PARTICLE SHAPES FOR MAXIMIZING LOW FREQUENCY ABSORPTION

T.B.A. Senior and H. Weil
Radiation Laboratory
Department of Electrical and Computer Engineering
The University of Michigan
Ann Arbor, Michigan 48109

ABSTRACT

The effects of particle shape and refractive index on the absorption efficiency of leigh particles is investigated analytically using spheroidal particles as prototypes.

The effect of particle shape on absorption by Rayleigh particles is studied by ıg spheroidal particles as prototypes. Since spheroidal Rayleigh particles can be dled analytically one can determine the relative absorption efficiency of clouds of dle-like, thin plate-like and spherical particles containing the same mass of particle volume of the cloud.

We begin with the theoretical expression for Rayleigh scattering of a plane wave

$$\bar{E}_x^{inc} = \hat{a}\, e^{i(\bar{k}\cdot\bar{r}-\omega t)}$$

ırbitrary elliptical polarization $\hat{a}$ ($\hat{a} \cdot \hat{a}^* = 1$) by a single homogeneous non-magnetizable :icle.

The scattered field can be written as

$$\bar{E}^s = -\frac{e^{ikr-i\omega t}}{kr}\,\bar{S}(\hat{r})$$

'e

$$\bar{S} = -\frac{(k\ell)^3}{4\pi\varepsilon_0}\,\bar{r} \times (\bar{r} \times \frac{\bar{p}}{\ell^3})$$

$\bar{p}$ is the electric dipole moment of the scatterer. In terms of the polarization tensor

$$\bar{p} = \varepsilon_0 \bar{\bar{P}}(\varepsilon) \cdot \hat{a} \;.$$

ε_r is the particle dielectric constant relative to ε_0, the permittivity of the dding medium. For magnetizable materials $\bar{S}$ has an additional term involving the ced magnetic moment $\bar{m}$ which is expressible in terms of the same tensor as a function of eability instead of permittivity; i.e., $\bar{P}(\mu)$.

We will obtain the absorption via the forward scattering theorem which states that

$$\sigma_{ext} = \frac{4\pi}{k^2} \mathrm{Im}(\hat{a}^* \cdot \bar{S})$$

the approximation, valid in the Rayleigh scattering approximations

$$\sigma_{ext} \simeq \sigma_a \ .$$

These formulas and procedures to determine the $\bar{\bar{P}}$ matrix are found in Ref. 1.

For forward scattering $\hat{a}^* \cdot \hat{r} = 0$. Hence

$$\hat{a}^* \cdot [\bar{r} \times (\bar{r} \times \bar{p})] = -\epsilon_0 a^* \cdot (\bar{\bar{P}} \cdot a)$$

$$= -\epsilon_0 (P_{11}|\hat{a} \cdot \hat{x}|^2 + P_{22}|\hat{a} \cdot \hat{y}|^2 + P_{33}|\hat{a} \cdot \hat{z}|^2) \ .$$

scatterers rotationally symmetric about the z axis $P_{11} = P_{22}$ and so we end up with

$$\sigma_a = kI_m[P_{11} + (P_{33} - P_{11})|\bar{a} \cdot \hat{z}|^2] \ .$$

For magnetizable particles, permeability $\mu \neq \mu_0$, there are additional terms due to induced magnetic dipole which may be found by a similar analysis. There are no terms ·esenting magnetic dipole-electric dipole interaction.

To make use of these results for particle distributions (clouds) we must average · particle orientations. Three types of averaging are of interest:

$$<|\hat{a} \cdot \hat{z}|^2> = \frac{1}{3}$$

$$\sigma_a = \frac{k}{3} \mathrm{Im}(2P_{11} + P_{33}) \ .$$

; situation occurs for purely random particle orientations.

$$<|\hat{a} \cdot \hat{z}|^2> = 0 \ ,$$

$$\sigma_a = k \ \mathrm{Im} \ P_{11} \ .$$

; situation could occur for example when the incident radiation is vertical and the ıetry axes of the particles cluster about the vertical. This can be the case for ıds of thin flat discs under some conditions; alternatively horizontal propagation, :ical polarization and horizontal needles.

$$\langle|\hat{a} \cdot \hat{x}_3|^2\rangle = \frac{1}{2} \ , \qquad \sigma_a = \frac{k}{2} \operatorname{Im}(P_{11} + P_{33}) \ .$$

s situation would be generated, again assuming the incident radiation is vertical in ection and plane polarized, by particles such as spindles whose symmetry axes are domly oriented in a horizontal plane, or by such particles not randomly oriented but adiated with circular polarization.

