FEBI Users and Theory Manual

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1 General Code Description

The Two Dimensional Finite Element Boundary Integral computer code (febi.f) finds the bistatic/backscatter scattering (TE or TM polarization) by a 2D cylinder of arbitrary shape, which can consist of any combination of the following:

1) metal
2) dielectric/magnetic material
3) anisotropic material (with in-plane anisotropy)
4) surfaces with impedance boundary conditions (IBCs)
5) resistive sheets for modeling thin, penetrable dielectric sheets
6) capacitive/resistive sheets for modeling frequency dependent resistive sheets

The following two codes have been supplied with this manual:

febi.f	**– computes the bistatic/backscatter echowidth vs observation angle**
febi-fsweep.f	**– computes echowidth at specific values of angle of incidence and angle of observation vs frequency**

Four parameter files (param1.h, param2.h, param3.h, and param4.h which include some of the variable declarations required by the code) and sample input files (mesh files, etc.) have also been supplied with the code.

The infinite region surrounding the cylinder is terminated via a Boundary Integral (BI). The BI surface is assumed to be placed in air at a distance of at least one finite element edge size away from the outermost surface of the cylinder. The integration surface coincides with the BI surface. The solution region is discretized using triangular elements and Tangential Vector/Edge-Based Finite Elements are applied. Thus, the unknowns are associated with electric (TE case) or magnetic (TM case) field values along the triangular element edges.

The computer code febi.f is written in FORTRAN 77 and uses an input file named "2d-fem-bi.in" ("febi-fsweep.in" for the code febi-fsweep.f) which contains some of the parameters associated with the scattering problem to be solved. If an input file already exists when the code is run, the existing input file can be used. If an input file does not exist or if the user wishes to create a new input file, the program will prompt the user to enter the necessary parameters and create a new input file.

During execution, the program will inform the user about the current state of the program, several parameters of interest to (some) users, and so on. After the execution is complete, the computed Echowidth per unit wavelength in dBs is stored in an output file whose name is specified by the user.

The code also generates an info file, which contains the information regarding a particular problem solved by the code. The information about the geometry is stored in a file called "2d-fem-bi.geo" ("febi-fsweep.geo" for the code febi-fsweep.f).

The system matrix is stored in a form that takes advantage of the sparsity of the global equation system. The matrix will initially be represented by (Avec,Cvec,Rvec) where :

- Avec(i) : Vector containing non-zero matrix element # i
- Cvec(i) : Vector pointing to the column of a matrix element # i
- Rvec(i) : Vector pointing to the row of a matrix element # i

This matrix representation is convenient when assemblying (and possibly condencing) the global matrix system. Before solving the global matrix system, however, the matrix is converted to an alternative representation by (AAvec,CCvec,RRvec) where :

- AAvec(i) : Vector containing the non-zero matrix elements (row-wise)
- CCvec(i) : Vector pointing to the column of a matrix element # i
- RRvec(j) : Vector describing the number of non-zero matrix elements in row # j

This matrix representation is convenient when solving the global system. The global matrix system is solved using the Quasi Minimal Residual (QMR) method.

2 Input Files

The computer code requires some or all of the following input files depending upon the material composition of the scatterer (2D cylinder):

1) **2d-fem-bi.in** *(always required)*
   an input file generated by the user from screen prompts
2) **meshfile** *(always required)*
   a user supplied input mesh file (the name of the mesh file is specified by the user)
3) **material.txt** *(sometimes required)*
   a user generated input file containing information on the material composition of the scatterer
4) **ibc-sproperties.txt** *(sometimes required)*
   a user generated input file containing information on the surface impedance of the IBC surfaces existing within the computational domain
5) **resis-sproperties.txt** *(sometimes required)*
   a user generated input file containing information on the parameters of the resistive and/or capacitive/resistive surfaces existing within the computational domain

3 Input File Format

meshfile

This is a user supplied input mesh file and the name of the file is also specified by the user. The 2D-FEBI code can accept any one of the following two types of format for the mesh files (the code asks for the mesh file format to be used):

1) Mesh file format generated through I-DEAS Master Series Version 6 *(filename.unv)*
2) A preset mesh file format with mesh data to be filled in by the user

I-DEAS Master Series Version 6 Mesh File

The mesh is written to a universal file that is read by the code. The name of the mesh file is specified by the user. Nodes on various types of surfaces are assumed to be grouped in I-DEAS under the group names given below:

- PEC surface - a group name starting with the letter "P"
- IBC surface - a group name starting with the letter "E"
- Resistive surface - a group name starting with the letter "R"
- Capacitive/Resistive surface - a group name starting with the letter "C"
- BI surface (assumed to be situated in free space) - a group name starting with the letter "B"
- Integration surface (assumed to be situated in free space) - a group name starting with the letter "I"

Several sample universal files (*.unv files) have been included (soft copies on a disk) with this user manual, which will give the user a clear idea what kind of mesh information the code requires. The code does not accept more than one resistive or IBC surface from the I-DEAS mesh file.

Preset Mesh File Format

The name of the mesh file is specified by the user and the user is required to fill the mesh related data in the mesh file using a specified fixed format. The code can accept any number of resistive or IBC surfaces from this mesh file.

Note:

Three nodes on the integration surface must not form an element and no edge normal vector must rotate more than 90 degrees between the neighboring elements. Also, **the element node numbers are required to be entered in a clockwise order in the connectivity table.**

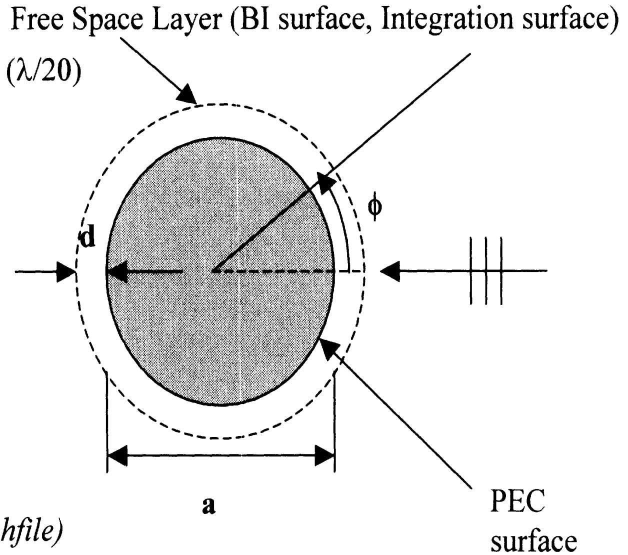
Elements in free space are assumed to have material code # 2 and the elements lying in the dielectric/magnetic material regions are assumed to have material code # 4,5,... **All dimensions are assumed to be in cm.**

4 Running the Code

The best way to explain how to run the code is to go through some specific examples illustrating the use of various input files. These specific examples with the required format of the corresponding input files are discussed next.

