q)\right)$$

By envelop theorem, we can differentiate the equilibrium condition with respect to θ to get $\frac{\mathrm{d}Q}{\mathrm{d}\theta}(\frac{\mathrm{d}H_i}{Q_i} + 2\frac{\mathrm{d}H_i}{Q_t}) + (1-\theta_s)(Pr(D_i < Q, D_t > 2Q) + Pr(D_i > Q, D_t < 2Q) = 0$; hence, we must have $\frac{\mathrm{d}Q}{\mathrm{d}\theta} > 0$. Also, $-\frac{r}{v-w} + 1 - \theta_s Pr(D_i < Q) - (1-\theta_s)Pr(D_t < 2Q)$ is decreasing in Q. Hence, optimal Π_{B_i} is quasi-concave in θ and the optimal θ satisfies the first order condition.

Therefore, either $\theta_s = 1$ and θ does not matter, or $\theta_s < 1$ and at optimal θ, Q must satisfy $\frac{r}{v-w} = 1 - \theta_s Pr(D_i < Q) - (1-\theta_s)Pr(D_t < 2Q)$.

Combined with equilibrium condition, we can conclude:

$\theta = \frac{Pr(D_i > Q, D_t < 2Q)}{Pr(D_i < Q, D_t > 2Q) + Pr(D_i > Q, D_t < 2Q}$), and $1 - \frac{r}{v-w} = \theta_s Pr(D_i < Q) + (1-\theta_s)Pr(D_t < 2Q)$. ■

Proof of Theorem 8. The supplier's profit function is:

$$\Pi_s = (r-h)(2Q) + (w-c)(2Q - \int_{-\infty}^{2Q} Pr(D_t < x)\,\mathrm{d}x) + (v-w)\theta_s(T_i(Q,2Q) + T_j(Q,2Q))\,.$$

Define $z = \frac{Q-\mu}{\sigma}$. Then we can simplify the supplier's profit function:

$$\frac{1}{2\sigma(v-c)}\Pi_s = (\frac{r}{v-w} - 1)m_r(z+\frac{\mu}{\sigma}) + s_l(z+\frac{\mu}{\sigma}) - (1-m_r)\int_{-\infty}^{z}\Phi(\frac{x}{\alpha})\,\mathrm{d}x + m_r\theta_s(\int_{-\infty}^{z}\Phi(x) - \Phi(\frac{x}{\alpha})\,\mathrm{d}x)\,.$$

We can replace the equilibrium condition for Q in the above expression and simplify $\frac{1}{2\sigma(v-c)}\Pi_s = \Pi_{0\theta*}(z) + m_r\theta_s f_{\theta^*}(z)$, where

$$\Pi_{0\theta*}(z) = s_l(z+\frac{\mu}{\sigma}) - m_r\Phi(\frac{z}{\alpha})(z+\frac{\mu}{\sigma}) - (1-m_r)\int_{-\infty}^{z}\Phi(\frac{x}{\alpha})\,\mathrm{d}x$$

$$f_{\theta^*}(z) = \int_{-\infty}^{z}\Phi(x) - \Phi(\frac{x}{\alpha})\,\mathrm{d}x - (\Phi(z) - \Phi(\frac{z}{\alpha}))(z+\frac{\mu}{\sigma})\,.$$

The supplier maximizes his profit by choosing r and θ_s or equivalently by maximizing for z and θ_s. Next, we characterize the solution of the function $f_{\theta^*} = 0$ which does not depend on s_l or m_r.

Lemma A.10.

1. *$f_{\theta^*}(z) = 0$ has a unique solution for $z > -\frac{\mu}{\sigma}$. Let $z^{f_{\theta^*}}$ represent this unique solution.*
2. *$f'_{\theta^*}(z^{f_{\theta^*}}) > 0$.*
3. *for $z < z^{f_{\theta^*}}$, we have $f_{\theta^*}(z) < 0$ and for $z > z^{f_{\theta^*}}$, we have $f_{\theta^*}(z) > 0$.*
4. *$z^{f_{\theta^*}} < 0$.*