The elements P_{11} and P_{33} for a homogeneous spheroid of volume V and complex lectric constant ε, are (Ref. 1)

$$P_{11} = 2V \left(q + \frac{2}{\varepsilon_r - 1} \right)^{-1}$$

$$P_{33} = V \left(-q + \frac{\varepsilon_r}{\varepsilon_r - 1} \right)^{-1}$$

'e for a prolate spheroid of length ℓ and width w

$$q = -\frac{1}{2} \xi(\xi^2 - 1) \ln \frac{\xi + 1}{\xi - 1} + \xi^2 \ ,$$

$$\xi = (\ell/w)[(\ell/w)^2 - 1]^{-1/2} \ .$$

an oblate spheroid

$$q = \xi(\xi^2 + 1) \tan^{-1} \frac{1}{\xi} - \xi^2$$

e

$$\xi = (\ell/w)[1 - (\ell/w)^2]^{-1/2} \ .$$

ξ varies continuously from $\xi = 0$ for a very thin lozenge or disk through $\xi = 1$ a spindle (approximately an eyeless needle) to $\xi = \infty$ for a sphere.

Writing $\varepsilon_r = 1 + a + ib$ with a and b real and non-negative yields

$$\operatorname{Im} P_{11} = bV \Gamma_1$$

$$\operatorname{Im} P_{33} = bV \Gamma_3$$

e

$$\Gamma_1 = 4[(2 + aq)^2 + (bq)^2]^{-1}$$

$$\Gamma_3 = \{[1 + a(1 - q)]^2 + [b(1 - q)]^2\}^{-1} \quad .$$

Then

$$\sigma_a = kVb\Gamma \quad ,$$

re, for the three types of averaging

(a) $\Gamma = (2\Gamma_1 + \Gamma_3)/3$,

(b) $\Gamma = \Gamma_1$,

(c) $\Gamma = (\Gamma_1 + \Gamma_3)/2$.

s for a disk (q = 0)

$$\Gamma_1 = 1 \quad , \quad \Gamma_3 = [(1 + a)^2 + b^2]^{-1} \quad ,$$

for a needle (q = 1),

$$\Gamma_1 = 4[(2 + a)^2 + b^2)]^{-1} \quad , \quad \Gamma_3 = 1 \quad .$$

a sphere (q = 2/3)

$$\Gamma_1 = 9[(3 + a)^2 + b^2]^{-1} = \Gamma_3$$

that the same value for σ_a is found for each type of averaging.

In Figure 1 we plot 3Γ vs. q for the two cases.

a = 0.56 , b = 0.83 implying n = 1.29 + i0.32

a = b = 3.0 implying n = 2.12 + i0.71 .

For type (a) averaging we see that for both materials, discs give the most orption while spheres give the least.

For type (c) averaging the spindles have a slight edge over discs in their orption ability, and again spheres are poorest.

For type (b) averaging both cases give the same result for disks and disks are siderably more efficient absorbers than spheres or spindles.

