INTEGRAL EQUATIONS FOR PERMEABLE BODIES BASED ON THE METHOD OF EQUIVALENT POLARIZED CURRENTS

The integral equations for both electrically and magnetically permeable bodies which we derived previously (RL Report No. 771) can also be obtained by the method of equivalent polarized currents. This method was used by Harrington. We reformulate here by using the method of dyadic Green's functions instead of potential theory. We write

$$\nabla \times \bar{E} = i\omega\mu\bar{H} = i\omega\mu_o\bar{H} + i\omega(\mu - \mu_o)\bar{H}$$

$$= i\omega\mu_o\bar{H} - \bar{J}_m \tag{1}$$

$$\nabla \times \bar{H} = -i\omega\varepsilon\bar{E} = -i\omega\varepsilon_o\bar{E} - i\omega(\varepsilon - \varepsilon_o)\bar{E}$$

$$= -i\omega\varepsilon_o\bar{E} + \bar{J}_e \tag{2}$$

where

$$\bar{J}_e = -i\omega(\varepsilon - \varepsilon_o)\bar{E} \tag{3}$$

$$\bar{J}_m = -i\omega(\mu - \mu_o)\bar{H} \tag{4}$$

being the equivalent polarized currents. Let

$$\bar{E} = \bar{E}_e + \bar{E}_m \ , \quad \bar{H} = \bar{H}_e + \bar{H}_m \tag{5}$$

where

$$\nabla \times \bar{E}_e = i\omega\mu_o\bar{H}_e \tag{6}$$

$$\nabla \times \bar{H}_e = -i\omega\varepsilon_o\bar{E}_e + \bar{J}_e \tag{7}$$

and

$$\nabla \times \bar{E}_m = i\omega\mu_o\bar{H}_m - \bar{J}_m \tag{8}$$

$$\nabla \times \bar{H}_m = -i\omega\varepsilon_o\bar{E}_m \tag{9}$$

The scattered fields due to these equivalent polarized currents are

$$\bar{E}_e^{(s)}(\bar{R}) = i\omega\mu_o\iiint\bar{\bar{G}}_o(\bar{R},\bar{R}') \cdot \bar{J}_e(\bar{R}')dv' \tag{10}$$

$$\bar{H}_m^{(s)}(\bar{R}) = i\omega\varepsilon_o\iiint\bar{\bar{G}}_o(\bar{R},\bar{R}') \cdot \bar{J}_m(\bar{R}')dv' \tag{11}$$

hence

$$\bar{E}^{(s)} = \bar{E}_e^{(s)} + \bar{E}_m^{(s)} = \bar{E}_e^{(s)} + \frac{1}{-i\omega\varepsilon_o} \nabla \times \bar{H}_m^{(s)}$$

$$= i\omega\mu_o\iiint \bar{\bar{G}}_o(\bar{R},\bar{R}') \cdot [-i\omega(\varepsilon - \varepsilon_o)\bar{E}]dv'$$

$$+ \iiint \nabla \times \bar{\bar{G}}_o(\bar{R},\bar{R}') \cdot [i\omega(\mu - \mu_o)\bar{H}]dv'$$

$$= \omega^2\mu_o(\varepsilon - \varepsilon_o) \iiint\bar{\bar{G}}_o(\bar{R},\bar{R}') \cdot \bar{E}\, dv'$$

$$- i\omega(\mu - \mu_o)\iiint\bar{H} \cdot \widetilde{\nabla \times G}_o(\bar{R},\bar{R}')dv'$$

$$= \omega^2\mu_o(\varepsilon - \varepsilon_o)\iiint\bar{\bar{G}}_o(\bar{R},\bar{R}') \cdot \bar{E}\, dv'$$

$$+ i\omega(\mu - \mu_o)\iiint\bar{H} \cdot \nabla' \times \bar{\bar{G}}_o(\bar{R},\bar{R}')dv' \tag{12}$$

The last term in the above equation can be decomposed into two terms as a result of the dyadic identity

$$\bar{H} \cdot \nabla' \times \bar{\bar{G}}_o = (\nabla' \times \bar{H}) \cdot \bar{\bar{G}}_o - \nabla' \cdot (\bar{H} \times \bar{\bar{G}}_o)$$

$$= -i\omega\varepsilon\bar{E} \cdot \bar{\bar{G}}_o - \nabla' \cdot (\bar{H} \times \bar{\bar{G}}_o)$$

$$= -i\omega\varepsilon\bar{\bar{G}}_o \cdot \bar{E} - \nabla' \cdot (\bar{H} \times \bar{\bar{G}}_o) \qquad (13)$$

