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MINIMAL THICKNESS COATINGS

by

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We wish to determine how the application of a very thin layer of highly conducting material affects the scattering behavior of a dielectric body. Since the only shape for which the analysis can be carried out exactly is a sphere, the specific problem treated is that of a sphere composed of a pure (lossless) homogeneous isotropic dielectric covered with a uniform metallic layer whose thickness d is less than the skin depth δ in the metal. The body is illuminated by a plane electromagnetic wave and attention will be confined to the far zone scattered fields in the back and forward directions for sphere sizes close to resonance: $k_o a \leq 2$ where k_o is the free space propagation constant and a is the radius of the dielectric core.

General Formulation

It is convenient to examine first the general problem in which no assumptions are made about the constitutive parameters of the sphere or its coating. To this end, consider a sphere of radius a whose permittivity and permeability are ϵ_1 and μ_1 respectively which is coated with a uniform layer of thickness d whose constitutive parameters are ϵ_2 and μ_2. The whole is immersed in free space and illuminated by a plane electromagnetic wave polarised in the x direction and propagating along the -z axis (see Fig. 1). We can then write

$$\underline{E}^i = \hat{x}\, e^{-ik_o z} \quad , \quad \underline{H}^i = -Y_o \hat{y} e^{-ik_o z} \tag{1}$$

where $Y_o = {}^1/Z_o$ is the intrinsic admittance of free space. Mks units are employed and a time factor $e^{-i\omega t}$ suppressed.

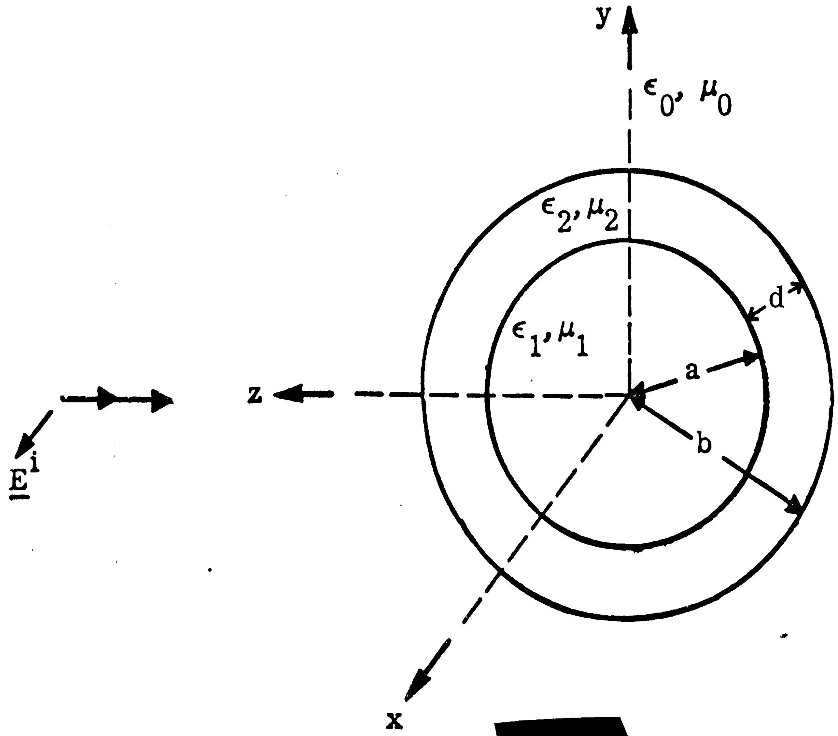

FIG. 1: The geometry.

General expressions for the scattered field are readily available in the literature (see, for example, Goodrich et al, 1961; p. 13 et seq.) and when these are particularised to the cases of far zone scattering in the backward ($\theta = 0$) and forward ($\theta = \pi$) directions, it is found that

$$S(0) = i \sum_{n=1}^{\infty} (-1)^n (n+\frac{1}{2})(A_n - B_n) \quad , \tag{2}$$

$$S(\pi) = i \sum_{n=1}^{\infty} (n+\frac{1}{2})(A_n + B_n) \quad , \tag{3}$$

where S is the far field amplitude, i.e. the coefficient of $\hat{x}\,\frac{e^{ik_o r}}{k_o r}$ in the far zone. The coefficients A_n and B_n can be written as

