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TECHNICAL REPORT

Dielectric-Covered, Cavity–Backed Apertures

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DIELECTRIC-COVERED, CAVITY-BACKED APERTURES

1. General formulation of the problem

An important special case of a cavity-backed aperture is that where the cavity is a finite rectangular waveguide. The aperture exists at one end of the the cylindrical waveguide and is covered by a dielectric slab. The other end of the waveguide is completely covered by a conductor. The cavity can then be viewed as a short-circuited waveguide and waveguide theory can be applied. Figure 1 represents a typical problem, where the excitation is by an incident plane wave from the half-space region. The equivalence principle is used to divide the problem into two equivalent problems as shown in figure 2. In region a, the field is produced by the incident field E^i, $\vec{H}^i$ and the equivalent magnetic current

$$\vec{M} = \hat{n} \times \vec{E} \qquad (1)$$

over the aperture region, with the aperture covered by an electric conductor. In region c, the field is produced by the equivalent magnetic current $-\vec{M}$ over the aperture region, with the aperture covered by an electric conductor. The fact that the equivalent current in region c is the negative of that in region b ensures that the tangential component of the electric field is continuous across the aperture. The remaining boundary condition to be applied is continuity of the tangential component of the magnetic field across the aperture.

The tangential component $\vec{H}_t^b$ of the magnetic field in region b over the aperture is the sum of that due to the impressed sources

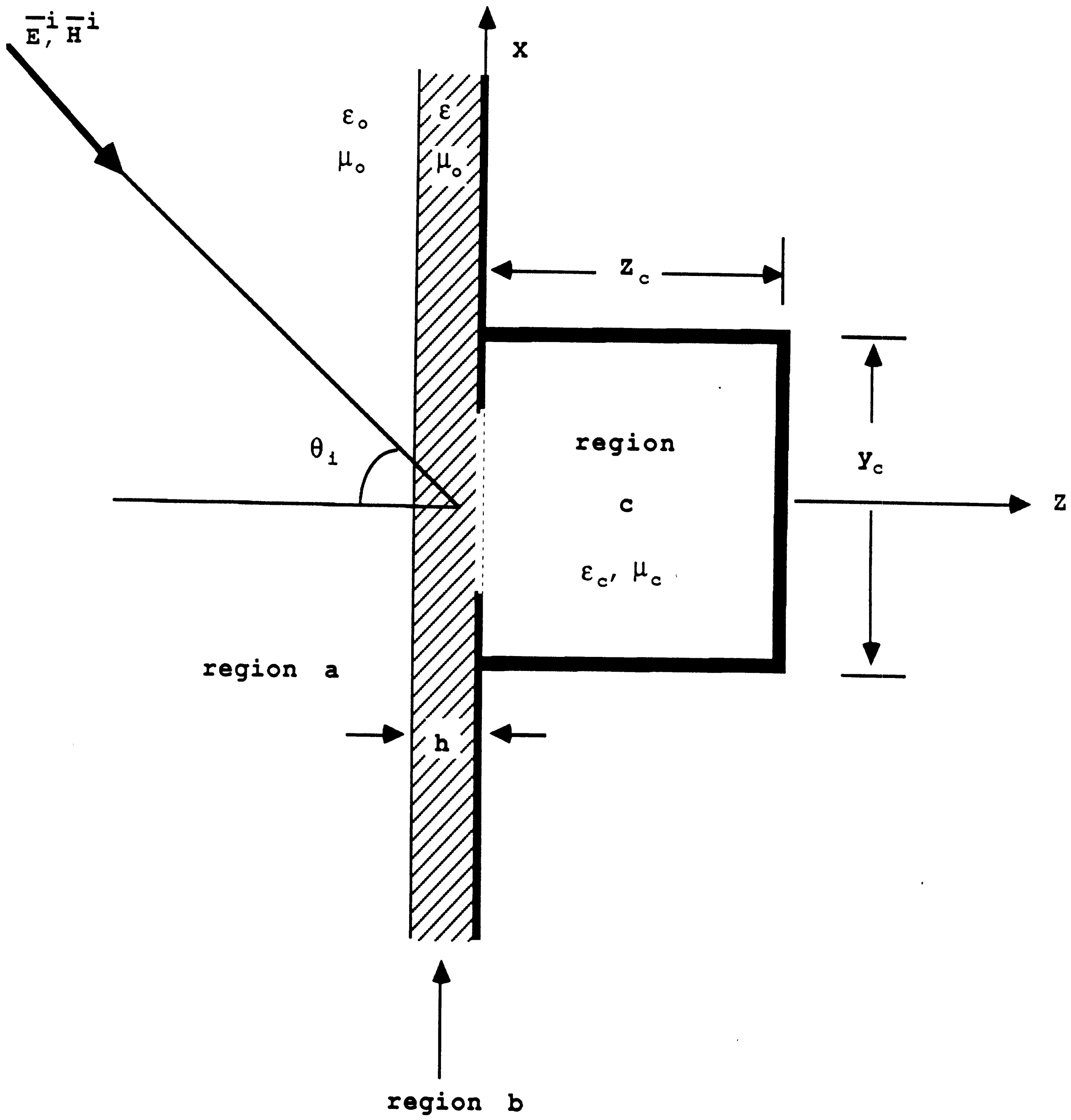
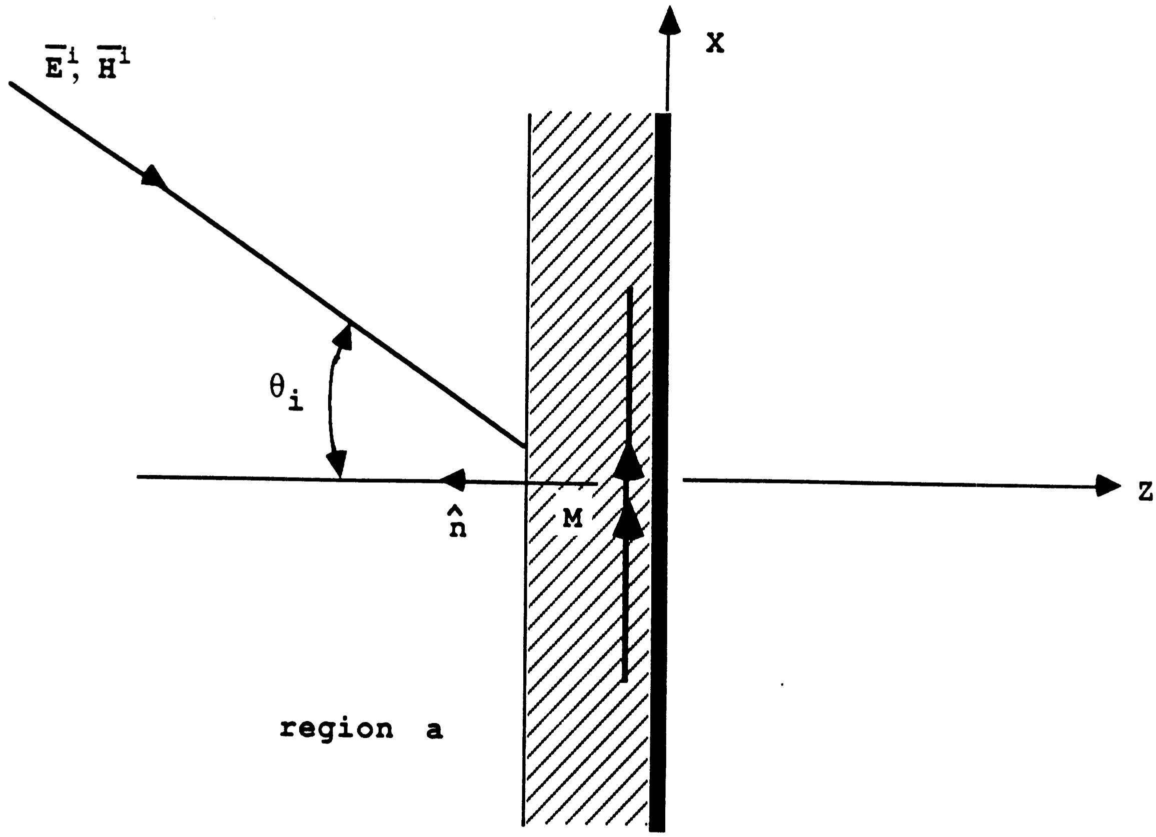
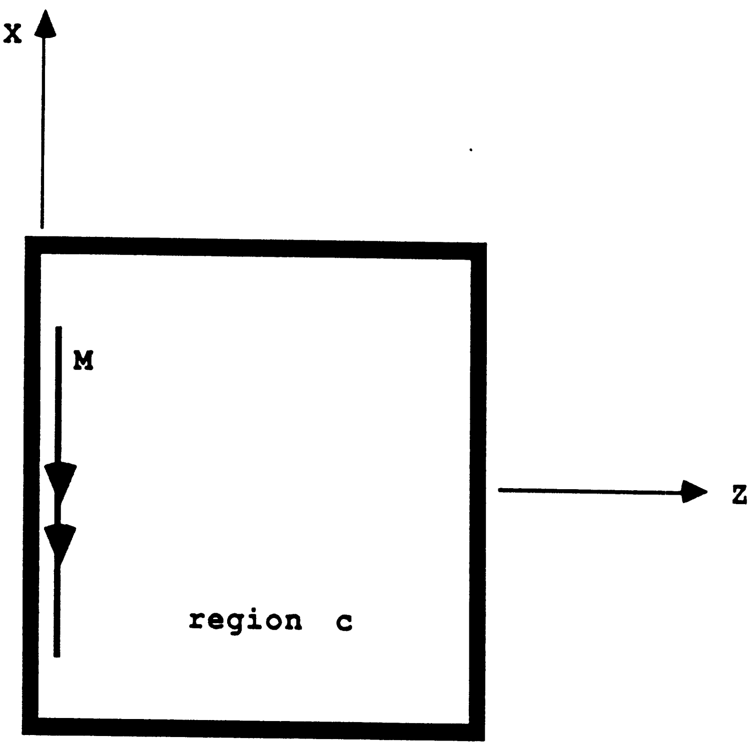


Figure 1



(a) Equivalent problem b



(b) Equivalent problem c

Figure 2

$\vec{H}_t^i$ plus that due to the equivalent source $\vec{M}$, denoted by $\vec{H}_t^b(M)$.

Therefore

$$\vec{H}_t^b = \vec{H}_t^i + \vec{H}_t^b(\vec{M}) \tag{2}$$

Both $\vec{H}_t^i$ and $\vec{H}_t^b(\vec{M})$ are computed with a conductor covering the aperture. In region c the equivalent source $-\vec{M}$ is the only source. Hence, the tangential component of magnetic field in region c over the aperture is

$$\vec{H}_t^c = \vec{H}_t^c(-\vec{M}) = -\vec{H}_t^c(\vec{M}) \tag{3}$$

Since $\vec{H}_t^b$ should be equal to $\vec{H}_t^c$, equations (2) and (3) give

$$\vec{H}_t^b(\vec{M}) + \vec{H}_t^c(\vec{M}) = -\vec{H}_t^i \tag{4}$$

or

$$\hat{n} \times \left(\vec{H}^b(\vec{M}) + \vec{H}^c(\vec{M}) \right) = -\hat{n} \times \vec{H}^i \tag{5}$$

Equation (5) is the basic operator equation for determining the equivalent magnetic current $\vec{M}$.

The magnetic fields $\vec{H}^b$ and $\vec{H}^c$ are given in terms of the magnetic current $\vec{M}$ as shown in the following two integral equations.

$$\vec{H}^b = \iint_{slot} \frac{j}{\omega\varepsilon_d\mu_d} \left[k_d^2 \bar{\bar{I}} + \bar{\nabla}\,\bar{\nabla} \right] \cdot \bar{\bar{F}}(\vec{r}/\vec{r}\,') \cdot \vec{M}(\vec{r}\,')ds' \tag{6}$$

and

$$\vec{H}^c = \iint_{slot} \frac{j}{\omega\varepsilon_c\mu_c} \left[k_c^2 \bar{\bar{I}} + \bar{\nabla}\,\bar{\nabla} \right] \cdot \bar{\bar{G}}(\vec{r}/\vec{r}\,') \cdot \vec{M}(\vec{r}\,')ds' \tag{7}$$

In equation (6), ε_d is the dielectric constant of the dielectric substrate, μ_d is the magnetic permeability and $\bar{\bar{F}}(\vec{r}/\vec{r}\,')$ is the dyadic Green's function for the equivalent problem of Fig. 2a. Similarly, ϵ_c is the dielectric constant of the material in the cavity, μ_c is the magnetic permeability and $\bar{\bar{G}}(\vec{r}/\vec{r}\,')$ is the dyadic Green's function for the equivalent problem b. In view of (6) and (7), equation (5) gives the following coupled integral equations

$$\iint_{slot} \left[K_{xx}(\vec{r}/\vec{r}\,')\ M_x(\vec{r}\,') + K_{xy}(\vec{r}/\vec{r}\,')M_y(\vec{r}\,')\right]ds' = -H_x^i \quad (z=-h) \tag{13}$$

and

$$\iint_{slot} \left[K_{yx}\left(\vec{r}/\vec{r}\,'\right)M_x\left(\vec{r}\,'\right) + K_{yy}\left(\vec{r}/\vec{r}\,'\right)M_y\left(\vec{r}\,'\right)\right]ds' = -H_y^1 \quad (z=-h) \tag{14}$$

where

$$K_{ij}\left(\vec{r}\,'/\vec{r}\,'\right) = j\omega\ \hat{i} \cdot \left\{ (\bar{\bar{F}} + \bar{\bar{G}}) + \bar{\nabla}\,\bar{\nabla} \cdot \left(\frac{\bar{\bar{F}}}{k_d^2} + \frac{\bar{\bar{G}}}{k_c^2} \right) \right\} \cdot \hat{j} \tag{15}$$

and i, j = x, y.

For a thin slot which is open at the center of the cavity wall as shown in Figure 3, the transverse component of the electric field is much smaller than the longitudinal $E_x \ll E_y$. This results in one directional magnetic current and therefore a single integral equation:

$$\iint_{slot} K_{xx}\left(\vec{r}/\vec{r}\,'\right)\ M_x\left(\vec{r}\,'\right)ds' = -H_x^i \qquad (z=-h) \tag{16}$$

In this case, $K_{xx}\left(\vec{r}/\vec{r}'\right)$ takes a simpler form as shown below

$$K_{xx} = \left(F_{xx} + G_{xx}\right) + \frac{\partial}{\partial x^2}\left(\frac{F_{xx}}{k_d^2} + \frac{G_{xx}}{k_c^2}\right) + \frac{\partial}{\partial x \partial x}\frac{F_{zx}}{k_d^2} \tag{17}$$

with [1]

$$F_{xx} = -\frac{j\omega\varepsilon_d}{2\pi k_d^2}\int_o^\infty J_o\left(\lambda\left|\vec{r}-\vec{r}'\right|\right)\frac{u\cosh(uz) - \varepsilon_{rd}u_o\sinh(uz)}{\varepsilon_{rd}u_o\cosh(uh) + u\sinh(uh)}\frac{\lambda}{u}\,d\lambda \tag{18}$$

$$F_{zx} = -\frac{j\omega\varepsilon_d\left(1-\varepsilon_{rd}\right)}{2\pi\ k_d^2}\cos\phi\int_o^\infty J_1\left(\lambda\left|\vec{r}-\vec{r}'\right|\right)\frac{\lambda^2}{\varepsilon_{rd}u_o\ \cosh(uh) + u_o\ \sinh(uh)}$$

$$\frac{\sinh\ [u(h+z)]}{u\ \cosh(uh) + u_o\ \sinh(uh)}\,d\lambda \tag{19}$$

and (Appendix A)

$$G_{xx}\left(\vec{r}/\vec{r}'\right) = j\ \frac{4\omega\varepsilon_c}{ab}\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\varepsilon_n\varepsilon_m\frac{1}{k_z\sin\left(k_z c\right)}$$

$$\cdot\sin\left[\frac{n\pi}{a}\left(x + x_o\right)\right]\sin\left[\frac{n\pi}{a}\left(x' + x_o\right)\right]\cdot \tag{20}$$

$$\cdot\cos\left[\frac{m\pi}{b}\left(y - y_o\right)\right]\cos\left[\frac{m\pi}{b}\left(y' - y_o\right)\right]\cdot$$

$$\cdot\cos\left[k_z\left(c + z + h\right)\right].$$

2. Method of Moments

To obtain the unknown magnetic current $M_x\left(\vec{r}'\right)$ in equation (16), we apply the method of moments. The unknown current is assumed to be in the form:

$$M_x\left(\vec{r}'\right) = \sum_n V_n\, f_n(x')g(y') \qquad (21)$$

where

$$g(y') = \frac{2}{w\pi}\int_{-w/2}^{w/2} \frac{1}{\sqrt{1-\left(\frac{2y'}{w}\right)^2}} \qquad (22)$$

and $\left\{f_n(x'),\ n=1,\ 2...,\ N\right\}$ is a set of expansion functions overlapping in nature. These expansion functions are given by

$$f_n(x') = \begin{cases} \dfrac{\sin\left[k\left(x'-x_{n-1}\right)\right]}{\sin(kl_x)} & X_{n-1} \le x' \le X_n \\ \dfrac{\sin\left[k\left(x_{n+1} - x'\right)\right]}{\sin(kl_x)} & X_n \le x \le X_{n+1} \\ 0 & \text{elsewhere} \end{cases} \qquad (23)$$

In view of equation (21), equation (16) takes the form

$$\sum_n V_n \int_o^L dx' \int_{-w/2}^{w/2} dy'\ K_{xx}(\vec{r}/\vec{r}')\ f_n(x')g(y') = -H_x^i + \Delta H_x^i \qquad (24)$$

In order to minimize the error ΔH_x^i, we impose the following condition

$$\left\langle \Delta H_x^i,\ \phi_m \right\rangle = 0 \qquad m = 1,2,\ldots,N \tag{25}$$

or

$$\int_{x_{m-1}}^{x_{m+1}} \phi_m(x)\ \Delta H_x^i\ dx = 0 \qquad \text{at } y=0 \tag{26}$$

The testing functions $\phi_m(x)$, in equation (26), have been chosen equal to the expansion functions (Galerkin's method). The above condition imposed on equation (24) reduce the integral equation into a set of linear equations which can determine the coefficents V_n and the magnetic current $\vec{M}$ according to equation (21). Once $\vec{M}$ is known, the fields and field-related parameters may be computed by standard methods.

The above solution can be put into a matrix notation as follows:

$$\left[V_n\right] = \left[Y\right]^{-1} \left[I_m\right] \tag{26}$$

where $\left[I_m\right]$ is the excitation vector given by

$$I_m = \left\langle -H_x^i,\ f_m \right\rangle \tag{27}$$

The admittance matrix [Y] can be split into two parts:

$\left[Y_{mn}\right]_b$ = aperture admittance matrix for equivalent problem b

and

$\left[Y_{mn}\right]_c$ = aperture admittance matrix for equivalent problem c.

In this manner, equation (26) takes the form:

$$\left[V_n\right] = \left(\left[Y_{mn}\right]_b + \left[Y_{mn}\right]_c\right)^{-1} \left[I_m\right] \tag{28}$$

with

$$Y^b_{mn} = \iint_{slot} \left\langle f_m, K^b_{xx} \right\rangle f_n \, g \; dx'dy' \tag{29}$$

$$Y^c_{mn} = \iint_{slot} \left\langle f_m, K^c_{mm} \right\rangle f_n \, g \; dx'dy' \tag{30}$$

and

$$K^b_{xx} = \left(1 + \frac{1}{k_d^2} \frac{\partial^2}{\partial x^2}\right) F_{xx} + \frac{1}{k_d^2} \frac{\partial^2}{\partial x \partial z} F_{zx} \tag{31}$$

$$K^c_{xx} = \left(1 + \frac{1}{k_c^2} \frac{\partial^2}{\partial x^2}\right) F_{xx} + \frac{1}{k_c^2} \frac{\partial^2}{\partial x \partial z} F_{zx} \tag{32}$$

It is important to note that computation of $[Y]_b$ involves only regions a, b and computation of $[Y]_c$ involves only region c. Therefore we have divided the problem into two parts each of which can be formulated independently. The elements of the two admittance matrices are given by:

$$Y^b_{mn} = \frac{2}{\pi w} \cdot \frac{1}{\sin^2\left(k_d l_x\right)} \int_{-w/2}^{w/2} dy' \, \frac{1}{\sqrt{1 - \left(\frac{2y'}{w}\right)^2}} \, .$$

$$\left\{ k_d^2 \int_0^{lx} dx \int_0^{lx} dx' \, \sin\left[k_d\left(l_x - x'\right)\right] \, \sin\left[k_d\left(l_x - x\right)\right] . \, D\left(|\vec{r} - \vec{r}'|\right)\right.$$

$$+ k_d \int_0^{l_x} dx \int_0^{l_x} dx' \, \sin\left[k_d\left(l_x - x\right)\right] \, S\left(\left|\vec{r}-\vec{r}\,'\right|\right) \Bigg\} _{\substack{z=z'=-h \\ y=0}} \tag{33}$$

and

$$Y^c_{mn} = j \, \frac{64}{\eta_c ab} \, \frac{k_d^2}{k_c} \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \varepsilon_n \varepsilon_m \;\; J_o\left(\frac{m\pi w}{2b}\right) \cos\left(\frac{m\pi Y_o}{b}\right) \cdot$$

$$\frac{k_c^2 - \left(\frac{n\pi}{a}\right)^2}{\left[k_d^2 - \left(\frac{n\pi}{a}\right)^2\right]^2} \; \frac{\cos\left(k_z c\right)}{k_z} \, .$$

$$\sin^2\left[\frac{l_x}{2}\left(k_d + \frac{n\pi}{a}\right)\right] \sin^2\left[\frac{l_x}{2}\left(k_d - \frac{n\pi}{a}\right)\right] .$$

$$\sin\left[\frac{n\pi}{a}\left(x_m + X_o\right)\right] \cdot \sin\left[\frac{n\pi}{a}\left(x_n + X_o\right)\right] . \tag{34}$$

In equation (33), the functions $D\left(\left|\vec{r}-\vec{r}\,'\right|\right)$ and $S\left(\left|\vec{r}-\vec{r}\,'\right|\right)$ have the following expressions

$$D\left(\left|\vec{r}-\vec{r}\,'\right|\right) = -\frac{j\omega\varepsilon_d}{2\pi k_d} \int_o^{\infty} \frac{u\cosh(uh) + \varepsilon_{rd} u_o \sinh(uh)}{\varepsilon_{rd} u_o \cosh(uh) + u\sinh(uh)} \, \frac{\lambda}{u} \sum_{i=1}^{4} J_o\left(\lambda \rho_i\right) d\lambda \tag{35}$$

$$S\left(\left|\vec{r}-\vec{r}\,'\right|\right) = -\frac{j\omega\varepsilon_d}{2\pi k_d} \int_o^{\infty} \frac{\lambda}{\varepsilon_{rd} u_o \cosh(uh) + u\sinh(uh)} \, .$$

$$\left\{\cosh(uh) + \varepsilon_{rd}\frac{u_o}{u}\sinh(uh) + \frac{u(\varepsilon_{rd}-1)}{u\cosh(uh) + u_o\sinh(uh)}\right\}\cdot$$

$$\cdot\sum_{i=1}^{2} J_o\left(\lambda\rho_i\right)\left[\delta\left(x'+l_x\right) + \delta\left(x'-l_x\right) - 2\cos\left(k_d l_x\right)\delta(x')\right] \tag{36}$$

with

$$\begin{aligned}
\rho_1 &= \left\{\left[(x+x') + (x_m-x_n)\right]^2 + y'^2\right\}^{1/2} \\
\rho_2 &= \left\{\left[(-x+x') + (x_m-x_n)\right]^2 + y'^2\right\}^{1/2} \\
\rho_3 &= \left\{\left[(x-x') + (x_m-x_n)\right]^2 + y'^2\right\}^{1/2} \\
\rho_4 &= \left\{\left[(-x-x') + (x_m-x_n)\right]^2 + y'^2\right\}^{1/2}
\end{aligned} \tag{37}$$

3. Excitation of the Slot

As it was mentioned in section 1, the slot is excited by an incident plane wave. This wave can be either parallel or vertically polarized with respect to the plane of incidence. The x-component of the incident magnetic field for both polarizations will be in the form:

$$H_x^i = H_o^{\parallel\perp} e^{-jk_d x \sin\theta_t} \tag{38}$$

with

$$H_o^{\parallel} = \frac{1}{\eta_d} \frac{2E_o^i \cos[k_d (z+h) \cos\theta_t] \sin\phi_i}{\frac{\eta_o}{\eta_d} \cos(k_d h\cos\theta_t) + j \frac{\cos\theta_t}{\cos\theta_i} \sin(k_d h\cos\theta_t)} \tag{39}$$

$$H_o^{\perp} = \frac{1}{\eta_d} \frac{2 E_o^i \cos [k_d(z+h) \cos\theta_t] \cos\phi_i}{\frac{\eta_o}{\eta_d} \frac{\cos\theta_t}{\cos\theta_i} \cos(k_d h\cos\theta_t) + j \sin(k_d h\cos\theta_t)} . \tag{40}$$

In view of equations (38)-(40), the elements of the excitation vector [I] are given by the following expression

$$I_m = 2 H_o^{\parallel\perp} \left[\frac{\cos\left(k_d l_x \sin\theta_t\right) - \cos\left(k_d l_x\right)}{k_d \left(1-\sin^2\theta_t\right)} \right] e^{-jk\sin\theta_t x_j} \tag{41}$$

4. Electric Field Distribution on the Slot-Numerical Results

Based on the formulation shown in previous sections, the electric field distribution on cavity-backed slot is shown in figures 1-5 for various slot, substrate and cavity characteristics.