Proof of Lemma A.10. Note that we can simplify $f_{\theta^*}(z) = -(\Phi(z) - \Phi(\frac{z}{\alpha}))\frac{\mu}{\sigma} + (\phi(z) - \alpha\phi(\frac{z}{\alpha}))$. Also derivative of $f_{\theta^*}(z)$ is $\frac{\mathrm{d}f_{\theta^*}(z)}{\mathrm{d}z} = -(\frac{\mu}{\sigma} + z)\frac{1}{\alpha}\phi(z)(\alpha - \sqrt{2\pi}\phi(\sqrt{\frac{1-\alpha^2}{\alpha}}z))$.
Therefore $\frac{\mathrm{d}f_{\theta^*}(z)}{\mathrm{d}z} = 0$ has exactly two solutions. Also, $\frac{\mathrm{d}f_{\theta^*}(z)}{\mathrm{d}z} < 0$ when $z \to -\infty$ and $\frac{\mathrm{d}f_{\theta^*}(z)}{\mathrm{d}z} < 0$ when $z \to \infty$. Hence, $f_{\theta^*}(z)$ is decreasing, then increasing and then decreasing. Also, $\lim_{z\to\pm\infty} f_{\theta^*}(z) = 0$. Hence, $f_{\theta^*}(z) = 0$ has exactly one solution and at this unique solution, $f_{\theta^*}(z)$ is strictly increasing. Hence, for $z < z^{f_{\theta^*}}$, we have $f_{\theta^*}(z) < 0$ and for $z > z^{f_{\theta^*}}$, we have $f_{\theta^*}(z) > 0$. Also, since $f(0) = \frac{1-\alpha}{\sqrt{2\pi}} > 0$, we must have $z^{f_{\theta^*}} < 0$. ∎

Next, we characterize the solution to optimization $\max_z\{\Pi_{0\theta^*}(z) \,|\, z \in [\frac{\mu_t}{2\sigma_i}, z^{f_{\theta^*}}]\}$.

Lemma A.11.

1. *For $z \in [\frac{\mu}{\sigma}, z^{f_{\theta^*}}]$, $\Pi_{0\theta^*}(z)$ is concave and hence has a unique maximizer in the interval.*
2. *If $\Pi'_{0\theta^*}(z^{f_{\theta^*}}) \geq 0$, then $\arg\max_z\{\Pi_{0\theta^*}(z) \,|\, z \in [\frac{\mu_t}{2\sigma_i}, z^{f_{\theta^*}}]\} = z^{f_{\theta^*}}$.*
3. *If $\Pi'_{0\theta^*}(z^{f_{\theta^*}}) < 0$, then $\arg\max_z\{\Pi_{0\theta^*}(z) \,|\, z \in [\frac{\mu_t}{2\sigma_i}, z^{f_{\theta^*}}]\}$ satisfies $\Pi'_{0\theta^*}(z) = 0$ and $z < z^{f_{\theta^*}}$.*

We label the solution to $\Pi'_{0\theta^*}(z) = 0$ *as* $z^{0\theta^*}$*.*

4. $z^{0\theta^*}$ *is decreasing in* m_r *and increasing in* s_l*. When* s_l *large enough and* m_r *small enough,* $z^{0\theta^*}$ *can be arbitrarily large (e.g., larger than* $z^{f_{\theta^*}}$*); when* s_l *is small enough,* $z^{0\theta^*}$ *can be arbitrarily small (e.g., smaller than* $z^{f_{\theta^*}}$*).*

Proof of Lemma A.11. Note that:

$$\frac{1}{1-\Phi(\frac{z}{\alpha})}\frac{\mathrm{d}\Pi_{0\theta^*}(z)}{\mathrm{d}z} = 1-(1-s_l)\frac{1}{1-\Phi(\frac{z}{\alpha})} - m_r\frac{\frac{1}{\alpha}\phi(\frac{z}{\alpha})(z+\frac{\mu}{\sigma})}{1-\Phi(\frac{z}{\alpha})}$$

Since $\frac{1}{1-\Phi(\frac{z}{\alpha})}$ and $\frac{\frac{1}{\alpha}\phi(\frac{z}{\alpha})(z+\frac{\mu}{\sigma})}{1-\Phi(\frac{z}{\alpha})}$ are increasing in z, $\frac{1}{1-\Phi(\frac{z}{\alpha})}\frac{\mathrm{d}\Pi_{0\theta^*}(z)}{\mathrm{d}z}$ is decreasing in z.

Also, since when $z \to -\frac{\mu}{\sigma}$, we have $\frac{\mathrm{d}\Pi_{0\theta^*}(z)}{\mathrm{d}z} \to s_l > 0$, and when $z \to \infty$, we have $\frac{\mathrm{d}\Pi_{0\theta^*}(z)}{\mathrm{d}z} \to -(1-s_l) < 0$, we must have that $\frac{\mathrm{d}\Pi_{0\theta^*}(z)}{\mathrm{d}z}$ has a unique solution. As a result, the first three statements of the lemma follows.