$$A_n = \frac{\psi_n(k_o b) + \Gamma_n^{(1)} \psi'_n(k_o b)}{\xi_n(k_o b) + \Gamma_n^{(1)} \xi'_n(k_o b)} \tag{4}$$

$$B_n = \frac{\psi'_n(k_o b) - \Gamma_n^{(2)} \psi_n(k_o b)}{\xi'_n(k_o b) - \Gamma_n^{(2)} \xi_n(k_o b)} \tag{5}$$

where

$$\psi_n(z) = z j_n(z) = \sqrt{\frac{\pi z}{2}}\, J_{n+1/2}(z) \ ,$$
$$\xi_n(z) = z h_n^{(1)}(z) = \sqrt{\frac{\pi z}{2}}\, H^{(1)}_{n+1/2}(z) \tag{6}$$

are modified spherical Bessel and Hankel functions. The primes denote differentiation with respect to the entire argument. In eqs. (4) and (5)

$$\Gamma_n^{(1)} = -\frac{Z_2}{Z_o}\,\frac{Z_1 P_1 - Z_2 P_3}{Z_1 P_2 - Z_2 P_4} \tag{7}$$

$$\Gamma_n^{(2)} = \frac{Z_2}{Z_o}\,\frac{Y_1 P_2 - Y_2 P_4}{Y_1 P_1 - Y_2 P_3} \tag{8}$$

where

$$P_1 = \psi_n(k_1 a) \left\{ \psi'_n(k_2 a)\xi_n(k_2 b) - \xi'_n(k_2 a)\psi_n(k_2 b) \right\} ,$$
$$P_2 = \psi_n(k_1 a) \left\{ \psi'_n(k_2 a)\xi'_n(k_2 b) - \xi'_n(k_2 a)\psi'_n(k_2 b) \right\} ,$$
$$P_3 = \psi'_n(k_1 a) \left\{ \psi_n(k_2 a)\xi_n(k_2 b) - \xi_n(k_2 a)\psi_n(k_2 b) \right\} , \tag{9}$$

$$P_4 = \psi'_n(k_1a) \left\{ \psi_n(k_2a)\,\xi'_n(k_2b) - \xi_n(k_2a)\,\psi'_n(k_2b) \right\} .$$

$Z_i = 1/Y_i$ and k_i are the characteristic impedance and propagation constant, respectively, of the ith medium and have the usual definitions

$$Z_i = \sqrt{\frac{\mu_i}{\epsilon_i}} \quad , \quad k_i = \omega\sqrt{\mu_i \epsilon_i} \tag{10}$$

where ϵ_i and μ_i are the electromagnetic constitutive parameters which may be complex.

It is of interest to examine the above results in certain special cases. If $b = a$, i.e. the layer thickness $d = b - a$ is zero,

$$P_2 = P_3 = 0$$

and

$$\frac{P_1}{P_4} = -\frac{\psi_n(k_1a)}{\psi'_n(k_1a)} .$$

Hence

$$\left.\begin{aligned} \Gamma_n^{(1)} &= -\frac{Z_1}{Z_0}\,\frac{\psi_n(k_1a)}{\psi'_n(k_1a)} , \\ \Gamma_n^{(2)} &= \frac{Z_1}{Z_0}\,\frac{\psi'_n(k_1a)}{\psi_n(k_1a)} \end{aligned}\right\} \tag{11}$$

and the expressions for A_n and B_n reduce to those for a homogeneous sphere of radius a, as required. On the other hand, if the core is perfectly conducting ($\mathrm{Im}\ \epsilon_1 = \infty$), $Z_1 = 0$ and

$$\Gamma_n^{(1)} = -\frac{Z_2}{Z_0}\,\frac{P_3}{P_4} , \quad \Gamma_n^{(2)} = \frac{Z_2}{Z_0}\,\frac{P_2}{P_1} \tag{12}$$

These ratios are independent of k_1, as expected, and the resulting expressions for A_n and B_n are those for a coated metal sphere (see, for example, Scharfman, 1954.) Finally, we note that if the coating is perfectly conducting ($\mathrm{Im}\ \epsilon_2 = \infty$), $Z_2 = 0$ and

$$\Gamma_n^{(1)} = \Gamma_n^{(2)} = 0 \quad . \tag{13}$$

The expressions for A_n and B_n are then identical to those for a perfectly conducting sphere of radius b. This last is a limiting case of the situation we are called upon to investigate, and a particular virtue of the expressions we have adopted is that as the layer thickness increases, $\Gamma_n^{(1)}$ and $\Gamma_n^{(2)}$ (whose computation is the crux of the problem) both approach zero.