Figures 1-3

Substrate

$\epsilon r = 2.35$

$h = 0.1\lambda_o$

Cavity

$X_c = 0.4\lambda_o$

$y_c = 0.25\lambda_o$

$z_c = 0.124\lambda_o$

$\epsilon_{rc} = 4.0$

$\mu_{rc} = 1.0$

$\delta_\epsilon = 0.0$

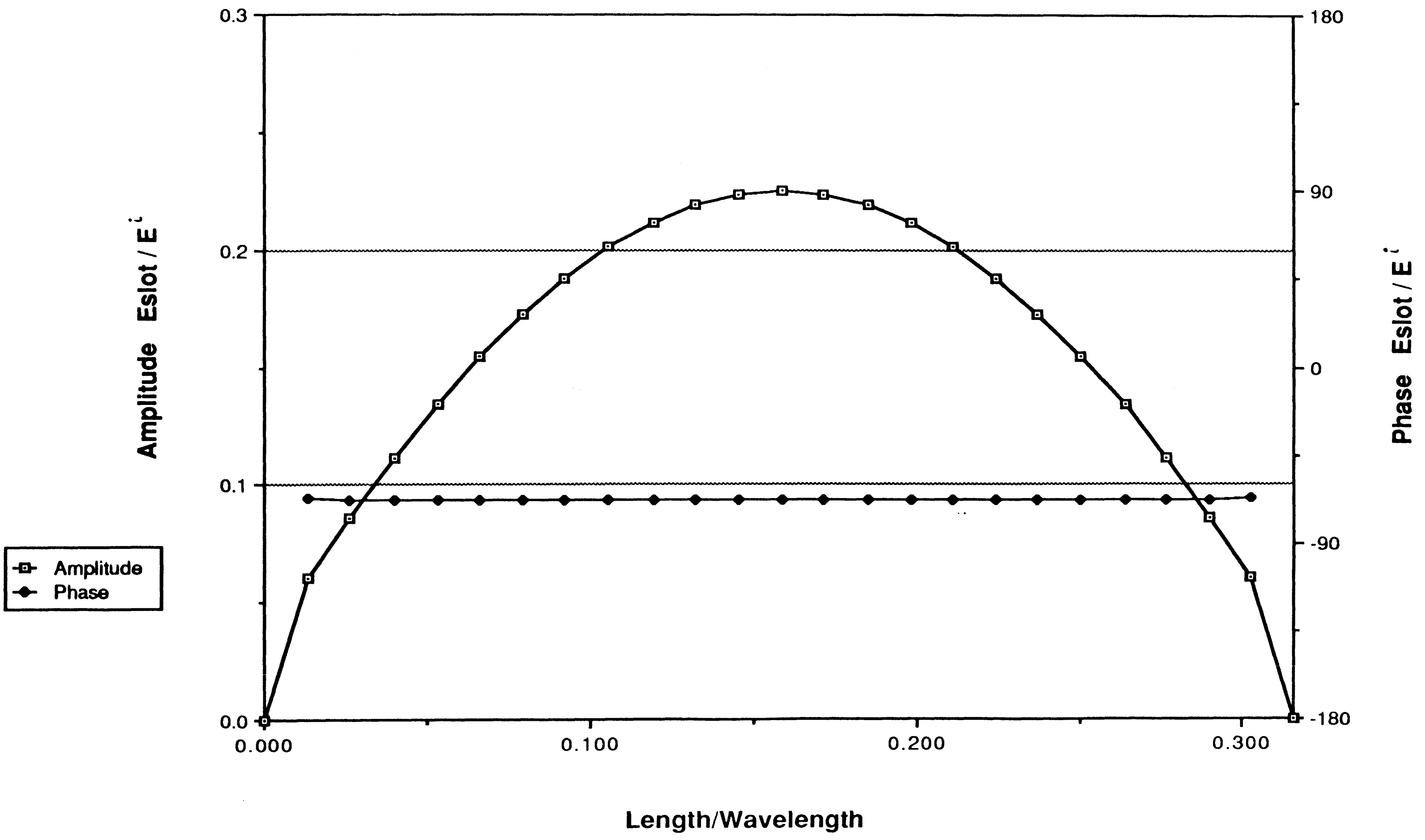
$\delta_\mu = 0.0$

Figure 1

Electric Field Distribution on a

Cavity-Backed Slot

($\phi = 0°$, $\theta = 0°$)

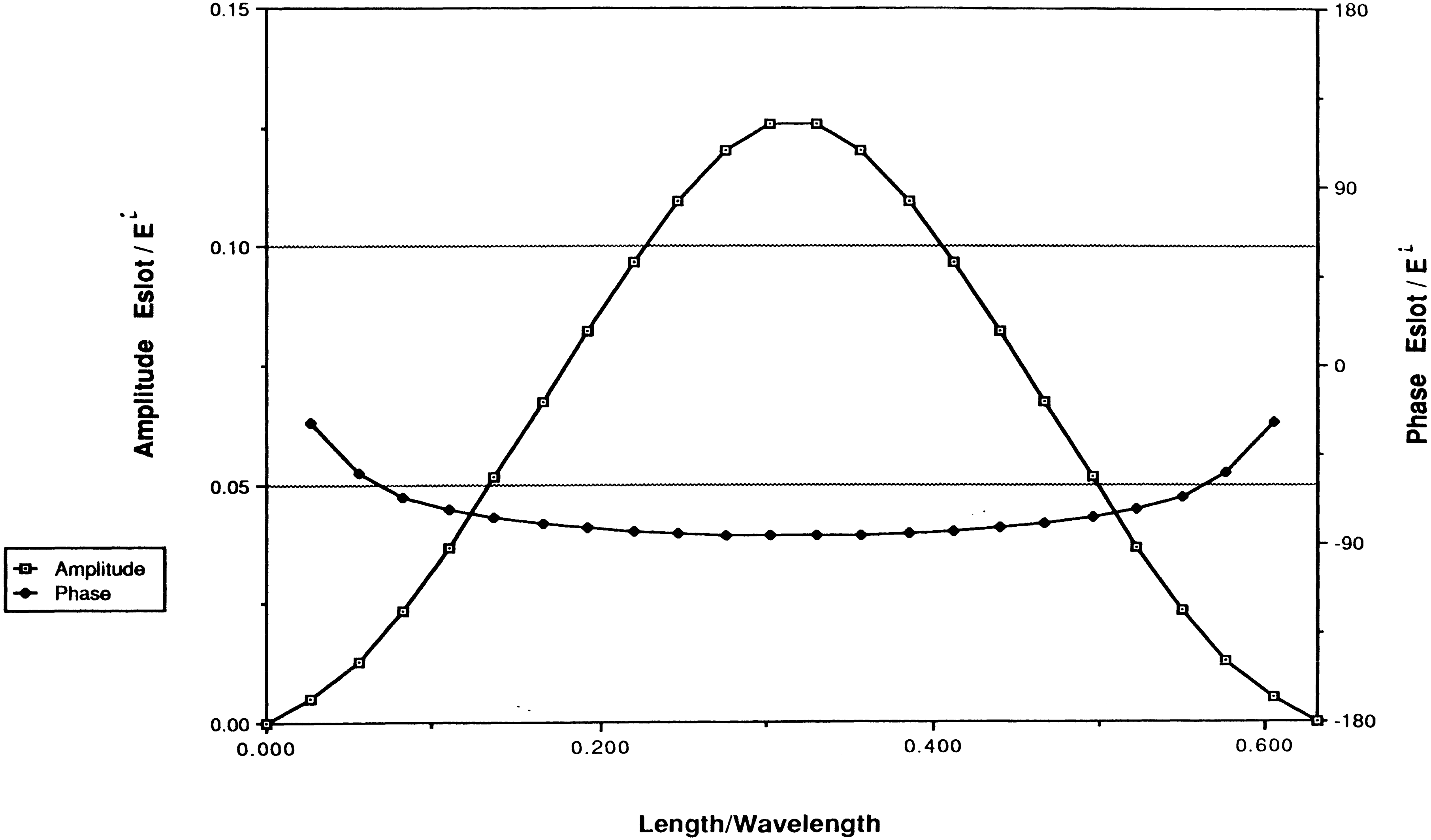


Electric Field Distribution on a

Cavity-Backed Slot

Figure 2

($\phi = 0°, \theta = 0°$)

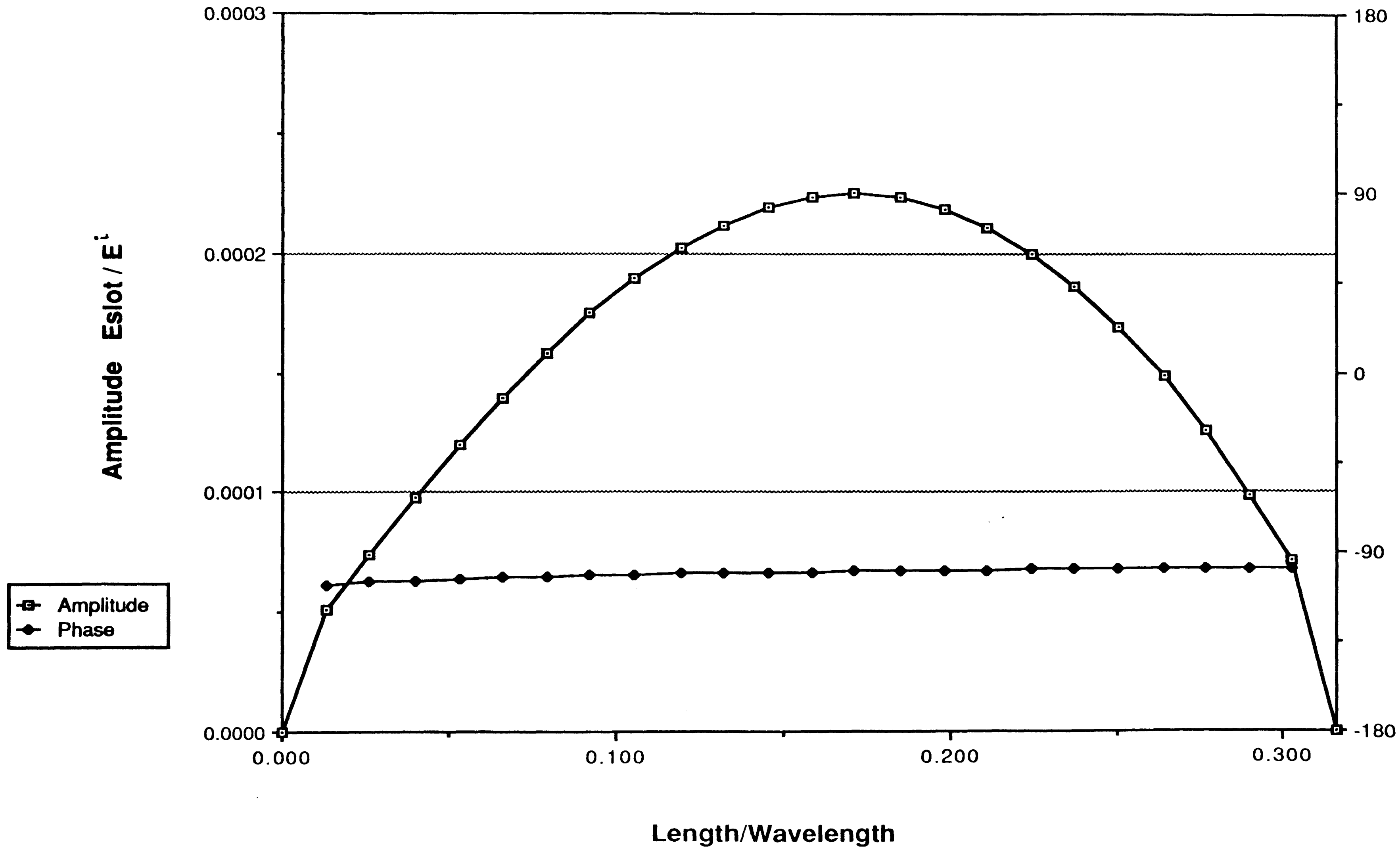


Electric Field Distribution on a

Cavity-Backed Slot

Figure 3

($\phi = 90°, \theta = 60°$)



Figures 4, 5

Substrate

$\epsilon_r = 2.35$

$h = 0.1\lambda_o$

Cavity

$x_c \doteq 0.4\lambda_o$

$y_c = 0.25\lambda_o$

$\epsilon_{rc} = 4.0$

$\mu_{rc} = 1.0$

$\delta_\epsilon = 0.0$

$\delta_\mu = 0.0$

Incident Wave

$\phi = 12°$

$\theta = 0°$

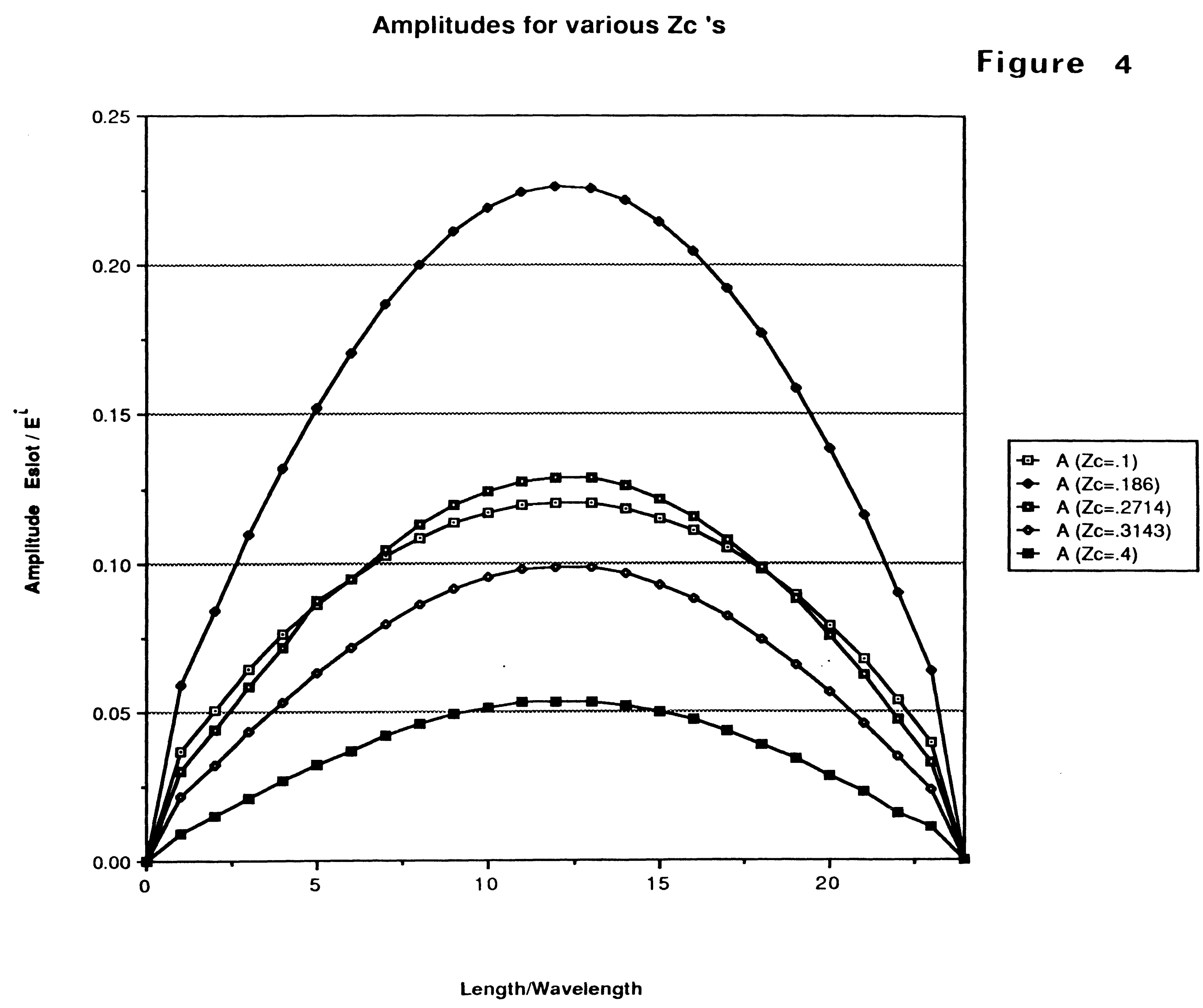
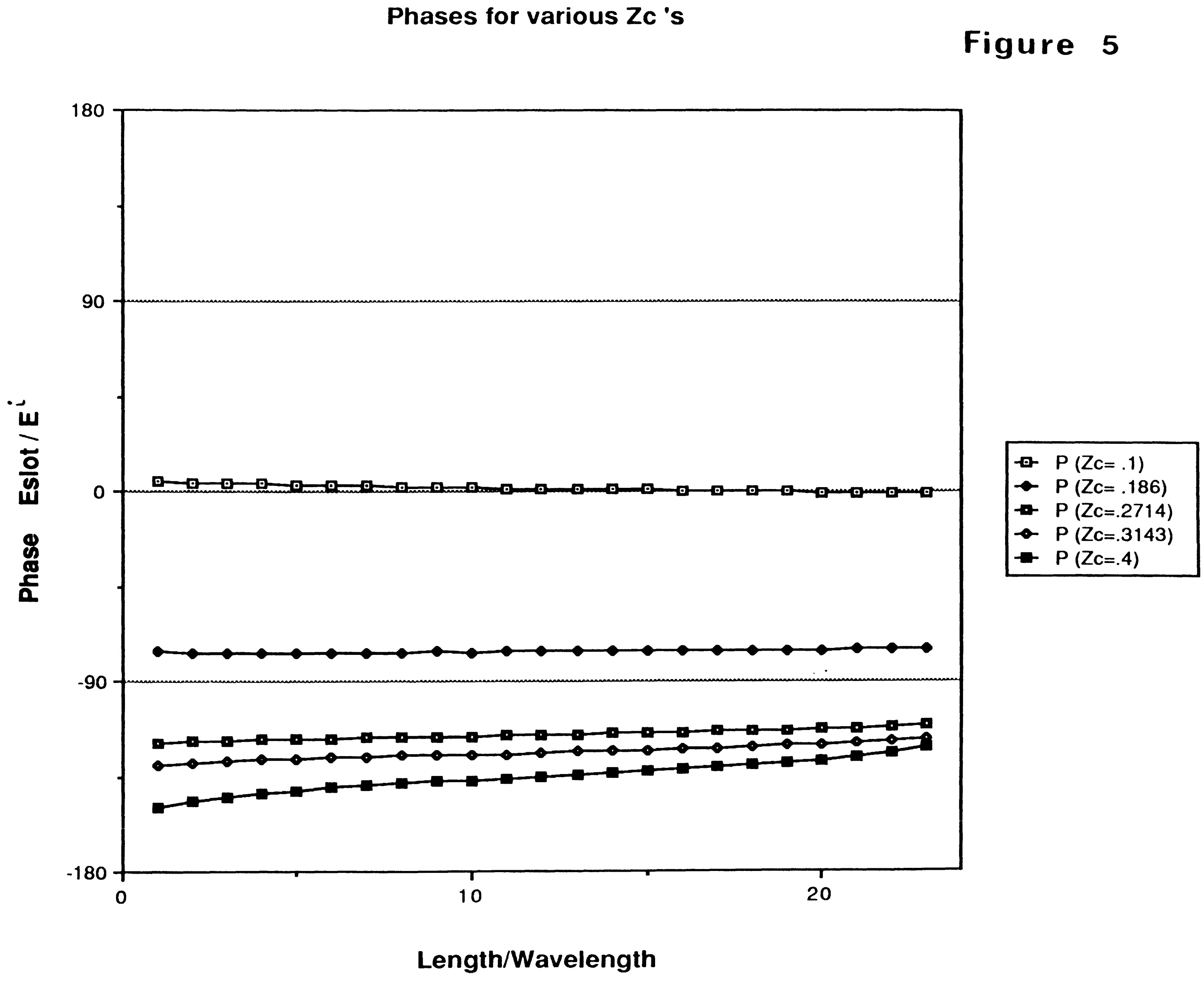

Amplitudes for various Zc 's
0.25
0.20
0.15
0.10
0.05
0.00
Amplitude Eslot / E^i
0
5
10
15
20
Length/Wavelength
A (Zc=.1)
A (Zc=.186)
A (Zc=.2714)
A (Zc=.3143)
A (Zc=.4)

Figure 4

Figure 5


Phases for various Zc 's
180
90
0
-90
-180
0
10
20
Phase Eslot / E
Length/Wavelength
P (Zc= .1)
P (Zc= .186)
P (Zc=.2714)
P (Zc=.3143)
P (Zc=.4)

