A STUDY OF SPHERICAL CAP ANTENNAS

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Introduction

Where space is limited, or at low frequencies where large antennas are required, reduced size antennas are often used. One method of achieving the necessary size reduction is through so-called capacitive or top loading [1]. Here we present analysis, and computed and measured impedance characteristics of one such antenna, specifically the spherical cap antenna. A variational technique [2] is used in the analysis.

Analytical Procedure

The geometry of the antenna is shown in Fig. 1. The antenna consists of a biconical section terminated by a pair of spherical caps. For analysis, two regions are defined, one inside the spherical shells (Region I), and the other outside (Region II). In the two regions electric fields are represented by equations (1) and (2), where Y_t is the effective outward admittance defined at the interior surface of the caps [3]. The $L_n(\theta)$ is a class of Legendre functions which go to zero at θ_o, $\pi/2$ and $\pi-\theta_o$. In general, for arbitrary θ_o, n are not integers. The K is the characteristic impedance of the biconical transmission line and a_n and b_k are constants to be determined.

$$\bar{E}^I(\theta) = \hat{\theta} \frac{Z_o I_o}{2\pi R \sin\theta} [KY_t \sin\beta(\ell - R) - j\cos\beta(\ell - R)] - \frac{jZ_o}{2\pi R} \sum_n \frac{a_n}{n(n+1)} \frac{S_n'(\beta R)}{S_n(\beta\ell)} \frac{\partial L_n(\theta)}{\partial\theta}, \quad (n = n_1, n_2, n_3 \ldots;\ R < \ell) \tag{1}$$

$$\bar{E}^{II}(\theta) = -\hat{\theta} \frac{jZ_o}{2\pi R} \sum_k \frac{b_k}{k(k+1)} \frac{R_k'(\beta R)}{R_k(\beta\ell)} \frac{\partial P_k(\theta)}{\partial\theta}, \quad (k = 1,3,5\ldots;\ R > \ell), \tag{2}$$

where I_o is the driving point current, $K = \frac{Z_o}{\pi} \ln \cot \frac{\theta_o}{2}$,

$$S_n(\beta R) = (\beta R)^{1/2} J_{n+1/2}(\beta R), \quad S_n'(\beta R) = \frac{dS_n(\beta R)}{d(\beta R)},$$

$$R_k(\beta R) = (\beta R)^{1/2} H^{(2)}_{k+1/2}(\beta R), \quad R_k'(\beta R) = \frac{dR_k(\beta R)}{d(\beta R)}$$

Introducing boundary aperture field $E_a(\theta)$, which is still unknown, variational expression for Y_t is obtained from equations (1) and (2). Thus,

$$Y_t = \frac{j2\pi}{Z_o} \frac{1}{\left[\int_{\theta_1}^{\pi-\theta_1} E_a(\theta)d\theta\right]^2} \left\{ \sum_n \frac{1}{n(n+1)N_n J_{nn}} \cdot \left[\int_{\theta_1}^{\pi-\theta_1} E_a(\theta)L'_n(\theta)\sin\theta\, d\theta\right]^2 - \sum_k \frac{1}{k(k+1)M_k I_{kk}} \cdot \left[\int_{\theta_1}^{\pi-\theta_1} E_a(\theta)P_k(\theta)\sin\theta\, d\theta\right]^2 \right\} \quad (3)$$

where

$$N_n = \frac{S_n'(\beta\ell)}{S_n(\beta\ell)}, \quad J_{nn} = \int_{\theta_o}^{\pi-\theta_o} [L_n(\theta)]^2 \sin\theta\, d\theta$$

$$M_k = \frac{R_k'(\beta\ell)}{R_k(\beta\ell)}, \quad I_{kk} = \int_0^{\pi} [P_k(\theta)]^2 \sin\theta\, d\theta$$

$$(n = n_1, n_2, n_3, \ldots; \; k = 1,3,5,\ldots.)$$

To avoid non-integer Legendre functions in computation of interior modes, θ_o is assumed to be small, and in such cases n are close to integers and integration pertaining to eigenfunctions $L_n(\theta)$ can be approximated by integration of selected Legendre functions $P_k(\theta)$ of odd integer order.

When taking the dominant mode, or the dominant plus one complimentary mode for the aperture field $E_a(\theta)$, the stationary property of equation (3) yields zeroth or first order solutions for Y_t, respectively. We found that for a wide-angle cap (small gap) the $Re[Y_t]$ is almost the same as that obtained by Stratton and Chu [4] in their study of spherical antennas with infinitesimal gap using spherical harmonic expansion.