Next, we show the last statement. Note that $\frac{\mathrm{d}\Pi_{0\theta^*}(z)}{\mathrm{d}z}$ is increasing in s_l and decreasing in m_r and decreasing in z. Hence, when s_l increases, the solution to $\frac{\mathrm{d}\Pi_{0\theta^*}(z)}{\mathrm{d}z} = 0$ should increase as well; and when m_r increases, the solution to $\frac{\mathrm{d}\Pi_{0\theta^*}(z)}{\mathrm{d}z} = 0$ should decrease as well.

Furthermore, when $s_l \to 1$ and $m_r \to 0$, $\frac{\mathrm{d}\Pi_{0\theta^*}(z)}{\mathrm{d}z} \to 1-\Phi(\frac{z}{\alpha})$. Hence, $z^{0\theta^*} \to +\infty$. Also, when $s_l \to 0$, $\frac{\mathrm{d}\Pi_{0\theta^*}(z)}{\mathrm{d}z} \to -\Phi(\frac{z}{\alpha}) - m_r\frac{1}{\alpha}\phi(\frac{z}{\alpha})(z+\frac{\mu}{\sigma})$. Hence, $z^{0\theta^*} \to -\infty$. That is, we have the last statement of the Lemma. ∎

Next, we characterize the solution to optimization $\max_z\{\Pi_{1\theta^*}(z)\,|\,z \geq z^{f_{\theta^*}}\}$.

Lemma A.12.

1. *For* $z \geq z^{f_{\theta^*}}$*, we have* $\Pi_{1\theta^*}(z) \overset{def}{=} \Pi_{1\theta^*}(z) + m_r f_{\theta^*}(z)$ *is quasi-concave and hence has a unique maximizer in the interval.*
2. *If* $\Pi'_{1\theta^*}(z^{f_{\theta^*}}) \leq 0$*, then* $\arg\max_z\{\Pi_{1\theta^*}(z)\,|\,z \geq z^{f_{\theta^*}}\} = z^{f_{\theta^*}}$*.*
3. *If* $\Pi'_{1\theta^*}(z^{f_{\theta^*}}) > 0$*, then* $\arg\max_z\{\Pi_{1\theta^*}(z)\,|\,z \geq z^{f_{\theta^*}}\}$ *satisfies* $\Pi'_{1\theta^*}(z) = 0$ *and* $z > z^{f_{\theta^*}}$*. We label the solution to* $\Pi'_{1\theta^*}(z) = 0$ *as* $z^{1\theta^*}$*.*
4. $z^{1\theta^*}$ *is decreasing in* m_r *and increasing in* s_l*. When* s_l *large enough and* m_r *small enough,* $z^{1\theta^*}$ *can be arbitrarily large (e.g., larger than* $z^{f_{\theta^*}}$*); when* s_l *is small enough,* $z^{1\theta^*}$ *can be arbitrarily small (e.g., smaller than* $z^{f_{\theta^*}}$*).*

Proof of Lemma A.12. Note that:

$$\frac{1}{1-\Phi(\frac{z}{\alpha})}\frac{\mathrm{d}\Pi_{1\theta^*}(z)}{\mathrm{d}z} = 1-(1-s_l)\frac{1}{1-\Phi(\frac{z}{\alpha})} - m_r\left(\frac{\phi(z)(z+\frac{\mu}{\sigma})}{1-\Phi(\frac{z}{\alpha})}\right)$$

Next we show that $\frac{\phi(z)(z+\frac{\mu}{\sigma})}{1-\Phi(\frac{z}{\alpha})}$ is increasing in z.