Modal Impedances

The quantities $\Gamma_n^{(1)}$ and $\Gamma_n^{(2)}$ have a certain physical interpretation an appreciation of which is important in understanding the effect of the coating. Specifically, $i\Gamma_n^{(1)}$ and $i\Gamma_n^{(2)}$ are the effective surface impedances of the nth magnetic and electric modes respectively. To make this point more clear, consider a sphere of radius b at the surface of which the boundary condition is

$$\underline{E} - (\underline{E}\cdot\hat{n})\,\hat{n} = \eta\, Z_o\, \hat{n}\wedge\underline{H} \tag{14}$$

where $\hat{n}$ is a unit normal in the outwards (radial) direction. On solving the boundary value problem, it is found that

$$A_n = \frac{\psi_n(k_o b) - i\eta\psi_n'(k_o b)}{\xi_n(k_o b) - i\eta\xi_n'(k_o b)} \quad , \tag{15}$$

$$B_n = \frac{\psi_n'(k_o b) + i\eta\psi_n(k_o b)}{\xi_n'(k_o b) + i\eta\xi_n(k_o b)} \tag{16}$$

and comparison of (15) and (16) with (4) and (5) then leads to the above interpretation of $\Gamma_n^{(1)}$ and $\Gamma_n^{(2)}$.

Let us now examine the effect on $\Gamma_n^{(1)}$ and $\Gamma_n^{(2)}$ of increasing the propagation constant k_2 of the layer material. Such an increase can be achieved either by increasing the real part of the permittivity (or permeability) or, as in the case of direct interest to us, by increasing the conductivity. Since

$$\xi_n(z) \sim (-i)^{n+1} e^{iz} \left\{1 - \frac{n(n+1)}{2iz}\right\},$$

$$\xi_n'(z) \sim i\,\xi_n(z)$$

$$\psi_n(z) = \frac{1}{2}\left\{\xi_n(z) + \overline{\xi_n(z)}\right\}$$

$$\sim \frac{1}{2}(-i)^{n+1} e^{iz}\left\{1 - \frac{n(n+1)}{2iz}\right\} + \frac{1}{2} i^{n+1} e^{-iz}\left\{1 + \frac{n(n+1)}{2iz}\right\}$$

$$\psi_n'(z) \sim \frac{i}{2}\left\{\xi_n(z) - \overline{\xi_n(z)}\right\}$$

as $|z| \to \infty$, it follows that if $|k_2|a \gg 1$ implying $|k_2|b \gg 1$ a fortiori,

$$\psi_n(k_2 b)\,\xi_n(k_2 a) \sim \frac{1}{2} e^{-ik_2 d}\left\{1 - \frac{n(n+1)\,d/a}{2ik_2 b}\right\}$$

$$+ \frac{1}{2}(-1)^{n+1} e^{ik_2(a+b)}\left\{1 - \frac{n(n+1)\,(2 + d/a)}{2ik_2 b}\right\}$$

with similar expressions for the other combinations of functions occuring in the formulae for $P_1, \ldots, P_4$. Hence

$$P_1 \sim -i \left\{ \cos k_2 d + \frac{n(n+1)\, d/a}{2 k_2 b} \sin k_2 d \right\} \psi_n(k_1 a)$$

$$P_2 \sim i \left\{ \sin k_2 d - \frac{n(n+1)\, d/a}{2 k_2 b} \cos k_2 d \right\} \psi_n(k_1 a)$$

$$P_3 \sim i \left\{ \sin k_2 d - \frac{n(n+1)\, d/a}{2 k_2 b} \cos k_2 d \right\} \psi_n'(k_1 a) \qquad (17)$$

$$P_4 \sim i \left\{ \cos k_2 d + \frac{n(n+1)\, d/a}{2 k_2 b} \sin k_2 d \right\} \psi_n'(k_1 a)$$

