The Equivalent Current Method

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As Professor Deschamps suggested in his opening talk, this method of equivalent currents is a derived method having close connections with PTD and GTD and forming a sort of bridge between the two. It is also one whose origins go back many years indeed, and all who have observed that the diffracted field far from a straight edge appears to emanate from a line source coincident with the edge and having an appropriate polar diagram could well claim to be the originators of the method. For example, Braunbek who developed expressions for the field diffracted by a circular aperture using local approximations to the surface field in the vicinity of the edge, and more especially Millar who actually used the phrase "equivalent edge currents" and gave their expressions for a plane disk of arbitrary shape illuminated by a plane wave at normal incidence. This is back in 1955. And some of you may remember "circular wedge theory" from the same time period. This also is only the equivalent current method in disguise, and then there are Ryan and Peters who resurrected the method and its terminology a few years ago. These are but a few of the people who have explored the ECM and I have mentioned them to emphasize that what we are talking about is not some radically new theory.

Such new features as the present method has are, in my opinion, limited to

—the more general framework in which it is expressed

—and the explicit connection with GTD which is brought out in the derivation of the currents.

However, the method does have the advantages of usefulness and convenience as a practical cross section estimation tool, and it is these attributes that I would commend to you most of all.

I think it would give the right flavor to this talk if I were to indicate how we first got involved with this method. As all of you are aware, computer programs have been developed for predicting the scattering from quite complicated bodies using ordinary physical optics. These programs are quite useful for system design purposes and though the accuracy is not that great, it may be good enough. However, this is no longer true when you come to bodies like cones, cone-cylinder flares etc. which do not often present any specular contribution at the aspects of most interest, and this failure also coincided with the use of more advanced radars for which the phase and polarization characteristics are quantities of interest. Since diffraction is one of the main sources of de-polarization, what we needed to do is to incorporate diffraction effects into the program and our thoughts naturally turned to GTD.

Unfortunately, GTD is not very convenient for computation. Like all ray methods, there is the problem of locating the specular or flash points—and I am speaking here of a procedure appropriate to a class of bodies; then there are the ray divergence factors which can be difficult in a general case, and last but not least the caustic infinities, requiring the introduction of matching functions to provide a smooth transition into a finite wavelength dependent value at a caustic. In practice, you then have the problem of specifying the angle at which these functions must be introduced. Now just as p.o. can be validated only as a caustically-corrected version of g.o., so we might hope to overcome all of these difficulties by seeking an integration procedure aimed at the caustic correction of GTD, and since the wide angle GTD contributions can be attributed to the edges, it is natural to look to integration along the edges. This thinking leads automatically to the equivalent current method, but I would like you to notice two points:

—we focussed on GTD because of our confidence in the adequacy of this approximation for the task in hand

—and that rather than avoiding a numerical integration, we almost courted it because of its compatibility with the p.o. portion of the program.

To see how these equivalent currents are derived, consider the simple case of a circular ring discontinuity in slope, as at the base of a right circular cone. Then away from the axial caustic, first order GTD leads to a prescription of the wide angle diffracted field as the sum of contributions from two diametrically-opposed flash points, and the electric field can be written as

$$\underline{E}^{d} = -\frac{\Gamma \exp(iks)}{\sin^{2}\beta}\left\{(\underline{E}^{i}\cdot\hat{t})(X-Y)\,\hat{s}\times(\hat{s}\times\hat{t}) + Z_{0}(\underline{H}^{i}\cdot\hat{t})(X+Y)\,\hat{s}\times\hat{t}\right\}$$

where $X \pm Y$ are the scalar diffraction coefficients for a wedge, $\hat{t}$ is a unit vector tangent to the edge, $\hat{s}$ is a unit vector in the direction of observation, Γ is the ray divergence factor and β is the half angle of the Keller cone. This far field can also be expressed in terms of the Hertz vectors

$$\bar{\pi}_e = \frac{\hat{t}(\underline{E}^i \cdot \hat{t})(X - Y)\Gamma \exp(iks)}{k^2 \sin^3\beta}$$

$$\bar{\pi}_e = \frac{\hat{t}(\underline{H}^i \cdot \hat{t})(X + Y)\Gamma \exp(iks)}{k^2 \sin^3\beta}$$

but so far this is only GTD. We now seek to attribute these to filamentary currents coincident with the edge, and since we have electric and magnetic Hertz vectors, we are forced to postulate electric and magnetic currents $\underline{I}_e$ and $\underline{I}_m$ in terms of which (of course)

$$\bar{\pi}_e = \frac{1}{ik} \int_C Z_0 \underline{I}_e \frac{\exp(ikr)}{4\pi r} \, d\ell$$

$$\bar{\pi}_m = \frac{1}{ik} \int_C Y_0 \underline{I}_m \frac{\exp(ikr)}{4\pi r} \, d\ell \quad .$$