For $z \leq 0$, $\phi(z)$ is increasing in z; hence $\frac{\phi(z)(z+\frac{\mu}{\sigma})}{1-\Phi(\frac{z}{\alpha})}$ is increasing in z. For $z > 0$, (i) $\frac{\phi(z)}{1-\Phi(z)}$ is hazard rate of normal distribution and it is positive, increasing and convex; (ii) $(z+\frac{\mu}{\sigma})$ is positive and increasing in z; (iii) $\frac{1-\Phi(z)}{1-\Phi(\frac{z}{\alpha})}$ is positive and it is increasing in z because:

$$\frac{1-\Phi(\frac{z}{\alpha})}{1-\Phi(z)} \times \frac{\mathrm{d}}{\mathrm{d}z}(\frac{1-\Phi(z)}{1-\Phi(\frac{z}{\alpha})}) = -\frac{\phi(z)}{1-\Phi(z)} + \frac{1}{\alpha}\frac{\phi(\frac{z}{\alpha})}{1-\Phi(\frac{z}{\alpha})}.$$

Since $z > 0$ and $0 \leq \alpha \leq 1$, and $\frac{\phi(z)}{1-\Phi(z)}$ is convex, we must have $\frac{1-\Phi(z)}{1-\Phi(\frac{z}{\alpha})}$ is increasing in z for $z > 0$.
As a result, $\frac{\phi(z)(z+\frac{\mu}{\sigma})}{1-\Phi(\frac{z}{\alpha})} = \frac{\phi(z)}{1-\Phi(z)} \times (z+\frac{\mu}{\sigma}) \times \frac{1-\Phi(z)}{1-\Phi(\frac{z}{\alpha})}$ is increasing in z for $z > 0$.
In summary, $\frac{\phi(z)(z+\frac{\mu}{\sigma})}{1-\Phi(\frac{z}{\alpha})}$ is increasing in z for any z.
Since $\frac{1}{1-\Phi(\frac{z}{\alpha})}$ and $\frac{\phi(z)(z+\frac{\mu}{\sigma})}{1-\Phi(\frac{z}{\alpha})}$ are increasing in z, $\frac{1}{1-\Phi(\frac{z}{\alpha})}\frac{\mathrm{d}\Pi_{1\theta^*}(z)}{\mathrm{d}z}$ is decreasing in z.
Also, since when $z \to -\frac{\mu}{\sigma}$, we have $\frac{\mathrm{d}\Pi_{1\theta^*}(z)}{\mathrm{d}z} \to s_l > 0$, and when $z \to \infty$, we have $\frac{\mathrm{d}\Pi_{1\theta^*}(z)}{\mathrm{d}z} \to -(1-s_l) < 0$, we must have that $\frac{\mathrm{d}\Pi_{1\theta^*}(z)}{\mathrm{d}z}$ has a unique solution. As a result, the first three statements of the lemma follows.

Next, we show the last statement. Note that $\frac{1}{1-\Phi(\frac{z}{\alpha})}\frac{\mathrm{d}\Pi_{1\theta^*}(z)}{\mathrm{d}z}$ is increasing in s_l and decreasing in m_r and decreasing in z. Hence, when s_l increases, the solution to $z^{1\theta^*}$ should increase as well; and when m_r increases, the solution $z^{1\theta^*}$ should decrease as well.

Furthermore, when $s_l \to 1$ and $m_r \to 0$, $\frac{\mathrm{d}\Pi_{1\theta^*}(z)}{\mathrm{d}z} \to 1-\Phi(\frac{z}{\alpha})$. Hence, $z^{1\theta^*} \to +\infty$. Also, when $s_l \to 0$, $\frac{\mathrm{d}\Pi_{1\theta^*}(z)}{\mathrm{d}z} \to -\Phi(\frac{z}{\alpha}) - m_r\phi(z)(z+\frac{\mu}{\sigma})$. Hence, $z^{1\theta^*} \to -\infty$. That is, we have the last statement of the Lemma. ■

Note that the supplier's problem is to find optimal z and θ_s as specified before. More specifically, the supplier's problem is $\max_{(z,\theta_s)}\{\Pi_{0\theta^*} + m_r\theta_s f_{\theta^*}(z)|\theta_s \in [0,1]\}$. The feasible region of the supplier's problem can be divided to two regions: (i) Region 0 where $z \in [-\frac{\mu}{\sigma}, z^{f_{\theta^*}}]$, and (ii) Region 1 where $z \geq z^{f_{\theta^*}}$. In region 0, either optimal $z < z^{f_{\theta^*}}$ and the optimal $\theta_s = 0$ or optimal $z = z^{f_{\theta^*}}$ and $\theta_s f_{\theta^*}(z) = 0$. That is, the optimization in Region 0 reduces to $\theta_s = 0$ and solving for optimal solution of the optimization problem discussed in Lemma A.11.