and we note that the terms $O\left(\frac{1}{k_2 b}\right)$ are all proportional to d/a. The eqs. (17) are valid regardless of the layer thickness and if $d/a \ll 1$, as is certainly true in the present problem, the second term in each of the above expressions can be neglected* in comparison with the first, yielding

$$\frac{P_1}{\psi_n(k_1 a)}, \quad -\frac{P_4}{\psi_n'(k_1 a)} \sim -i \cos k_2 d$$

$$\frac{P_2}{\psi_n(k_1 a)}, \quad \frac{P_3}{\psi_n'(k_1 a)} \sim i \sin k_2 d \qquad (18)$$

from which we obtain

$$\Gamma_n^{(1)} \sim \frac{Z_2}{Z_o} \cot\left(k_2 d - \gamma_n^{(1)}\right) \qquad (19)$$

$$\Gamma_n^{(2)} \sim \frac{Z_2}{Z_o} \cot\left(k_2 d + \gamma_n^{(2)}\right) \qquad (20)$$

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*An exception occurs if $k_2 d$ is an odd or even integer multiple of $\pi/2$, but this is possible only if k_2 is real.

where

$$\tan \gamma_n^{(1)} = \frac{Z_2}{Z_1} \frac{\psi_n'(k_1 a)}{\psi_n(k_1 a)} \quad , \tag{21}$$

$$\tan \gamma_n^{(2)} = \frac{Z_2}{Z_1} \frac{\psi_n(k_1 a)}{\psi_n'(k_1 a)} \quad . \tag{22}$$

As long as $\gamma_n^{(1)}$ and $\gamma_n^{(2)}$ differ from zero, the effective surface impedances of the electric and magnetic modes differ and vary with the mode number. As $Z_2/Z_1 \rightarrow 0$, however,

$$\gamma_n^{(1)}, \; \gamma_n^{(2)} \rightarrow 0$$

and

$$\Gamma_n^{(1)}, \; \Gamma_n^{(2)} \rightarrow \frac{Z_2}{Z_o} \cot k_2 d \tag{23}$$

corresponding to a single surface impedance

$$\eta = i \frac{Z_2}{Z_o} \cot k_2 d \quad . \tag{24}$$

This is identical to the surface impedance of a highly conducting slab of thickness d illuminated by a plane wave at normal incidence. It should be noted that as $Z_2/Z_1 \rightarrow 0$, the impedance η of eq. (24) also approaches zero unless $k_2 d$ is maintained small by progressively reducing the layer thickness. In the latter case,

$$\eta \rightarrow i \frac{\epsilon_o}{\epsilon_2} \frac{1}{k_o d} \quad . \tag{25}$$

Low Frequency Scattering

The only situation in which we can arrive at an analytical expression for the scattered field is the low frequency one in which $k_1 a,\ k_o b \ll 1$. Since

$$\xi_n(z) = O(z^{-n}) \ , \qquad \psi_n(z) = O(z^{n+1})$$

for $z \ll 1$, it is evident from eqs. (4) and (5) that only the lowest order ($n = 1$) modes now contribute to the leading term in the expansion for the far zone scattered field. This is the Rayleigh scattering regime in which the scattered field is attributable to the induced electric and magnetic dipoles; and since

$$\xi_1(z) = -\frac{i}{z}\left\{1 + O(z^2)\right\} \ , \qquad \psi_1(z) = \frac{z^2}{3}\left\{1 + O(z^2)\right\}, \tag{26}$$

we have

$$A_1 = -\frac{2i}{3}(k_o b)^3 \left\{ \frac{\Gamma_1^{(1)} + \frac{1}{2} k_o b}{\Gamma_1^{(1)} - k_o b} + O(\overline{k_o b}^2) \right\} \ , \tag{27}$$

$$B_1 = -\frac{2i}{3}(k_o b)^3 \left\{ \frac{1 - \frac{1}{2} k_o b\, \Gamma_1^{(2)}}{1 + k_o b\, \Gamma_1^{(2)}} + O(\overline{k_o b}^2) \right\} . \tag{28}$$

