RAYLEIGH SCATTERING

RL 645

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It is well known that in Rayleigh scattering the far-zone scattered fields can be attributed to radiating electric and magnetic dipoles of moments $\underline{p}$ and $\underline{m}$. Electric and magnetic polarizability tensors $\bar{\bar{P}}$ and $\bar{\bar{M}}$ are introduced such that $\underline{p} = \varepsilon_o \bar{\bar{P}} \cdot \hat{a}$ and $m = Y_o \bar{\bar{M}} \cdot \hat{b}$ where ε_o and Y_o are the permittivity and intrinsic admittance of free space and $\hat{a}$ and $\hat{b}$ are unit vectors specifying the directions of the electric and magnetic fields of an incident plane wave. A knowledge of these two tensors is sufficient to determine the far-zone scattered fields for plane waves of arbitrary polarization and angle of incidence.

The tensor elements are representable as weighted integrals of certain potential functions and their normal derivatives over the surface of the scatterer. Using the boundary conditions for a linear homogeneous isotropic dielectric particle, integral equations are derived for both the potentials and their normal derivatives. In this case $\bar{\bar{P}}$ and $\bar{\bar{M}}$ are merely special cases of a general polarizability tensor $\bar{\bar{X}}$ which is a function only of the particle geometry and a material parameter τ representing either the permittivity or permeability of the scattering medium; and a knowledge of either the potentials or their normal derivatives is sufficient to determine the tensor elements. The integral equations obtained using the boundary conditions for perfect conductivity are found to be limiting cases of the dielectric equations as $\varepsilon \to \infty$ and $\mu \to o$, i.e., for a perfectly conducting body $\bar{\bar{P}} = \bar{\bar{X}}(\infty)$ and $\bar{\bar{M}} = \bar{\bar{X}}(o)$. $\bar{\bar{X}}(\tau)$ is a real (for real τ) symmetric second rank tensor, and for real values of τ in the range $0 < \tau < \infty$ several geometric and isoperimetric inequalities are obtained which provide bounds on the tensor elements. Since an exact analytical solution is obtainable only for a very few geometries, e.g. spheroids, these bounds are of great utility in checking the results of numerical computations.

Computer programs have been written to compute $\bar{\bar{X}}(\tau)$ for rotationally symmetric scatterers, rectangular parallelepipeds, and hexagonal crystal structures. In each of these geometries, the symmetry of the body serves to diagonalize the tensor. The data show the tensor elements to be slowly increasing functions of τ $(o < \tau < \infty)$. As the length-to-width ratio of the scatterer increases for a fixed value of τ, the axial tensor element increases while the transverse elements decrease. The data support the conjecture that, geometrically, a sphere of equal volume provides a lower bound on the trace of the tensor. Although this can be rigorously proved for perfect conductors, for dielectric bodies it has been proved only for spheroids. For cylinders of equal length and cross-sectional area, the tensor elements are only slightly dependent upon cross-sectional geometry. The data obtained for perfectly conducting bodies finds application in radar,e.g. calculations of the back scattering behavior of aerospace vehicles, whereas the dielectric data provide the fields scattered by particle geometries occuring in ice crystal clouds and aerosol-laden atmospheres. The original computer programs have also been modified to treat lossy dielectrics by admitting complex values of τ.

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