In region 1, either optimal $z > z^{f_{\theta^*}}$ and the optimal $\theta_s = 1$ or optimal $z = z^{f_{\theta^*}}$ and $\theta_s f_{\theta^*}(z) = 0 = f_{\theta^*}(z)$. That is, the optimization in Region 1 reduces to $\theta_s = 1$ and solving for optimal solution of the optimization problem discussed in Lemma A.12.

The following Lemma shows that at $z = z^{f_{\theta *}}$ is never optimal for the supplier.

Lemma A.13. *It is impossible that both $\Pi'_{0\theta^*}(z^{f_{\theta^*}}) \geq 0$ and $\Pi'_{1\theta^*}(z^{f_{\theta^*}}) \leq 0$.*

Proof of Lemma A.13. Assume $\Pi'_{0\theta^*}(z^{f_{\theta^*}}) \geq 0$ and $\Pi'_{1\theta^*}(z^{f_{\theta^*}}) \leq 0$. Then $m_r f'_{\theta^*}(z^{f_{\theta^*}}) = \Pi'_{1\theta^*}(z^{f_{\theta^*}}) - \Pi'_{0\theta^*}(z^{f_{\theta^*}}) \leq 0$, which contradicts Lemma A.10. ■

Since, the profit function of the supplier is continuous, Lemma A.13 implies that either $z^{1\theta^*}$ or $z^{0\theta^*}$ is the optimal solution of the supplier's problem which are different from $z^{f_{\theta *}}$ because $z^{0\theta^*} < z^{f_{\theta *}} < z^{1\theta^*}$. Finally, note that, when s_l is arbitrary close to 1 and m_r is arbitrary close to zero, by Lemma A.11 and Lemma A.12, both $z^{0\theta^*}$ and $z^{0\theta^*}$ get arbitrary larger than $z^{f_{\theta *}}$ and hence $\theta_s = 1$ is the optimal solution of the supplier's problem. Similarly, when s_l gets arbitrary close to zero, by Lemma A.11 and Lemma A.12, both $z^{0\theta^*}$ and $z^{0\theta^*}$ get arbitrary smaller than $z^{f_{\theta *}}$ and hence $\theta_s = 0$ is the optimal solution of the supplier's problem. ■

A.3 Robustness Check: Underinvestment with Breach Remedy

In this section, we compare the setting in **?** by allowing the supplier to underinvest in capacity with a breach remedy with our setting in which the supplier always invests in sufficient capacity.

Similar to **?**, we assume that the supplier exercises a breach remedy. More specifically, in case the supplier underinvests in capacity and cannot deliver the reserved capacity of buyers, he would compensate the buyers for their losses that can be due to (i) inability to satisfy their own demand with their own reserved capacity, (ii) inability to transfer their excess reserved capacity to another buyer in need of capacity for a fee, or (iii) inability to receive transfers from another buyer who could have excess capacity. In summary, similar to **?**, the buyers do not lose profit if the supplier decides to underinvest in capacity.

Denote the capacity that the supplier reserves by C. Assume the supplier is allowed to reserve capacity C less than what the buyers have asked for, i.e., $C \leq Q_t$. The supplier decides r, θ, and θ_s before the buyers decide the capacity to reserve Q_i and Q_j. After the buyers order to reserve capacity Q_i and Q_j, the supplier decides how much capacity $C \leq Q_i + Q_j$ to invest. We focus on buyers with symmetric normal demand distributions.

Proposition A.1. *The supplier never underinvests in capacity. That is, the supplier never invests* *in less capacity than requested by buyers, i.e., C is never less than Q_t.*

This proposition implies that even if we allow underinvestment like the setting in **?**, the supplier would not underinvest in capacity.

Proof of Proposition A.1. The supplier's profit in this model is

$$\Pi_s = rQ_t - hC + (w-c)(C - \int_{-\infty}^{C} Pr(D_t < x)\,\mathrm{d}x) + \theta_s(v-w)(T_i(Q_i,Q_j) + T_j(Q_j,Q_i))$$
$$- (v-w)\mathbb{E}[\min\{Q_t - C, D_t - C\} \times \mathbb{1}(D_t > C)]$$

We can simplify the supplier's profit as follows:

$$\begin{aligned}\frac{1}{v-c}\Pi_s &= (\frac{r}{v-w} - 1)m_rQ_t + s_lC - (1-m_r)\int_{-\infty}^{C} Pr(D_t < x)\,\mathrm{d}x \\ &+ \theta_s m_r\left(\int_{-\infty}^{Q_i} Pr(D_i < x)\,\mathrm{d}x + \int_{-\infty}^{Q_j} Pr(D_j < x)\,\mathrm{d}x - \int_{-\infty}^{Q_t} Pr(D_t < x)\,\mathrm{d}x\right) \\ &+ m_r\int_{C}^{Q_t} Pr(D_t < x)\,\mathrm{d}x\end{aligned}$$