4.1 PEC Circular Cylinder

Bistatic Echo-width of a PEC Circular Cylinder
a = 6 cm (2 λ)
d = 0.15 cm ($\lambda/20$)
FEM element edge size δ = 0.15 cm ($\lambda/20$)
Frequency = 10 GHz
Angle of incidence: 0.0 deg



Input files required:

pecccyl.unv	*(if using I-DEAS meshfile)*
pecccyl.txt	*(if using user-supplied meshfile with mesh data to be filled in by the user)*

Format of pecccyl.txt:

```
Enter total number of nodes below
258
Enter x and y coordinates of these nodes below: x(i),y(i)
  3.15000000000000     0.000000000000000E+000
  3.00000000000000     0.000000000000000E+000
  3.14642961359480     0.149935608491471
          .                      .
          .                      .
          .                      .
Enter total number of elements below
258
Connectivity Table: For each element,
global node#1    node#2    node#3   element group#
           2          3          1          2
```

```
      133        2          1           2
      134        3          2           2
                   .                   .
                   .                   .
                   .                   .
Enter total number of PEC surface nodes below
126
Enter the global node numbers of PEC surface nodes below
     2
    134
    135
      .
      .
      .
Enter total number of integration surface nodes below
132
Enter the global node numbers of integration surface nodes below
     1
     3
     4
      .
      .
Enter total number of BI surface nodes below
132
Enter the global node numbers of BI surface nodes below
     1
     3
     4
      .
      .
      .
```

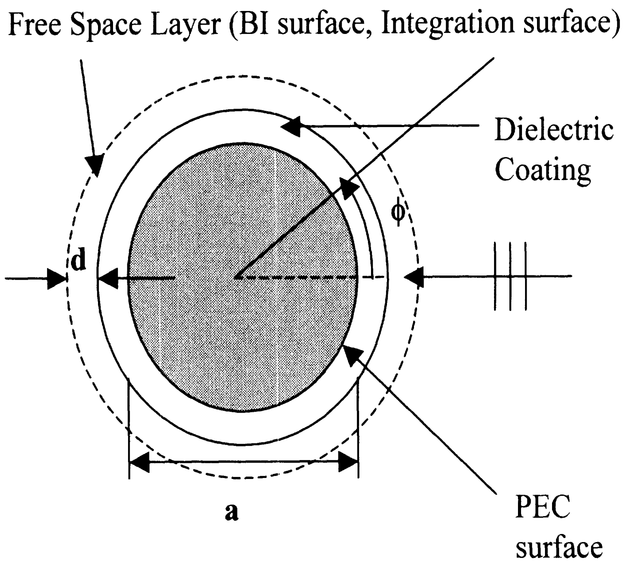
Comments:

The format of the mesh file "pecccyl.txt" given above is self-explanatory. Note that the discretization is only done for the region outside the PEC cylinder. The coinciding BI and integration surfaces have been placed at a distance of 0.15 cm (one finite element edge size) away from the surface of the PEC cylinder. It is important to note that in the connectivity table the element (material) group # is 2 which shows that the element lies in free space. For the elements lying in different dielectric/magnetic material regions the element (material) group # vary from 4,5,6, ……

4.2 Dielectric Coated PEC Circular Cylinder

Bistatic Echo-width of a Dielectric Coated PEC Circular Cylinder
$a = 6$ cm ($2\ \lambda$)
$d = 0.15$ cm ($\lambda/20$)
Dielectric region thickness = 0.45 cm ($0.15\ \lambda$)
Dielectric region $\varepsilon = 3 - j3$, $\mu = 1.0$
FEM element edge size $\delta = 0.15$ cm ($\lambda/20$)
Frequency = 10 GHz

Angle of incidence: 0.0 deg



Input files required:

coatccyl.unv	*(if using I-DEAS meshfile)*
coatccyl.txt	*(if using user-supplied meshfile with mesh data to be filled in by the user)*
material.txt	*(contains information on the ε and μ of the dielectric/magnetic material regions)*

Format of coatccyl.txt:

```
Enter total number of nodes below
737
Enter x and y coordinates of these nodes below: x(i),y(i)
  3.45000000000000     0.000000000000000E+000
  3.00000000000000     0.000000000000000E+000
  3.44675939258491    -0.149498125833379
        .                      .
        .                      .
Enter total number of elements below
1197
Connectivity Table: For each element,
global  node#1     node#2     node#3    element group#
             1          3        543          4
             1        542        146          4
             1        543        542          4
                  .                       .
                  .                       .
                  .                       .
Enter total number of PEC surface nodes below
126
Enter the global node numbers of PEC surface nodes below
        2
```

```
     147
     148
       .
       .
       .
Enter total number of integration surface nodes below
151
Enter the global node numbers of integration surface nodes below
     587
     588
     589
       .
       .
Enter total number of BI surface nodes below
151
Enter the global node numbers of BI surface nodes below
     587
     588
     589
       .
       .
       .
```

Format of material.txt:

In case of a single isotropic material region (see material.txt):

```
For this dielectric/magnetic material region:
Enter 1 below if this dielectric/magnetic material is isotropic
Enter 2 below if this dielectric/magnetic material is anisotropic
1
Enter relative permittivity below:
(3.0,-3.0)
Enter relative permeability below:
(1.0,0.0)
```

Comments:

The format of the mesh file "coatccyl.txt" given above is self-explanatory. Note that the discretization is only done for the region (dielectric and air layers) outside the PEC cylinder. The coinciding BI and integration surfaces have been placed at a distance of 0.15 cm (one finite element edge size) away from the surface of the coated cylinder. It is important to note that in the connectivity table the element (material) group # 2 shows that the element lies in free space. For the elements lying in different dielectric/magnetic material regions the element (material) group # vary from 4,5,6, ...

Some Other Examples of the Format of material.txt:

In case of a single anisotropic material region (see anisotropic-material2.txt):

```
For this dielectric/magnetic material region:
Enter 1 below if this dielectric/magnetic material is isotropic
```

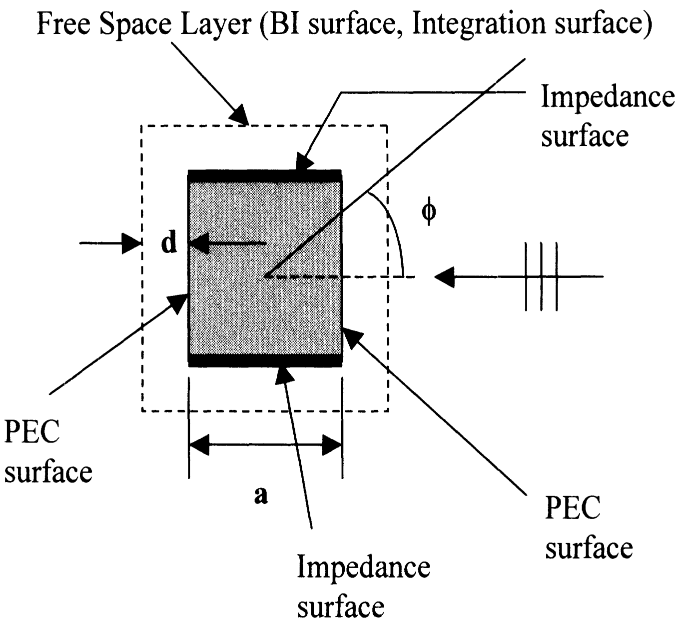
```
Enter 2 below if this dielectric/magnetic material is anisotropic
2
Enter relative permittivity tensor below:
Enter epsilon_xx:
(3.1415927,-0.5)
Enter epsilon_xy:
(0.0,0.0)
Enter epsilon_yx:
(0.0,0.0)
Enter epsilon_yy:
(1.5707963,-1.0)
Enter epsilon_zz:
(1.0,0.0)
Enter relative permeability tensor below:
Enter mu_xx:
(1.0,0.0)
Enter mu_xy:
(0.0,0.0)
Enter mu_yx:
(0.0,0.0)
Enter mu_yy:
(1.0,0.0)
Enter mu_zz:
(1.5707963,0.0)
```