If we assume that the phase of the currents is that of the incident field, the integrals can be evaluated by the stationary phase method. The SP points are, of course, the flash points of GTD and comparison with the GTD expression then gives

$$\underline{I}_e = -\frac{2\hat{t}(\underline{E}^i \cdot \hat{t})(X - Y)}{ikZ_0 \sin^2\beta}$$

$$\underline{I}_m = -\frac{2\hat{t}(\underline{H}^i \cdot \hat{t})(X + Y)}{ikY_0 \sin^2\beta} \quad .$$

These reduce precisely to Millar's currents in the special case of axial incidence on a circular disk, when the diffraction coefficients are those for a half plane, and to Ryan and Peters' currents for axial incidence on a cone—in both cases, of course, $\beta = \pi/2$.

So far we have established the currents only at the flash points, but since these can be rotated around the ring by displacing the point of observation, the equations in fact specify the currents all the way around. There is certainly some similarity to the physical optics currents—even extending to the factor 2—but through forcing the currents to be confined to the edge, we have paid a penalty: X, Y and β depend on the direction of observation as well as incidence as well as position on the edge, and are therefore non-physical in character.

We'll examine some of the implications of these currents in a moment, but let's first see how we could use them in conjunction with the physical optics program. Thinking in terms of rotationally symmetric bodies composed of cones, cylinders, spheres, etc., we might try just adding ring currents at each of the junctions to the p.o. current over the rest of the surface. However, the diffraction coefficients $X \mp Y$ already contain a physical optics contribution, which must therefore be subtracted from the X and Y to avoid being included twice. This is precisely what Ufimtsev does in his separation of the uniform current. The functions which must be subtracted are easily found—they are just trigonometric ratios—and the modified diffraction coefficients then have the pleasant property of remaining finite even in the direction of the incident and reflected wave boundaries. In other words, the ring currents as well as the physical optics currents are finite for all directions of incidence and diffraction.

We now have a manageable prescription of a surface field and since the axial integration of the p.o. currents can be carried out analytically, we are left with just the azimuthal integration, which is done numerically. The resulting program is reasonably efficient, quite effective and handles bodies with a rather wide variety of profiles. Just as an illustration, there is the H plane backscattering pattern for a sphere-cone frustum: dots are computed, solid line measured, and though the radius of the front edge singularity is only $\lambda/4$, the agreement is quite good. Notice that there are no discontinuities such as you would normally get with GTD when a flash point becomes shadowed. The field is automatically continuous with this method though there is, of course, no justification for the particular smooth transition that this procedure produces.

At this stage we had completed what we set out to do. It is obvious that what we have is something very close to Ufimtsev's approximation, but which is also intimately connected to GTD through the form of the filamentary currents. For any linear diffracting element for which the GTD wide angle expression and, hence, the diffraction coefficients are known, you can write the currents and thereby determine the scattered field for arbitrary directions of incidence and observation, for example

$$S = \frac{1}{\lambda} \int_L \frac{e^{ik\underline{r}\cdot(\hat{i}-\hat{s})}}{\sin^2\beta} \left\{ e_{it}\, e_{rt} (X - Y) + h_{it}\, h_{rt} (X + Y) \right\} d\ell$$

where this is the far field amplitude measured by a receiver whose electric and magnetic polarizations have components e_{rt}, h_{rt} along the element. A stationary phase evaluation is guaranteed to reproduce the GTD expression. For a more accurate result you can always turn to a numerical evaluation, but experience with p.o. suggests that the improvement in accuracy will be a valid one only under circumstances where the stationary phase method cannot be used: in other words, near to a caustic.

In seeking to explore this method of equivalent currents further, it is natural to go back to the simple case of a circular ring singularity. The integration is then w.r.t. an azimuthal variable ϕ and you can obtain an approximate analytical evaluation of the integral for all directions of incidence and observation. To do this it is necessary to expand X and Y as Fourier series in ϕ. When you retain the terms which are dominant in ka, you find that the coefficients involve X_1, X_2, etc., namely the diffractions coefficients evaluated at the flash points in wide angle scattering. The integral is then

$$S = -\frac{ka}{2\sin^2\beta} \Big\{ (e_{it1}\, e_{rt1} + h_{it1}\, h_{rt1})(X_1 + X_2) J_2(kaT)$$

$$+ (e_{it1}\, e_{rt1} - h_{it1}\, h_{rt1}) \left[(Y_1 + Y_2) J_0(kaT) - i(Y_1 - Y_2) J_1(kaT) \right] \Big\}$$

where

$$T = \left\{ \sin^2\theta_i + 2\sin\theta_i \sin\theta_s \cos(\phi_s - \phi_i) + \sin^2\theta_s \right\}^{1/2} .$$

