

RESONANT FREQUENCY OF A TUNABLE
RECTANGULAR PATCH ANTENNA

Dipak L. Sengupta
Radiation Laboratory
Department of Electrical Engineering and Computer Science
The University of Michigan
Ann Arbor, MI 48109 U.S.A.

Abstract: The uniform transmission line model is applied to determine the resonant frequency of a coaxial probe fed rectangular patch antenna tuned by a number of passive metallic posts suitably placed within the antenna's boundary. An approximate expression is given for the resonant frequency as a function of the post location and number, and of the other characteristic parameters of the antenna. Theoretical results are compared with available measured values.

It has been reported^{1,2} that the resonant or operating frequency of a rectangular patch antenna can be increased by placing a number of passive metallic posts within the antenna's boundary. Depending on the number and location of the posts the operating frequency can be increased in excess of 50 percent.² The resonant frequency of a coaxial probe excited rectangular patch antenna has been previously³ determined by using the TEM transmission line model. In the present communication the same model is modified to obtain the operating frequency of a tunable rectangular patch antenna.

Consider a tunable rectangular patch antenna of length ℓ , width a , having a substrate of thickness d ($d \ll a$) and permittivity ϵ_r . It is assumed that the antenna is excited from the back with a coaxial probe of radius r_0 located at one of the radiating edges; the antenna is tuned by

a metallic post of radius r_0 placed inside and normal to the patch, and along the center line of the antenna at a distance ℓ_1 from the feed edge. The equivalent transmission line circuit of the antenna is shown in Fig. 1 (also showing a sketch of the patch) where G and B are the normalized conductance and capacitive susceptance appropriate for each radiating aperture, and X_L , B_L ($=1/X_L$) are the normalized reactance and susceptance of the exciting probe and the tuning post, respectively. Y_c ($=1/Z_c$) and β are the characteristic admittance and propagation constants of the transmission line. It should be noted that the transverse field distribution inside the antenna being uniform, with more than one post located along the transverse direction at $\ell = \ell_1$ the B_L in Fig. 1 should be multiplied by the number of posts used; with posts located at variable distances in the longitudinal direction appropriate susceptances be located at the corresponding locations of the posts. The transmission line parameters in Fig. 1 are:³

$$Y_c = \frac{a\alpha \sqrt{\epsilon_e}}{\eta_0 d}, \quad \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}, \quad (1)$$

$$\beta = \frac{\omega}{c} \sqrt{\epsilon_e}, \quad c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \quad (2)$$

where

$$\epsilon_e = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left(1 + \frac{10d}{a}\right)^{-1/2} \quad (3)$$

$$\alpha = 1 + 1.393 \frac{d}{a} + 0.667 \frac{d}{a} \ln \left(\frac{a}{d} + 1.444\right) \quad (4)$$

and $\omega (= 2\pi f)$, c are the radian frequency and the velocity of light in free space, respectively.

The other parameters in Fig. 1 are:³

$$G = \frac{\beta d}{2\alpha \epsilon_e} \quad (5)$$

$$B = \frac{\beta d}{\pi \alpha \epsilon_e} \ln \left(\frac{\sqrt{\epsilon_e} e^{2\pi e}}{\gamma \beta d} \right) \quad (6)$$

$$X_L = \frac{1}{B_L} = \frac{\beta a \alpha}{2\pi} \ln \left(\frac{2}{\gamma \beta r_0} \right) \quad (7)$$

where

$$\gamma = 1.78107$$

$$e = 2.71828$$

and for simplicity it has been assumed that the radius of the tuning post is the same as that of the probe.

