THE TRANSMISSION LINE MODEL FOR RECTANGULAR PATCH ANTENNAS

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Summary

Early analyses of rectangular patch antennas have been based on the transmission line model [1,2]. Although this model provides results adequate for engineering purposes, it predicts a resonant frequency which is lower [2] and also it cannot account for the dependence of the resonant frequency on the patch aspect ratio and the substrate thickness [3]. Later, the cavity model [4] and its modification [2,3] have been developed to analyze such antennas; these methods provide accurate results but require elaborate computation.

The present paper re-examines the transmission line model for a coaxial probe fed rectangular patch antenna with the goal of correcting the abovementioned defects of the model, and to develop simple but accurate expressions so that the design of such antennas may be carried out without elaborate computation. The model treats the patch antenna as a section of transmission line (sustaining the dominant TEM mode) terminated at both ends by admittances appropriate for the two radiating edges. Approximate expressions for the latter are obtained from the known aperture admittance of a parallel plate waveguide radiating into space [5]. The equivalent transmission line circuit for a rectangular patch antenna of length ℓ and width a, using a substrate of relative permittivity ϵ_{r} and thickness d, and fed at one edge by a coaxial probe of radius r_{0} is shown in Fig. 1, where the transmission line parameters are [6].

$$\gamma_{c} = \frac{1}{Z_{c}} = \frac{a\alpha\sqrt{\epsilon_{e}}}{\eta_{o}d}, \quad \eta_{o} = \sqrt{\mu_{o}/\epsilon_{o}}$$

$$\beta = \omega\sqrt{\mu_{o}\epsilon_{o}\epsilon_{e}} = k_{o}\sqrt{\epsilon_{e}}$$

$$\epsilon_{e} = \frac{\epsilon_{r}+1}{2} + \frac{\epsilon_{r}-1}{2} \frac{1}{\left(1 + \frac{10d}{a}\right)^{1/2}}$$

$$\alpha = 1 + 1.393 \left(\frac{d}{a}\right) + 0.667 \left(\frac{d}{a}\right) \ln \left(\frac{a}{d} + 1.444\right)$$
(1)

The normalized aperture admittance parameters are [5,7]:

$$G = \frac{G_T}{Y_C} = \frac{\beta d}{2\alpha \varepsilon_e}$$

$$B = \frac{B_T}{Y_C} = \frac{\beta d}{\pi \alpha \varepsilon_e} \ln \left(\frac{\sqrt{\varepsilon_e} 2\pi e}{\gamma \beta d}\right)$$

$$\gamma = 1.78107$$

$$e = 2.71828$$
(2)

and the normalized inductive reactance of the coaxial probe is [7]:

$$\frac{X_L}{Z_C} = \frac{\beta a\alpha}{2\pi} \ln \left(\frac{2}{\gamma \beta r_0}\right). \tag{3}$$

Using the equivalent circuit given, it can be shown that in the absence of the probe reactance the resonant frequency of the patch antenna is

$$f_{r} = \begin{bmatrix} \frac{1 - \frac{2d}{\varepsilon_{e}^{l \pi \alpha}}}{1 + \frac{2d}{\varepsilon_{e}^{l \pi \alpha}} \ln\left(\frac{\sqrt{\varepsilon_{e}^{2}l}}{\gamma d}\right)} \end{bmatrix} f_{0}, \qquad (4)$$

where

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$$f_0 = \frac{c}{2\ell\sqrt{\epsilon_e}}, \quad c = \frac{1}{\sqrt{\mu_0\epsilon_0}}.$$
 (5)

The accuracy of Eq. (4) has been found to be within one percent of the measured values. It can be shown that the relative shift in the frequency caused by the probe is

$$\frac{\Delta f_{r}}{f_{r}} \simeq \frac{\pi}{2} \left(\frac{d}{\ell} \right)^{2} \left(\frac{a}{\ell} \right) \frac{1}{\alpha \epsilon_{e}^{2}} \ln \left(\frac{2}{\gamma \pi r_{o}} \right) . \tag{6}$$

Simple expressions for the resonant resistance and Q of the antenna have also been obtained. The present work establishes the transmission line model as a valid representation of rectangular patch antennas.

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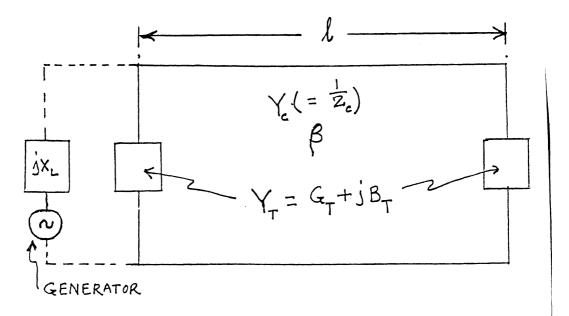


Fig. 1: Equivalent transmission line circuit for a rectangular patch antenna fed at one edge.

References

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