Figures 6-9

Substrate

$\epsilon_r = 2.35$

$h = 0.1\lambda_o$

Cavity

$x_c = 0.4\lambda_o$

$y_c = 0.25\lambda_o$

$z_c = 0.125\lambda_o$

$\mu_{rc} = 1.0\lambda_o$

$\delta_\epsilon = 0.0$

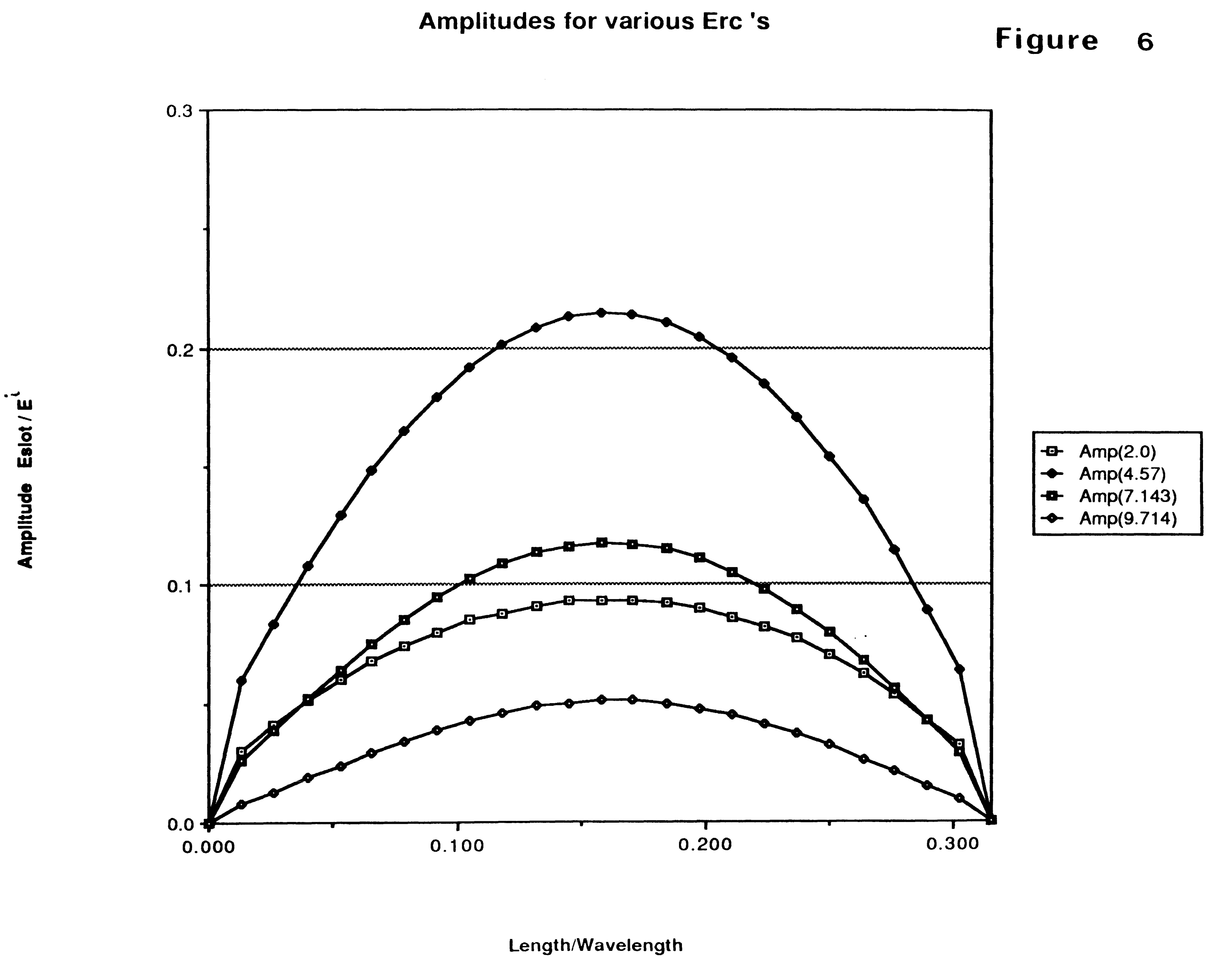
$\delta_\mu = 0.0$

Incident Wave

$\phi = 12°$

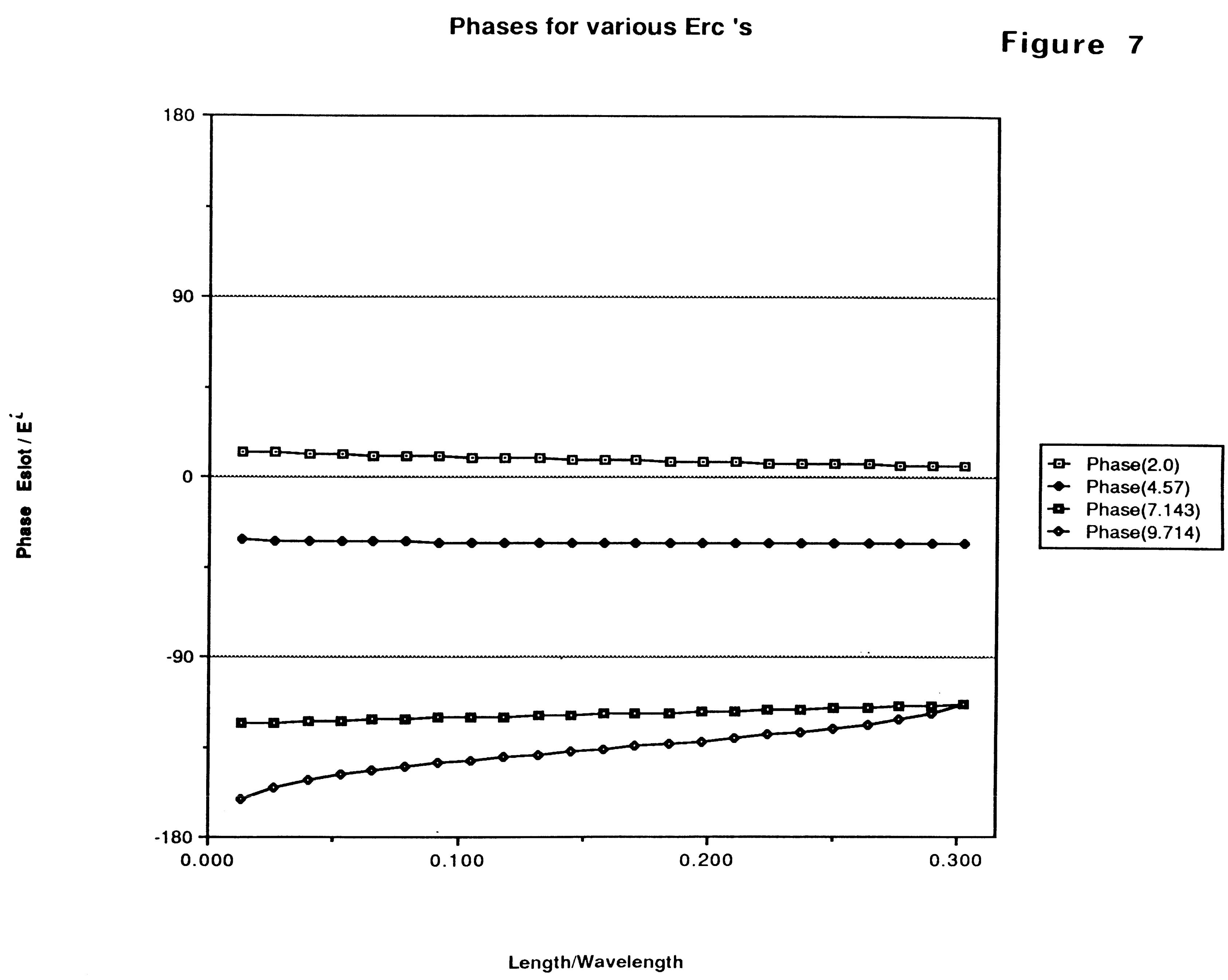
$\theta = 0°$

Figure 6



Amplitudes for various Erc 's
0.3
0.2
0.1
0.0
0.000
0.100
0.200
0.300
Amplitude Eslot / E^i
Length/Wavelength
Amp(2.0)
Amp(4.57)
Amp(7.143)
Amp(9.714)

Figure 7



Phases for various Erc 's
180
90
0
-90
-180
0.000
0.100
0.200
0.300
Phase Eslot / Ei
Length/Wavelength
Phase(2.0)
Phase(4.57)
Phase(7.143)
Phase(9.714)

Figure 8

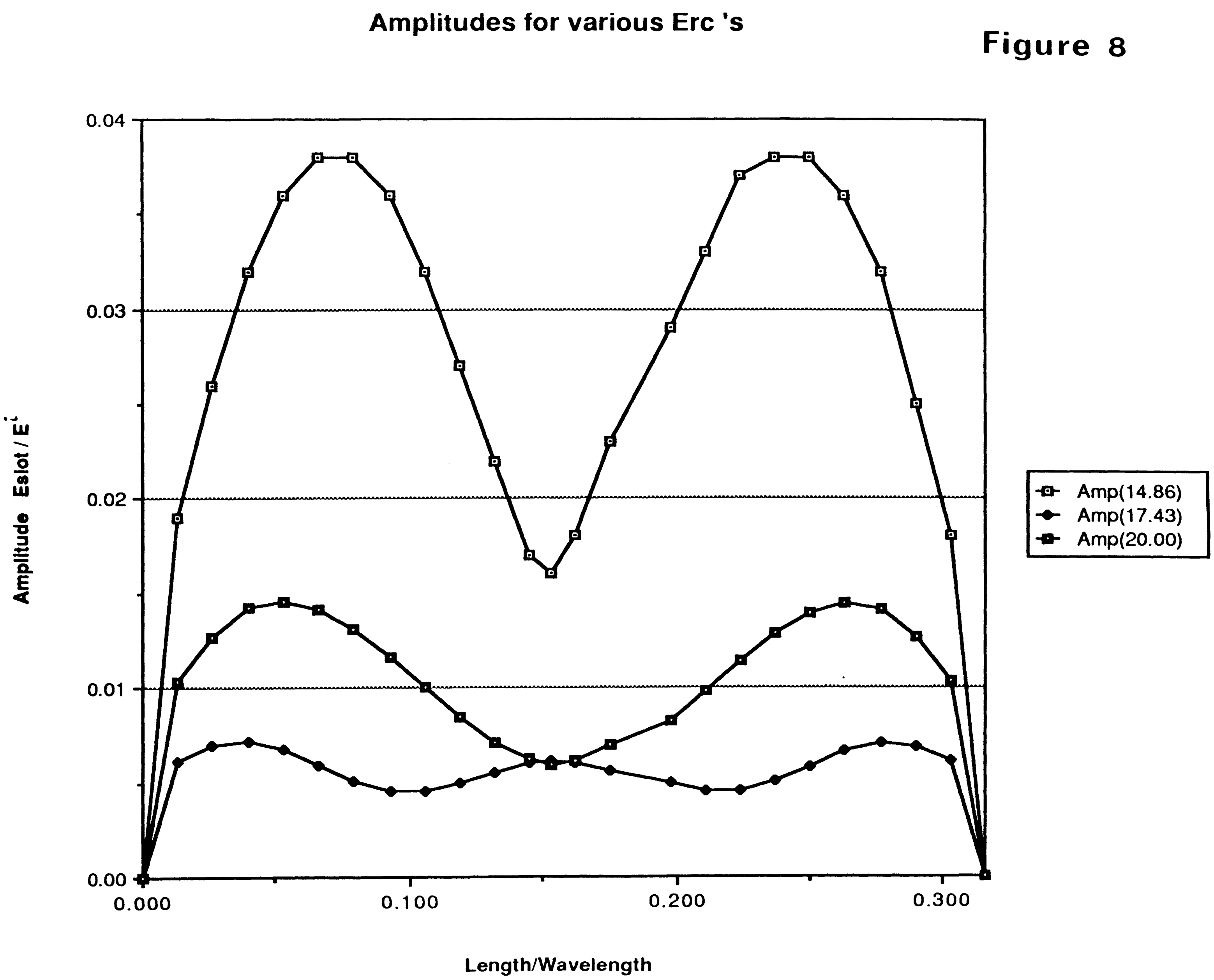
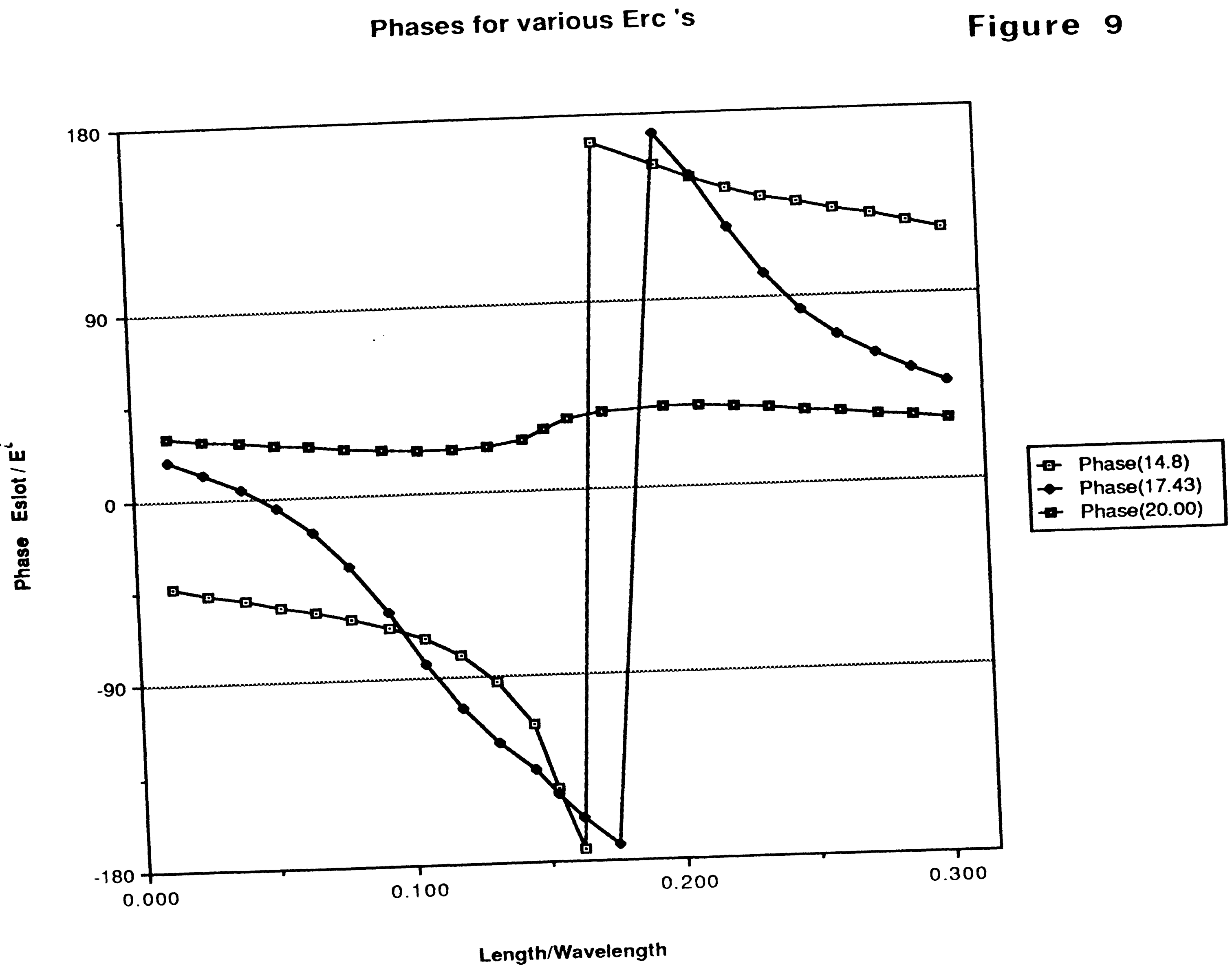

Amplitudes for various Erc 's
Amplitude Eslot / E^i
0.04
0.03
0.02
0.01
0.00
0.000
0.100
0.200
0.300
Length/Wavelength
Amp(14.86)
Amp(17.43)
Amp(20.00)

Figure 9



Phases for various Erc 's
180
90
0
-90
-180
Phase Eslot / E^i
0.000
0.100
0.200
0.300
Length/Wavelength
Phase(14.8)
Phase(17.43)
Phase(20.00)

References

[1] P.B. Katehi, "The Green's Function for a slot cut on the Ground of a Dielectric Substrate," Rad. Lab. Report.

Appendix A

Green's Function for a Magnetic Current on the Wall of a Rectangular Cavity

Let us begin by considering an x-directed Hertzian magnetic dipole (HMD) at the position (x',y',z') as shown in figure (A.1).

With an assumed $e^{j\omega t}$ time dependency, Maxwell's equations take the form,

$$\vec{\nabla} \times H = j\omega \vec{D}$$
$$\nabla \times \vec{E} = -j\omega \vec{B} - \vec{M} \tag{1}$$
$$\vec{\nabla} \cdot \vec{D} = 0$$
$$\vec{\nabla} \cdot \vec{B} = 0$$

and

$$\vec{D} = \varepsilon_c \vec{E}$$
$$\vec{B} = \mu_c \vec{H} \tag{2}$$

with

$$\varepsilon_c = \varepsilon_c^{'}(1-j\tan\delta_\varepsilon) \tag{3}$$

$$\mu_c = \mu_c^{'}(1-j\tan\delta_\mu). \tag{4}$$

We can now introduce an electric vector potential function F such that

$$\vec{D} = \vec{\nabla} \times \vec{F}. \tag{5}$$

When we substitute equation (5) into (1a), the following relation is derived

$$\vec{\nabla} \times \left[\vec{H} - j\omega\vec{F}\right] = 0. \tag{6}$$

Equation (6) is equivalent to

$$\vec{H} = j\omega\vec{F} + \nabla\phi_m . \tag{7}$$

From equations (7) and (1b), we can get the following result

$$\vec{\nabla}\times\vec{\nabla}\times\vec{F} = \vec{\nabla}\,(\vec{\nabla}\cdot\vec{F}) - \nabla^2\vec{F} = k_c^2\vec{F} - j\omega\mu_c\varepsilon_c\vec{\nabla}\phi_m - \varepsilon_c\vec{M}. \tag{8}$$

Since equation (5) alone cannot specify $\vec{F}$, we can select its divergence so that

$$\vec{\nabla}\cdot\vec{F} = -j\omega\varepsilon\mu\phi_{m.} \tag{9}$$

Equations (8) and (9) then give

$$\left(\nabla^2 + k_c^2\right)\vec{F} = \varepsilon_c\vec{M}. \tag{10}$$

The electric and magnetic fields, in terms of $\vec{F}$, are given as follows

$$\vec{E} = \frac{1}{\varepsilon_c}\vec{\nabla}\times\vec{F} \tag{11}$$

$$\vec{H} = \frac{j}{\omega\varepsilon_c\mu_c}\left[k_c^2\vec{F} + \vec{\nabla}\vec{\nabla}\cdot\vec{F}\right] \tag{12}$$

with

$$k_c^2 = \omega^2\varepsilon_c\mu_c.$$

For a magnetic current $\vec{M}$ with more than one component, the electric potential can be consider as a dyadic function of the form

$$\overline{\overline{F}} = \hat{x}\hat{x}\,F_{xx} + \hat{y}\hat{y}\,F_{yy} + \hat{z}\hat{z}\,F_{zz}. \tag{11}$$

For the case of Hertzian dipoles $\vec{F}$ is being considered as the Green's function of the problem and satisfies the following differential equation

$$\left(\nabla^2 + k_c^2\right) \overline{\overline{G}} = \varepsilon_c \overline{\overline{I}} \tag{12}$$

where $\overline{\overline{I}}$ is the unit dyadic. For the problem of figure A.1, $G_{yy}=G_{zz}=0$

and

$$E_x = 0$$
$$E_y = \frac{1}{\varepsilon_c} \frac{\partial G_{xx}}{\partial z} \tag{13}$$
$$E_z = -\frac{1}{\varepsilon_c} \frac{\partial G_{xx}}{\partial y}$$

Also, the magnetic field is given by

$$H_x = \frac{j}{\omega \varepsilon_c \mu_c} \left(k_c^2 + \frac{\partial^2}{\partial x^2} \right) G_{xx}$$

$$H_y = \frac{j}{\omega \varepsilon_c \mu_c} \frac{\partial^2}{\partial y \partial x} G_{xx} \tag{14}$$

$$H_z = \frac{j}{\omega \varepsilon_c \mu_c} \frac{\partial^2}{\partial z \partial x} G_{xx}$$

The boundary conditions for this problem are:

$$E_y^{I} = 0 \qquad z = 0 \qquad x = 0, a$$
$$E_z^{I} = 0 \qquad z = 0, a \qquad y = 0, b$$
$$\tag{15}$$
$$E_y^{II} = 0 \qquad z = c \qquad x = 0, a$$
$$E_z^{II} = 0 \qquad x = 0, a \qquad y = 0, b$$

The solution to equation (12) which satisfies the above boundary conditions is in the form:

region I

$$G^{I}_{xx} = \frac{4\varepsilon_c}{ab} \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \varepsilon_n \varepsilon_m \quad \sin\left(\frac{n\pi x'}{a}\right) \cos\left(\frac{m\pi y'}{b}\right) \cdot \sin\left(\frac{n\pi x}{a}\right) \cos\left(\frac{m\pi y}{b}\right) \cdot \frac{\cos\left[k_z\left(c-z'\right)\right]}{k_z} \frac{\cos(k_z z)}{\sin(k_z c)} \tag{16}$$

region II

$$G^{II}_{xx} = \frac{4\varepsilon_c}{ab} \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \varepsilon_n \varepsilon_m \quad \sin\left(\frac{n\pi x'}{a}\right) \cos\left(\frac{m\pi y'}{b}\right) \cdot \sin\left(\frac{n\pi x}{a}\right) \cos\left(\frac{m\pi y}{b}\right) \cdot \frac{\cos\left(k_z z'\right)}{k_z} \cdot \frac{\cos\left[k_z\left(c-z\right)\right]}{\sin\left(k_z c\right)} \tag{17}$$

with

$$k_z^2 = k_c^2 - \left(\frac{n\pi}{a}\right)^2 - \left(\frac{m\pi}{b}\right)^2 \tag{18}$$

Using the above expressions and for the case of a magnetic dipole M_x at $z' = 0$, the x-component of the magnetic field in the cavity takes the form

$$H_x^c = \iint_S \left[1 + \frac{1}{k_c^2} \frac{\partial^2}{\partial x^2} \right] G_{xx}^{II} \,(x,y,z;x',y') \quad M_x(x',y')\,dx'dy'. \qquad (19)$$

```
C*********************************************************************
C     ...................  Slot.ftn    ..............................
C
C     This program evaluates the elements of the admittance matrix
C         in the the following problem:
C
C      "Scattering by two slots on infinite dielectric substrate with
C             cavities at the back."
C
C
C         This program is good for any substrate thickness h, er and
C                any dimensions of the cavities.
C
C*********************************************************************
      SUBROUTINE SLOT
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*4 CONST,GSK,GS1S2K
      COMPLEX ZS,ZS1S2,CI,BMATR,CUR

      COMMON/CTAIL/S1(4,205,7),D1(4,205,7),D2(4,205,7),
     *T1(4,205,7),T2(4,205,7),T3(4,205,7),T4(4,205,7)

      COMMON/COMP/ZS(50),ZS1S2(350),NS,NS1S2

      COMMON/OUT/GS(50),GS1S2(350)

      COMMON/MAT/PLI,AI,TI,V(3),IY

      COMMON/PUT/SSJ0(250,7),SAJ0(250,7),YSIN,YCOS

      COMMON/ADON/DIST(250,7,10),RCOE(20,250,7,10),AX,SERS(5),SERA(5),
     *DARG(10,4),S(10,2),WREAL,NSER,NMAX(7)

      COMMON/DAT/ER,H,T,DLX,XC,YC,ZC,ERC,RMC,X0,Y0,A,TPI,TPI2,PI,W,E1,
     *E2,EER,AK0,AK,AKK,FA,OFFSET(7),WDELTA,OFFLIM,ERROR,THI,FI,
     *NS1,NS2,NSS2,NOFF

      COMMON/DATT/COAL(20),POINT(20),CN(51),BM(51),POLTM(20),
     *POLTE(20),AM(41),DM(41),POLES(40),VXXM(20),VZXM(20),VZXE(20),
     *BPOINT(10),BCOAL(10),MPOINT,NPOINT,NK0,MA,NTM,NTE,NK0K,IFIRST


      COMMON/COEF/RX,XX,RZ,XZ,FRX,FRZ,F1X,F1Z

      COMMON/IOFF/INS,INS1S2

      COMMON/B01/BJ0,BJ1


      CALL DATA_SLOT

      Subroutine POLES evaluates the poles of the Green's function
      and orders them according to their magnitude

      IFIRST=0  :dominant mode is a TM wave
             1  :dominant mode is a TE wave
             2  :only one TM wave

      CALL SPOLES
      PRINT *,'SPOLES has run'
      H=H*DSQRT(ER)

      CI=(0.00,1.00)

      NS=NS1
      IF (NS1.LT.NS2) NS=NS2
```

```
      MS=NS
      IF (NOFF.EQ.1) GO TO 50
          NS1S2=NS2+NSS2-1
          MS1S2=NS1S2
          IF (NS1S2.GT.200) NS1S2=200
C
  50  IF(NMAX(INS).LE.(NS+2)) NMAX(INS)=NS+2
      IF (NOFF.EQ.1) GO TO 51
          IF(NMAX(INS1S2).LE.(NS1S2+2)) NMAX(INS1S2)=NS1S2+2
C
C
  51  ADL=AKK*DLX
      YSIN=DSIN(ADL)
      YCOS=DCOS(ADL)
C
C     For the normalization of the current along the y axis
C
                  CVON=W*PI/2.D0
C
C     Computation of lamda-integration limits between 0 and A
C
C
      CALL LIMIT
C
C     Evaluation of the Green's function at different points
C     in the interval [0,A]. The Bessel function has been excluded
C
      CALL GREEN
C
C     Evaluation of the tail contribution (from a to infinity)
C
      CALL TAIL
C
      CONST=(1.D0/CVON)*DSQRT(ER)/(480.D0*(PI**3)*YSIN*YSIN)
      WRITE(6,9)
   9  FORMAT('-------------------- ZS ---------------------')
      WRITE(6,10) MS
  10  FORMAT(11X,I4)
      DO 11 K=1,MS
            ZS(K)=ZS(K)*CONST
            GSK=REAL(GS(K))*CONST
C           WRITE (6,30) K,ZS(K),GSK
  30        FORMAT(1X,I4,2X,'ZS=',E14.7,2X,E14.7,2X,
     *             'GSK=',E14.7)
            ZS(K)=(ZS(K)+GSK)*CI
C           WRITE (6,12) ZS(K)
  12        FORMAT(68X,E14.7,1X,E14.7)
  11  CONTINUE
      IF (NOFF.EQ.1) GO TO 52
            WRITE (6,15)
  15        FORMAT('--------------------  ZS1S2  -----------------')
            WRITE (6,10) MS1S2
            DO 16 K=1,MS1S2
                  ZS1S2(K)=ZS1S2(K)*CONST
                  GS1S2K=REAL(GS1S2(K))*CONST
                   WRITE (6,32) K,ZS1S2(K),GS1S2K
  32              FORMAT(1X,I4,2X,'ZS1S2=',E14.7,2X,E14.7,2X,
     *                   'GS1S2K=',E14.7)
                   WRITE (6,12) ZS1S2(K)
                  ZS1S2(K)=(ZS1S2(K)+GS1S2K)*CI
  16        CONTINUE
  52  CONTINUE

1000  CONTINUE
      RETURN
      END
```

```
C...........................................................
C...........................................................
C     This subroutine evaluates the limits of integration in
C     the interval [0,A].
C     Specifically:
C          1) It divides the interval [0,k0] to 10 equal
C             subsections and then apply fixed-point Gaussian
C             Quadrature
C          2) It divides the interval [k0,k] into so many
C             subsections as the number of poles and in
C             such a way that each subsection includes one
C             pole only away from the ends of the subsection
C          3) It divides the interval [k,A] into 20 equal
C             subsections and then apply fixed-point Gaussian
C             Quadrature
C............................................................
      SUBROUTINE LIMIT
      IMPLICIT REAL*8 (A-H,O-Z)
      EXTERNAL WSPE,WTPE,WSPM
C
      COMMON/DAT/ER,H,T,DLX,XC,YC,ZC,ERC,RMC,X0,Y0,A,TPI,TPI2,PI,W,E1,
     *E2,EER,AK0,AK,AKK,FA,OFFSET(7),WDELTA,OFFLIM,ERROR,THI,FI,
     *NS1,NS2,NSS2,NOFF
C
      COMMON/DATT/COAL(20),POINT(20),CN(51),BM(51),POLTM(20),
     *POLTE(20),AM(41),DM(41),POLES(40),VXXM(20),VZXM(20),VZXE(20),
     *BPOINT(10),BCOAL(10),MPOINT,NPOINT,NK0,MA,NTM,NTE,NK0K,IFIRST
C
C-------------------------------------------------+
C     Step 1 : Evaluation of vector CN            |
C              it gives the end points of the     |
C              intervals considered in (0,k0)     |
C-------------------------------------------------+
      DELTA=AK0/FLOAT(NK0)
      CN(1)=0.D0
      DO 1 I=1,NK0
           CN(I+1)=DELTA*FLOAT(I)
  1   CONTINUE
C-------------------------------------------------+
C     Step 2 : Evaluation of vector BM            |
C              it gives the end points of the     |
C              intervals considered in (k,A)      |
C-------------------------------------------------+
      DELTA=(A/DSQRT(EER)-AK)/FLOAT(MA)
      BM(1)=AK
      DO 2 I=1,MA
           BM(I+1)=DELTA*FLOAT(I)+AK
  2   CONTINUE
C-------------------------------------------------+
C     Step 3 : Evaluation of the vectors AM,DM    |
C              "AM" gives the end points around   |
C                   the TM poles                  |
C              "DM" gives the end points around   |
C                   the TE poles                  |
C                                                 |
C     IFIRST=   2   only one TM pole              |
C               1   TE0<TM0                       |
C               0   TM0<TE0                       |
C-------------------------------------------------+
      AM(1)=AK0
      DM(1)=AK0
      NMAX=NTE+NTM-1
      IF (IFIRST.EQ.2) GO TO 3
      DO 4 I=1,NMAX
           AM(I+1)=(POLES(I+1)+POLES(I))/2.D0
           DM(I+1)=AM(I+1)
```

```
C
C     ---- Dielectric constant ---
C
      2.35
C
C     ---- Substrate Thickness ---
C
      0.1
C
C     ---- Conductor Thickness ---
C
      0.0001
C
C     ---- Dimensions of the Cavity ----
C
      0.4
      0.25
      0.124
C
C     ---- Dielectric Permittivity and Permeability of the Medium in the Cavity ----
C
      4.00
      1.00
      0.00
      0.00
C
C     ---- Slot width ----
C
      0.1
C
C     ---- Position of the Center of the Leading Edge of the Slot ----
C
      0.072796
      0.075
C
C     ---- Subsection Length ----
C
      0.022
C
C     ---- Lower Limit of the Tail Contribution ----
C
      100.0
C
C     ---- Number of Points on Each Slot ----
C
      23
      0
C
C     ---- Number of Offsets ----
C
      1
C
C     ---- Offset Between the Two Slots ----
C
      0.0
C
C     ---- Longitudinal Displacement Between the Two Slots ----
C
      0
C
C     ---- Theta angle ----
C
      0.0
C
C     ---- Fi angle ----
C
```