This result — agrees (to the first order in ka) with the known value on the axial caustic

— agrees with the wide angle GTD result when you replace each Bessel function by the leading term in its asymptotic expansion

— predicts that for backscattering the cross polarized return is proportional to the Bessel function J_2, and the values obtained are in good agreement with experimental data for a wide variety of bodies like cones, cone-spheres, torii, wire rings, etc.

— is virtually indistinguishable from data provided by a numerical evaluation of the integral expression

— agrees quite well with measured data.

This last fact is illustrated in the next slide, where I show comparison of theory and experiment for a 40^o half angle cone (ka = 18.4) as a function of θ_s ($= 30^o - \theta_i$) for bistatic scattering in the E- and H-planes. The agreement is quite reasonable, certainly out to 20^o.

It is obvious that the formula is very similar to a generalized form of that which Ufimtsev obtained for a plane disk when the scattering is in the plane of incidence. But there is a subtle difference: J_0 instead of J_2.

The difference is zero on axis and at wide angles. It is unclear which, if either, is correct, or even if the question has any meaning in the first place!

The expression for the first order diffracted field can be used as a new incident field to arrive at an expression for the re-diffracted field in the form of a double line integral. We have explored this in some detail for a circular ring discontinuity in the general bistatic scattering case. Though I don't want to go into this here, I will remark that if you use the stationary phase method of evaluation, the stationary points are just the coupled flash points of second order GTD, and you recover the precise wide-angle GTD expressions; whereas an approximate analytic evaluation yields a uniform expression valid even in the caustic regions. It is found that the functions describing this behavior are Fresnel integrals and Ufimtsev has now revised his second order results in accordance with this fact. But you must understand that I don't make this statement with any intent to claim theoretical superiority of one method over the other; in practice I believe them to be so similar that at most it is a question of convenience.

So far I have avoided bringing up one nagging uncertainty in the equivalent current method and its specification of the currents. This is evidenced by the angle β which occurs in the form $\sin^2\beta$ in the definitions of the currents and the scattered field, and the fact is that a knowledge of the GTD result determines the current only at those places on the edge where the Keller cone includes the scattering direction. But should β refer to the scattering direction or the incident direction, or to some combination of the two? For reasons of symmetry we have generally taken

$$\sin^2\beta = \sin\beta_i \sin\beta_s$$

where $\cos\beta_i = \hat{i}\cdot\hat{t}$, $\cos\beta_s = \hat{s}\cdot\hat{t}$, but until a specific choice can be justified, this is a conceptual flaw in the method. I do not know of any way to pin it down and it is perhaps fortunate the choice does not seem to matter very much in practice.

This is just one facet of the equivalent current method that still merits exploration, but even in its present state we have found it useful and convenient, both analytically and numerically. It may well be the common ground on which GTD and Ufimtsev theory can come together.

THE EQUIVALENT CURRENT METHOD

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This is a method for the prediction of edge diffraction which occupies a place midway between the geometrical and physical theories of diffraction. Currents are prescribed in such a way as to reproduce precisely the GTD far field scattering at wide angles, i.e., away from any caustic, but because the currents are filamentary ones confined to the edge, they are non-physical and, indeed, dependent on the direction of observation. In theory at least, the method is merely a caustically-corrected GTD whose accuracy diminishes at angles away from the Keller cones of the edge-diffracted rays. Nevertheless, it does have advantages, particularly from a computational viewpoint. It is readily combined with physical optics and is then almost indistinguishable from the physical theory of diffraction, and is convenient for practical cross section estimation purposes.

The genesis of the method is described, and its virtues and limitations illustrated and discussed. For a few selected geometries, its predictions are compared with those of GTD and PTD as well as with experimental data, and some possible developments of the method are proposed.

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ALTERNATIVE REPRESENTATIONS OF THE DYADIC GREEN'S FUNCTION FOR A RECTANGULAR CAVITY

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The different representations of the dyadic Green's function of the electric type will be presented in this paper. The work supplements the incomplete version previously discussed by Morse and Feshbach. In one of the representations, the construction of the dyadic Green's function is based on the modal functions for a cavity; in another representation use is made of the waveguide modes. The equivalence between the different representations can be proved by means of some not too well known series. The relationship between the dyadic Green's function of the electric type and that of the vector potential type will also be pointed out.

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