The operating frequency of the antenna is defined to be the frequency at which the imaginary part of the admittance seen by the generator is zero. Using the equivalent circuit given in Fig. 1 and standard transmission line theory the operating frequency of the antenna has been determined numerically for a variety of post arrangements under the assumption of no mutual interaction between the individual posts and negligible effects³ of the probe. For design purposes, it is of considerable interest to obtain an expression for the operating frequency showing explicitly the influence of various antenna parameters. In the absence of the generally negligible³

effects of the probe inductance, it can be shown⁴ that for a single post tuned antenna shown in Fig. 1 the propagation constant at the operating frequency is a solution of the following equation

$$\beta l = \beta_r l + B_L \Big|_{\beta=\pi/l} \cos^2\left(\frac{\pi l_1}{l}\right) \quad (8)$$

where β_r is the resonant propagation constant in the absence of the probe reactance, and is given by³

$$\beta_r l \approx (\pi - 2B) \Big|_{\beta=\pi/l} . \quad (9)$$

Using Eqs. (1), (2), (6) through (9), it can be shown that the operating frequency of a tunable patch antenna having one tuning post is

$$f_{op} = f_r \left[1 + \frac{2l}{\pi a \alpha} \frac{1}{\ln \frac{2l}{\gamma \pi r_o}} \cos^2\left(\frac{\pi l_1}{l}\right) \right] , \quad (10)$$

where f_r is the resonant frequency in the absence of the post and is given by³

$$f_r = f_o \left[\frac{1 - \frac{2d}{\epsilon_e l \pi \alpha}}{1 + \frac{2d}{\epsilon_e l \pi \alpha} \ln \left(\frac{\sqrt{\epsilon_e} 2l}{\gamma d} \right)} \right] \quad (11)$$

with

$$f_o = \frac{c}{2l \sqrt{\epsilon_e}} . \quad (12)$$

Assuming no mutual interaction between the tuning posts and N similar tuning posts located along the center line of the antenna at $\ell = \ell_n$, $n = 1, 2, \dots, N$, Eq. (11) can be generalized to

$$f_{op} = f_r \left[1 + \frac{1}{\pi} \sum_{n=1}^N \sin^{-1} \left\{ \frac{2\ell}{a\alpha} \frac{1}{\ln \left(\frac{2\ell}{\gamma\pi r_0} \right)} \cos^2 \left(\frac{\pi\ell_n}{\ell} \right) \right\} \right] \quad (13)$$

It is interesting to observe in Eq. (10) that $f_{op} = f_r$ when $\ell_1 = \ell/2$, a fact confirmed by experiment also.²

The variation of the operating frequency as a function of the post location, obtained numerically, and by using Eq. (10), for a single post tuned rectangular patch antenna is shown in Table 1 where the corresponding measured values² are also shown for comparison. The results indicate that Eq. (10) approximates the numerically obtained values well, and that the agreement between the calculated and measured results is satisfactory. Further investigation indicated that Eq. (13) can be used satisfactorily for antennas using up to six tuning posts, beyond that it is preferable to use a numerical method.

It should be noted the calculated values in Table 1 are symmetrical about $\ell_1/\ell = 0.5$ whereas the measured values exhibit a slight asymmetry. This is due to the fact that the calculations ignored the effects of the probe inductance which introduces the above asymmetry. Detailed numerical studies of the resonant frequency and input VSWR for a number of tunable antennas and comparison with available measured values indicated that the above model is valid for up to ten tuning posts. Detailed derivation

of the expressions given and further discussion of the resonant behavior of tunable patch antennas are discussed elsewhere.⁴ This work was supported by the U. S. Army Research Office under Contract No. DAAG29-82-K-0076.

References

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3. Sengupta, D. L.: 'Appropriate expression for the resonant frequency of a rectangular patch antenna,' Electron. Lett, 1983, 19, pp. 834-835.
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Table 1: Calculated and Measured Value² of Resonant Frequency for a Tunable Rectangular Patch

Antenna Using One Post

ℓ_1 / ℓ	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
f_{op} Numerical	1.645	1.640	1.600	1.545	1.501	1.485	1.501	1.545	1.600	1.640	1.645
f_{op} Eq. 10	1.656	1.640	1.597	1.544	1.501	1.485	1.501	1.544	1.597	1.640	1.656
f_{op} Measured ²	1.588	1.594	1.573	1.533	1.485	1.466	1.480	1.525	1.562	1.590	1.580

($\ell = 6.2$ cm, $a = 9.0$ cm, $\epsilon_r = 2.55$, $d = 0.16$ cm, $r_o = 0.064$ cm)

Figure Captions

Figure

- 1 Equivalent transmission line circuit for a tunable rectangular patch antenna with one tuning post at ℓ_1 , and fed by a coaxial probe at one edge.