0.0

```
C.............................................................
C                              MAIN_SLOT.FTN
C      This program evaluates the reflection coefficient of
C      cavity-backed slot as a function of the characteristics
C      of the cavity and the incident wave
C..............................................................
      IMPLICIT REAL*8 (A-H,O-Z)
%INCLUDE '/SYS/INS/CAL.INS.FTN'
      INTEGER*4 CPU_SECONDS
      INTEGER*2 TIMEDATE_REC(6)
C
      COMPLEX R0,REFL,R_RATIO,CUR,BMATR,ZS,ZS1S2
C
      CHARACTER*10 PAR1,PAR2
      LOGICAL THFLAG1,FIFLAG1,XCFLAG1,YCFLAG1,ZCFLAG1,ERCFLAG1
      LOGICAL THFLAG2,FIFLAG2,XCFLAG2,YCFLAG2,ZCFLAG2,ERCFLAG2
C
      COMMON/CTAIL/S1(4,205,7),D1(4,205,7),D2(4,205,7),
     *T1(4,205,7),T2(4,205,7),T3(4,205,7),T4(4,205,7)
C
      COMMON/COMP/ZS(50),ZS1S2(350),NS,NS1S2
C
      COMMON/OUT/GS(50),GS1S2(350)
C
      COMMON/MAT/PLI,AI,TI,V(3),IY
C
      COMMON/PUT/SSJ0(250,7),SAJ0(250,7),YSIN,YCOS
C
      COMMON/ADON/DIST(250,7,10),RCOE(20,250,7,10),AX,SERS(5),SERA(5),
     *DARG(10,4),S(10,2),WREAL,NSER,NMAX(7)
C
      COMMON/DAT/ER,H,T,DLX,XC,YC,ZC,ERC,RMC,X0,Y0,A,TPI,TPI2,PI,W,E1,
     *E2,EER,AK0,AK,AKK,FA,OFFSET(7),WDELTA,OFFLIM,ERROR,THI,FI,
     *NS1,NS2,NSS2,NOFF
C
      COMMON/DATT/COAL(20),POINT(20),CN(51),BM(51),POLTM(20),
     *POLTE(20),AM(41),DM(41),POLES(40),VXXM(20),VZXM(20),VZXE(20),
     *BPOINT(10),BCOAL(10),MPOINT,NPOINT,NK0,MA,NTM,NTE,NK0K,IFIRST
C

      COMMON/COEF/RX,XX,RZ,XZ,FRX,FRZ,F1X,F1Z

      COMMON/IOFF/INS,INS1S2

      COMMON/MAN/BMATR(260,260),IA(260),IB(260)

      COMMON/PAT/CUR(260)

      COMMON/B01/BJ0,BJ1

      COMMON/LOSS/TLOS_E,TLOS_M

      OPEN(UNIT=01,FILE='OUT.PLOT',STATUS='OLD')
      OPEN(UNIT=05,FILE='DATA.SLOT',STATUS='OLD')
      OPEN(UNIT=06,FILE='OUT.SLOT',STATUS='OLD')

      CALL PROC1_$GET_CPUT(CLOCK)
      CPU_SECONDS=CAL_$CLOCK_TO_SEC(CLOCK)

      THFLAG1=.FALSE.
      FIFLAG1=.FALSE.
      XCFLAG1=.FALSE.
      YCFLAG1=.FALSE.
      ZCFLAG1=.FALSE.
      ERCFLAG1=.FALSE.
```

```
      THFLAG2=.FALSE.
      FIFLAG2=.FALSE.
      XCFLAG2=.FALSE.
      YCFLAG2=.FALSE.
      ZCFLAG2=.FALSE.
      ERCFLAG2=.FALSE.
C
      PRINT *, 'You can plot the reflection coeff. as a function of:'
      PRINT *, 'THETA,FI,XC,YC,ZC,ERC'
      PRINT *, 'The reflection coefficient will vary as a function of:'
      PRINT *, 'First parameter:'
      READ (*,1) PAR1
      PRINT *, 'Second parameter:'
      READ (*,1) PAR2
  1   FORMAT(A10)
C
      PRINT *, 'What is the maximum value of  ', PAR1
      READ (*,2) PAR1MX
  2   FORMAT(E14.7)
      PRINT *, 'What is the minimum value of  ',PAR1
      READ (*,2) PAR1MN
      PRINT *, 'How many points should be considered'
      PRINT *, '(number of points >=1)'
      READ (*,23) NPAR1
      PRINT *, 'What is the maximum value of  ', PAR2
      READ (*,2) PAR2MX
      PRINT *, 'What is the minimum value of  ',PAR2
      READ (*,2) PAR2MN
      PRINT *, 'How many points should be considered'
      PRINT *, '(number of points >=1)'
      READ (*,23) NPAR2
 23   FORMAT (I5)
:
:     Subroutine DATA reads the given values for different
:     variables
:
      CALL DATA
:
      PRINT *,'Subroutine DATA run'
:
:     Subroutine slot evaluates the parts of the elements of
:     the impedance matrix which are coming from the dielectric
:     substrate

      WREAL=W
      W=W*(1.D0+2.D0*WDELTA/W)

      CALL SLOT

      PRINT *,'Subroutine SLOT run'

      DELTA1=0.D0
      DELTA2=0.D0
      IF (NPAR1.GT.1) DELTA1=(PAR1MX-PAR1MN)/FLOAT(NPAR1-1)
      IF (NPAR2.GT.1) DELTA2=(PAR2MX-PAR2MN)/FLOAT(NPAR2-1)

      NTHI=-1
      NFI=-1
      NXC=-1
      NYC=-1
      NZC=-1
      NERC=-1

      IF ((PAR1.EQ.'THETA').OR.(PAR1.EQ.'theta')) THFLAG1=.TRUE.
      IF ((PAR2.EQ.'THETA').OR.(PAR2.EQ.'theta')) THFLAG2=.TRUE.
      IF ((PAR1.EQ.'FI').OR.(PAR1.EQ.'fi')) FIFLAG1=.TRUE.
```

```
      IF ((PAR2.EQ.'FI').OR.(PAR2.EQ.'fi')) FIFLAG2=.TRUE.
      IF ((PAR1.EQ.'XC').OR.(PAR1.EQ.'xc')) XCFLAG1=.TRUE.
      IF ((PAR2.EQ.'XC').OR.(PAR2.EQ.'xc')) XCFLAG2=.TRUE.
      IF ((PAR1.EQ.'YC').OR.(PAR1.EQ.'yc')) YCFLAG1=.TRUE.
      IF ((PAR2.EQ.'YC').OR.(PAR2.EQ.'yc')) YCFLAG2=.TRUE.
      IF ((PAR1.EQ.'ZC').OR.(PAR1.EQ.'zc')) ZCFLAG1=.TRUE.
      IF ((PAR2.EQ.'ZC').OR.(PAR2.EQ.'zc')) ZCFLAG2=.TRUE.
      IF ((PAR1.EQ.'ERC').OR.(PAR1.EQ.'erc')) ERCFLAG1=.TRUE.
      IF ((PAR2.EQ.'ERC').OR.(PAR2.EQ.'ERC')) ERCFLAG2=.TRUE.
C
      IF (THFLAG1) THEN
           THIMX=PAR1MX*PI/180.D0
           THIMN=PAR1MN*PI/180.D0
           DELTHI=DELTA1*PI/180.D0
           NTHI=NPAR1
           ID_THI=1
      END IF
      IF (THFLAG2) THEN
           THIMX=PAR2MX*PI/180.D0
           THIMN=PAR2MN*PI/180.D0
           DELTHI=DELTA2*PI/180.D0
           NTHI=NPAR2
           ID_THI=2
      END IF
      IF (FIFLAG1) THEN
           FIMX=PAR1MX*PI/180.D0
           FIMN=PAR1MN*PI/180.D0
           DELFI=DELTA1*PI/180.D0
           NFI=NPAR1
           ID_FI=1
      END IF
      IF (FIFLAG2) THEN
           FIMX=PAR2MX*PI/180.D0
           FIMN=PAR2MN*PI/180.D0
           DELFI=DELTA2*PI/180.D0
           NFI=NPAR2
           ID_FI=2
      END IF
      IF (XCFLAG1) THEN
           XCMX=PAR1MX
           XCMN=PAR1MN
           DELXC=DELTA1
           NXC=NPAR1
           ID_XC=1
      END IF
      IF (XCFLAG2) THEN
           XCMX=PAR2MX
           XCMN=PAR2MN
           DELXC=DELTA2
           NXC=NPAR2
           ID_XC=2
      END IF
      IF (YCFLAG1) THEN
           YCMX=PAR1MX
           YCMN=PAR1MN
           DELYC=DELTA1
           NYC=NPAR1
           ID_YC=1
      END IF
      IF (YCFLAG2) THEN
           YCMX=PAR2MX
           YCMN=PAR2MN
           DELYC=DELTA2
           NYC=NPAR2
           ID_YC=2
      END IF
```

```
      IF (ZCFLAG1) THEN
           ZCMX=PAR1MX
           ZCMN=PAR1MN
           DELZC=DELTA1
           NZC=NPAR1
           ID_ZC=1
      END IF
      IF (ZCFLAG2) THEN
           ZCMX=PAR2MX
           ZCMN=PAR2MN
           DELZC=DELTA2
           NZC=NPAR2
           ID_ZC=2
      END IF
      IF (ERCFLAG1) THEN
           ERCMX=PAR1MX
           ERCMN=PAR1MN
           DELERC=DELTA1
           NERC=NPAR1
           ID_ERC=1
      END IF
      IF (ERCFLAG2) THEN
           ERCMX=PAR2MX
           ERCMN=PAR2MN
           DELERC=DELTA2
           NERC=NPAR2
           ID_ERC=2
      END IF
C
      WRITE (1,100) PAR1,PAR2
 100  FORMAT(5X,A10,5X,A10,5X,'Rel.Refl.Coefficient-Real',5X,
     *'Rel.Refl.Coefficient-Imaginary'//)
C
      SLENG=(NS1+1)*DLX
      IF (NXC.EQ.-1) THEN
          XCMN=XC
          NXC=1
      END IF
      DO 3 IXC=1,NXC
          XC=XCMN+FLOAT(IXC-1)*DELXC
          X0=(XC-SLENG)/2.D0
          IF(ID_XC.EQ.1) WPAR1=XC
          IF(ID_XC.EQ.2) WPAR2=XC
          IF (NYC.EQ.-1) THEN
              YCMN=YC
              NYC=1
          END IF
          DO 4 IYC=1,NYC
               YC=YCMN+FLOAT(IYC-1)*DELYC
               Y0=YC/2.D0
               IF(ID_YC.EQ.1) WPAR1=YC
               IF(ID_YC.EQ.2) WPAR2=YC
               IF (NZC.EQ.-1) THEN
                   ZCMN=ZC
                   NZC=1
               END IF
               DO 5 IZC=1,NZC
                    ZC=ZCMN+FLOAT(IZC-1)*DELZC
                    IF(ID_ZC.EQ.1) WPAR1=ZC
                    IF(ID_ZC.EQ.2) WPAR2=ZC
                    IF (NERC.EQ.-1) THEN
                        ERCMN=ERC
                        NERC=1
                    END IF
                    DO 6 IERC=1,NERC
                         ERC=ERCMN+FLOAT(IERC-1)*DELERC
```

```
                        IF(ID_ERC.EQ.1) WPAR1=ERC
                        IF(ID_ERC.EQ.2) WPAR2=ERC
                        CALL CAV_INV(NOR)
C
                        PRINT*, 'Subroutine CAV_INV run'
C
                        IF (NTHI.EQ.-1) THEN
                            THIMN=THI
                            NTHI=1
                        END IF
                        DO 7 ITHI=1,NTHI
                             THI=THIMN+FLOAT(ITHI-1)*DELTHI
                             IF(ID_THI.EQ.1) WPAR1=THI*180.D0/PI
                             IF(ID_THI.EQ.2) WPAR2=THI*180.D0/PI
                             IF (NFI.EQ.-1) THEN
                                 FINM=FI
                                 NFI=1
                             END IF
                             DO 8 IFI=1,NFI
                                  FI=FIMN+FLOAT(IFI-1)*DELFI
                                  IF(ID_FI.EQ.1) WPAR1=FI*180.D0/PI
                                  IF(ID_FI.EQ.2) WPAR2=FI*180.D0/PI
                                  CALL MULT_EXC(NOR,R0)

                                  PRINT *,'Subroutine MULT_EXC run'

                                  CALL REFLEC(WREAL,R0,REFL)

                                  PRINT *,'Subroutine REFLEC run'

                                  R_RATIO=REFL/R0

                                  WRITE (1,10) WPAR1,WPAR2,R_RATIO
 10                               FORMAT(5X,E14.7,5X,E14.7,5X,(E14.7,
     *                            1X,E14.7),5X,(E14.7,1X,E14.7))
  8   CONTINUE
  7   CONTINUE
  6   CONTINUE
  5   CONTINUE
  4   CONTINUE
  3   CONTINUE

      WRITE (6,9000) CPU_SECONDS
 9000 FORMAT(1X,'CPU TIME=',I10,'SECS')
      STOP
      END
.............................................................
      The name of this subroutine is        DATA
      and gives all the data used by the main program and the other
      subroutines.
..............................................................
      SUBROUTINE DATA
      IMPLICIT REAL*8 (A-H,O-Z)

      COMMON/DAT/ER,H,T,DLX,XC,YC,ZC,ERC,RMC,X0,Y0,A,TPI,TPI2,PI,W,E1,
     *E2,EER,AK0,AK,AKK,FA,OFFSET(7),WDELTA,OFFLIM,ERROR,THI,FI,
     *NS1,NS2,NSS2,NOFF

      COMMON/IOFF/INS,INS1S2

      COMMON/LOSS/TLOS_E,TLOS_M

      PI=3.141592653589D0


      ---- Dielectric constant ---
```

```
      READ (5,1) ER
  1   FORMAT (///6X,D16.9)
      EER=ER
      AER=DSQRT(ER)
      WRITE (6,2) ER
  2   FORMAT(10X,'Dielectric Constant of the Substrate'/10X,E14.7/)
C
C     ---- Substrate Thickness ---
C
      READ (5,1) H
      WRITE (6,3) H
  3   FORMAT(10X,'Substrate Thickness'/10X,E14.7/)
C
C     ---- Conductor Thickness ---
C
      READ (5,1) T
      WRITE (6,4) T
  4   FORMAT(10X,'Conductor Thickness'/10X,E14.7/)
      T=T*AER
C
C     ---- Dimensions of the Cavity ----
C
      READ (5,1) XC
      READ (5,10) YC
      READ (5,10) ZC
 10   FORMAT(6X,D14.7)
      WRITE (6,5) XC,YC,ZC
  5   FORMAT(10X,'Dimensions of the Cavity'/10X,'XC=',E14.7/
     *10X,'YC=',E14.7/10X,'ZC=',E14.7/)
      XC=XC*AER
      YC=YC*AER
      ZC=ZC*AER
C
C     ---- Relative Permittivity and Permeability of the Cavity ----
C
      READ (5,1) ERC
      READ (5,10) RMC
      READ (5,10) TLOS_E
      READ (5,10) TLOS_M
      WRITE (6,6) ERC,RMC,TLOS_E,TLOS_M
  6   FORMAT(10X,'Dielectric of the Cavity'/10X,'ERC=',E14.7,2X,'RMC='
     *,E14.7/10X,'TLOS_E=',E14.7,2X,'TLOS_M=',E14.7)

      ---- Slot width ----

      READ (5,1) W
      WRITE (6,7) W
  7   FORMAT(10X,'Slot Width'/10X,E14.7/)
      W=W*AER

      ---- Position of the Center of the Slot ----

      READ (5,1) X0
      READ (5,10) Y0
      WRITE (6,8) X0,Y0
  8   FORMAT(10X,'Position of the Center of the Slot'/10X,'X0=',
     *E14.7/10X,'Y0=',E14.7/)
      X0=X0*AER
      Y0=Y0*AER


      ---- Subsection Length ----

      READ (5,1) DLX
      WRITE (6,9) DLX
  9   FORMAT(10X,'Subsection Length'/10X,E14.7/)
```

```
C
C     ---- Lower Limit of the Tail Contribution ----
C
      READ (5,1) A
      WRITE (6,11) A
   11 FORMAT(10X,'Lower Limit of Tail Contribution'/10X,E14.7/)
C
C     ---- Number of Points on Each Slot ----
C
      READ (5,20) NS1
      READ (5,21) NS2
   20 FORMAT(///6X,I4)
   21 FORMAT(6X,I4)
      WRITE (6,12) NS1,NS2
   12 FORMAT (10X,'Number of Points on Each Slot'/10X,'NS1=',I4/
     *10X,'NS2=',I4)
C
C     ---- Number of Offsets ----
C
      READ (5,20) NOFF
      WRITE (6,15) NOFF
   15 FORMAT(10X,'Number of Offsets'/10X,I4/)
C
C     ---- Offset Between the Two Slots ----
C
      READ (5,1) OFFSET(1)
      WRITE (6,13) OFFSET(1)
   13 FORMAT (10X,'Offset Between the Two Slots'/10X,E14.7/)
      OFFSET(1)=OFFSET(1)*AER
      DO 16 I=2,NOFF
            READ (5,10) OFFSET(I)
            WRITE (6,17) OFFSET(I)
   17       FORMAT(10X,E14.7/)
            OFFSET(I)=OFFSET(I)*AER
   16 CONTINUE
C
C     ---- Order of Offsets ----
C
      INS=1
      INS1S2=2
C
C     ---- Longitudinal Displacement Between the Two Slots ----
C
      READ (5,20) NSS2
      WRITE (6,14) NSS2
   14 FORMAT(10X,'Longitudinal Displacement Between the Two Slots'/
     *10X,I4/)
C
C     ---- Theta angle ----
C
      READ (5,1) THI
      WRITE (6,23) THI
   23 FORMAT(10X,'Theta Angle'/10X,E14.7/)
C
C     ---- Fi Angle ----
C
      READ (5,1)FI
      WRITE (6,22) FI
   22 FORMAT(10X,'Fi Angle'/10X,E14.7/)
      RETURN
      END
```

```
      COMMON/IOFF/INS,INS1S2
C
      CI=(0.0,1.0)
      NCON=0
      X=AUP-ALOW
      Y=AUP+ALOW
      AK02=AK0*AK0
      AK2=AK*AK
      ER1=1.D0-ER
      NPOL=N
      IF (IFIRST.EQ.2) NPOL=1
      IF (IAD.GT.2) GO TO 1
         ALI=0.5D0*(TI*X+Y)
         GCONX=AI*X*0.5D0
         FCONX=GCONX
         GCONZ=GCONX*ER1
         FCONZ=FCONX
         AIMA=1.D0
         CALL GREI(ALI,0.D0,0.D0,IAD,0.D0)
         GO TO 10
 1    IF (IAD.NE.3) GO TO 2
         ALI=0.5D0*(TI*X+Y)
         XTM=POLTM(NPOL)
         TMTM=(2.D0*XTM-Y)/X
         GCONX=AI/(TI-TMTM)
         GCONZ=GCONX*ER1
         FCONX=GCONX
         FCONZ=FCONX
         AIMA=0.D0
         CALL GREI(ALI,XTM,0.D0,IAD,0.D0)
         GO TO 10
 2    IF (IAD.NE.4) GO TO 3
         ALI=POLTM(N)
         TM=(2.D0*ALI-Y)/X
         GCONX=-AI/(TI-TM)
         GCONZ=GCONX*ER1
         FCONX=0.D0
         FCONZ=0.D0
         AIMA=0.D0
         RX=VXXM(N)
         RZ=VZXM(N)
         GO TO 10
 3    IF (IFIRST.EQ.2) GO TO 5
      IF (IAD.NE.5) GO TO 4
         ALI=0.5D0*(TI*X+Y)
         XTE=POLTE(N)
         TMTE=(2.D0*XTE-Y)/X
         GCONX=AI*X*0.5D0
         GCONZ=AI*ER1/(TI-TMTE)
         FCONX=GCONX
         FCONZ=FCONX
         AIMA=0.D0
         CALL GREI(ALI,0.D0,XTE,IAD,TMTE)
         GO TO 10
 4    IF (IAD.NE.6) GO TO 5
         NCON=6
         ALI=POLTE(N)
         TM=(2.D0*ALI-Y)/X
         GCONX=0.D0
         GCONZ=-AI*ER1/(TI-TM)
         FCONX=0.D0
         FCONZ=0.D0
         AIMA=0.D0
         RZ=VZXE(N)
         GO TO 10
 5    IF (IAD.NE.7) GO TO 6
```

```
        ALI=0.5D0*(TI*X+Y)
        GCONX=AI*X*0.5D0
        GCONZ=GCONX*ER1
        FCONX=GCONX
        FCONZ=FCONX
        AIMA=0.D0
        CALL GREI(ALI,0.D0,0.D0,IAD,0.D0)
        GO TO 10
 6      NCON=8
        ALI=POLTM(N)
        TM=(2.D0*ALI-Y)/X
        FCONX=0.D0
        FCONZ=0.D0
        AIMA=0.D0
        RX=VXXM(N)
        RZ=VZXM(N)
        GO TO 28
C
 10   CONTINUE
        GXXR=GCONX*RX-FCONX*FRX
        GXXX=AIMA*GCONX*XX
        GZXR=GCONZ*RZ-FCONZ*FRZ
        GZXX=AIMA*GCONZ*XZ
 27   CONTINUE
        VARX=AK2*GZXR
        VARZ=AKK*(GXXR-GZXR)
        GXXR=VARX
        GZXR=VARZ
        VARX=AK2*GZXX
        VARZ=AKK*(GXXX-GZXX)
        GXXX=VARX
        GZXX=VARZ
      PLI=ALI
C
      CALL ADONIS
      DO 13 K=1,NS
            S1=REAL(GXXR*SSJ0(K,INS)+GZXR*SAJ0(K,INS))
            S2=REAL(GXXX*SSJ0(K,INS)+GZXX*SAJ0(K,INS))
            ZS(K)=ZS(K)+S1-CI*S2
 13   CONTINUE
      DO 20 K=1,NS1S2
            S1=REAL(GXXR*SSJ0(K,INS1S2)+GZXR*SAJ0(K,INS1S2))
            S2=REAL(GXXX*SSJ0(K,INS1S2)+GZXX*SAJ0(K,INS1S2))
            ZS1S2(K)=ZS1S2(K)+S1-CI*S2
 20   CONTINUE
 28   IF (NCON.EQ.0) GO TO 24
         GCONX1=0.0
         GCONX2=0.0
         GCONZ1=ER1*DLOG((1.D0-TM)/(1.D0+TM))
         GCONZ2=ER1*PI
         IF (NCON.EQ.6) GO TO 29
             GCONX1=GCONZ1/ER1
             GCONX2=GCONZ2/ER1
 29      CONTINUE
             GXXR=GCONX1*RX
             GXXX=GCONX2*RX
             GZXR=GCONZ1*RZ
             GZXX=GCONZ2*RZ
 25      CONTINUE
         NCON=0
         GO TO 27
 24      CONTINUE
         RETURN
         END
.....................................................................
      This subroutine evaluates the integrand of the green's
```

```
  4  CONTINUE
     AM(NMAX+2)=AK
     DM(NMAX+2)=AK
     IF (IFIRST.EQ.1) GO TO 5
         DM(NMAX+1)=AM(NMAX+2)
         DO 6 I=1,NMAX
              DM(NMAX-I+1)=AM(NMAX-I+2)
  6      CONTINUE
         GO TO 7
  5      AM(NMAX+1)=DM(NMAX+2)
         DO 8 I=1,NMAX
              AM(NMAX-I+1)=DM(NMAX-I+2)
  8      CONTINUE
         GO TO 7
C
  3      DELTA=(AK-AK0)/FLOAT(NK0K)
         AM(1)=AK0
         DO 9 I=1,NK0K
              AM(I+1)=DELTA*FLOAT(NK0K)+AK0
  9      CONTINUE
  7      CONTINUE
C-----------------------------------------------+
C     Step 4 : evaluation of vectors VZXE       |
C-----------------------------------------------+
     IF (IFIRST.EQ.2) GO TO 10
         DO 11 I=1,NTE
               ARG=POLTE(I)
               VZXE(I)=HZXE(ARG)
 11      CONTINUE
 10  CONTINUE
C-----------------------------------------------+
C     Step 5 : evaluation of vector VXXM,VZXM   |
C-----------------------------------------------+
     DO 12 I=1,NTM
           ARG=POLTM(I)
           VXXM(I)=GXXM(ARG)
           VZXM(I)=GZXM(ARG)
 12  CONTINUE
     RETURN
     END
:..............................................................
:..............................................................