In case of two different isotropic material regions existing in the solution domain:

```
For this dielectric/magnetic material region:
Enter 1 below if this dielectric/magnetic material is isotropic
Enter 2 below if this dielectric/magnetic material is anisotropic
1
Enter relative permittivity below:
(3.0,-3.0)
Enter relative permeability below:
(1.0,0.0)
For this dielectric/magnetic material region:
Enter 1 below if this dielectric/magnetic material is isotropic
Enter 2 below if this dielectric/magnetic material is anisotropic
1
Enter relative permittivity below:
(8.0,-1.0)
Enter relative permeability below:
(1.0,0.0)
```

4.3 Impedance-PEC Square Cylinder

Bistatic/Backscatter Echo-width of an Impedance-PEC Square Cylinder
a = 3 cm (1 λ)
d = 0.15 cm (λ/20)
Surface Impedance = 753.46 – j 753.46 (Ohms)
Frequency = 10 GHz
Angle of incidence: 0.0 deg



Input files required:

ibcpec.unv *(if using I-DEAS meshfile)*
ibcpec.txt *(if using user-supplied meshfile with mesh data to be filled in by the user)*
ibc-sproperties.txt *(contains information on the parameters of impedance surfaces)*

Format of ibcpec.txt:

```
Enter total number of nodes below
160
Enter x and y coordinates of these nodes below: x(i),y(i)
  -1.65000000000000      1.65000000000000
  -1.65000000000000     -1.65000000000000
   1.65000000000000      1.65000000000000
         .                      .
         .                      .
         .                      .
Enter total number of elements below
160
Connectivity Table: For each element,
global  node#1     node#2     node#3    element group#
            8          9          1          2
           48          8          1          2
           28          5          2          2
                  .                     .
                  .                     .
                  .                     .
Enter total number of PEC surface nodes below
```

```
40
Enter the global node numbers of PEC surface nodes below
        5
        6
        7
        8
        107
        108
        109
          .
          .
          .
Enter total number of IBC surface nodes below
40
Enter the global node numbers of IBC surface nodes below
        5
        6
        7
        8
        89
        90
        91
          .
          .
          .
Enter total number of integration surface nodes below
84
Enter the global node numbers of integration surface nodes below
        1
        2
        3
          .
          .
Enter total number of BI surface nodes below
84
Enter the global node numbers of BI surface nodes below
        1
        2
        3
          .
          .
          .
```

Format of ibc-sproperties.txt:

In case of a single impedance surface:

```
Enter the value of surface impedance (in Ohms) for
the impedance surface #1
(753.46,-753.46)
```

Comments:

The format of the mesh file "ibcpec.txt" given above is self-explanatory. Note that the discretization is only done for the region outside the Impedance-PEC square cylinder.

The first four nodes of the IBC and the PEC surfaces in the mesh file are the same. This is because they are the nodes located at the four corners of the square cylinder and, hence, are shared by both the IBC and the PEC surfaces. The coinciding BI and integration surfaces have been placed at a distance of 0.15 cm (one finite element edge size) away from the surface of the cylinder. It is important to note that in the connectivity table the element (material) group # 2 shows that the element lies in free space. For the elements lying in different dielectric/magnetic material regions the element (material) group # vary from 4,5,6, ...

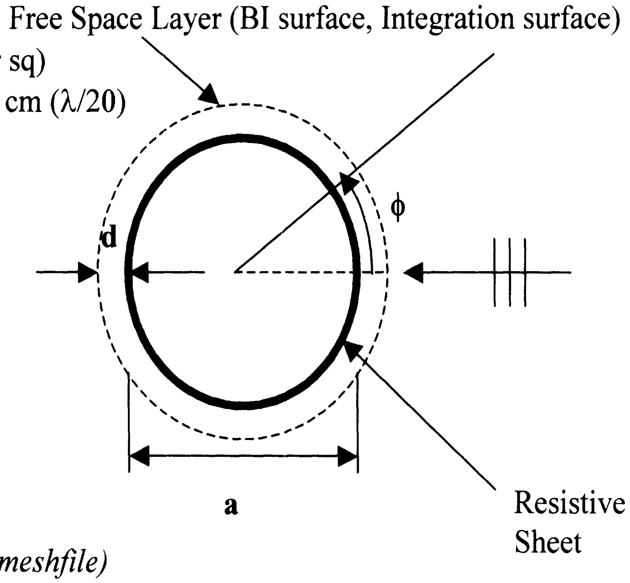
Some Other Examples of the Format of ibc-sproperties.txt:

In case of two impedance surfaces with different surface impedances:

```
Enter the value of surface impedance (in Ohms) for
the impedance surface #1
(753.46,-753.46)
Enter the value of surface impedance (in Ohms) for
the impedance surface #2
(200.0,-300.0)
```

4.4 Resistive Sheet Circular Cylinder (TE-Polarization)

Bistatic Echo-width of a Hollow Resistive Sheet Circular Cylinder
a = 3 cm (1 λ)
d = 0.15 cm (λ/20)
Resistivity = 1998.05 (Ohms per sq)
FEM element edge size δ = 0.15 cm (λ/20)
Frequency = 10 GHz
Angle of incidence: 0.0 deg



Input files required:

resccyl.unv *(if using I-DEAS meshfile)*
resccyl.txt *(if using user-supplied meshfile with mesh data to be filled in by the user)*
resis-sproperties.txt *(contains information on the parameters of resistive, capacitive/resistive surfaces)*

Format of resccyl.txt:

```
Enter total number of nodes below
466
Enter x and y coordinates of these nodes below: x(i),y(i)
```

```
  1.50000000000000     0.000000000000000E+000
  1.49253704566726     0.149442856339304
  1.47020939458285     0.297463167602863
        .                    .
        .                    .
        .                    .
Enter total number of elements below
861
Connectivity Table: For each element,
global  node#1     node#2     node#3    element group#
          360         2          1           2
          356       358         63           2
          358         1         63           2
                 .                           .
                 .                           .
                 .                           .
Enter total number of resistive sheet nodes below
63
Enter the global node numbers of resistive sheet nodes below
        1
        2
        3
        .
        .
        .
Enter total number of integration surface nodes below
69
Enter the global node numbers of integration surface nodes below
      398
      399
      400
        .
        .
Enter total number of BI surface nodes below
69
Enter the global node numbers of BI surface nodes below
      398
      399
      400
        .
        .
        .
```

Format of resis-sproperties.txt:

In case of a single resistive sheet surface:

```
Sheet info for sheet #1
Enter 0 if it is a resistive sheet
Enter 1 if it is a capacitive/resistive sheet
0
Do you want to enter the value of resistivity
directly (enter 0 if yes) or do you want to
calculate it (enter 1 if yes)?
0
Enter the value of resistivity (in Ohms per square)
```

```
for the resistive sheet #1
(1998.05,0.0)
```

Comments:

The format of the mesh file "resccyl.txt" given above is self-explanatory. Note that the discretization is done for the region inside and outside the resistive sheet circular cylinder. The coinciding BI and integration surfaces have been placed at a distance of 0.15 cm (one finite element edge size) away from the surface of the cylinder. It is important to note that in the connectivity table the element (material) group # 2 shows that the element lies in free space. For the elements lying in different dielectric/magnetic material regions the element (material) group # vary from 4,5,6, ...