We use backward induction. In the last stage, the supplier determines the quantity C to reserve, given quantities Q_i and Q_j reserved by buyers, and given the values of r and θ, with the constraint that $C \le Q_t$, where $Q_t = Q_i + Q_j$. Note that $\frac{\mathrm{d}\Pi_s}{\mathrm{d}C} = s_l - Pr(D_t < C)$. As a result, the optimal C^* satisfies the follow: (i) if $Pr(D_t < Q_t) > s_l$, then C^* satisfies $s_l = Pr(D_t < C^*)$; (ii) if $Pr(D_t < Q_t) \le s_l$, then $C^* = Q_t$. If $C^* = Q_t$, we have the proof. Suppose $C^* < Q_t$ which means $s_l = Pr(D_t < C^*)$. In this case, C^* does not depend on Q_t. That is $\frac{\mathrm{d}C^*}{\mathrm{d}Q_t} = 0$.

In the next stage, the buyers optimize for the quantity of capacity to reserve. The buyer's profit function is:

$$\Pi_{B_i} = -rQ_i + (v-w)(Q_i - \int_{-\infty}^{Q_i} Pr(D_i < x)\,\mathrm{d}x) + (1-\theta_s)(v-w)\left((1-\theta)T_i(Q_i,Q_j) + \theta T_j(Q_j,Q_i)\right)$$

The buyers' problem is similar to our original model. Hence, for symmetric buyers, we can conclude that at equilibrium, the buyers each reserve the same quantity Q^* that satisfies: $H(Q^*) = \frac{r}{v-w}$, where, $H(Q) = 1 - Pr(D_i < Q) - (1-\theta_s)\theta Pr(D_i > Q, D_t < 2Q) + (1-\theta_s)(1-\theta)Pr(D_i < Q, D_t > 2Q)$.

In the next stage, the supplier optimizes for (r,θ,θ_s) at the same time. Define function $z(Q) = \frac{Q-\mu}{\sigma}$ (or in short z). Since there is a one to one correspondence between (r,θ,θ_s) and $(z(Q^*),\theta,\theta_s)$, the supplier can instead optimize his profit by choosing (z,θ,θ_s), and then optimal r is obtained by equilibrium condition.

Replacing the equilibrium condition in the profit function of the supplier we have

$$\tfrac{1}{v-c}\Pi_s = \big((-Pr(D_i < Q^*) - (1-\theta_s)\theta Pr(D_i > Q^*, D_t < 2Q^*)$$
$$+ (1-\theta_s)(1-\theta)Pr(D_i < Q^*, D_t > 2Q^*)\big)m_r(2Q^*) + s_l C^* - (1-m_r)\int_{-\infty}^{C^*} Pr(D_t < x)\,\mathrm{d}x$$
$$+ \theta_s m_r\left(2\int_{-\infty}^{Q^*} Pr(D_i < x)\,\mathrm{d}x - \int_{-\infty}^{2Q^*} Pr(D_t < x)\,\mathrm{d}x\right) + m_r\int_{C^*}^{2Q^*} Pr(D_t < x)\,\mathrm{d}x.$$

Note that $\frac{\mathrm{d}\Pi_s}{\mathrm{d}\theta} = -2Q(v-c)m_r(1-\theta_s)\left(Pr(D_i > Q, D_t < 2Q) + Pr(D_i < Q, D_t > 2Q)\right) < 0$. Hence, optimal $\theta = 0$. The supplier's profit reduces to

$$\tfrac{1}{v-c}\Pi_s = (-\Phi(z) + (1-\theta_s)(\Phi(z) - \Phi_\alpha(z)))\,m_r(2(\mu+\sigma z)) + s_l C^* - (1-m_r)\int_{-\infty}^{C^*} Pr(D_t < x)\,\mathrm{d}x +$$
$$\theta_s m_r\left(2\int_{-\infty}^{(\mu+\sigma z)} Pr(D_i < x)\,\mathrm{d}x - \int_{-\infty}^{2(\mu+\sigma z)} Pr(D_t < x)\,\mathrm{d}x\right) + m_r\int_{C^*}^{2(\mu+\sigma z)} Pr(D_t < x)\,\mathrm{d}x.$$

Hence, $\frac{1}{v-c}\frac{1}{2\sigma m_r}\frac{\mathrm{d}\Pi_s}{\mathrm{d}z} = -(1-\theta_s)(\Phi'_\alpha(z)(z+\frac{\mu}{\sigma}) + \Phi_\alpha(z) - \Phi(\frac{z}{\alpha})) - \theta_s\phi(z)(z+\frac{\mu}{\sigma})$.