:    This subroutine evaluates the values of the integrand of
:    the Green's function at different points in the interval
:    [0,A]. Then it evaluetes the space integrals of the Bessel
:    function at the same points and multiply these values with
:    the corresponding values of the Green's function.
:    Finally , it multiplies these products with known coeffic.
     and it adds them up. This way, the moments'-method
     space integrals of the first part of the Green's function are
     evaluated and are stored in the complex vectors ZS,ZS1S2
 ............................................................
 ............................................................
     SUBROUTINE GREEN
     IMPLICIT REAL*8 (A-H,O-Z)
     COMPLEX ZS,ZS1S2,CI

     COMMON/COMP/ZS(50),ZS1S2(350),NS,NS1S2

     COMMON/MAT/PLI,AI,TI,V(3),IY

     COMMON/PUT/SSJ0(250,7),SAJ0(250,7),YSIN,YCOS

     COMMON/ADON/DIST(250,7,10),RCOE(20,250,7,10),AX,SERS(5),SERA(5),
    *DARG(10,4),S(10,2),WREAL,NSER,NMAX(7)
```

```
      COMMON/DAT/ER,H,T,DLX,XC,YC,ZC,ERC,RMC,X0,Y0,A,TPI,TPI2,PI,W,E1,
     *E2,EER,AK0,AK,AKK,FA,OFFSET(7),WDELTA,OFFLIM,ERROR,THI,FI,
     *NS1,NS2,NSS2,NOFF
C
      COMMON/DATT/COAL(20),POINT(20),CN(51),BM(51),POLTM(20),
     *POLTE(20),AM(41),DM(41),POLES(40),VXXM(20),VZXM(20),VZXE(20),
     *BPOINT(10),BCOAL(10),MPOINT,NPOINT,NK0,MA,NTM,NTE,NK0K,IFIRST
C
      COMMON/COEF/RX,XX,RZ,XZ,FRX,FRZ,F1X,F1Z
C
      COMMON/IOFF/INS,INS1S2
C
C-----------------------------------------------+
C     Evaluation of the coefficients for the    |
C            FF's functions                     |
C-----------------------------------------------+
      F1X=1.D0
      F1Z=(1.D0-ER)/((1.D0+ER)*(1.D0+E2)*(1.D0+0.5D0*E1))
C
      CALL ARIS
C
      DO 1 I=1,NPOINT
            IY=I
            AI=COAL(I)
            TI=POINT(I)
C
C     evaluation of intervals 1 and 2
C
            IAD=1
            DO 2 N=1,NK0
                 AUP=CN(N+1)
                 ALOW=CN(N)
                 CALL FUNCT(IAD,AUP,ALOW,N)
   2        CONTINUE
C
C     evaluation of intervals 3 and 4
C
            NTTM=NTM
            IF (IFIRST.EQ.2) NTTM=NK0K
            DO 3 IAD=3,4
                 IFD=0
                 DO 4 N=1,NTTM
                      IFD=IFD+1
                      AUP=AM(IFD+1)
                      ALOW=AM(IFD)
                      CALL FUNCT(IAD,AUP,ALOW,N)
                      IFD=IFD+1
   4             CONTINUE
   3        CONTINUE
            IF (IFIRST.EQ.2) GO TO 9
C
C     evaluation of the intervals 5 and 6,9,11
C
            DO 5 IAD=5,6
                 IFD=0
                 DO 6 N=1,NTE
                      IFD=IFD+1
                      AUP=DM(IFD+1)
                      ALOW=DM(IFD)
                      CALL FUNCT(IAD,AUP,ALOW,N)
                      IFD=IFD+1
   6             CONTINUE
   5        CONTINUE
   9        CONTINUE

      evaluation of the interval 7
```

```
C
          IAD=7
          DO 7 N=1,MA
               AUP=BM(N+1)
               ALOW=BM(N)
               CALL FUNCT(IAD,AUP,ALOW,N)
 7        CONTINUE
 1    CONTINUE
C
C     evaluation of the intervals 8,10
C
      IAD=8
      IFD=0
      DO 8 N=1,NTM
           IFD=IFD+1
           AUP=AM(IFD+1)
           ALOW=AM(IFD)
           CALL FUNCT(IAD,AUP,ALOW,N)
           IFD=IFD+1
  8   CONTINUE
      RETURN
      END
C.................................................................
C             Functions :  GXXM,GZXM,HZXE
C
C     These functions evaluate the residues from the different poles
C.................................................................
      FUNCTION GXXM(X)
      IMPLICIT REAL*8 (A-H,O-Z)
C
      COMMON/DAT/ER,H,T,DLX,XC,YC,ZC,ERC,RMC,X0,Y0,A,TPI,TPI2,PI,W,E1,
     *E2,EER,AK0,AK,AKK,FA,OFFSET(7),WDELTA,OFFLIM,ERROR,THI,FI,
     *NS1,NS2,NSS2,NOFF
C
      X2=X*X
      AK02=AK0*AK0
      AK2=AK*AK
      RM=DSQRT(AK2-X2)
      RM0=DSQRT(X2-AK02)
      RMH=RM*H
      RM0H=RM0*H
      RMT=RM*(-H+T)
      SXN=RM*DCOS(RMT)-ER*RM0*DSIN(RMT)
      SXD=(ER+RM0H)*(RM/RM0)*DCOS(RMH)+(1.D0+ER*RM0H)*DSIN(RMH)
      GXXM=SXN/SXD
      RETURN
      END

 .................................................................

      FUNCTION GZXM(X)
      IMPLICIT REAL*8 (A-H,O-Z)

      COMMON/DAT/ER,H,T,DLX,XC,YC,ZC,ERC,RMC,X0,Y0,A,TPI,TPI2,PI,W,E1,
     *E2,EER,AK0,AK,AKK,FA,OFFSET(7),WDELTA,OFFLIM,ERROR,THI,FI,
     *NS1,NS2,NSS2,NOFF

      X2=X*X
      AK02=AK0*AK0
      AK2=AK*AK
      RM=DSQRT(AK2-X2)
      RM0=DSQRT(X2-AK02)
      RMH=RM*H
      RM0H=RM0*H
      RMT=RM*T
      CST=DCOS(RMT)
```

```
      CSH=DCOS(RMH)
      SNH=DSIN(RMH)
      SXN=RM*CST
      SXD=(RM*CSH+RM0*SNH)*((ER+RM0H)*CSH/RM0+(1.D0+ER*RM0H)*SNH/RM)
      GZXM=SXN/SXD
      RETURN
      END
C
C
C.............................................................
C
      FUNCTION HZXE(X)
      IMPLICIT REAL*8 (A-H,O-Z)
C
      COMMON/DAT/ER,H,T,DLX,XC,YC,ZC,ERC,RMC,X0,Y0,A,TPI,TPI2,PI,W,E1,
     *E2,EER,AK0,AK,AKK,FA,OFFSET(7),WDELTA,OFFLIM,ERROR,THI,FI,
     *NS1,NS2,NSS2,NOFF
C
      X2=X*X
      AK02=AK0*AK0
      AK2=AK*AK
      RM=DSQRT(AK2-X2)
      RM0=DSQRT(X2-AK02)
      RMH=RM*H
      RMT=RM*T
      RM0H=RM0*H
      CSH=DCOS(RMH)
      CST=DCOS(RMT)
      SNH=DSIN(RMH)
      SXN=RM*CST
      SXD=(ER*RM0*CSH-RM*SNH)*(1.D0+RM0H)*(SNH/RM0-CSH/RM)
      HZXE=SXN/SXD
      RETURN
      END
C.............................................................
C     1) This subroutine evaluates the integrand of the Green's
C        function at different points (subroutine Grei).
C     2) It evaluates the space integrals comming from the
C        application of moments' method (subroutine adonis)
C     3) Multiply these two valueswith appropriate weighting
C        coefficients and it adds them upZXX2*SAJ0(K)
C.............................................................
      SUBROUTINE FUNCT(IAD,AUP,ALOW,N)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*4 S1,S2
      COMPLEX ZS,ZS1S2,CI
C
      COMMON/COMP/ZS(50),ZS1S2(350),NS,NS1S2
C
      COMMON/MAT/PLI,AI,TI,V(3),IY
C
      COMMON/PUT/SSJ0(250,7),SAJ0(250,7),YSIN,YCOS
C
      COMMON/ADON/DIST(250,7,10),RCOE(20,250,7,10),AX,SERS(5),
     *SERA(5),DARG(10,4),S(10,2),WREAL,NSER,NMAX(7)
C
      COMMON/DAT/ER,H,T,DLX,XC,YC,ZC,ERC,RMC,X0,Y0,A,TPI,TPI2,PI,W,E1,
     *E2,EER,AK0,AK,AKK,FA,OFFSET(7),WDELTA,OFFLIM,ERROR,THI,FI,
     *NS1,NS2,NSS2,NOFF
C
      COMMON/DATT/COAL(20),POINT(20),CN(51),BM(51),POLTM(20),
     *POLTE(20),AM(41),DM(41),POLES(40),VXXM(20),VZXM(20),VZXE(20),
     *BPOINT(10),BCOAL(10),MPOINT,NPOINT,NK0,MA,NTM,NTE,NK0K,IFIRST
C
      COMMON/COEF/RX,XX,RZ,XZ,FRX,FRZ,F1X,F1Z
C
```

```
C     function at different points
C.............................................................
      SUBROUTINE GREI(X,XFM,XFE,IAD,TM)
      IMPLICIT REAL*8 (A-H,O-Z)
C
      COMMON/DAT/ER,H,T,DLX,XC,YC,ZC,ERC,RMC,X0,Y0,A,TPI,TPI2,PI,W,E1,
     *E2,EER,AK0,AK,AKK,FA,OFFSET(7),WDELTA,OFFLIM,ERROR,THI,FI,
     *NS1,NS2,NSS2,NOFF
C
      COMMON/COEF/RX,XX,RZ,XZ,FRX,FRZ,F1X,F1Z
C
      X2=X*X
      AK2=AK*AK
      AK02=AK0*AK0
      RM=DSQRT(DABS(AK2-X2))
      RM0=DSQRT(DABS(X2-AK02))
      RMH=RM*H
      RMT=RM*T
      RMHT=RM*(-H+T)
C
      CSH=DCOS(RMH)
      SNH=DSIN(RMH)
      CST=DCOS(RMT)
      SNT=DSIN(RMT)
      CSHT=DCOS(RMHT)
      SNHT=DSIN(RMHT)
C
      RM2=RM*RM
      RM02=RM0*RM0
      CSH2=CSH*CSH
      ERM0=ER*RM0
      ERM02=ERM0*ERM0
C
      EXX=DEXP(-X*T/FA)/FA
      EXZ=DEXP(-X*(2.D0*H-T)/FA)/FA
      IF (IAD.NE.7) GO TO 100
          EX=DEXP(RMH)
          TANH=(EX-1.D0/EX)/(EX+1.D0/EX)
          CSHH=(EX+1.D0/EX)/2.D0
          EX=DEXP(RMT)
          CSHT=0.5D0*(EX+1.D0/EX)
          SNHT=0.5D0*(EX-1.D0/EX)
          TANT=SNHT/CSHT
          EX=DEXP(RMHT)
          CSHHT=0.5D0*(EX+1.D0/EX)
          SNHHT=0.5D0*(EX-1.D0/EX)
          TANHT=SNHHT/CSHHT
C
  100 IF (IAD.NE.1) GO TO 1
          DEN=RM2+(ERM02-RM2)*CSH2
          RNOM=-RM2*SNT+(RM2-ERM02)*CSH*SNHT
          XNOM=ER*RM*RM0*CST
          C1=X/RM

          RX=C1*RNOM/DEN
          XX=C1*XNOM/DEN
          FRX=F1X*EXX

          DEN=DEN*(RM02+AK02*(ER-1.D0)*CSH2)
          RNOM=CST*(RM2+ER*RM02)*CSH*SNH
          XNOM=CST*RM*RM0*(1.D0-(1.D0+ER)*CSH2)
          C1=X*RM
          RZ=-C1*RNOM/DEN
          XZ=C1*XNOM/DEN
          FRZ=F1Z*EXZ
          RETURN
```

```
  1    IF (IAD.NE.3) GO TO 2
           C1=X-XFM
           IF (DABS(AK-X).LT.1.D-6) GO TO 10
                DEN=ERM0*CSH-RM*SNH
                RNOM=(RM*CSHT-ERM0*SNHT)
                C2=X/RM
                RX=C1*C2*RNOM/DEN
:
                DEN=DEN*(RM*CSH+RM0*SNH)
                RNOM=CST
                C3=X*RM
                RZ=C1*C3*RNOM/DEN
:
                FRX=F1X*EXX
                FRZ=F1Z*EXZ
                RETURN
:
   10      RNOM=1.D0-ERM0*(-H+T)
           RX=C1*X*RNOM/ERM0
           FRX=F1X*EXX
:
           RZ=X*C1/(ERM0*(1.D0+RM0*H))
           FRZ=F1Z*EXZ
           RETURN
  2    IF (IAD.NE.5) GO TO 4
           C1=X-XFE
           IF (DABS(AK-X).LT.1.D-6) GO TO 13
                RNOM=RM*CSHT-ERM0*SNHT
                DEN=ERM0*CSH-RM*SNH
                RX=(X/RM)*RNOM/DEN
                FRX=F1X*EXX

                RNOM=RM*CST
                DEN=DEN*(RM*CSH+RM0*SNH)
                RZ=X*C1*RNOM/DEN
                FRZ=F1Z*EXZ
                RETURN
   13      RX=X*(1.D0-ERM0*(-H+T))/ERM0
           FRX=F1X*EXX

           RZ=X*C1/(ERM0*(1.D0+RM0*H))
           FRZ=F1Z*EXZ
           RETURN
  4    IF (IAD.NE.7) GO TO 6
           IF (DABS(X-AK).LT.1.D-6) GO TO 15
                DEN=ERM0+RM*TANH
                RNOM=(RM-ERM0*TANHT)*(CSHT-TANH*SNHT)
                RX=(X/RM)*RNOM/DEN
                FRX=F1X*EXX

                RNOM=X*(RM*CSHT)/(CSHH*CSHH)
                DEN=DEN*(RM+RM0*TANH)
                RZ=RNOM/DEN
                FRZ=F1Z*EXZ
                RETURN
 15    RX=X*(1.D0-ERM0*(-H+T))/ERM0
       FRX=F1X*EXX
       RZ=(X/ERM0)/(1.D0+RM0*H)
       FRZ=F1Z*EXZ
  6    CONTINUE
       RETURN
       END
..................................................................
                                   ARIS
..................................................................
       SUBROUTINE ARIS
```

```
      IMPLICIT REAL*8 (A-H,O-Z)
C
      COMMON/DAT/ER,H,T,DLX,XC,YC,ZC,ERC,RMC,X0,Y0,A,TPI,TPI2,PI,W,E1,
     *E2,EER,AK0,AK,AKK,FA,OFFSET(7),WDELTA,OFFLIM,ERROR,THI,FFI,
     *NS1,NS2,NSS2,NOFF
C
      COMMON/DATT/COAL(20),POINT(20),CN(51),BM(51),POLTM(20),
     *POLTE(20),AM(41),DM(41),POLES(40),VXXM(20),VZXM(20),VZXE(20),
     *BPOINT(10),BCOAL(10),MPOINT,NPOINT,NK0,MA,NTM,NTE,NK0K,IFIRST
C
      COMMON/ADON/DIST(250,7,10),RCOE(20,250,7,10),AX,SERS(5),
     *SERA(5),DARG(10,4),S(10,2),WREAL,NSER,NMAX(7)
C
      COMMON/COEF/RX,XX,RZ,XZ,FRX,FRZ,F1X,F1Z
C
C     +-------------------------------------------+
C     | Formation of the matrices:  DIST,         |
C     |     DARG,RCOE                             |
C     |                                           |
C     +-------------------------------------------+
      W2=W/2.D0
      U=WREAL/W
      THMIN=DATAN(DSQRT(1.D0/(U*U)-1.D0))
      THMAX=PI-THMIN
      AX=(THMAX-THMIN)/2.D0
      BX=(THMAX+THMIN)/2.D0
      X=PI/4.D0
      DO 1 J=1,NOFF
           MAX=NMAX(J)
           LPOINT=MPOINT
           IF (OFFSET(J).LE.OFFLIM) LPOINT=NPOINT
           DO 2 I=1,LPOINT
                POIN=BPOINT(I)
                IF (OFFSET(J).LE.OFFLIM) POIN=POINT(I)
                FI=X*(POIN+1.D0)
                THETA=AX*POIN+BX
                AS=DSIN(FI)
                AC=DCOS(FI)
                DARG(I,1)=W2*AC
                DARG(I,2)=AC
                DARG(I,3)=AS
                DARG(I,4)=X
                DO 3 N=1,MAX
                     AXN=FLOAT(N-2)*DLX
                     IF (OFFSET(J).GT.OFFLIM) GO TO 4
                        DIST(N,J,I)=AXN*AS
                        GO TO 5
  4                  AXN2=AXN*AXN
                     BXN=OFFSET(J)-W*DCOS(THETA)/2.D0
                     BXN2=BXN*BXN
                     DIST(N,J,I)=DSQRT(AXN2+BXN2)
                     SIG=DIST(N,J,I)
                     SIG2=SIG*SIG
                     SIG3=SIG2*SIG
                DSIG=DABS(AXN)/SIG
                DSIG2=BXN2/SIG3
                DSIG3=-3.D0*DSIG*DSIG2/SIG
                DSIG4=-3.D0*DSIG2*(DSIG2-4.D0*DSIG**2/SIG)/SIG
                DSIG5=-3.D0*(-15.D0*DSIG2**2*DSIG+(20.D0/SIG)*
     *                 DSIG2*DSIG**3)/SIG2
                DSIG6=-3.D0*(-15.D0*DSIG2**3+(180.D0/SIG)*DSIG2
     *                 **2*DSIG**2-(120.D0/SIG2)*DSIG2*DSIG**4)/
     *                 SIG2
                DSIG7=-3.D0*(525.D0*DSIG2**3*DSIG-(2100.D0/SIG)*
     *                 DSIG2**2*DSIG**3+(840.D0/SIG2)*DSIG2*DSIG
     *                 **5)/SIG3
```

```
              DSIG8=-3.D0*(525.D0*DSIG2**4-(12600.D0/SIG)*DSIG2
     *              **3*DSIG**2+(25200.D0/SIG2)*DSIG2**2*DSIG**4
     *              -(6720.D0/SIG3)*DSIG2*DSIG**6)/SIG3
C
C           Evaluation of the coefficients Gij
C
              G21=DSIG2
              G22=DSIG**2
C----------------
              G41=DSIG4
              G42=4.D0*DSIG3*DSIG+3.D0*DSIG2**2
              G43=6.D0*DSIG2*DSIG**2
              G44=DSIG**4
C----------------
              G61=DSIG6
              G62=6.D0*DSIG5*DSIG+15.D0*DSIG4*DSIG2+10.D0*DSIG3**2
              G63=15.D0*DSIG4*DSIG**2+60.D0*DSIG3*DSIG2*DSIG+15.D0
     *           *DSIG2**3
              G64=20.D0*DSIG3*DSIG**3+45.D0*DSIG2**2*DSIG**2
              G65=15.D0*DSIG2*DSIG**4
              G66=DSIG**6
C----------------
              G81=DSIG8
              G82=8.D0*DSIG7*DSIG+28.D0*DSIG6*DSIG2+56.D0*DSIG5
     *           *DSIG3+35.D0*DSIG4**2
              G83=28.D0*DSIG6*DSIG**2+168.D0*DSIG5*DSIG2*DSIG+
     *           280.D0*DSIG4*DSIG3*DSIG+210.D0*DSIG4*DSIG2**2+
     *           280.D0*DSIG3**2*DSIG2
              G84=56.D0*DSIG5*DSIG**3+420.D0*DSIG4*DSIG2*DSIG**2
     *           +280.D0*DSIG3**2*DSIG**2+840.D0*DSIG3*DSIG2**2
     *           *DSIG+105.D0*DSIG2**4
              G85=70.D0*DSIG4*DSIG**4+560.D0*DSIG3*DSIG2*DSIG**3
     *           +420.D0*DSIG2**3*DSIG**2
              G86=56.D0*DSIG3*DSIG**5+210.D0*DSIG2**2*DSIG**4
              G87=28.D0*DSIG2*DSIG**6
              G88=DSIG**8
C----------------
              RCOE(2,N,J,I)=-0.5D0*(G22+SIG*G21)
              RCOE(1,N,J,I)=0.5D0*(G22-SIG*G21)
C----------------
              SX=0.5D0*SIG*(G42-SIG*G41)
              S30=-0.5D0*SIG*(G42+SIG*G41)
              S31=0.25D0*(SX+3.D0*G43)
              S33=0.25D0*(SX-G43)
              RCOE(3,N,J,I)=0.5D0*(SIG*S33/3.D0+G44/4.D0)
              RCOE(4,N,J,I)=0.5D0*(SIG*S31+SIG*S33/3.D0-G44)
              RCOE(5,N,J,I)=0.5D0*(SIG*S31+3.D0*G44/4.D0)
              RCOE(6,N,J,I)=SIG*S30
C----------------
              SX=SIG*S33/3.D0+G64/4.D0
              ST=SIG*S31+SIG*S33/3.D0-G64
              S5M3=SIG2*S30
              S5M1=0.5D0*SIG*(SIG*S31+3.D0*G64/4.D0)
              S51=0.25D0*(0.5D0*SIG*ST-5.D0*G65/2.D0)
              S53=0.25D0*(0.5D0*SIG*ST+0.25D0*SIG*SX+0.5D0*G65/
     *           4.D0)
              S55=0.125D0*(0.5D0*SIG*SX-0.5*G65)
              RCOE(7,N,J,I)=0.5D0*(SIG*S55/5.D0+G66/16.D0)
              RCOE(8,N,J,I)=0.5D0*(SIG*S53/3.D0+SIG*S55/5.D0-
     *                       6.D0*G66/16.D0)
              RCOE(9,N,J,I)=0.5D0*(SIG*S51+SIG*S53/3.D0+15.D0*
     *                       G66/16.D0)
              RCOE(10,N,J,I)=0.5D0*(SIG*S51-10.D0*G66/16.D0)
              RCOE(11,N,J,I)=SIG*S5M1
              RCOE(12,N,J,I)=SIG*S5M3
C----------------
```

```
               S7M5=SIG2*S5M3
               S7M3=SIG2*S5M1
               S7M1=0.5D0*SIG*(SIG*S51-10.D0*G86/16.D0)
               S71=0.5D0*(0.25D0*SIG*(SIG*S51+SIG*S53/3.D0+
     *             15.D0*G86/16.D0)+35.D0*G87/32.D0)
               S73=0.5D0*(0.25D0*SIG*(SIG*S51+SIG*S53/3.D0+15.D0
     *             *G86/16.D0)+0.125D0*SIG*(SIG*S53/3.D0+SIG*
     *             S55/5.D0-6.D0*G86/16.D0)-21.D0*G87/32.D0)
               S75=0.5D0*(0.125D0*SIG*(SIG*S53/3.D0+SIG*S55/5.D0-
     *             6.D0*G86/16.D0)+(SIG/12.D0)*(SIG*S55/5.D0+
     *             G86/16.D0)+7.D0*G87/32.D0)
               S77=0.5D0*((SIG/12.D0)*(SIG*S55/5.D0+G86/16.D0)-
     *             G87/32.D0)
               RCOE(13,N,J,I)=0.5D0*(SIG*S77/7.D0+G88/64.D0)
               RCOE(14,N,J,I)=0.5D0*(SIG*S75/5.D0+S77*SIG/7.D0
     *                        -8.D0*G88/64.D0)
               RCOE(15,N,J,I)=0.5D0*(SIG*S73/3.D0+SIG*S75/5.D0
     *                        +28.D0*G88/64.D0)
               RCOE(16,N,J,I)=0.5D0*(SIG*S71+SIG*S73/3.D0-56.D0
     *                        *G88/64.D0)
               RCOE(17,N,J,I)=0.5D0*(SIG*S71+70.D0*G88/64.D0)
               RCOE(18,N,J,I)=SIG*S7M1
               RCOE(19,N,J,I)=SIG*S7M3
               RCOE(20,N,J,I)=SIG*S7M5
  5                 CONTINUE
  3            CONTINUE
  2        CONTINUE
  1    CONTINUE
:
:             Formation of the series s(dlx) . Storage in
:                 vectors SERS(5),SERA(5)
:
       U1=2.D0*THMIN/FLOAT(NSER)
       DO 6 JN=1,NSER
           S2=(2.D0*FLOAT(JN)-1.D0)
           S2=S2/(2.D0*FLOAT(NSER))
           S3=DCOS(S2*THMIN)
           S(JN,2)=S3*W/2.D0
           S(JN,1)=U1
  6    CONTINUE
       ADL=AKK*DLX
       ADL2=ADL*ADL