Some Other Examples of the Format of resis-sproperties.txt:

In case of two resistive surfaces with different resistivities:

```
Sheet info for sheet #1
Enter 0 if it is a resistive sheet
Enter 1 if it is a capacitive/resistive sheet
0
Do you want to enter the value of resistivity
directly (enter 0 if yes) or do you want to
calculate it (enter 1 if yes)?
0
Enter the value of resistivity (in Ohms per square)
for the resistive sheet #1
(1998.05,0.0)
Sheet info for sheet #2
Enter 0 if it is a resistive sheet
Enter 1 if it is a capacitive/resistive sheet
0
Do you want to enter the value of resistivity
directly (enter 0 if yes) or do you want to
calculate it (enter 1 if yes)?
0
Enter the value of resistivity (in Ohms per square)
for the resistive sheet #2
(1998.05,0.0)
```

In case of a single dielectric layer simulated as a resistive sheet:

```
Sheet info for sheet #1
Enter 0 if it is a resistive sheet
Enter 1 if it is a capacitive/resistive sheet
0
Do you want to enter the value of resistivity
directly (enter 0 if yes) or do you want to
calculate it (enter 1 if yes)?
1
```

Enter the value of dielectric constant for the
resistive sheet #1
(3.0,-0.5)
Enter the thickness (in cm) of the dielectric layer
to be modeled as a resistive sheet #1
0.005

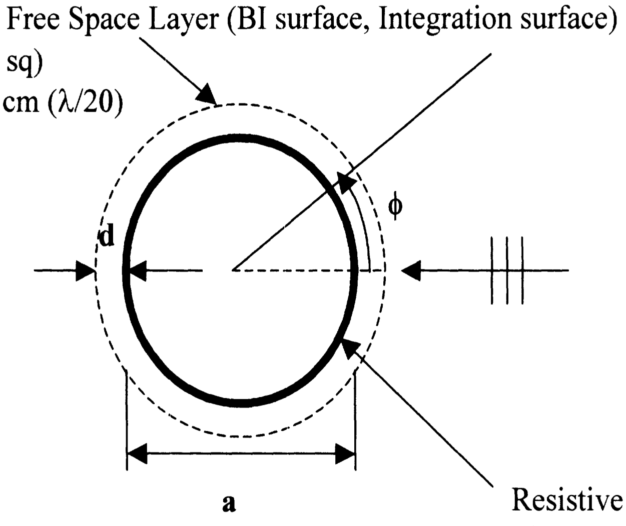
In case of a single capacitive/resistive sheet:

Sheet info for sheet #1
Enter 0 if it is a resistive sheet
Enter 1 if it is a capacitive/resistive sheet
1
Enter the value of resistivity (in Ohms per square)
for the capacitive/resistive sheet #1
(1998.05,0.0)
Enter the value of the capacitance (in nano Farads)
for the capacitive/resistive sheet #1
3.0

Note: If there are both impedance and resistive surfaces present in the computational domain, the nodes of all the impedance surfaces are entered first in the user-supplied mesh file. The nodes of the resistive surfaces are entered after that.

4.5 Resistive Sheet Circular Cylinder (TM-Polarization)

Bistatic Echo-width of a Hollow Resistive Sheet Circular Cylinder
a = 3 cm (1 λ)
d = 0.15 cm ($\lambda/20$)
Resistivity = 1998.05 (Ohms per sq)
FEM element edge size δ = 0.15 cm ($\lambda/20$)
Frequency = 10 GHz
Angle of incidence: 0.0 deg



Input files required:

resis22.txt *(if using user-supplied meshfile with mesh data to be filled in by the user)*
resis-sproperties.txt *(contains information on the parameters of resistive, capacitive/resistive surfaces)*

Format of resis22.txt:

Enter total number of nodes below
96

```
Enter x and y coordinates of these nodes below: x(i),y(i)
 0.650000000000000      0.000000000000000E+000
 0.500000000000000      0.000000000000000E+000
 0.632449718061997      0.150024511741583
        .                       .
        .                       .
        .                       .
Enter total number of elements below
121
Connectivity Table: For each element,
global  node#1     node#2     node#3    element group#
            2          3          1           2
           28          2          1           2
           48          3          2           2
                .                           .
                .                           .
                .                           .
Enter total number of resistive sheet nodes below
42
Enter the global node numbers of resistive sheet nodes below
        2
       29
       30
        .
        .
        .
Enter total number of element pairs for the resistive sheet below
21
Enter the element numbers for the top and bottom elements below
   3  50
   7  82
   9  81
        .
        .
        .
Enter total number of integration surface nodes below
27
Enter the global node numbers of integration surface nodes below
        1
        3
        4
        .
        .
Enter total number of BI surface nodes below
27
Enter the global node numbers of BI surface nodes below
        1
        3
        4
        .
        .
        .
```

Format of resis-sproperties.txt:

In case of a single resistive sheet surface:

```
Sheet info for sheet #1
Enter 0 if it is a resistive sheet
Enter 1 if it is a capacitive/resistive sheet
0
Do you want to enter the value of resistivity
directly (enter 0 if yes) or do you want to
calculate it (enter 1 if yes)?
0
Enter the value of resistivity (in Ohms per square)
for the resistive sheet #1
(1998.05,0.0)
```

Comments:

The format of the mesh file "resis22.txt" given above is self-explanatory. Note that the discretization is done for the region inside and outside the resistive sheet circular cylinder. The coinciding BI and integration surfaces have been placed at a distance of 0.15 cm (one finite element edge size) away from the surface of the cylinder. It is important to note that in the connectivity table the element (material) group # 2 shows that the element lies in free space. For the elements lying in different dielectric/magnetic material regions the element (material) group # vary from 4,5,6, ...

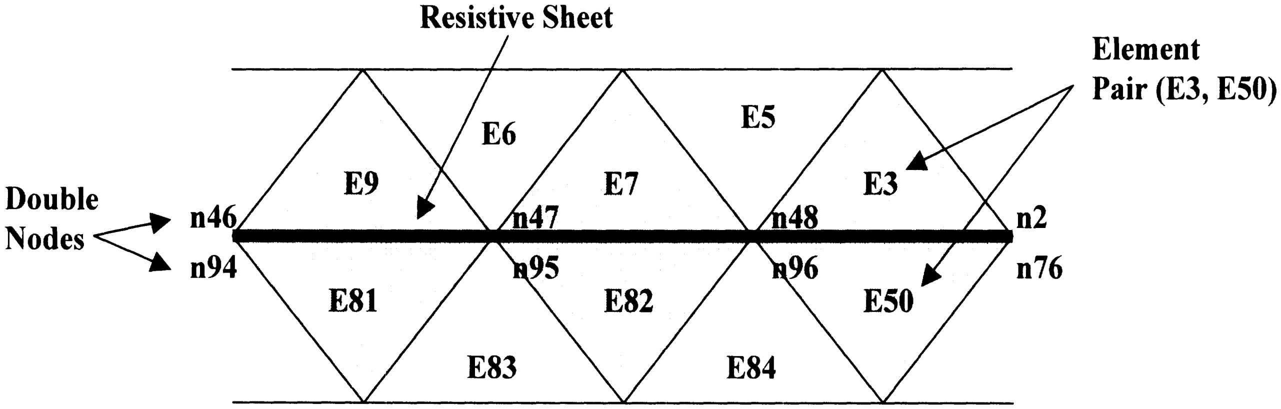
Introduction of Double Nodes:

It is important to realize here that the TM polarization in the case of a resistive sheet requires a special treatment. The reason for this is that for the TM polarization our working variable (in the weak form of the wave equation) is the magnetic field, which is discontinuous across the resistive sheet (the magnetic field has different values above and below the resistive sheet). This forces us to introduce co-located double nodes across the resistive sheet (above and below the resistive sheet as shown in the figure below).