Using Lemma A.2, the last expression is negative. That is, Π_s is decreasing in z or equivalently increasing in capacity reservation fee $r < v - w$, if the supplier underinvests in capacity. As a result, $r^* = v - w$, which implies $Q^* = 0$. But we had assumed $C* < Q_t = 2Q^* = 0$ which is a contradiction. Hence, if the supplier determines C, we must have $C^* = Q_t$. ■

A.4 Robustness Check: θ is set by one of the buyers after demand is realized.

In this section, we check the robustness of our result by extending our analysis to cases in which the value of θ is set by one of the buyers, *after* their demand is realized and they recognize their shortage and excess capacities. More specifically, in two different models, we allow (1) the buyer with an excess capacity, and (2) the buyer with a shortage of capacity to set the value of θ. We focus on two symmetric buyers. We are interested to how the solution to the model changes in this new scenarios. The sequence of events for this scenario is shown in Figure 1.

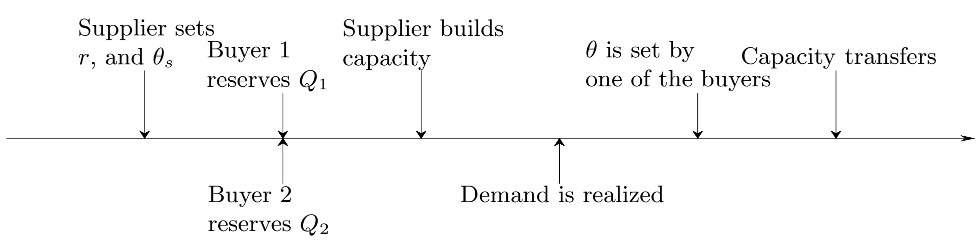

Figure 1: The sequence of events when θ is set by the buyer with an excess or a shortage of capacity.

Let us focus on the case in which the buyer with an excess capacity sets the value of θ. Note that, if capacity is not transferred, value of θ is not relevant.

Theorem A.1. *The model in which the buyer with an excess capacity sets the value of θ is equivalent to a model in which the supplier sets the value of θ. More specifically, in both models, at equilibrium, θ, θ_s, and r are the same, and the supplier's expected profit and the buyers' expected profits are the same.*

Proof of Theorem A.1. Consider the model in which the buyer with an excess capacity sets the value of θ. We use backward induction to solve the problem.

Suppose demand of Buyer i and Buyer j is realized and they are d_i, and d_j respectively. Without loss of generality, assume buyer i has excess capacity and buyer j has shortage of capacity. That is, $Q_i > d_i$ and $Q_j < d_j$. Then, the profit of the Buyer i is:

$$\Pi_{B_i} = -rQi + (v-w)(Q_i - d_i) + (1-\theta_s)(1-\theta)(\min\{Q_i - d_i, d_j - Q_j\}) .$$

This profit function is decreasing in value of θ. Hence, for the buyer with an excess capacity, optimal $\theta = 0$. As a result, the expected profit of buyer i is $\mathbb{E}\big[\Pi_{B_i}\big] = -rQi + (v-w)(Q_i - \int_{-\infty}^{Q_i})Pr(D_i < x)\,\mathrm{d}x + (1-\theta_s)T_j(Q_j, Q_i)$.

Therefore, using Lemma A.1, we have

$$\begin{aligned}\frac{1}{v-w}\frac{\mathrm{d}\Pi_{B_i}}{\mathrm{d}Q_i} &= -\frac{r}{v-w} + Pr(D_i > Q_i) + (1-\theta_s)\frac{\mathrm{d}T_j(Q_j, Q_i)}{\mathrm{d}Q_i} \\ &= -\frac{r}{v-w} + Pr(D_i > Q_i) + (1-\theta_s)Pr(D_i < Q_i, D_t > Q_i + Q_j) .\end{aligned}$$