       ADL3=ADL2*ADL
       ADL4=ADL3*ADL
       ADL5=ADL4*ADL
       ADL6=ADL5*ADL
       YSIN=DSIN(ADL)
       YCOS=DCOS(ADL)

       SER1=(1.D0-YCOS)*2.D0/AKK

       SER2=-YSIN/3.D0+ADL*YCOS/4.D0+ADL2*YSIN/10.D0-ADL3*YCOS/36.D0
     *     -ADL4*YSIN/168.D0+ADL5*YCOS/960.D0+ADL6*YSIN/6480.D0

       SER3=YSIN/60.D0-ADL*5.D0*YCOS/360.D0-ADL2*YSIN/168.D0+ADL3
     *     *YCOS/560.D0+ADL4*YSIN/2592.D0-ADL5*YCOS/12960.D0-ADL6
     *     *YSIN/95040.D0

       SER4=-YSIN/2520.D0+ADL*YCOS/2880.D0+ADL2*YSIN/6480.D0-ADL3
     *     *YCOS/21600.D0-ADL4*YSIN/95040.D0+ADL5*YCOS/518400.D0

       SER5=YSIN/181440.D0-ADL*YCOS/201600.D0-ADL2*YSIN/443520.D0+
     *     ADL3*YCOS/1442775.9D0

       SERS(1)=SER1*SER1
       SERS(2)=DLX*2.D0*SER1*SER2
```

```
      SERS(3)=DLX*(DLX*SER2*SER2+2.D0*SER1*SER3)
      SERS(4)=DLX*(2.D0*SER1*SER4+2.D0*DLX*SER2*SER3)
      SERS(5)=DLX*(DLX*SER3*SER3+2.D0*DLX*SER2*SER4)
C
      SERA(1)=SER1
      SERA(2)=DLX*SER2
      SERA(3)=DLX*SER3
      SERA(4)=DLX*SER4
      SERA(5)=DLX*SER5
  111 CONTINUE
      RETURN
      END
C.......................................................................
C                          ADONIS
C     This subroutine evaluates the space integarls of the bessel
C         function
C
C
C....................................................................
      SUBROUTINE ADONIS
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION BJ(10,2),DERIV(9,3)
C
      COMMON/ADON/DIST(250,7,10),RCOE(20,250,7,10),AX,SERS(5),
     *SERA(5),DARG(10,4),S(10,2),WREAL,NSER,NMAX(7)
C
      COMMON/PUT/SSJ0(250,7),SAJ0(250,7),YSIN,YCOS
C
      COMMON/DAT/ER,H,T,DLX,XC,YC,ZC,ERC,RMC,X0,Y0,A,TPI,TPI2,PI,W,E1,
     *E2,EER,AK0,AK,AKK,FA,OFFSET(7),WDELTA,OFFLIM,ERROR,THI,FI,
     *NS1,NS2,NSS2,NOFF
C
      COMMON/DATT/COAL(20),POINT(20),CN(51),BM(51),POLTM(20),
     *POLTE(20),AM(41),DM(41),POLES(40),VXXM(20),VZXM(20),VZXE(20),
     *BPOINT(10),BCOAL(10),MPOINT,NPOINT,NK0,MA,NTM,NTE,NK0K,IFIRST
C
      COMMON/BSS/ARG(10),AARG
C
      COMMON/MAT/PLI,AI,TI,V(3),IY
C
      COMMON/COEF/RX,XX,RZ,XZ,FRX,FRZ,F1X,F1Z
C
      ARX=W*AX/2.D0
      W1=2.D0*YCOS
      PR1=PLI*DLX
      PR2=PR1*PR1
      PR4=PR2*PR2
      PR6=PR4*PR2
      PR8=PR6*PR2
      DO 1 J=1,NOFF
           MAX=NMAX(J)
           DO 2 N=1,MAX
                SSJ0(N,J)=0.D0
                SAJ0(N,J)=0.D0
  2        CONTINUE
  1   CONTINUE

      DO 11 J=1,NOFF
            LPOINT=MPOINT
            IF (OFFSET(J).GT.OFFLIM) GO TO 12
                LPOINT=NPOINT
                DO 13 I=1,NPOINT
                      ARG(I)=PLI*DARG(I,1)
  13            CONTINUE
                CALL BESS1(BJ)
  12        DO 14 I=1,LPOINT
                  DO 17 NK=1,5
```

```
                  DERIV(NK,1)=0.D0
                  DERIV(NK,2)=0.D0
 17          CONTINUE
             ASIN=ARX*BCOAL(I)
             IF (OFFSET(J).GT.OFFLIM) GO TO 15
                  ASIN=W*DARG(I,4)*COAL(I)
                  AROF=PLI*OFFSET(J)*DARG(I,2)
                  COFF=DCOS(AROF)
                  SSUM=0.D0
                  DO 16 JN=1,NSER
                       ARAF=PLI*S(JN,2)*DARG(I,2)
                       CAFF=DCOS(ARAF)
                       SSUM=SSUM+S(JN,1)*CAFF
 16               CONTINUE
 15          CONTINUE
             KMAX=NMAX(J)
             DO 18 K=1,KMAX
                  DO 20 NK=1,5
                       DERIV(NK,1)=DERIV(NK,2)
                       DERIV(NK,2)=DERIV(NK,3)
 20               CONTINUE
                  IF (OFFSET(J).GT.OFFLIM) GO TO 21
                       SIN1=DARG(I,3)
                       SIN2=SIN1*SIN1
                       COS1=DCOS(PLI*DIST(K,J,I))
                       TERM=COFF*(BJ(I,1)-SSUM/PI)*COS1
                       DERIV(1,3)=TERM
                       SIN1=SIN2
                       DERIV(2,3)=-TERM*SIN1
                       SIN1=SIN1*SIN2
                       DERIV(3,3)=TERM*SIN1
                       SIN1=SIN1*SIN2
                       DERIV(4,3)=-TERM*SIN1
                       SIN1=SIN1*SIN2
                       DERIV(5,3)=TERM*SIN1
                       GO TO 22
 21             AARG=PLI*DIST(K,J,I)
                ARG2=AARG*AARG
                ARG4=ARG2*ARG2
                ARG6=ARG4*ARG2
                CALL BESS2(BJ)
                DERIV(1,3)=BJ(1,2)
                DERIV(2,3)=RCOE(1,K,J,I)*BJ(3,2)+
     *                 RCOE(2,K,J,I)*BJ(1,2)
                DERIV(3,3)=RCOE(3,K,J,I)*BJ(5,2)+
     *                 RCOE(4,K,J,I)*BJ(3,2)+(RCOE(5,K,J,I)
     *                 +RCOE(6,K,J,I)/ARG2)*BJ(1,2)
                DERIV(4,3)=RCOE(7,K,J,I)*BJ(7,2)+
     *                 RCOE(8,K,J,I)*BJ(5,2)+RCOE(9,K,J,I)*
     *                 BJ(3,2)+(RCOE(10,K,J,I)+RCOE(11,K,
     *                 J,I)/ARG2+RCOE(12,K,J,I)/ARG4)*
     *                 BJ(1,2)
                DERIV(5,3)=RCOE(13,K,J,I)*BJ(9,2)+
     *                 RCOE(14,K,J,I)*BJ(7,2)+RCOE(15,K,J,
     *                 I)*BJ(5,2)+RCOE(16,K,J,I)*BJ(3,2)+
     *                 (RCOE(17,K,J,I)+RCOE(18,K,J,I)/ARG2
     *                 +RCOE(19,K,J,I)/ARG4+RCOE(20,K,J,I)
     *                 /ARG6)*BJ(1,2)
 22             IF (K.LT.3) GO TO 18
                SUMS=SERS(1)*DERIV(1,2)-PR2*SERS(2)*DERIV(2,2)
     *               +PR4*SERS(3)*DERIV(3,2)-PR6*SERS(4)*DERIV
     *               (4,2)+PR8*SERS(5)*DERIV(5,2)

                CH1=SERA(1)*(DERIV(1,1)+DERIV(1,3)-W1*DERIV
     *              (1,2))
                CH2=SERA(2)*(DERIV(2,1)+DERIV(2,3)-W1*DERIV
```

```
     *                       (2,2))*PR2
                        CH3=SERA(3)*(DERIV(3,1)+DERIV(3,3)-W1*DERIV
     *                       (3,2))*PR4
                        CH4=SERA(4)*(DERIV(4,1)+DERIV(4,3)-W1*DERIV
     *                       (4,2))*PR6
                        CH5=SERA(5)*(DERIV(5,1)+DERIV(5,3)-W1*DERIV
     *                       (5,2))*PR8
                        SUMA=CH1-CH2+CH3-CH4+CH5
                        KJ=K-2
                        SSJ0(KJ,J)=SSJ0(KJ,J)+ASIN*SUMS
                        SAJ0(KJ,J)=SAJ0(KJ,J)+ASIN*SUMA
CCCC
C                       IF (KJ.EQ.1)WRITE (6,665) KJ,J,SSJ0(KJ,J),
C    *                  SUMS,SAJ0(KJ,J),SUMA
C665                    FORMAT(10X,'KJ=',I4,2X,'J=',I4/10X,'SSJ0=',
C    *                  E14.7,2X,'SUMS=',E14.7/10X,'SAJ0=',E14.7,
C    *                  2X,'SUMA=',E14.7/)
CCCC
  18              CONTINUE
  14         CONTINUE
  11    CONTINUE
        RETURN
        END
C...................................................................
C                                 BESS1
C       This subroutine gives values for the zeroth order
C       Bessel functions. It is used for small offsets
C...................................................................
        SUBROUTINE BESS1(BJ)
        IMPLICIT REAL*8 (A-H,O-Z)
        DIMENSION BJ(10,2)
C
        COMMON/COEF/RX,XX,RZ,XZ,FRX,FRZ,F1X,F1Z
C
        COMMON/ADON/DIST(250,7,10),RCOE(20,250,7,10),AX,SERS(5),
       *SERA(5),DARG(10,4),S(10,2),WREAL,NSER,NMAX(7)
C
        COMMON/BSS/ARG(10),AARG
C
        COMMON/DATT/COAL(20),POINT(20),CN(51),BM(51),POLTM(20),
       *POLTE(20),AM(41),DM(41),POLES(40),VXXM(20),VZXM(20),VZXE(20),
       *BPOINT(10),BCOAL(10),MPOINT,NPOINT,NK0,MA,NTM,NTE,NK0K,IFIRST
C
        PI=3.141592653589D0
        DO 1 IJ=1,NPOINT
             X=ARG(IJ)
             IF (X.GT.0.001D0) GO TO 10
                X3=X/3.D0
                X32=X3*X3
                X34=X32*X32
                X36=X34*X32
                BJ0=1.D0-2.2499997D0*X32+1.2656208D0*X34-0.3163866D0
     *                *X36
                BJ(IJ,1)=BJ0
                GO TO 1
  10         IF (X.GT.3.D0) GO TO 12
                X3=X/3.D0
                X32=X3*X3
                X34=X32*X32
                X36=X34*X32
                X38=X36*X32
                X310=X38*X32
                X312=X310*X32
                BJ0=1.D0-2.2499997D0*X32+1.2656208D0*X34-0.3163866D0
     *               *X36+0.0444479D0*X38-0.0039444D0*X310+0.00021000
     *               D0*X312
```

```
            BJ(IJ,1)=BJ0
            GO TO 1
  12      CONTINUE
          X3=3.D0/X
          X32=X3*X3
          X33=X32*X3
          X34=X33*X3
          X35=X34*X3
          X36=X35*X3
          FJ0=0.79788456D0-0.00000077D0*X3-0.00552740D0*X32-0.0000
     *        9512D0*X33+0.00137237D0*X34-0.00072805D0*X35+0.00014
     *        476D0*X36
          TJ0=X-0.78539816D0-0.04166397D0*X3-0.00003954D0*X32+0.00
     *        262573D0*X33-0.00054125D0*X34-0.00029333D0*X35+0.000
     *        13558D0*X36
          WCON=DSQRT(1.D0/X)
          BJ(IJ,1)=WCON*FJ0*DCOS(TJ0)
  1       CONTINUE
      RETURN
      END
C..............................................................
C                                     TAIL
C     This subroutine evaluates the tail contribution
C..............................................................
      SUBROUTINE TAIL
      IMPLICIT REAL*8 (A-H,O-Z)
      COMPLEX ZS,ZS1S2
      DIMENSION MAX(8,2)
C
      COMMON/CTAIL/S1(4,205,7),D1(4,205,7),D2(4,205,7),
     *T1(4,205,7),T2(4,205,7),T3(4,205,7),T4(4,205,7)
C
      COMMON/COMP/ZS(50),ZS1S2(350),NS,NS1S2
C
      COMMON/DAT/ER,H,T,DLX,XC,YC,ZC,ERC,RMC,X0,Y0,A,TPI,TPI2,PI,W,E1,
     *E2,EER,AK0,AK,AKK,FA,OFFSET(7),WDELTA,OFFLIM,ERROR,THI,FI,
     *NS1,NS2,NSS2,NOFF
C
      COMMON/INT/XNS(40),CNS(40),XND(20,2),CND(20),XNT(40,3),
     *CNT(40),NDP,NTP,NSP
C
      COMMON/ADON/DIST(250,7,10),RCOE(20,250,7,10),AX,SERS(5),
     *SERA(5),DARG(10,4),S(10,2),WREAL,NSER,NMAX(7)
C
      COMMON/OUT/GS(50),GS1S2(350)
C
      COMMON/IOFF/INS,INS1S2
C
C     This vector contains the values of t in the integrals h0
C
      Z1=T
      Z2=2.D0*H-T

      This vector contains the values of the coefficient C in
           the integrals h0

      C1=FA

       This vector contains the number of elements of the
       matrices ZS,ZS1S2,....

      MAX(1,1)=NS
      MAX(2,1)=NS1S2

      MAX(1,2)=INS
      MAX(2,2)=INS1S2
```

```
;
;
;    This vector contains the values of the coefficient A in
;             the integrals h0
;
     AKK=TPI
     W2=W/2.D0
     THMIN=WREAL/W
     THMIN=DATAN(DSQRT(1.D0/THMIN**2-1.D0))
     THMAX=PI-THMIN
     PI2=PI/2.D0
     PI4=PI/4.D0
     DLX2=DLX/2.D0
     DLX4=DLX2*DLX2
;
     YCOS=DCOS(AKK*DLX)
     CCS=DCOS(2.D0*AKK*DLX)
     YSIN=DSIN(AKK*DLX)
     SSN=DSIN(2.D0*AKK*DLX)
;
;    +---------------------------------+
;    | Evaluation of S1,S2,S3,S4,S5,S6 |
;    |       (Single Integrals)        |
;    +---------------------------------+
;
;
     DO 201 J=1,7
            DO 202 K=1,205
                   DO 203 JK=1,4
                          S1(JK,K,J)=0.D0
                          D1(JK,K,J)=0.D0
                          D2(JK,K,J)=0.D0
                          T1(JK,K,J)=0.D0
                          T2(JK,K,J)=0.D0
                          T3(JK,K,J)=0.D0
                          T4(JK,K,J)=0.D0
 203               CONTINUE
 202        CONTINUE
 201 CONTINUE

     ZP1=Z1/C1
     ZP2=Z2/C1

     ZP12=ZP1*ZP1
     ZP22=ZP2*ZP2
     DO 1 J=1,NOFF
          KMAX=NMAX(J)+2
          IF (OFFSET(J).LT.1.D-6) THMAX=PI
          DSP=(THMAX-THMIN)/4.D0
          DDP=DSP*DLX2
          DTP=DSP*DLX4
          COEF1=(THMAX-THMIN)/2.D0
          IF (OFFSET(J).LT.1.D-6) COEF1=(PI/2.D0-THMIN)/2.D0
          COEF2=(THMAX+THMIN)/2.D0
          IF (OFFSET(J).LT.1.D-6) COEF2=(PI/2.D0+THMIN)/2.D0
          DO 10 I=1,NSP
                THI=COEF1*XNS(I)+COEF2
                C1=DCOS(THI)
                C2=W2*C1
                C2=OFFSET(J)-C2
                CW=C2*C2
                AASIN=CNS(I)*DSP
                DO 11 K=1,KMAX
                      XN=FLOAT(K-3)*DLX
                      RAD2=XN*XN+CW
                      TRAD1=DSQRT(RAD2+ZP12)
```

```
                    TRAD2=DSQRT(RAD2+ZP22)
                    S1(1,K,J)=S1(1,K,J)+DLOG(2.D0*(TRAD1+XN))*AASIN
                    S1(2,K,J)=S1(2,K,J)+DLOG(2.D0*(TRAD2+XN))*AASIN
  11            CONTINUE
  10        CONTINUE
C
C     +------------------------------------------------------------+
C     | EVALUATION OF D1,D2,D4,D5                                  |
C     +------------------------------------------------------------+
        DO 20 I=1,NDP
            THI=COEF1*XND(I,1)+COEF2
            XI=DLX2*(XND(I,2)+1.D0)
            C1=DCOS(THI)
            C2=W2*C1
            C2=OFFSET(J)-C2
            CW=C2*C2
            AASIN=CND(I)*DDP
            SV1=DSIN(AKK*(DLX-XI))
            SV2=-SV1
            SV4=DSIN(AKK*XI)
            C2=DCOS(AKK*(DLX-XI))
            DO 21 K=1,KMAX
                XNP=XI+FLOAT(K-2)*DLX
                XNM=-XI+FLOAT(K-2)*DLX
                RADP2=XNP*XNP+CW
                RADM2=XNM*XNM+CW
                TRAP1=DSQRT(RADP2+ZP12)
                TRAP2=DSQRT(RADP2+ZP22)

                TRAM1=DSQRT(RADM2+ZP12)
                TRAM2=DSQRT(RADM2+ZP22)

                XA1=AKK*XNP
                XA2=AKK*XNM
                XAP=DSIN(XA1)
                XAM=DSIN(XA2)

                SANP1=XAP*DLOG(2.D0*(TRAP1+XNP))
                SANP2=XAP*DLOG(2.D0*(TRAP2+XNP))

                SANM1=XAM*DLOG(2.D0*(TRAM1+XNM))
                SANM2=XAM*DLOG(2.D0*(TRAM2+XNM))

                XAP=DSIN(XA1/2.D0)
                XAM=DSIN(XA2/2.D0)
                SONP1=XAP/TRAP1
                SONP2=XAP/TRAP2

                SONM1=XAM/TRAM1
                SONM2=XAM/TRAM2

                Y1=-XNM/2.D0-DLX
                Y2=-XNP/2.D0+DLX
                CY1=DCOS(AKK*Y1)
                CY2=DCOS(AKK*Y2)
                SY1=DSIN(AKK*Y1)
                SY2=DSIN(AKK*Y2)

                D1(1,K,J)=D1(1,K,J)+(SANP1+SANM1)*SV2*AASIN
                D2(1,K,J)=D2(1,K,J)+(CY1*SONP1-CY2*SONM1)*AASIN
                D1(2,K,J)=D1(2,K,J)+(SANP2+SANM2)*SV2*AASIN
                D2(2,K,J)=D2(2,K,J)+(CY1*SONP2-CY2*SONM2)*AASIN
 21             CONTINUE
 20         CONTINUE

      evaluation of T1,T2,T3,T4
```

```
C
          DO 30 I=1,NTP
               THI=COEF1*XNT(I,1)+COEF2
               XI=DLX2*(XNT(I,2)+1.D0)
               XIP=DLX2*(XNT(I,3)+1.D0)
               C1=DCOS(THI)
               C2=W2*C1
               C2=OFFSET(J)-C2
               CW=C2*C2
               SV1=DSIN(AKK*(DLX-XI))
               SV2=-SV1
               SV3=DSIN(AKK*(DLX-XIP))
               AASIN=DTP*CNT(I)
               DO 31 K=1,KMAX
                    XNPP=(XI+XIP)+FLOAT(K-1)*DLX
                    XNPM=(XI-XIP)+FLOAT(K-1)*DLX
                    XNMP=(-XI+XIP)+FLOAT(K-1)*DLX
                    XNMM=(-XI-XIP)+FLOAT(K-1)*DLX
                    RADPP2=XNPP*XNPP+CW
                    RADPM2=XNPM*XNPM+CW
                    RADMP2=XNMP*XNMP+CW
                    RADMM2=XNMM*XNMM+CW
                    TAPP1=DSQRT(RADPP2+ZP12)
                    TAPP2=DSQRT(RADPP2+ZP22)
                    TAPM1=DSQRT(RADPM2+ZP12)
                    TAPM2=DSQRT(RADPM2+ZP22)
                    TAMP1=DSQRT(RADMP2+ZP12)
                    TAMP2=DSQRT(RADMP2+ZP22)
                    TAMM1=DSQRT(RADMM2+ZP12)
                    TAMM2=DSQRT(RADMM2+ZP22)
                    CST1=DCOS(AKK*(XNPM/2.D0+DLX))*DSIN(AKK*XNPP
     *                   /2.D0)
                    CST2=DCOS(AKK*(-XNMP/2.D0+DLX))*DSIN(AKK*XNMM
     *                   /2.D0)
                    CST3=DCOS(AKK*(XNMM/2.D0+DLX))*DSIN(AKK*XNMP
     *                   /2.D0)
                    CST4=DCOS(AKK*(-XNPP/2.D0+DLX))*DSIN(AKK*XNPM
     *                   /2.D0)
                    T1(1,K,J)=T1(1,K,J)+SV2*AASIN*CST1/TAPP1
                    T2(1,K,J)=T2(1,K,J)+SV1*AASIN*CST2/TAMM1
                    T3(1,K,J)=T3(1,K,J)+SV1*AASIN*CST3/TAMP1
                    T4(1,K,J)=T4(1,K,J)+SV2*AASIN*CST4/TAPM1
                    T1(2,K,J)=T1(2,K,J)+SV2*AASIN*CST1/TAPP2
                    T2(2,K,J)=T2(2,K,J)+SV1*AASIN*CST2/TAMM2
                    T3(2,K,J)=T3(2,K,J)+SV1*AASIN*CST3/TAMP2
                    T4(2,K,J)=T4(2,K,J)+SV2*AASIN*CST4/TAPM2
 31            CONTINUE
 30       CONTINUE
  1  CONTINUE

     Evaluation of GS,GS1S2

     CZX=(1.D0-ER)/((1.D0+ER)*(1.D0+E2)*(1.D0+0.5D0*E1))
     CXX=1.D0
     CS=TPI2*CZX/FA
     CAX=TPI*CXX/FA
     CAZ=TPI*CZX/FA
     DO 4 JM=1,NOFF
          NJMAX=MAX(JM,1)
          J=MAX(JM,2)
          DO 62 N=1,NJMAX
               NP1=N+2
               N0=N+1
               NM1=N
```

```
                ST=-D1(2,NP1,J)+2.D0*YCOS*D1(2,N0,J)-D1(2,NM1,J)
     *             +2.D0*(T1(2,N,J)+T2(2,N,J)-T3(2,N,J)-T4(2,N,J))
                MP2=N+4
                MP1=N+3
                M0=N+2
                MM1=N+1
                MM2=N
                SINP2=DSIN(AKK*FLOAT(N+1)*DLX)
                SINP1=DSIN(AKK*FLOAT(N)*DLX)
                SIN0=DSIN(AKK*FLOAT(N-1)*DLX)
                SINM1=DSIN(AKK*FLOAT(N-2)*DLX)
                SINM2=DSIN(AKK*FLOAT(N-3)*DLX)
                ATX=SINP2*S1(1,MP2,J)-4.D0*YCOS*SINP1*S1(1,MP1,J)
     *             +2.D0*(2.D0+CCS)*SIN0*S1(1,M0,J)-4.D0*YCOS
     *             *SINM1*S1(1,MM1,J)+SINM2*S1(1,MM2,J)
                ATZ=SINP2*S1(2,MP2,J)-4.D0*YCOS*SINP1*S1(2,MP1,J)
     *             +2.D0*(2.D0+CCS)*SIN0*S1(2,M0,J)-4.D0*YCOS
     *             *SINM1*S1(2,MM1,J)+SINM2*S1(2,MM2,J)
                AAX=-2.D0*(D2(1,NP1,J)-2.D0*YCOS*D2(1,N0,J)
     *             +D2(1,NM1,J))
                AAZ=-2.D0*(D2(2,NP1,J)-2.D0*YCOS*D2(2,N0,J)