Thus

$$\iiint \bar{H} \cdot \nabla' \times \bar{\bar{G}}_o \, dv'$$

$$= -i\omega\varepsilon \iiint \bar{\bar{G}}_o \cdot \bar{E} \, dv' - \iint \hat{n} \cdot (\bar{H} \times \bar{\bar{G}}_o) ds'$$

$$= -i\omega\varepsilon \iiint \bar{\bar{G}}_o \cdot \bar{E} \, dv' - \iint (\hat{n} \times \bar{H}) \cdot \bar{\bar{G}}_o \, ds'$$

$$= i\omega\varepsilon \iiint \bar{\bar{G}}_o \cdot \bar{E} \, dv' - \frac{1}{i\omega\mu} \iint (\hat{n} \times \nabla' \times \bar{E}) \cdot \bar{\bar{G}}_o \, ds' \qquad (14)$$

Therefore,

$$\bar{E}^{(s)} = [\omega^2\mu_o(\varepsilon - \varepsilon_o) + \omega^2\varepsilon(\mu - \mu_o)] \iiint \bar{\bar{G}}_o \cdot \bar{E} \, dv'$$

$$- \left(\frac{\mu - \mu_o}{\mu}\right) \oiint [\hat{n} \times \nabla' \times \bar{E}] \cdot \bar{\bar{G}}_o \, ds'$$

$$= (k^2 - k_o) \iiint \bar{\bar{G}}_o \cdot \bar{E} \, dv'$$

$$- \left(\frac{\mu - \mu_o}{\mu}\right) \oiint \bar{\bar{G}}_o \cdot [\hat{n} \times \nabla' \times \bar{E}] \, ds' \qquad (15)$$

which is the same as the integral equation derived in Report No. 771 since $\bar{E}^{(s)} = \bar{E} - \bar{E}^{(i)}$. For completeness the integral equation for $\bar{H}^{(s)}$ is also included here.

$$\bar{H}^{(s)} = \omega^2\varepsilon_0(\mu - \mu_0) \iiint \bar{\bar{G}}_0 \cdot \bar{H}\, dv' - i\omega(\varepsilon - \varepsilon_0) \iiint \nabla \times \bar{\bar{G}}_0 \cdot \bar{E}\, dv'$$

$$= (k^2 - k_0^2) \iiint \bar{\bar{G}}_0 \cdot \bar{H}\, dv' - \left(\frac{\varepsilon - \varepsilon_0}{\varepsilon}\right) \oiint \bar{\bar{G}}_0 \cdot [\hat{n} \times \nabla \times \bar{H}]ds' \qquad (16)$$

For numerical analysis it may be desirable to solve two simultaneous integral equations involving $\bar{E}$ and $\bar{H}$, i.e.,

$$\bar{E} = \bar{E}^{(i)} + \omega^2\mu_0(\varepsilon - \varepsilon_0) \iiint \bar{\bar{G}}_0 \cdot \bar{E}\, dv' + i\omega(\mu - \mu_0) \iiint (\nabla \times \bar{\bar{G}}_0) \cdot \bar{H}\, dv' \qquad (17)$$

$$\bar{H} = \bar{H}^{(i)} + \omega^2\varepsilon_0(\mu - \mu_0) \iiint \bar{\bar{G}}_0 \cdot \bar{H}\, dv' - i\omega(\varepsilon - \varepsilon_0) \iiint (\nabla \times \bar{\bar{G}}_0) \cdot \bar{E}\, dv' \qquad (18)$$

instead of solving a single integral equation for $\bar{E}$ that involves $\bar{E}$ and its derivatives $(\nabla \times \bar{E})$ like Eq. (15). It is interesting to observe that in the method of equivalent polarized currents the boundary conditions required for the scatterer are automatically satisfied.

REFERENCES

1. Roger Harrington. Field Computation by Moment Method, MacMillan, N.Y. 1968.
2. K. M. Chen. Interaction of Electromagnetic Fields with Biological Bodies, Chapter 13 in Research Topics in EM Wave Theory edited by J. A. Kong, John Wiley, N.Y. 1980.