Since, ex-ante, Buyer i and Buyer j are identical, we can find the equilibrium reserved capacities $Q_i = Q_j$ by solving the following equation $\frac{r}{v-w} = Pr(D_i > Q_i) + (1-\theta_s)Pr(D_i < Q_i, D_t > 2Q_i)$. Notice that this equilibrium condition is the same condition as the model in which the supplier sets the value of $\theta = 0$. As a result, in both models, where the supplier sets the value of θ or the buyer with an excess demand sets the value of θ, the supplier's problem to find the optimal θ_s and r is the same. That is, the two models result in the same equilibrium θ, θ_s, r, and profits. ■

Theorem A.2. *In the model in which the buyer with a shortage of capacity sets the value of θ, $\theta = 1$, and $\theta_s = 1$. That is, the supplier gets all the benefits from the transfers.*

Proof of Theorem A.2. Consider the model in which the buyer with a shortage of capacity sets the value of θ. We use backward induction to solve the problem.

Suppose demand of Buyer i and Buyer j is realized and they are d_i, and d_j respectively. Without loss of generality, assume buyer i has shortage of capacity and buyer j has excess of capacity. That is, $Q_i < d_i$ and $Q_j > d_j$. Then, the profit of the Buyer i is $\Pi_{B_i} = -rQi + (v-w)Q_i + (1-\theta_s)\theta(\min\{d_i - Q_i, Q_j - d_j\})$. This profit function is increasing in value of θ. Hence, for the buyer with a shortage of capacity, optimal $\theta = 1$.

As a result, the expected profit of buyer i is $\mathbb{E}\big[\Pi_{B_i}\big] = -rQi + (v-w)(Q_i - \int_{-\infty}^{Q_i})Pr(D_i < x)\,\mathrm{d}x + (1-\theta_s)T_i(Q_i, Q_j)$. Therefore, using Lemma A.1, we have

$$\begin{aligned}\frac{1}{v-w}\frac{\mathrm{d}\Pi_{B_i}}{\mathrm{d}Q_i} &= -\frac{r}{v-w} + Pr(D_i > Q_i) + (1-\theta_s)\frac{\mathrm{d}T_i(Q_i,Q_j)}{\mathrm{d}Q_i} \\ &= -\frac{r}{v-w} + Pr(D_i > Q_i) - (1-\theta_s)Pr(D_i > Q_i, D_t < Q_i + Q_j)\,.\end{aligned}$$

Since, ex-ante, Buyer i and Buyer j are identical, we can find the equilibrium reserved capacities $Q_i = Q_j$ by solving the following equation $\frac{r}{v-w} = 1 - Pr(D_i < Q_i) - (1-\theta_s)Pr(D_i > Q_i, D_t < 2Q_i)$. The supplier's expected profit is $\Pi_s = (r-h)Q_t + (w-c)\mathbb{E}\big[\min(Q_t, D_t)\big] + \theta_s(v-w)(T_i(Q_i, Qj) + T_j(Q_j, Q_i))$. Hence, at equilibrium,

$$\begin{aligned}\frac{1}{2(v-c)}\Pi_s &= s_l Q_i - m_r\left(1 - \frac{r}{v-w}\right)Q_i - (1-m_r)\int_{-\infty}^{Q_i} Pr(D_t < 2x)\,\mathrm{d}x + \theta_s m_r T_i(Q_i, Q_i) \\ &= s_l Q_i - m_r\left(Pr(D_i < Q_i) + Pr(D_i > Q_i, D_t < 2Q_i)\right)Q_i - (1-m_r)\int_{-\infty}^{Q_i} Pr(D_t < 2x)\,\mathrm{d}x \\ &\quad + \theta_s m_r\left(T_i(Q_i, Q_i) + Pr(D_i > Q_i, D_t < 2Q_i)Q_i\right)\end{aligned}$$

where Q_i satisfies the equilibrium condition $\frac{r}{v-w} = 1 - Pr(D_i < Q_i) - (1-\theta_s)Pr(D_i > Q_i, D_t < 2Q_i$. Since based on the equilibrium condition, there is a one to one correspondence between (θ_s, r) and (θ_s, Q_i), instead of optimizing for (θ_s, r), the supplier can optimize for (θ_s, Q_i) and find the optimal value of r from the equilibrium condition.

Notice that, for any given Q_i, the expected profit function of the supplier is increasing in θ_s. Therefore, optimal $\theta_s = 1$ for any given Q_i. As a result, in equilibrium, optimal $\theta_s = 1$. ■