     *             +D2(2,NM1,J))
                AX=ATX+AAX
                AZ=ATZ+AAZ
                ZW=W*(CS*ST+CAX*AX-CAZ*AZ)
                IF (JM.EQ.1) GS(N)=ZW
                IF (JM.EQ.2) GS1S2(N)=ZW
  62        CONTINUE
   4  CONTINUE
      RETURN
      END
C...............................................................
C  This subroutine evaluates the higher order bessel functions using
C  the ascenting series expression or hankel's expansion.
C...............................................................
      SUBROUTINE BESS2 (BJ)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION BJ(10,2),U(4),RBJ(50,2)
      COMMON/B01/BJ0,BJ1
      COMMON/BSS/ARG(10),X
C
      PI=3.141592653589
C
C     Evaluation of J0,J1
C
      CALL BSJ0(X)
      RBJ(1,2)=BJ0
      RBJ(2,2)=BJ1
C
      NCON=1
      N=IDINT(2.4D0*X)
      IF (N.LT.10) N=10
      IF (X.LT.3.D0) GO TO 10
C
C     EVALUATION OF HIGHER ORDER BESSEL FUNCTIONS UP TO
C     ORDER LESS THEN THE ARGUMENT
C
      NIMAX=IDINT(X)-1
      IF (NIMAX.GT.9) NIMAX=9
      DO 1 I=2,NIMAX
           NJ1=I
           NJ2=I-1
           NB=I+1
           RBJ(NB,2)=FLOAT(2*NJ2)*RBJ(NJ1,2)/X-RBJ(NJ2,2)
  1   CONTINUE
      IF (NIMAX.EQ.9) GO TO 20
```

```
      NCON=NIMAX
:
:     DEBYE'S ASYMPTOTIC EXPANSION-EVALUATION OF JN
:
 10   DO 11 J=1,2
            JN=N-J+1
            XA=X/FLOAT(JN)
            XA=1.D0/XA
            XE=XA+DSQRT(XA*XA-1.D0)
            A=DLOG(XE)
            CTH=(XE+1.D0/XE)/(XE-1.D0/XE)
            CALL F(CTH,U)
            TNH=1.D0/CTH
            R1=DEXP(FLOAT(JN)*(TNH-A))
            R2=DSQRT(2.D0*PI*FLOAT(JN)*TNH)
            BN1=JN
            BN2=JN*JN
            BN3=BN2*JN
            BN4=BN3*JN
            RBJ(JN+1,2)=(R1/R2)*(1.D0+U(1)/BN1+U(2)/BN2+U(3)/BN3+
     *                 U(4)/BN4)
 11   CONTINUE
:
:     EVALUATION OF HIGHER ORDER BESSEL FUNCTIONS WHEN X<10
:
      NJMAX=N-2-NCON
      DO 2 I=1,NJMAX
           NJB=N-I
           NJB1=NJB+1
           NJB2=NJB1+1
           RBJ(NJB,2)=2.D0*FLOAT(NJB)*RBJ(NJB1,2)/X-RBJ(NJB2,2)
 2    CONTINUE
 20   CONTINUE
      DO 3 I=1,9
           BJ(I,2)=RBJ(I,2)
 3    CONTINUE
      RETURN
      END
..................................................................
..................................................................
      SUBROUTINE BSJ0(X)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/B01/BJ0,BJ1


    Evaluation of J0 using the series expansion given in
    Abramowitz.

      PI=3.141592653589D0
      IF (X.GT.3.D0) GO TO 20
      X3=X/3.D0
      X32=X3*X3
      X34=X32*X32
      X36=X32*X34
      X38=X32*X36
      X310=X38*X32
      X312=X310*X32
      BJ0=1.D0-2.2499997D0*X32+1.2656208D0*X34-0.3163866D0*X36+
     *    0.0444479D0*X38-0.0039444D0*X310+0.00021000D0*X312
      BJ1=X*(0.5D0-0.56249985D0*X32+0.21093573D0*X34-0.03954289D0
     *    *X36+0.00443319D0*X38-0.00031761D0*X310+0.00001109D0
     *    *X312)
      GO TO 21

   20 X3=3.D0/X
      X32=X3*X3
```

```
      X33=X32*X3
      X34=X33*X3
      X35=X34*X3
      X36=X35*X3
      FJ0=0.79788456D0-0.00000077D0*X3-0.00552740D0*X32-0.00009512D0
     *    *X33+0.00137237D0*X34-0.00072805D0*X35+0.00014476D0*X36
      FJ1=0.79788456D0+0.00000156D0*X3+0.01659667D0*X32+0.00017105D0
     *    *X33-0.00249511D0*X34+0.00113653D0*X35-0.00020033D0*X36
      TJ0=X-0.78539816D0-0.04166397D0*X3-0.00003954D0*X32+0.00262573D0
     *    *X33-0.00054125D0*X34-0.00029333D0*X35+0.00013558D0*X36
      TJ1=X-2.35619449D0+0.12499612D0*X3+0.00005650D0*X32-0.00637879D0
     *    *X33+0.00074348D0*X34+0.00079824D0*X35-0.00029166D0*X36
      WCON=DSQRT(1.D0/X)
      BJ0=WCON*FJ0*DCOS(TJ0)
      BJ1=WCON*FJ1*DCOS(TJ1)
  21  CONTINUE
      RETURN
      END
C..................................................................
C................................................................
      SUBROUTINE F(X,U)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION U(4)
      X2=X*X
      X3=X2*X
      X4=X3*X
      X5=X4*X
      X6=X5*X
      X7=X6*X
      X8=X7*X
      X9=X8*X
      X10=X9*X
      X11=X10*X
      X12=X11*X
C
      U(1)=(3.D0*X-5.D0*X3)/24.D0
      U(2)=(81.D0*X2-462.D0*X4+385.D0*X6)/1152.D0
      U(3)=(30375.D0*X3-369603.D0*X5+765765.D0*X7-425425.D0*X9)/
     *      414720.D0
      U(4)=(4465125.D0*X4-94121676.D0*X6+349922430.D0*X8-446185740.D0*
     *      X10+185910725.D0*X12)/39813120.D0
      RETURN
      END
C..................................................................
C             SUBROUTINE DATA_SLOT
C     This subroutine gives all the data for integration used in
C     subroutine SLOT.FTN
C
C..................................................................
      SUBROUTINE DATA_SLOT
      IMPLICIT REAL*8 (A-H,O-Z)

      COMMON/DAT/ER,H,T,DLX,XC,YC,ZC,ERC,RMC,X0,Y0,A,TPI,TPI2,PI,W,E1,
     *E2,EER,AK0,AK,AKK,FA,OFFSET(7),WDELTA,OFFLIM,ERROR,THI,FI,
     *NS1,NS2,NSS2,NOFF

      COMMON/DATT/COAL(20),POINT(20),CN(51),BM(51),POLTM(20),
     *POLTE(20),AM(41),DM(41),POLES(40),VXXM(20),VZXM(20),VZXE(20),
     *BPOINT(10),BCOAL(10),MPOINT,NPOINT,NK0,MA,NTM,NTE,NK0K,IFIRST

      COMMON/INT/XNS(40),CNS(40),XND(20,2),CND(20),XNT(40,3),
     *CNT(40),NDP,NTP,NSP

      COMMON/ADON/DIST(250,7,10),RCOE(20,250,7,10),AX,SERS(5),
     *SERA(5),DARG(10,4),S(10,2),WREAL,NSER,NMAX(7)

      COMMON/IOFF/INS,INS1S2
```

```
C
      PI=3.141592653589D0
C
C
C
      TPI=2.D0*PI
      TPI2=TPI*TPI
C     +------------------+
C     | ERROR FUNCTIONS  |
C     +------------------+
C
      A1=A*A-TPI2
      A2=TPI2-TPI2/ER
      E1=0.5D0*A2/A1
      E2=ER*E1/(1.D0+ER)
      FA=DSQRT(1.D0+TPI2/A1)
      AK0=2.D0*PI/DSQRT(EER)
      AKK=2.D0*PI
      AK=AK0*DSQRT(ER)
C     +-----------------------------------------------------+
C     |          Data for the poles                         |
C     |  IFIRST= 0 : dominant mode is TM wave (many poles)  |
C     |          1 : dominant mode is TE wave (many poles)  |
C     |          2 : only one TM surface wave               |
C     +-----------------------------------------------------+
C   +---------------------------+
C   |  Data for the Integration |
C   +---------------------------+
      NK0=20
      NK0K=1
      MA=20
      NSER=10
C
      NPOINT=10
C-----------
      Vector COAL
C-----------
      COAL(1)=0.0666713443D0
      COAL(2)=0.14945134915D0
      COAL(3)=0.21908636251D0
      COAL(4)=0.26926671931D0
      COAL(5)=0.29552422471D0
      COAL(6)=COAL(5)
      COAL(7)=COAL(4)
      COAL(8)=COAL(3)
      COAL(9)=COAL(2)
      COAL(10)=COAL(1)
      ------------
      Vector POINT
      ------------
      POINT(1)=0.973906528517D0
      POINT(2)=0.865063366688D0
      POINT(3)=0.679409568299D0
      POINT(4)=0.433395394129D0
      POINT(5)=0.148874338981D0
      POINT(6)=-POINT(5)
      POINT(7)=-POINT(4)
      POINT(8)=-POINT(3)
      POINT(9)=-POINT(2)
      POINT(10)=-POINT(1)

      MPOINT=5
      ------------
      Vector BCOAL
      ------------
      BCOAL(1)=0.2369268851D0
      BCOAL(2)=0.4786286705D0
```

```
      BCOAL(3)=0.5688888888D0
      BCOAL(4)=BCOAL(2)
      BCOAL(5)=BCOAL(1)
C     -------------
C     Vector BPOINT
C     -------------
      BPOINT(1)=0.9061798459D0
      BPOINT(2)=0.5384693101D0
      BPOINT(3)=0.D0
      BPOINT(4)=-BPOINT(2)
      BPOINT(5)=-BPOINT(1)
C     ------------------
C     Single integration
C     ------------------
C
      NSP=31
      RS1=0.99708748181D0
      RS2=0.98468590966D0
      RS3=0.96250392509D0
      RS4=0.93075699789D0
      RS5=0.88976002994D0
      RS6=0.83992032014D0
      RS7=0.78173314841D0
      RS8=0.71577678458D0
      RS9=0.64270672292D0
      RS10=0.56324916140D0
      RS11=0.47819378204D0
      RS12=0.38838590160D0
      RS13=0.29471806998D0
      RS14=0.19812119933D0
      RS15=0.09955531215D0
      RS16=0.D0
C
      XNS(1)=RS1
      XNS(2)=RS2
      XNS(3)=RS3
      XNS(4)=RS4
      XNS(5)=RS5
      XNS(6)=RS6
      XNS(7)=RS7
      XNS(8)=RS8
      XNS(9)=RS9
      XNS(10)=RS10
      XNS(11)=RS11
      XNS(12)=RS12
      XNS(13)=RS13
      XNS(14)=RS14
      XNS(15)=RS15
      XNS(16)=RS16
      XNS(17)=-RS15
      XNS(18)=-RS14
      XNS(19)=-RS13
      XNS(20)=-RS12
      XNS(21)=-RS11
      XNS(22)=-RS10
      XNS(23)=-RS9
      XNS(24)=-RS8
      XNS(25)=-RS7
      XNS(26)=-RS6
      XNS(27)=-RS5
      XNS(28)=-RS4
      XNS(29)=-RS3
      XNS(30)=-RS2
      XNS(31)=-RS1

      CNS(1)=0.0074708315792D0
```

```
      CNS(2)=0.0173186207903D0
      CNS(3)=0.0270090191849D0
      CNS(4)=0.0364322739123D0
      CNS(5)=0.0454937075272D0
      CNS(6)=0.0541030824249D0
      CNS(7)=0.0621747865610D0
      CNS(8)=0.0696285832354D0
      CNS(9)=0.0763903865987D0
      CNS(10)=0.0823929917615D0
      CNS(11)=0.0875767406084D0
      CNS(12)=0.0918901138936D0
      CNS(13)=0.0952902429123D0
      CNS(14)=0.0977433353863D0
      CNS(15)=0.0992250112266D0
      CNS(16)=0.0997205447934D0
      CNS(17)=CNS(15)
      CNS(18)=CNS(14)
      CNS(19)=CNS(13)
      CNS(20)=CNS(12)
      CNS(21)=CNS(11)
      CNS(22)=CNS(10)
      CNS(23)=CNS(9)
      CNS(24)=CNS(8)
      CNS(25)=CNS(7)
      CNS(26)=CNS(6)
      CNS(27)=CNS(5)
      CNS(28)=CNS(4)
      CNS(29)=CNS(3)
      CNS(30)=CNS(2)
      CNS(31)=CNS(1)
C
C
C
C     2)  Double Integration
C         ------------------
C
      NDP=16
      R1=DSQRT((15.D0-2.D0*DSQRT(30.D0))/35.D0)
      R2=-R1
      S1=DSQRT((15.D0+2.D0*DSQRT(30.D0))/35.D0)
      S2=-S1
      A1=4.D0*(59.D0+6.D0*DSQRT(30.D0))/864.D0
      A2=4.D0*(59.D0-6.D0*DSQRT(30.D0))/864.D0
      A3=4.D0*49.D0/864.D0
C
      XND(1,1)=R1
      XND(1,2)=R1
      CND(1)=A1
C
      XND(2,1)=R2
      XND(2,2)=R1
      CND(2)=A1
C
      XND(3,1)=R1
      XND(3,2)=R2
      CND(3)=A1
C
      XND(4,1)=R2
      XND(4,2)=R2
      CND(4)=A1

      XND(5,1)=S1
      XND(5,2)=S1
      CND(5)=A2

      XND(6,1)=S1
```

```
      XND(6,2)=S2
      CND(6)=A2
C
      XND(7,1)=S2
      XND(7,2)=S1
      CND(7)=A2
C
      XND(8,1)=S2
      XND(8,2)=S2
      CND(8)=A2
C
      XND(9,1)=R1
      XND(9,2)=S1
      CND(9)=A3
C
      XND(10,1)=R1
      XND(10,2)=S2
      CND(10)=A3
C
      XND(11,1)=S1
      XND(11,2)=R1
      CND(11)=A3
C
      XND(12,1)=S2
      XND(12,2)=R1
      CND(12)=A3
C
      XND(13,1)=R2
      XND(13,2)=S1
      CND(13)=A3
C
      XND(14,1)=R2
      XND(14,2)=S2
      CND(14)=A3
C
      XND(15,1)=S1
      XND(15,2)=R2
      CND(15)=A3
C
      XND(16,1)=S2
      XND(16,2)=R2
      CND(16)=A3
C
C     3)  Triple Integration
C         ------------------
C
      NTP=34
      RS1=0.9317380000D0
      RS2=-RS1
      UU1=0.9167441779D0
      UU2=-UU1
      SS1=0.4086003800D0
      SS2=-SS1
      TT1=0.7398529500D0
      TT2=-TT1
      B1=8.D0*0.03558180896D0
      B2=8.D0*0.01247892770D0
      B3=8.D0*0.05286772991D0
      B4=8.D0*0.02672752182D0
C
      XNT(1,1)=RS1
      XNT(1,2)=0.D0
      XNT(1,3)=0.D0
      CNT(1)=B1
C
      XNT(2,1)=RS2
```

```
      XNT(2,2)=0.D0
      XNT(2,3)=0.D0
      CNT(2)=B1
C
      XNT(3,1)=0.D0
      XNT(3,2)=RS1
      XNT(3,3)=0.D0
      CNT(3)=B1
C
      XNT(4,1)=0.D0
      XNT(4,2)=RS2
      XNT(4,3)=0.D0
      CNT(4)=B1
C
      XNT(5,1)=0.D0
      XNT(5,2)=0.D0
      XNT(5,3)=RS1
      CNT(5)=B1
C
      XNT(6,1)=0.D0
      XNT(6,2)=0.D0
      XNT(6,3)=RS2
      CNT(6)=B1
C
      XNT(7,1)=UU1
      XNT(7,2)=UU1
      XNT(7,3)=0.D0
      CNT(7)=B2
C
      XNT(8,1)=UU2
      XNT(8,2)=UU1
      XNT(8,3)=0.D0
      CNT(8)=B2
C
      XNT(9,1)=UU1
      XNT(9,2)=UU2
      XNT(9,3)=0.D0
      CNT(9)=B2
C
      XNT(10,1)=UU2
      XNT(10,2)=UU2
      XNT(10,3)=0.D0
      CNT(10)=B2
C
      XNT(11,1)=UU1
      XNT(11,2)=0.D0
      XNT(11,3)=UU1
      CNT(11)=B2
C
      XNT(12,1)=UU1
      XNT(12,2)=0.D0
      XNT(12,3)=UU2
      CNT(12)=B2

      XNT(13,1)=UU2
      XNT(13,2)=0.D0
      XNT(13,3)=UU1
      CNT(13)=B2

      XNT(14,1)=UU2
      XNT(14,2)=0.D0
      XNT(14,3)=UU2
      CNT(14)=B2

      XNT(15,1)=0.D0
      XNT(15,2)=UU1
```

```
      XNT(15,3)=UU1
      CNT(15)=B2
C
      XNT(16,1)=0.D0
      XNT(16,2)=UU1
      XNT(16,3)=UU2
      CNT(16)=B2
C
      XNT(17,1)=0.D0
      XNT(17,2)=UU2
      XNT(17,3)=UU1
      CNT(17)=B2
C
      XNT(18,1)=0.D0
      XNT(18,2)=UU2
      XNT(18,3)=UU2
      CNT(18)=B2

      XNT(19,1)=SS1
      XNT(19,2)=SS1
      XNT(19,3)=SS1
      CNT(19)=B3

      XNT(20,1)=SS1
      XNT(20,2)=SS1
      XNT(20,3)=SS2
      CNT(20)=B3

      XNT(21,1)=SS1
      XNT(21,2)=SS2
      XNT(21,3)=SS1
      CNT(21)=B3

      XNT(22,1)=SS1
      XNT(22,2)=SS2
      XNT(22,3)=SS2
      CNT(22)=B3

      XNT(23,1)=SS2
      XNT(23,2)=SS1
      XNT(23,3)=SS1
      CNT(23)=B3

      XNT(24,1)=SS2
      XNT(24,2)=SS1
      XNT(24,3)=SS2
      CNT(24)=B3

      XNT(25,1)=SS2
      XNT(25,2)=SS2
      XNT(25,3)=SS1
      CNT(25)=B3

      XNT(26,1)=SS2
      XNT(26,2)=SS2
      XNT(26,3)=SS2
      CNT(26)=B3

      XNT(27,1)=TT1
      XNT(27,2)=TT1
      XNT(27,3)=TT1
      CNT(27)=B4

      XNT(28,1)=TT1
      XNT(28,2)=TT1
      XNT(28,3)=TT2
```

```
      CNT(28)=B4
C
      XNT(29,1)=TT1
      XNT(29,2)=TT2
      XNT(29,3)=TT1
      CNT(29)=B4
C
      XNT(30,1)=TT1
      XNT(30,2)=TT2
      XNT(30,3)=TT2
      CNT(30)=B4
C
      XNT(31,1)=TT2
      XNT(31,2)=TT1
      XNT(31,3)=TT1
      CNT(31)=B4
C
      XNT(32,1)=TT2
      XNT(32,2)=TT1
      XNT(32,3)=TT2
      CNT(32)=B4
C
      XNT(33,1)=TT2
      XNT(33,2)=TT2
      XNT(33,3)=TT1
      CNT(33)=B4
C
      XNT(34,1)=TT2
      XNT(34,2)=TT2
      XNT(34,3)=TT2
      CNT(34)=B4
C
      RETURN
      END
```

```
C***********************************************************************
C     The name of this file is ........... POLES.FTN ................
C***********************************************************************
      SUBROUTINE SPOLES
      IMPLICIT REAL*8(A-H,O-Z)
C.......................................................................
C
C     ER      :....Dielectric constant
C
C     H       :....Height of the dielectric substrate
C
C     NE      :....Number of TE surface waves
C
C     NM      :....Number of tm surface waves
C
C     XS      :....Matrix of poles contributing to TE surface waves
C
C     XR      :....Matrix of poles contributing to TM surface waves
C
C     ERR     :....Error in the computation of the poles
C
C.......................................................................
      DIMENSION XS(40),XR(40),LOR(40)
C
      COMMON/DAT/ER,H,T,DLX,XC,YC,ZC,ERC,RMC,XQ,YQ,A,TPI,TPI2,PI,W,E1,
     *E2,EER,BK0,BK,BKK,FA,OFFSET(7),WDELTA,OFFLIM,ERROR,THI,FI,
     *NS1,NS2,NSS2,NOFF
C
      COMMON/DATT/COAL(20),POINT(20),CN(51),BM(51),TMP(20),TEP(20),
     *AM(41),DM(41),TPO(40),VXXM(20),VZXM(20),VZXE(20),BPOINT(10),
     *BCOAL(10),MPOINT,NPOINT,NK0,MA,NM,NE,NK0K,IFIRST

      AER=DSQRT(ER)
      ER2=ER*ER
      PI2=PI*PI
      MAXE=5
      ERR=0.0000001D0
      DP=H

      PART I : TE MODES

      AK0=2.D0*PI
      AK=DSQRT(ER)*AK0
      X0=DP*DSQRT(AK**2-AK0**2)

      WRITE (6,300) AK0,AK,X0,PI
  300 FORMAT(10X,'AK0=',E14.7,2X,'AK=',E14.7,2X,'X0=',E14.7,
     *2X,'PI=',E14.7/)

      AN=X0/PI+0.5D0
      NE=AN
      IF (NE.EQ.0) GO TO 310
      DO 2 I=1,NE
            IF (X0-(2.D0*FLOAT(I)+1.D0)*PI/2.D0) 3,3,4
    4       XS0=(2.D0*FLOAT(I)-1.D0)*PI/2.D0+ERR
            XS1=(2.D0*FLOAT(I)+1.D0)*PI/2.D0-ERR
            GO TO 5
    3       XS0=(2.D0*FLOAT(I)-1.D0)*PI/2.D0+ERR
            XS1=X0
    5       CONTINUE
            IF (DABS(XS0-XS1)-ERR) 22,7,7
    7       XSA=(XS0+XS1)/2.D0
            Y=-DTAN(XSA)*DSQRT(X0**2-XSA**2)-XSA
            IF (Y) 8,9,10
    9       XS(I)=XSA
            GO TO 222
```

```
   8       XS1=XSA
           GO TO 5
  10       XS0=XSA
           GO TO 5
  22       XS(I)=(XS0+XS1)/2.D0
 222       XS(I)=DSQRT(AK**2-XS(I)**2/DP**2)
   2  CONTINUE
C
C     WRITE (6,301) ER,H
C301  FORMAT(//10X,' Dielectric Constant=',D16.9/10X,'Substrate '
C    *,'Thickness',D16.9///)
C
 310  IF (NE.EQ.0) WRITE (6,304)
 304  FORMAT(/////10X,'No TE waves excited in the substrate'//)
      IF (NE.EQ.0) GO TO 312
      IF (NE.GT.0) WRITE (6,305) NE
 305  FORMAT(///10X,'There are',I4,
     *' TE waves excited in the substrate'//)
      DO 302 I=1,NE
             TEP(I)=XS(I)
            WRITE (6,303) I,TEP(I)
 303        FORMAT (10X,I4,2X,D16.9)
            TEP(I)=TEP(I)/AER
 302  CONTINUE
 312  CONTINUE
C
C     END OF PART I
C
C
C     PART II : TM MODES
C
      AN=X0/PI+1.D0
      NM=AN
      DO 13 I=1,NM
            IF (X0-(2.D0*FLOAT(I)+1.D0)*PI/2.D0) 14,14,15
  15        XS1=FLOAT(I)*PI-PI/3.D0-0.01D0
            GO TO 16
  14        XS1=X0
  16        XS0=FLOAT(I-1)*PI+ERR
  17        CONTINUE
            IF (DABS(XS0-XS1)-ERR) 113,19,19
  19        XRA=(XS0+XS1)/2.D0
C
C           WRITE (6,301) XRA
C 301       FORMAT(10X,'XRA=',E14.7/)
C
            Y=DSQRT(ER)**2*(1.D0/DTAN(XRA))*DSQRT(X0**2-XRA**2)-XRA
            IF (Y) 20,21,24
  21        XR(I)=XRA
            GO TO 333
  20        XS1=XRA
            GO TO 17
  24        XS0=XRA
            GO TO 17
 113        XR(I)=(XS0+XS1)/2.D0
 333        XR(I)=DSQRT(AK**2-XR(I)**2/DP**2)
  13  CONTINUE
      WRITE (6,307) NM
 307  FORMAT(///10X,'There are',I4,' TM waves excited in the substrate'/
     */)
      DO 308 I=1,NM
             TMP(I)=XR(I)
             WRITE (6,306) I,XR(I)