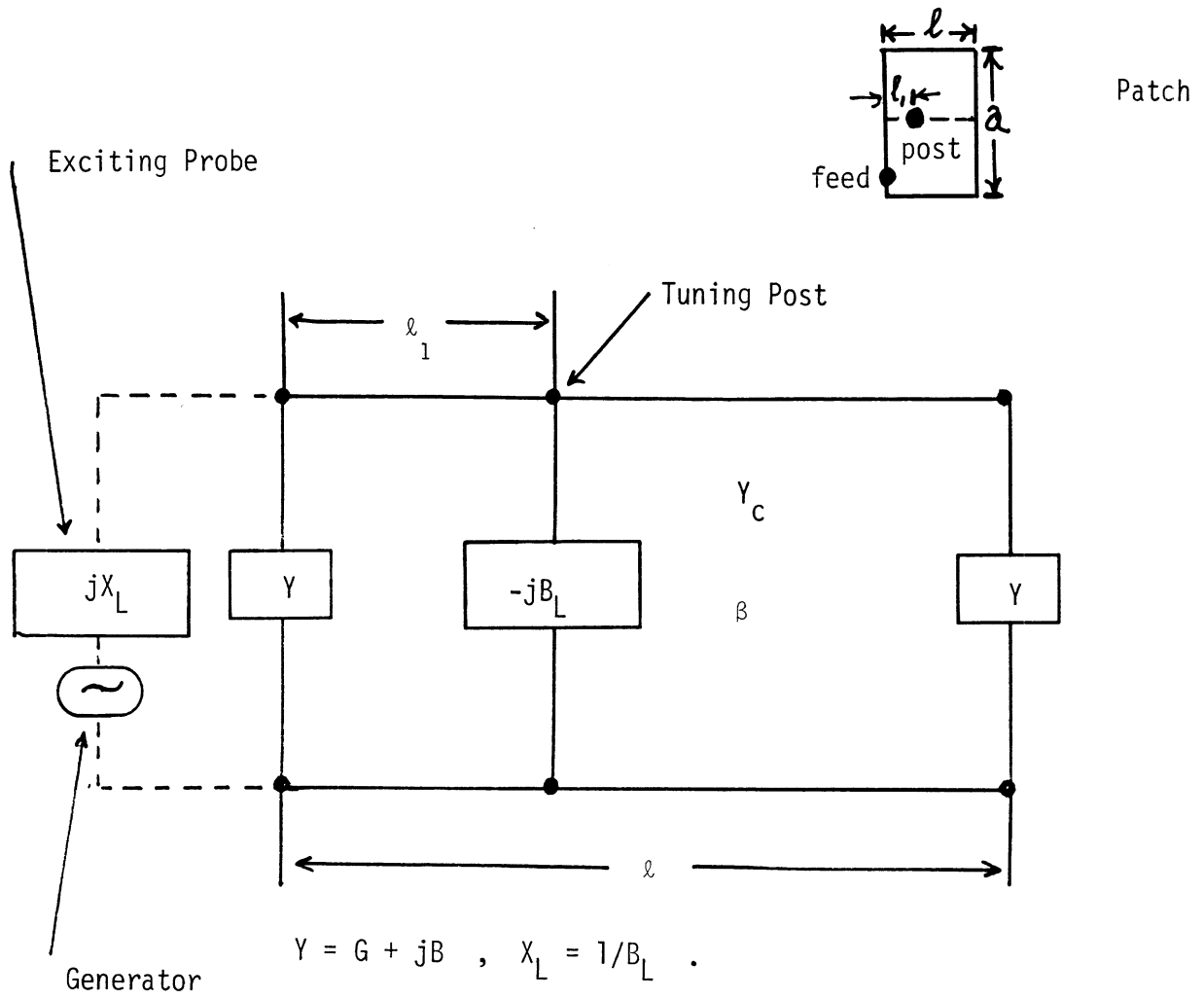


Fig. 1: Equivalent transmission line circuit for a tunable rectangular patch antenna with one tuning post at l_1 , and fed by a coaxial probe at one edge.