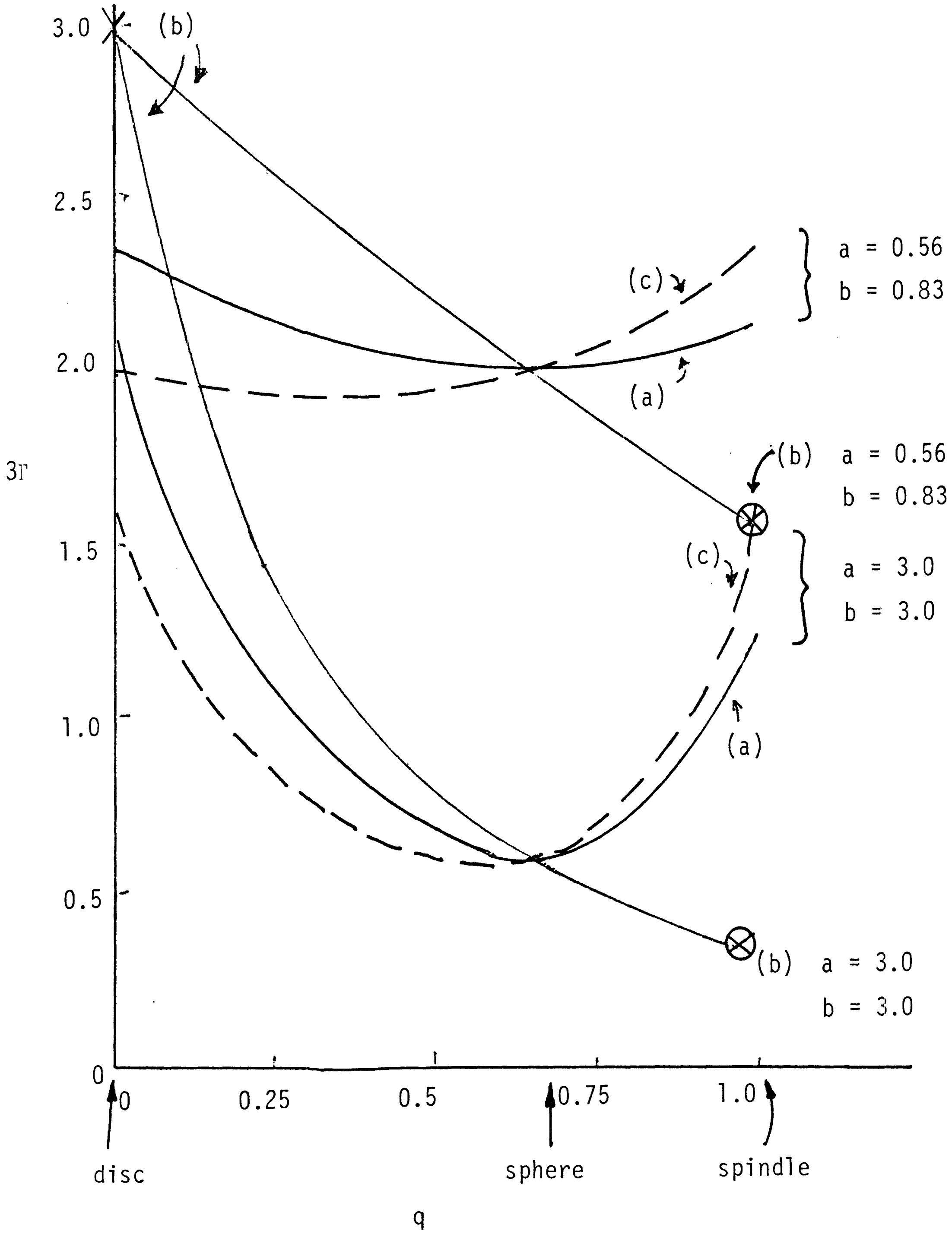
The type (a) results have been given, by a somewhat different derivation, in ref 2.

If magnetic dipole contributions had been retained the results would be far more
ıplex since not only complex ε_r but complex μ_r itself and the ratio μ_r/ε_r would
:er as parameters.

'erences

T.B.A. Senior, "Low-frequency scattering by a dielectric body," Radio Sci 11, 1976, pp. 447-482.

T.B.A. Senior, "Effect of particle shape on low-frequency scattering," Appl. Opt. 19, No. 15, 1 August 1980, pp. 2483-2485.



1: Particle shape influence on absorption for the three types of orientation averaging. (3Γ vs. q)