The driving point impedance or admittance of the antenna is obtained by applying transmission line transformation to Y_t. An example of such calculations is shown in Fig. 2.

Experimental Verification

Two monopole version antennas with θ_1 = 45° and 89° were constructed and impedance measurements were performed. In each case a good agreement with prediction was found. Figure 3 shows an example of the results.

Conclusions

Variational technique was used to solve for impedance of the spherical cap antenna and the results agree well with the measurements. As much as 88 percent reduction in the physical height of the antenna can be achieved, but bandwidth and the radiation resistance is lost in the trade-off. If needed, more precise results can be obtained by using higher order modes when prescribing the aperture field.

References

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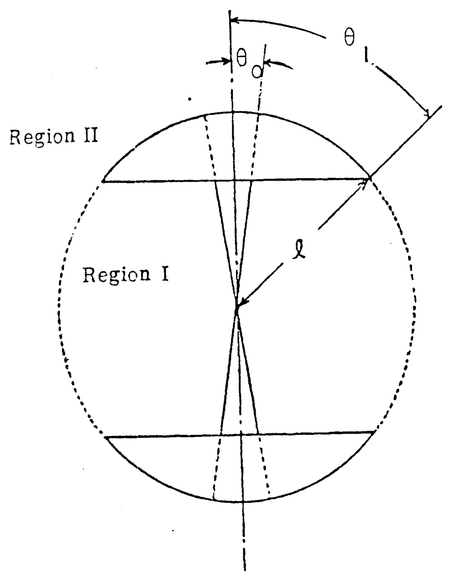

Fig. 1: `Geometry of the antenna.

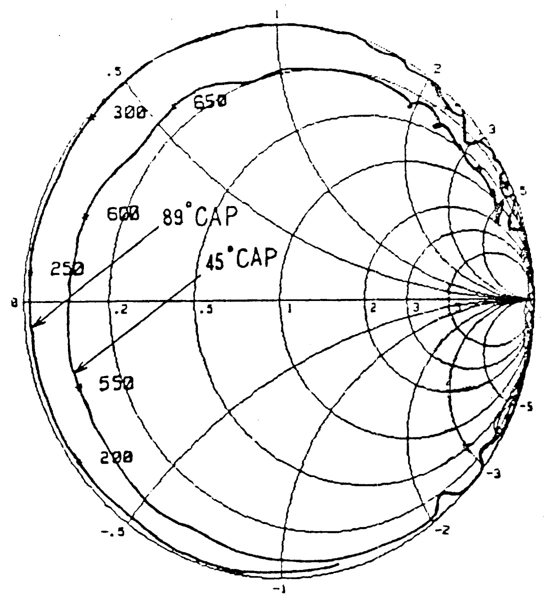

Fig. 3: Experimental results. Freq.; MHz

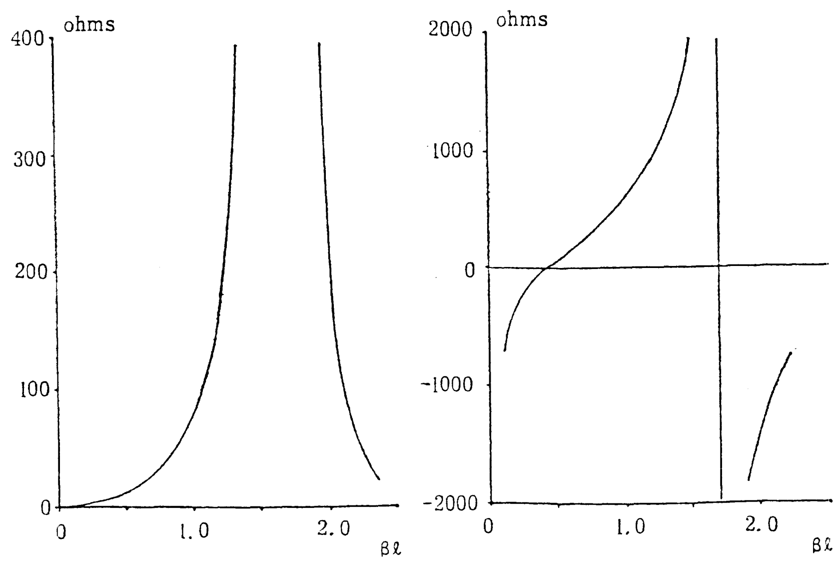

Fig. 2: Example of numerical calculation (1st order)
$\theta_0 = 1.77$ deg. $\theta_1 = 45$ deg.
Real part; Left Imag. part; Right