In the mesh file "resis22.txt" given above the total number of resistive sheet nodes (including the nodes above and below the resistive sheet) are entered first (equal to 42 as shown above). Then the global node numbers are entered listing the resistive sheet node numbers of the nodes belonging to the elements above the resistive sheet first (and then the resistive sheet node numbers of the nodes belonging to the elements below the resistive sheet are listed).

The next step is to specify the total number of element pairs corresponding to the resistive sheet (as shown in the mesh file "resis22.txt" above). An element pair consists of the two elements located on the two sides of the resistive sheet sharing one common edge belonging to the resistive sheet (see the figure shown below). After entering the total number of element pairs the element numbers for the top and the bottom elements

(elements above and below the resistive sheet sharing a common edge belonging to the resistive sheet) are entered as pairs.



5 Code febi-fsweep.f

This program is used for the computation of Echowidth vs frequency for a specific angle of incidence and observation. All the frequency dependent parameters like permittivity, permeability, impedance, resistivity, etc. are specified in the input files at the starting frequency and the ending frequency. Their values at other frequencies, lying between the starting and the ending frequencies, are calculated using linear interpolation.

6 Input Files

The computer code requires some or all of the following input files depending upon the material composition of the scatterer (2D cylinder):

1) **febi-fsweep.in** *(always required)*
   an input file generated by the user from screen prompts
2) **meshfile** *(always required)*
   a user supplied input mesh file (the name of the mesh file is specified by the user)
3) **material-f.txt** *(sometimes required)*
   a user generated input file containing information on the material composition of the scatterer
4) **ibc-sproperties-f.txt** *(sometimes required)*

a user generated input file containing information on the surface impedance of the IBC surfaces existing within the computational domain

5) **resis-sproperties-f.txt** *(sometimes required)*
   a user generated input file containing information on the parameters of the resistive and/or capacitive/resistive surfaces existing within the computational domain

Examples of input files material-f.txt, ibc-sproperties-f.txt, resis-sproperties-f.txt

material-f.txt

```
For this dielectric/magnetic material region:
Enter 1 below if this dielectric/magnetic material is isotropic
Enter 2 below if this dielectric/magnetic material is anisotropic
1
Enter relative permittivity below at fstart and fstop:
(3.0,-3.0)    (5.0,-5.0)
Enter relative permeability below at fstart and fstop:
(1.0,0.0)    (1.0,0.0)
```

ibc-sproperties-f.txt

```
Enter the value of surface impedance (in Ohms) for
the impedance surface #1 at fstart and fstop
(753.46,-753.46)    (800.46,-800.46)
```

resis-sproperties-f.txt

```
Sheet info for sheet #1
Enter 0 if it is a resistive sheet
Enter 1 if it is a capacitive/resistive sheet
0
Do you want to enter the value of resistivity
directly (enter 0 if yes) or do you want to
calculate it (enter 1 if yes)?
0
Enter the value of resistivity (in Ohms per square)
for the resistive sheet #1 at fstart and fstop
(1998.05,0.0)    (2500.0,0.0)
```

Note: The basic difference between the above input files and the ones for the febi.f is that in the above files all the frequency dependent parameters like permittivity, permeability, impedance, resistivity, etc. are specified at both the starting frequency and the ending frequency.

List of some of the important variables used in the code:

Nno : # of nodes

Nsno : # of PEC surface nodes
Nibcsno(i): # of IBC surface nodes for the impedance surface # i
Nrssno(i) : # of resistive sheet nodes for the resistive sheet # i
Nbino : # of boundary integral surface nodes
Nbno : # of boundary surface nodes
Nino : # of integration surface nodes
Ned : # of edges
Nsed : # of PEC surface edges
Nibced(i) : # of IBC surface edges for the impedance surface # i
Nrsed(i) : # of resistive sheet edges for the resistive sheet # i
Nbied : # of boundary integral surface edges
Nbed : # of boundary surface edges
Nied : # of integration surface edges
Nel : # of elements
Nmat : # of non-zero global matrix elements
x(i) : x coordinate of global node # i
y(i) : y coordinate of global node # i
conn(i,j) : Global node # for local node # j of element # i
grp(i) : Material group # for element # i
    (2=air, 4,5,6,...=dielectric/magnetic/anisotropic material)
edno(i,j) : Global node # for node # j of global edge # i
eled(i,j) : Global edge # for local edge # j of element # i
edel(i,j) : Element # for element having global edge # i as an edge
sign(i,j) : Sign for local edge # j of element # i. Positive/negative
    if the local edge is directed in/against the global edge
    direction
sno(i) : Global node # for local PEC surface node # i
sibcno(i,j): Global node # for IBC surface # i and local IBC surface node # j
srsno(i,j): Global node # for resistive sheet # i and local resistive sheet node # j
ino(i) : Global node # for local integration surface node # i
bino(i) : Global node # for local boundary integral surface node # i
bno(i) : Global node # for local boundary surface node # i
sed(i) : Global edge # for local PEC surface edge # i
sibced(i,j): Global edge # for IBC surface # i and local IBC surface edge # j
srsed(i,j): Global edge # for resistive sheet # i and local resistive sheet edge # j
ied(i) : Global edge # for local integration surface edge # i
bied(i) : Global edge # for local boundary integral surface edge # i
bed(i) : Global edge # for local boundary surface edge # i
eps(i,j) : Relative permittivity of material group # i, j=1,2,...,9
mu(i,j) : Relative permeability of materiel group # i, j=1,2,...,9
epsc(i,j) : Relative permittivity of coating material # i, j=1,2,...,9
muc(i,j) : Relative permeability of coating material # i, j=1,2,...,9
TETM : Polarization of electromagnetic field (0=TE, 1=TM)
TVFEcode : Code describing the type of TVFE used by the code :
    1 : Standard linear TVFE's (Whitney elements)
N1 : Order of element matrices

phi0 : Angle of incidence
phi : Angle of observation
Nff : Number of far field points where RCS is found
phimin : Minimum phi-value for which RCS is found
phimax : Maximum phi-value for which RCS is found
f : Frequency in Hz
eta(i,j) : Normalized surface impedance for each impedance surafce # i
and edge # j
Resis(i,j): Normalized resistivity for each resistive sheet # i
and edge # j

Note: The files "param1.h" and "param2.h", given below, contain information on the dimensioning of the arrays used in the code. If an electrically large structure is required to be solved the dimensioning of the arrays will be required to be changed in these two files. In addition to that, the dimensioning of the following arrays may also be required to be changed:

In subroutine convert(....)
v1(10000),v2(10000)

In subroutine sort(....)
v1(10000),v2(10000), v1h(10000),v2h(10000)

param1.h

```
c **********************************************************************
c Parameters for the code febi.f.
c **********************************************************************

c Variables
    Integer Nnomax,Nsnomax,Ninomax,Nbinomax,Nedmax,Nsedmax,Niedmax
  &      ,Nbiedmax,Nelmax,Nmatmax,Ncoatmax,Nbnomax,Nbedmax,Nunkmax
  &      ,Nshoedmax,Nbihoedmax,Nbhoedmax,Nhoelmax,Nhoedmax
  &      ,Nhobedmax,Nbimax,Nibcsnomax,Nibcedmax
  &      ,Nrssnomax,Nrsedmax

c Parameters
    Parameter(Nnomax=10000,Nsnomax=1500,Ninomax=1500,Nbinomax=1500
  &       ,Nibcsnomax=1500,Nibcedmax=1500
  &       ,Nrssnomax=1500,Nrsedmax=1500
  &       ,Nedmax=20000,Nsedmax=1500,Niedmax=1500,Nbiedmax=1500
  &       ,Nelmax=12000,Nmatmax=20*Nedmax,Ncoatmax=10
  &       ,Nbnomax=500,Nbedmax=500,Nunkmax=2*Nedmax+2*Nelmax
  &       ,Nshoedmax=Nsedmax,Nbihoedmax=Nbiedmax
  &       ,Nbhoedmax=Nbedmax,Nhoelmax=Nelmax,Nhoedmax=Nedmax
  &       ,Nhobedmax=500,Nbimax=2*Nbiedmax)
```

param2.h

```
c ********************************************************************
c Parameters for the code febi.f.
c ********************************************************************

c Include declarations and parameters from file 'param1.h'

     Include "param1.h"
     Include "param4.h"

c Variables

     Integer conn(Nelmax,3),grp(Nelmax),edno(Nedmax,2),eled(Nelmax,3)
     Integer sno(Nsnomax),sed(Nsedmax),sign(Nelmax,3),edel(Nedmax,2)
     Integer sibcno(10,Nibcsnomax),sibced(10,Nibcedmax)
     Integer srsno(10,Nrssnomax),srsed(10,Nrsedmax)
     Integer ieltop(10,1000),ielbottom(10,1000)
     Integer ino(Ninomax),bino(Nbinomax),ied(Niedmax),bied(Nbiedmax)
    &      ,bno(Nbnomax),bed(Nbedmax),hocode(Nelmax)
    &      ,hoel(Nhoelmax),hoed(Nhoedmax),hobed(Nhobedmax)
     Integer Nno,Nsno,Nino,Nbino,Ned,Nsed,Nied,Nbied,Nel,Nmat,Ncoat
    &      ,Nbno,Nbed,Nshoed,Nbihoed,Nbhoed,Nhoel,Nhoed,Nhobed
    &      ,Nibcsno(2000),Nibced(2000),Nrssno(2000),Nrsed(2000)
    &      ,Nrsepair(2000)
     Integer ltg1(Nelmax,N1),ltg2(Nelmax,N2),ltg3(Nelmax,N3)
    &      ,ltg4(Nelmax,N4)
     Real*8 x(Nnomax),y(Nnomax)
     Complex*16 eta(10,1500),Resis(10,1500)

c Commons

     Common/Global1/x,y,conn,grp,edno,eled,edel,sibcno,sibced
    &          ,srsno,srsed,eta,Resis
    &          ,sno,ino,bino,sed,ied,bied,sign,bno,bed,hocode
    &          ,ltg1,ltg2,ltg3,ltg4,hoel,hoed,hobed
     Common/Global2/Nno,Nsno,Nino,Nbino,Ned,Nsed,Nied,Nbied,Nel,Nmat
    &          ,Ncoat,Nbno,Nbed,Nshoed,Nbihoed,Nbhoed
    &          ,Nhoel,Nhoed,Nhobed,Nibcsno,Nibced,Nrssno,Nrsed
```

Theory Manual

Two-Dimensional Hybrid Finite Element Boundary Integral (FE-BI) Formulation for Anisotropic StructuresContaining PEC, Impedance, Resistive and Resistive/Capacitive Surfaces

1 Introduction

Integral Equation (IE) methods have for the past many years been widely used for the solution of electromagnetic problems. IE's are formulated in terms of the unknown currents or fields on the physical structure and they can be solved using, for example, the Method of Moments (MOM). IE methods are exact and provide efficient solutions for various electromagnetic problems. However, they suffer from large memory requirements (a full resulting system matrix) making the solution of the resulting equation system an unrealistic task for large-scale problems. Moreover, it is tedious if not impossible to treat different material compositions occurring in practical problems.

The Finite Element Method (FEM) has over the past few years emerged as an efficient alternative to the traditional IE approach. This method is based on the formulation of a Partial Differential Equation (PDE) instead of an IE. The PDE is solved via a discretization of the solution region (referred to as a "mesh") and an expansion of the unknown quantity within each element. Due to the local nature of all PDE formulations, the FEM benefits from a sparse resulting system matrix (i.e. most of the elements of the resulting system matrix are zero). Such a matrix has a significantly smaller memory demand than the full IE method system matrix and the resulting equation system can be efficiently solved using iterative matrix equation solvers. Furthermore, the presence of different materials and/or implementation of various boundary conditions can be elegantly incorporated in the formulation and, therefore, require no special treatment.

In this report, a two-dimensional hybrid Finite Element Boundary Integral (FE-BI) formulation is presented for electromagnetic scattering from a two-dimensional cylinder of arbitrary shape consisting of any combination of the following:

7) metal
8) dielectric/magnetic material
9) anisotropic material (with in-plane anisotropy)
10) surfaces with impedance boundary conditions (IBCs)
11) resistive sheet boundary conditions for modeling thin, penetrable dielectric sheets
12) capacitive/resistive sheets for modeling frequency dependent resistive sheets

The infinite region surrounding the cylinder is terminated via a Boundary Integral (BI). The solution region is discretized using triangular elements and Tangential Vector Finite Elements (TVFE's) or edge-based basis functions are applied. Thus, the unknowns are associated with electric (TE case) or magnetic (TM case) field values along triangle edges.

2 Formulation

2.1 TE Polarization

Consider the two-dimensional cylinder of arbitrary shape shown in figure 1. The cylinder, in general, can be composed of anisotropic material with any combination of PEC, impedance, resistive and resistive/capacitive surfaces. A plane wave is assumed to be incident on the cylinder and the observation point lies in the far zone.