 306         FORMAT (10X,I4,2X,D16.9)
             TMP(I)=TMP(I)/AER
 308  CONTINUE
```

```
 322  CONTINUE
C
      NK=NE+NM
      IF (NE.EQ.0) GO TO 350
      DO 411 IQW=1,NE
             TPO(IQW)=TEP(IQW)
             LOR(IQW)=1
  411 CONTINUE
  350 CONTINUE
      DO 412 IQW=1,NM
             TPO(NE+IQW)=TMP(IQW)
             LOR(NE+IQW)=0
  412 CONTINUE
C
      IF (NK.EQ.1) GO TO 416
      NNK=NK-1
      DO 415 IIP=1,NNK
             IK=IIP+1
             DO 413 IIF=IK,NK
                    QWR=TPO(IIP)
                    IIW=LOR(IIP)
                    IF (TPO(IIP).LT.TPO(IIF)) GO TO 413
                    TPO(IIP)=TPO(IIF)
                    LOR(IIP)=LOR(IIF)
                    TPO(IIF)=QWR
                    LOR(IIF)=IIW
  413        CONTINUE
  415 CONTINUE
      IF (LOR(1).EQ.0) IFIRST=0
      IF (LOR(1).EQ.1) IFIRST=1
      GO TO 417
C
  416 IFIRST=2
  417 CONTINUE
      RETURN
      END
```

```
C*******************************************************************
C     The name of this file is ............. CAV_INV.FTN................
C     It finds the inverse matrix for the case of scatering one or two
C      slots on the gound of a dielectric substrate.
C
C     The slots have cavities at the back for minimization of the
C     reflected field.
C*******************************************************************
      SUBROUTINE CAV_INV(NOR)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMPLEX CUR,BMATR,CAV_MAT(60)
      COMPLEX ZS,ZS1S2,CJ,CCAV,CAV,CINC,SUMC,DETA,CIN,RUN,CS1,
     *CADD,SUMN
C
      COMMON/COMP/ZS(50),ZS1S2(300),NS,NS1S2
C
      COMMON/DAT/ER,H,T,DLX,XC,YC,ZC,ERC,RMC,X0,Y0,A,TPI,TPI2,PI,W,E1,
     *E2,EER,AK0,AK,AKK,FA,OFFSET(7),WDELTA,OFFLIM,ERROR,THI,FI,
     *NS1,NS2,NSS2,NOFF
C
      COMMON/MAN/BMATR(260,260),IA(260),IB(260)
C
      COMMON/BESSEL/BJ0(6000)
C
      COMMON/CAV/RUN(1000),IRUN
C
      COMMON/LOSS/TLOS_E,TLOS_M
C...................................................................
C                   DATA
C...................................................................
      CJ=(0.0,1.0)
      NOEL1=NS1
      NOR=NS1
      IDEL=20
C
C     .....First Diagonal Matrix.........
C
      ARG1=PI*W/(2.D0*YC)
      ARG2=PI*Y0/YC
      CALL VBJ0(ARG1,ARG2)
      CALL CAVITY
      SIN=DSIN(2.D0*PI*DLX)
      CS1=SNGL((16.D0/120.D0)*DSQRT(ER)/(PI**3*XC*YC*RMC*SIN*SIN))/
     *     (1.0-CJ*SNGL(TLOS_M))
      CCAV=CJ*CS1
      C1=PI/XC
      IMIN=1
      IMAX=NOEL1
      DO 4 I=IMIN,IMAX
           IXN=0
           DO 5 KI=I,IMAX
                IXN=IXN+1
                XJ=(IXN*DLX+X0)*C1
                XI=(KI*DLX+X0)*C1
                SUMN=(0.0,0.0)
                DO 200 IN=1,IRUN
                       SINI=DSIN(FLOAT(IN)*XI)
                       SINJ=DSIN(FLOAT(IN)*XJ)
                       CADD=RUN(IN)*SNGL(SINI*SINJ)
                       SUMN=SUMN+CADD
 200            CONTINUE
                CAV=CCAV*SUMN
                IF (IXN.EQ.1) CAV_MAT(KI)=CAV
                BMATR(IXN,KI)=-ZS(I)+CAV
                BMATR(KI,IXN)=BMATR(IXN,KI)
 5         CONTINUE
```

```
  4   CONTINUE
  300 CONTINUE
C
      DO 400 I=1,NS1
             WRITE (6,401) I,CAV_MAT(I),ZS(I)
 401         FORMAT(10X,'I=',I4,2X,'CAV=',E14.7,2X,E14.7,2X,
     *        'ZS=',E14.7,2X,E14.7/)
 400  CONTINUE
C
      CALL MINVCD (NOR,NOR,DETA)
      RETURN
      END
C*************************************************************************
C     THIS SUBROUTINE INVERTS A SQUARE COMPLEX MATRIX
C*************************************************************************
      SUBROUTINE MINVCD (IA,MA,DETA)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMPLEX A,PIV,DETA,TEMP,PIV1
      COMMON/MAN/A(260,260),IR(260),IC(260)
      DO 1 I=1,MA
      IR(I)=0
    1 IC(I)=0
C     DETA=(1.00,0.00)
      S=0.00
      R=MA
    2 CALL SUBMCD(IA,IA,MA,MA,I,J)
      PIV=A(I,J)
C     DETA=PIV*DETA
      Y=CABS(PIV)
      IF (Y.EQ.0) GO TO 17
      IR(I)=J
      IC(J)=I
      PIV=(1.00,0.00)/PIV
      A(I,J)=PIV
      DO 5 K=1,MA
    5 IF (K.NE.J) A(I,K)=A(I,K)*PIV
      DO 9 K=1,MA
      IF (K.EQ.I) GO TO 9
      PIV1=A(K,J)
    6 DO 8 L=1,MA
    8 IF (L.NE.J) A(K,L)=A(K,L)-PIV1*A(I,L)
    9 CONTINUE
      DO 11 K=1,MA
   11 IF (K.NE.I) A(K,J)=-PIV*A(K,J)
      S=S+1.00
      IF (S.LT.R) GO TO 2
   12 DO 16 I=1,MA
      K=IC(I)
      M=IR(I)
      IF (K.EQ.I) GO TO 16
C     DETA=-DETA
      DO 14 L=1,MA
      TEMP=A(K,L)
      A(K,L)=A(I,L)
   14 A(I,L)=TEMP
      DO 15 L=1,MA
      TEMP=A(L,M)
      A(L,M)=A(L,I)
   15 A(L,I)=TEMP
      IC(M)=K
      IR(K)=M
   16 CONTINUE
      RETURN
   17 WRITE (6,18)I,J
   18 FORMAT (10X,'MATRIX IS SINGULAR'/10X,'I=',I4,5X,'J=',I4)
      RETURN
```

```
      END
C***********************************************************************
C.......................................................................
C***********************************************************************
      SUBROUTINE SUBMCD(IA,JA,MA,NA,I,J)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMPLEX  A
      COMMON/MAN/A(260,260),IR(260),IC(260)
      I=0
      J=0
      TEST=0.00
      DO 5 K=1,MA
      IF (IR(K).NE.0) GO TO 5
      DO 4 L=1,NA
      IF (IC(L).NE.0) GO TO 4
      X=CABS(A(K,L))
      IF(X.LT.TEST) GO TO 4
      I=K
      J=L
      TEST=X
    4 CONTINUE
    5 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CAVITY
      IMPLICIT REAL*8 (A-H,O-Z)
      COMPLEX RUN,CRNER,CK2,CI,COEF,SERM,CRNM,CSQR,CARG,CTERM,CCOT,
     *SERN
C
      COMMON/CAV/RUN(1000),IRUN
C
      COMMON/BESSEL/BJ0(6000)
C
      COMMON/DAT/ER,H,T,DLX,XC,YC,ZC,ERC,RMC,X0,Y0,A,TPI,TPI2,PI,W,E1,
     *E2,EER,AK0,AK,AKK,FA,OFFSET(7),WDELTA,OFFLIM,ERROR,THI,FI,
     *NS1,NS2,NSS2,NOFF
C
      COMMON/LOSS/TLOS_E,TLOS_M
C
      CI=(0.0,1.0)
      ERROR=1.D-4
      PI=3.141592653589D0
      SERN=(0.0,0.0)
      NTEST=0
      DO 1 N=1,1000
            EN=1.D0
            RN=FLOAT(N)/(2.D0*XC)
            RN2=RN*RN
            RNP1=1.D0+RN
            RNM1=1.D0-RN
            RNPM=RNP1*RNM1
            RNPM2=RNPM*RNPM
            CK2=(1.0-CI*SNGL(TLOS_E))*(1.0-CI*SNGL(TLOS_M))*SNGL(ERC*RMC
     *           /ER)
            CRNER=CK2-SNGL(RN2)
            SIN1=DSIN(PI*DLX*RNP1)
            SIN2=DSIN(PI*DLX*RNM1)
            SINP=SIN1*SIN2
            SINP2=SINP*SINP
            COEF=CRNER*SNGL(SINP2*EN/RNPM2)
            SERM=(0.0,0.0)
            ITEST=0
            M1=0
  2         M1=M1+1
                  M=M1-1
                  EM=0.5D0
                  IF(M.GT.0) EM=1.D0
                  BJ=BJ0(M1)
                  RM=FLOAT(M)/(2.D0*YC)
                  RM2=RM*RM
                  CRNM=CRNER-SNGL(RM2)
                  CABS1=CABS(CRNM)
                  IF (CABS1.LT.1.E-8) GO TO 100
                  CSQR=CSQRT(CRNM)
                  CARG=SNGL(2.D0*PI*ZC)*CSQR
                  IF (CABS1.LT.50.0) THEN
                        CTERM=SNGL(EM*BJ)*(CCOS(CARG)/(CSQR*CSIN(CARG)))
                  ELSE
                        RARG=REAL(CARG)
                        AARG=AIMAG(CARG)
                        COT=DCOS(RARG)/DSIN(RARG)
                        EX=DEXP(-2.D0*AARG)
                        COTH=(1.D0+EX)/(1.D0-EX)
                        CCOT=(-1.0-CI*SNGL(COT*COTH))/(SNGL(COT)-
     *                       CI*SNGL(COTH))
                        CTERM=SNGL(EM*BJ)*CCOT/CSQR
                  END IF
                  SERM=SERM+CTERM
                  RATIO=CABS(CTERM/SERM)
```

```
               IF (RATIO.LT.ERROR) GO TO 5
                         ITEST=0
                         GO TO 2
   5           ITEST=ITEST+1
               IF (ITEST.LT.4) GO TO 2
C
C                      WRITE (6,10) N,M,SERM
C  10                  FORMAT (//6X,'Maximum M for a given N:',2X,'N=',
C    *                 I4,3X,'Mmax=',I4,2X,'Serm=',E14.7,1X,E14.7)
C
          RUN(N)=COEF*SERM
          WRITE (6,40) N,RUN(N)
  40      FORMAT(10X,'N=',I4,3X,'RUN=',(E14.7,2X,E14.7))
          SERN=SERN+RUN(N)
          RATIO=CABS(RUN(N)/SERN)
          IF (RATIO.LT.ERROR) GO TO 8
              NTEST=0
              GO TO 1
  8       NTEST=NTEST+1
          IF (NTEST.LT.4) GO TO 1
              IRUN=N
              GO TO 101
   1  CONTINUE
      GO TO 101
C
C
  100 WRITE (6,20)
   20 FORMAT(///10X,'.........WARNING!........'/10X,'The eigenvalue Kz'
     *,' became equal to 0'///)
C
C
  101 CONTINUE
      RETURN
      END
C*********************************************************************
C     This function evaluates the zeroth order first kind Bessel
C                                Function J0
C*********************************************************************
      SUBROUTINE VBJ0(ARG1,ARG2)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/BESSEL/BJ0(6000)
      PI=3.141592653589D0
      DO 1 M1=1,6000
           M=M1-1
           X=FLOAT(M)*ARG1
           X1=FLOAT(M)*ARG2
           COS1=DCOS(X1)
           COS2=COS1*COS1
           IF (X.GT.0.001D0) GO TO 10
               X3=X/3.D0
               X32=X3*X3
               X34=X32*X32
               X36=X34*X32
               BSJ0=1.D0-2.2499997D0*X32+1.2656208D0*X34-0.3163866D0
     *              *X36
               BJ0(M1)=BSJ0*COS2
               GO TO 1
  10       IF (X.GT.3.D0) GO TO 12
               X3=X/3.D0
               X32=X3*X3
               X34=X32*X32
               X36=X34*X32
               X38=X36*X32
               X310=X38*X32
               X312=X310*X32
               BSJ0=1.D0-2.2499997D0*X32+1.2656208D0*X34-0.3163866D0
```

```
     *              *X36+0.0444479D0*X38-0.0039444D0*X310+0.00021000
     *              D0*X312
               BJ0(M1)=BSJ0*COS2
               GO TO 1
  12       CONTINUE
           X3=3.D0/X
           X32=X3*X3
           X33=X32*X3
           X34=X33*X3
           X35=X34*X3
           X36=X35*X3
           FJ0=0.79788456D0-0.00000077D0*X3-0.00552740D0*X32-0.0000
     *         9512D0*X33+0.00137237D0*X34-0.00072805D0*X35+0.00014
     *         476D0*X36
           TJ0=X-0.78539816D0-0.04166397D0*X3-0.00003954D0*X32+0.00
     *         262573D0*X33-0.00054125D0*X34-0.00029333D0*X35+0.000
     *         13558D0*X36
           WCON=DSQRT(1.D0/X)
           BSJ0=WCON*FJ0*DCOS(TJ0)
           BJ0(M1)=BSJ0*COS2
  1   CONTINUE
      RETURN
      END
C*****************************************************************
```

```
C*****************************************************************
C               The name of this file is:    MULT_EXC.FTN
C
C     This subroutine evaluates the excitation vector and
C     multiplies it with the inverse of the impedance matrix
C     to get the unknown field on the slot
C******************************************************************
      SUBROUTINE MULT_EXC(NOR,R0)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION Y(260),PHASE(260)
      COMPLEX CUR,BMATR,CJ,CCAV,CAV,CINC,SUMC,CIN,R0
C
      COMMON/DAT/ER,H,T,DLX,XC,YC,ZC,ERC,RMC,X0,Y0,A,TPI,TPI2,PI,W,E1,
     *E2,EER,AK0,AK,AKK,FA,OFFSET(7),WDELTA,OFFLIM,ERROR,THI,FI,
     *NS1,NS2,NSS2,NOFF
C
      COMMON/MAN/BMATR(260,260),IA(260),IB(260)
C
      COMMON/PAT/CUR(260)
C.................................................................
C                   DATA
C.................................................................
      CJ=(0.0,1.0)
C
      RN0=120.D0*PI
      RN=RN0/DSQRT(ER)
      RK=2.D0*PI
      RK0=RK/DSQRT(ER)
      SINI=DSIN(THI)
      COSI=DCOS(THI)
      SINT=SINI/DSQRT(ER)
      SINT2=SINT*SINT
      COST=DSQRT(1.D0-SINT2)
      ARGT=RK*COST*T
      ARGH=RK*COST*H
      ARGL1=RK*DLX
      ARGL2=ARGL1*SINT
      D1=4.D0*COSI*DCOS(FI)*DCOS(ARGT)*(DCOS(ARGL2)-DCOS(ARGL1))
     *    /(RK*(1.D0-SINT2))
      DC=RN0*COST*DCOS(ARGH)
      DS=RN*COSI*DSIN(ARGH)
      CINC=SNGL(D1)/(SNGL(DC)+CJ*SNGL(DS))
      ARG=ARGH
      DC=RN0*COST
      DS=RN*COSI*DTAN(ARG)
      R0=-(SNGL(DC)-CJ*SNGL(DS))/(SNGL(DC)+CJ*SNGL(DS))
      DO 70 IQ=1,NOR
            SUMC=(0.0,0.0)
  401       DO 170 JQ=1,NOR
                   ARG=ARGL2*FLOAT(JQ)+X0
                   EC=DCOS(ARG)
                   ES=DSIN(ARG)
                   CIN=CINC*(SNGL(EC)-CJ*SNGL(ES))

                   WRITE (6,778) IQ,JQ,CIN,BMATR(IQ,JQ)
  778              FORMAT(2X,I4,2X,I4,2X,E14.7,2X,E14.7,2X,E14.7,2X,
     *                   E14.7)

                   SUMC=SUMC-BMATR(IQ,JQ)*CIN
  170       CONTINUE
            CUR(IQ)=SUMC
   70 CONTINUE
      WRITE (6,71)
   71 FORMAT (///10X,'Electric Field Distribution on the Slots',////)
      IMIN=1
      IMAX=NOR
```

```
      DO 76 IQQ=IMIN,IMAX
            RECUR1=REAL(CUR(IQQ))
            Y(IQQ)=CABS(CUR(IQQ))
            AICUR1=AIMAG(CUR(IQQ))
            PHCUR1=ATAN2(AICUR1,RECUR1)
            PHASE(IQQ)=180.00*PHCUR1/PI
            IF (IQQ.EQ.1) WRITE (6,77)
   77       FORMAT(///10X,'Electric Field on the first Slot',///)
            IF (IQQ.EQ.(NS1+1)) WRITE (6,78)
   78       FORMAT(///10X,'Electric Filed on the Second Slot',///)
C
            WRITE (6,81) IQQ,CUR(IQQ),Y(IQQ),PHASE(IQQ)
   81       FORMAT (1X,'C(',I4,')=(',E14.7,',',E14.7,')',2X,
     *              '(',E14.7,',',E14.7,')',/)
C
C            WRITE (1,92) IQQ,CUR(IQQ),Y(IQQ),PHASE(IQQ)
C   92       FORMAT(I5,4E15.3)
C
   76 CONTINUE
      RETURN
      END
```

```
C*********************************************************************
C     The name of this file is ..... REFLEC.FTN .....................
C*********************************************************************
C     It computes the reflection coefficient for cavity-backed slots
C*********************************************************************
      SUBROUTINE REFLEC(WREAL,R0,REFL)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMPLEX ETH,EFI,CUR,REFL,R0,RS,RA
C
      COMMON/DAT/ER,H,T,DLX,XC,YC,ZC,ERC,RMC,X0,Y0,A,TPI,TPI2,PI,
     *WIDTH,E1,E2,EER,AK0,AK,AKK,FA,OFFSET(7),WDELTA,OFFLIM,ERROR,
     *THI,FI,NS1,NS2,NSS2,NOFF
C
      COMMON/PAR/NSLOT,ALONG(7),S(20,2),NSER
C
      COMMON/PAT/CUR(260)
C
      NSLOT=NOFF
C
      NSER=10
      U=(WREAL/WIDTH)
      U=DATAN(DSQRT(1.D0/(U*U)-1.D0))
      U1=U/FLOAT(NSER)
      DO 3 JN=1,NSER
           S2=2.D0*FLOAT(JN)-1.D0
           S2=S2/(2.D0*FLOAT(NSER))
           S3=COS(S2*U)
           S(JN,2)=S3*WIDTH/2.D0
           S(JN,1)=U1
 3    CONTINUE
      ATH1=WREAL/WIDTH
      ATH1=DATAN(DSQRT(1./(ATH1*ATH1)-1.0))
      ATH2=PI-ATH1
      CURIN=(1.D0/PI)*(ATH2-ATH1)
      CALL EFIELD (ETH,EFI)
      RS=(ETH*SNGL(DCOS(THI)*DSIN(FI))+EFI*SNGL(DCOS(FI)))*
     *SNGL(CURIN)
      RA=2.0*RS/SNGL(DSQRT(ER))
      REFL=R0+RA

      WRITE (6,100) THI,FI,R0,RA
 100  FORMAT(10X,'THI=',E14.7,2X,'FI=',E14.7/10X,'R0=',E14.7,1X,E14.7
     */10X,'RA=',E14.7,2X,E14.7)

      RETURN
      END
**********************************************************************
      The name of this subroutine is:       EFIELD
**********************************************************************
      It evaluates the far field of a dipole
**********************************************************************
      SUBROUTINE EFIELD(ETH,EFI)
      IMPLICIT REAL*8(A-H,O-Z)
      COMPLEX W,ETH,EFI,XNUM,XDEN,F1X,ZNUM,ZDEN,F1Z,PTH,PFI,
     *SUMC,WEXP,CUR,COEF

      COMMON/DAT/ER,H,T,DLX,XC,YC,ZC,ERC,RMC,X0,Y0,A,TPI,TPI2,PI,
     *WIDTH,E1,E2,EER,AK0,AK,AKK,FA,OFFSET(7),WDELTA,OFFLIM,ERROR,
     *THI,FI,NS1,NS2,NSS2,NOFF

      COMMON/PAR/NSLOT,ALONG(7),S(20,2),NSER

      COMMON/PAT/CUR(260)

      COMMON/B01/BJ0,BJ1
```

```
      CKK=2.D0*PI
      CK0=CKK/DSQRT(ER)
      SINFT=DSIN(FI)*DSIN(THI)
      ARG=(CK0*WIDTH/2.D0)*SINFT
      CALL BSJ0(ARG)
      SSUM=0.D0
      DO 5 JN=1,NSER
           ARAF=CK0*S(JN,2)*SINFT
           CAFF=DCOS(ARAF)
           SSUM=SSUM+S(JN,1)*CAFF
  5   CONTINUE
      TERMI=(BJ0-SSUM*2.D0/PI)
C
      ERTH=DSQRT(ER-DSIN(THI)**2)
      ERH=CK0*ERTH*H
      SINH=DSIN(ERH)
      SINTH=DSIN(THI)
      SINFI=DSIN(FI)
      COSH=DCOS(ERH)
      COSTH=DCOS(THI)
      COSFI=DCOS(FI)
C
      W=(0.0,1.0)
      XNUM=SNGL(COSTH)
      XDEN=SNGL(-ERTH*SINH)+W*SNGL(ER*COSTH*COSH)
      F1X=-SNGL((ER/(4.D0*PI)))*XNUM/XDEN
      ZNUM=SNGL(SINH*SINTH*COSTH)
      ZDEN=XDEN*(-SNGL(COSTH*SINH)+W*SNGL(ERTH*COSH))
      F1Z=SNGL((1.D0-ER)/(4.D0*PI))*ZNUM/ZDEN
      PTH=SNGL(COSFI)*(F1X*SNGL(COSTH)-F1Z*SNGL(SINTH))
      PFI=SNGL(SINFI)*F1X
C
C     For the single slot
C
      SUMC=(0.0,0.0)
      DO 1 I=1,NSLOT
           R8=CK0*(FLOAT(I)*DLX)*SINTH*COSFI
           SUMC=SUMC+CUR(I)*(SNGL(DCOS(R8))+SNGL(DSIN(R8))*W)
  1   CONTINUE
      R8=CK0*ALONG(1)*SINTH*COSFI
      R9=CK0*OFFSET(1)*SINFI*SINTH
      WEXP=(SNGL(DCOS(R8))+SNGL(DSIN(R8))*W)*(SNGL(DCOS(R9))+SNGL(
     *DSIN(R9))*W)
      COEF=WEXP*SNGL(TERMI)
      PTH=PTH*COEF
      PFI=PFI*COEF
      IF (ABS(FI).GT.1.E-4) GO TO 2
      THER=ABS(ABS(THETA)-PI/2.0)
      IF (THER.GE.1.E-4) GO TO 2
      IF (ABS(EER-1.00).LT.1.E-6) GO TO 3
  2   RNUM=DCOS(CK0*DLX*SINTH*COSFI)-DCOS(CKK*DLX)
      RDEN=DSIN(CKK*DLX)*(1.D0-(CK0*SINTH*COSFI/CKK)**2)
      RATIO=RNUM/RDEN
C
      ETH=PFI*SNGL(RATIO)
      EFI=PTH*SNGL(RATIO)
C
      ETH=ETH*SUMC
      EFI=EFI*SUMC
C
      WRITE (6,4) ETH,EFI
  4   FORMAT (5X,'ETH=',(E14.8,1X,E14.8),5X,'EFI=',
     *(E14.8,1X,E14.8)/)

      RETURN
```

```
3  RATIO=DLX*CK0/2.0
   ETH=PFI*SNGL(RATIO)*SUMC
   EFI=-PTH*SNGL(RATIO)*SUMC
   RETURN
   END
```