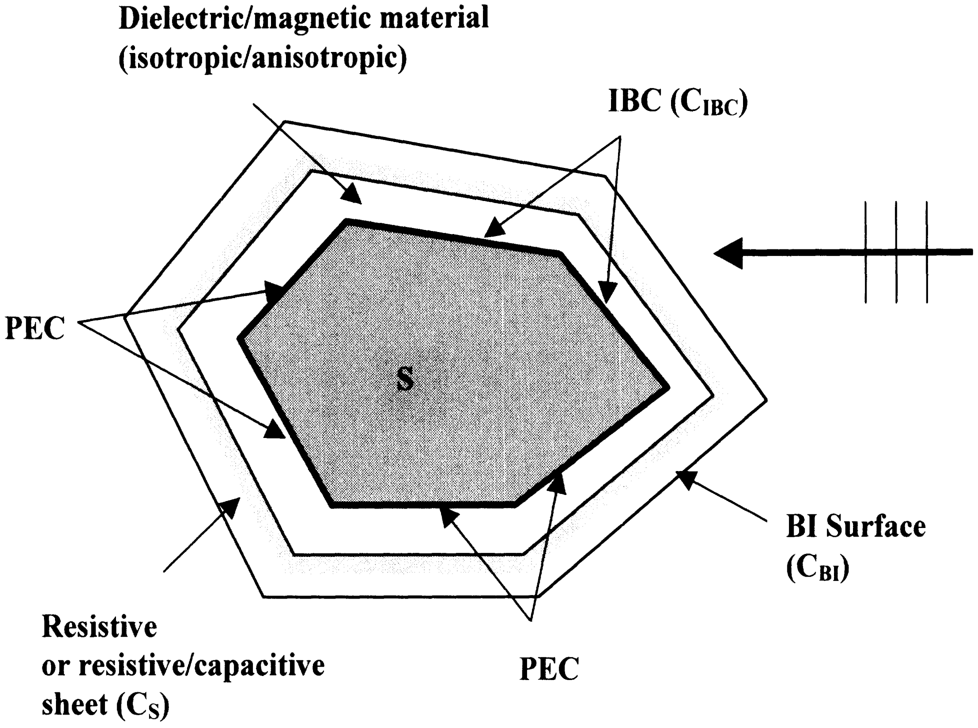


Figure 1: Geometry of a 2D cylinder of arbitrary shape

Following the residual method the weak form of the vector wave equation can be written as (exp(jωt) time convention has been assumed and suppressed throughout)

$$\iint_S \left[(\nabla_t \times \overline{W}_t) \cdot \left(\overline{\overline{\mu}}_r^{-1} \cdot \nabla_t \times \overline{E}_t \right) - k_o^2 \overline{W}_t \cdot (\overline{\overline{\varepsilon}}_r \cdot \overline{E}_t) \right] ds + \int_{C_{BI}} \overline{W}_t \cdot \left[\hat{n} \times (\overline{\overline{\mu}}_r^{-1} \cdot \nabla_t \times \overline{E}_t) \right] dl$$

$$+ \int_{C_{IBC}, C_S} \overline{W}_t \cdot \left[\hat{n} \times (\overline{\overline{\mu}}_r^{-1} \cdot \nabla_t \times \overline{E}_t) \right] dl = 0 \qquad (1)$$

where

$\overline{E}_t$ --- transverse electric field

$\overline{W}_t$ --- transverse vector testing function

∇_t --- transverse part of the ∇ operator

S --- represents the solution region

k_o --- free space wave number

C_{BI} --- corresponds to the boundary integral

C_{IBC} --- corresponds to the impedance surface

C_S --- corresponds to the resistive or resistive/capacitive sheet surface

$\overline{\overline{\varepsilon}}_r$ and $\overline{\overline{\mu}}_r$ represent the tensors shown below

$$\overline{\overline{\varepsilon}}_r = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}, \qquad \overline{\overline{\mu}}_r = \begin{bmatrix} \mu_{xx} & \mu_{xy} & 0 \\ \mu_{yx} & \mu_{yy} & 0 \\ 0 & 0 & \mu_{zz} \end{bmatrix}$$

Impedance Surface

From Maxwell's equations

$$\overline{\overline{\mu}}_r^{-1} \cdot \nabla \times \overline{E} = -jk_o Z_o \overline{H} \tag{2}$$

where Z_o is free space wave impedance.

This implies that the last term on the left hand side of equation (1) can be written as

$$\int_{C_{IBC}} \overline{W}_t \cdot \left[\hat{n} \times (\overline{\overline{\mu}}_r^{-1} \cdot \nabla_t \times \overline{E}_t) \right] dl = -jk_o Z_o \int_{C_{IBC}} \overline{W}_t \cdot (\hat{n} \times \overline{H}) dl \tag{3}$$

Now, from impedance boundary conditions, we have

$$\hat{n}_k \times (\hat{n}_k \times \overline{E}) = -\eta Z_o (\hat{n}_k \times \overline{H}) \tag{4}$$

where η is the normalized surface impedance ($\eta = Z_s / Z_o$) and $\hat{n}_k = -\hat{n}$. From (3) and (4) we get

$$\int_{C_{IBC}} \overline{W}_t \cdot \left[\hat{n} \times (\overline{\overline{\mu}}_r^{-1} \cdot \nabla_t \times \overline{E}_t) \right] dl = \frac{jk_o}{\eta} \int_{C_{IBC}} (\hat{n} \times \overline{W}_t) \cdot (\hat{n} \times \overline{E}_t) dl \tag{5}$$

Resistive or Capacitive/Resistive Sheet Surface

Using (2), the last term on the left hand side of equation (1) can be written as

$$\int_{C_S} \overline{W}_t \cdot \left[\hat{n} \times (\overline{\overline{\mu}}_r^{-1} \cdot \nabla_t \times \overline{E}_t) \right] dl = -jk_o Z_o \int_{C_S} \overline{W}_t \cdot (\hat{n} \times \overline{H}) dl \tag{6}$$

or

$$\int_{C_S} \overline{W}_t \cdot \left[\hat{n} \times (\overline{\overline{\mu}}_r^{-1} \cdot \nabla_t \times \overline{E}_t) \right] dl = -jk_o Z_o \int_{C_S} \overline{W}_t \cdot \hat{n} \times (\overline{H}^+ - \overline{H}^-) dl \tag{7}$$

Now, from resistive sheet boundary conditions, we have

$$\hat{n}_k \times \left[\hat{n}_k \times (\overline{E}^+ + \overline{E}^-) \right] = -2 Z_o R_e \hat{n}_k \times (\overline{H}^+ - \overline{H}^-) \tag{8}$$

and

$$\hat{n}_k \times (\overline{E}^+ - \overline{E}^-) = 0 \tag{9}$$

where R_e is the resistivity of the sheet and $\hat{n}_k = -\hat{n}$. The resistivity Re of a thin dielectric layer of thickness τ and dielectric constant ε_r can be computed from the following expression:

$$R_e = \frac{-j}{k_o \tau (\varepsilon_r - 1)} \tag{10}$$

From (7) - (9), we get

$$\int_{C_S} \overline{W}_t \cdot \left[\hat{n} \times (\overline{\overline{\mu}}_r^{-1} \cdot \nabla_t \times \overline{E}_t) \right] dl = \frac{jk_o}{R_e} \int_{C_S} (\hat{n} \times \overline{W}_t) \cdot (\hat{n} \times \overline{E}_t) dl \tag{11}$$

Equation (1) can now be written as

$$\begin{aligned} &\iint_S \left[(\nabla_t \times \overline{W}_t) \cdot \left(\overline{\overline{\mu}}_r^{-1} \cdot \nabla_t \times \overline{E}_t \right) - k_o^2 \overline{W}_t \cdot (\overline{\overline{\varepsilon}}_r \cdot \overline{E}_t) \right] ds + \int_{C_{BI}} \overline{W}_t \cdot \left[\hat{n} \times (\overline{\overline{\mu}}_r^{-1} \cdot \nabla_t \times \overline{E}_t) \right] dl \\ &+ \frac{jk_o}{\eta} \int_{C_{IBC}} (\hat{n} \times \overline{W}_t) \cdot (\hat{n} \times \overline{E}_t) dl + \frac{jk_o}{R_e} \int_{C_S} (\hat{n} \times \overline{W}_t) \cdot (\hat{n} \times \overline{E}_t) dl = 0 \end{aligned} \tag{12}$$

Boundary Integral

Using (2), the second term on the left hand side of equation (1) can be written as

$$\int_{C_{BI}} \overline{W}_t \cdot \left[\hat{n}\times(\overline{\overline{\mu}}_r^{-1} \cdot \nabla_t \times \overline{E}_t) \right] dl = -jk_o Z_o \int_{C_{BI}} \overline{W}_t \cdot (\hat{n}\times\overline{H}) dl \qquad (13)$$

For the field point on the boundary (C_{BI}), the above integral can be incorporated in the calculations by utilizing the integral equation given below

$$\begin{aligned} \overline{H}_{inc}(\overline{\rho}') = \overline{H}(\overline{\rho}') &+ \frac{k_o}{4Z_o} \int_{C_{BI}} H_o^{(2)}(k_o\,|\,\overline{\rho}-\overline{\rho}'\,|)\{\overline{E}_t(\overline{\rho}')\times\hat{n}'\}dl' \\ &- \frac{1}{4j}\int_{C_{BI}} \nabla\times\left[H_o^{(2)}(k_o\,|\,\overline{\rho}-\overline{\rho}'\,|)\{\hat{n}'\times\overline{H}(\overline{\rho}')\}\right]dl' \end{aligned} \qquad (14)$$

where $H_o^{(2)}$ is the Hankel function of zeroth order and second kind and $\overline{\rho}$ is the usual polar vector in a circular cylindrical coordinate system. The integral equation given above relates the unknown $\overline{H}$ and $\overline{E}_t$ fields on the boundary in terms of the known incident field $\overline{H}_{inc}$ on the boundary (C_{BI}).

Far-Field

Once the electric and magnetic fields are computed at the boundary (C_{BI}), the scattered field can then be found from the far-field expression given below

$$\overline{E}^s(\rho) \approx \sqrt{\frac{k_o}{8\pi}}\, e^{j\frac{\pi}{4}} \frac{e^{-jk_o\rho}}{\sqrt{\rho}} \int_{C_{INT}} (\hat{\rho}\times\overline{M} + Z_o\hat{\rho}\times\hat{\rho}\times\overline{J}) e^{jk_o\hat{\rho}\cdot\overline{\rho}'} dl' \qquad (15)$$

with $\overline{J} = \hat{n}\times\overline{H}$ and $\overline{M} = \overline{E}\times\hat{n}$ are the equivalent electric and magnetic currents and $Y_o = \frac{1}{Z_o}$.

Echowidth

Finally, the echowidth is computed using the following expression

$$\sigma_{2D} = \lim_{\rho\to\infty} 2\pi\rho \left| \frac{\overline{E}^s}{\overline{E}^i} \right|^2 \qquad (16)$$

2.2 TM Polarization

For the TM polarization case equation (1) can be written as

$$\begin{aligned}&\iint_S \left[(\nabla_t \times \overline{W}_t)\cdot\left(\overline{\overline{\varepsilon}}_r^{\,-1}\cdot\nabla_t\times\overline{H}_t\right) - k_o^2\overline{W}_t\cdot(\overline{\overline{\mu}}_r\cdot\overline{H}_t)\right]ds + \int_{C_{BI}} \overline{W}_t\cdot\left[\hat{n}\times(\overline{\overline{\varepsilon}}_r^{\,-1}\cdot\nabla_t\times\overline{H}_t)\right]dl \\ &+ \int_{C_{IBC},C_S} \overline{W}_t\cdot\left[\hat{n}\times(\overline{\overline{\varepsilon}}_r^{\,-1}\cdot\nabla_t\times\overline{H}_t)\right]dl = 0\end{aligned} \tag{17}$$

where

$\overline{H}_t$ --- transverse magnetic field
$\overline{W}_t$ --- transverse vector testing function
∇_t --- transverse part of the ∇ operator
S --- represents the solution region
k_o --- free space wave number
C_{BI} --- corresponds to the boundary integral
C_{IBC} --- corresponds to the impedance surface
C_S --- corresponds to the resistive or resistive/capacitive sheet surface

$\overline{\overline{\varepsilon}}_r$ and $\overline{\overline{\mu}}_r$ represent the tensors shown below

$$\overline{\overline{\varepsilon}}_r = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}, \qquad \overline{\overline{\mu}}_r = \begin{bmatrix} \mu_{xx} & \mu_{xy} & 0 \\ \mu_{yx} & \mu_{yy} & 0 \\ 0 & 0 & \mu_{zz} \end{bmatrix}$$

Impedance Surface

Following the formulation given for the TE polarization above, equation (5) can now be written as

$$\int_{C_{IBC}} \overline{W}_t\cdot\left[\hat{n}\times(\overline{\overline{\varepsilon}}_r^{\,-1}\cdot\nabla_t\times\overline{H}_t)\right]dl = jk_o\eta\int_{C_{IBC}}(\hat{n}\times\overline{W}_t)\cdot(\hat{n}\times\overline{H}_t)dl \tag{18}$$

Resistive or Capacitive/Resistive Sheet Surface

For the resistive sheet we follow the similar procedure as given above for the TE polarization and, therefore, can write the modified form of equation (11) as

$$\int_{C_S} \overline{W}_t \cdot \left[\hat{n} \times (\overline{\overline{\varepsilon}}_r^{\,-1} \cdot \nabla_t \times \overline{H}_t) \right] dl = jk_o R_e \int_{C_S} (\hat{n} \times \overline{W}_t) \cdot \left[\hat{n} \times (\overline{H}_t^{+} - \overline{H}_t^{-}) \right] dl \tag{19}$$

Note that for TM polarization the magnetic field is discontinuous across the resistive sheet. Equation (1) can now be written as

$$\begin{aligned} &\iint_{s} \left[(\nabla_t \times \overline{W}_t) \cdot \left(\overline{\overline{\varepsilon}}_r^{\,-1} \cdot \nabla_t \times \overline{H}_t \right) - k_o^2 \overline{W}_t \cdot (\overline{\overline{\mu}}_r \cdot \overline{H}_t) \right] ds + \int_{C_{BI}} \overline{W}_t \cdot \left[\hat{n} \times (\overline{\overline{\varepsilon}}_r^{\,-1} \cdot \nabla_t \times \overline{H}_t) \right] dl \\ &+ jk_o \eta \int_{C_{IBC}} (\hat{n} \times \overline{W}_t) \cdot (\hat{n} \times \overline{H}_t) dl + jk_o R_e \int_{C_S} (\hat{n} \times \overline{W}_t) \cdot \left[\hat{n} \times (\overline{H}_t^{+} - \overline{H}_t^{-}) \right] dl = 0 \end{aligned} \tag{20}$$

For the TM polarization, the rest of the formulation is quite similar to the one for the TE polarization and the relevant equations can be simply written by using duality.