Hence

$$S(0) = -(k_o b)^3 \left\{ \frac{\Gamma_1^{(1)} + \frac{1}{2} k_o b}{\Gamma_1^{(1)} - k_o b} - \frac{1 - \frac{1}{2} k_o b\, \Gamma_1^{(2)}}{1 + k_o b\, \Gamma_1^{(2)}} + O(\overline{k_o b}^2) \right\} \tag{29}$$

$$S(\pi) = (k_o b)^3 \left\{ \frac{\Gamma_1^{(1)} + \frac{1}{2} k_o b}{\Gamma_1^{(1)} - k_o b} + \frac{1 - \frac{1}{2} k_o b\, \Gamma_1^{(2)}}{1 + k_o b\, \Gamma_1^{(2)}} + O(\overline{k_o b}^2) \right\} \tag{30}$$

and in the particular case $\Gamma_1^{(1)} = \Gamma_1^{(2)} = 0$,

$$S(0) = \frac{3}{2}(k_o b)^3 \left\{ 1 + O(k_o b^2) \right\},$$

$$S(\pi) = \frac{1}{2}(k_o b)^3 \left\{ 1 + O(k_o b^2) \right\}, \tag{31}$$

in agreement with the known results (Bowman et al, 1969; p. 406) for a perfectly conducting sphere.

Let us now examine the expressions for $\Gamma_1^{(1)}$ and $\Gamma_1^{(2)}$ when the layer is highly conducting but very thin. Since $|k_2|\,a$ is then large, the formulae (19) and (20) for $\Gamma_n^{(1)}$ and $\Gamma_n^{(2)}$ apply, and when $k_1 a \ll 1$,

$$\tan \gamma_1^{(1)} = \frac{Z_2}{Z_1}\,\frac{2}{k_1 a}\left\{ 1 + O(k_1 a)^2 \right\},$$

$$\tan \gamma_1^{(2)} = \frac{Z_2}{Z_1}\,\frac{k_1 a}{2}\left\{ 1 + O(k_1 a^2) \right\}.$$

Hence

$$\Gamma_1^{(1)} = \frac{Z_2}{Z_o}\,\frac{Z_1 k_1 a + 2 Z_2 \tan k_2 d}{Z_1 k_1 a \tan k_2 d - 2 Z_2}$$

and if $|k_2 d|$ is sufficiently small for us to write $\tan k_2 d = k_2 d$,

$$\Gamma_1^{(1)} = -\frac{\mu_1}{\mu_o}\,\frac{k_o a}{2}\,\frac{1 + 2\dfrac{\mu_2}{\mu_1}\dfrac{d}{a}}{1 - \dfrac{1}{2}\dfrac{\mu_1}{\mu_2} k_2 a k_2 d}. \tag{32}$$

Similarly

$$\Gamma_1^{(2)} = \frac{\mu_1}{\mu_0}\,\frac{k_o}{k_1}\,\frac{2}{k_1 a}\;\frac{1-\frac{1}{2}\frac{\mu_2}{\mu_1}\,\text{[redacted]}\,k_1 a\, k_1 d}{1+2\frac{\mu_1}{\mu_2}\left(\frac{k_2}{k_1}\right)^2\frac{d}{a}}$$

$$\simeq \frac{\mu_1}{\mu_0}\,\frac{k_o}{k_1}\,\frac{2}{k_1 a}\;\frac{1}{1+2\frac{\mu_1}{\mu_2}\left(\frac{k_2}{k_1}\right)^2\frac{d}{a}} \qquad (33)$$

since $k_1 d$ is indeed negligible compared with unity.

If, for simplicity, it is assumed that $\mu_2 = \mu_1 = \mu_o$ the above expressions become

$$\Gamma_1^{(1)} = -\frac{k_o a}{2}\;\frac{1+2\frac{d}{a}}{1-\frac{1}{2}k_2 a\, k_2 d}\,, \qquad (34)$$

$$\Gamma_1^{(2)} = \frac{k_o}{k_1}\,\frac{2}{k_1 a}\;\frac{1}{\text{[redacted]}\;1+2\left(\frac{k_2}{k_1}\right)^2\frac{d}{a}} \qquad (35)$$

and since a and b are virtually identical for an electrically thin, highly conducting layer, substitution of (34) and (35) into (27) and (28) gives

$$A_1 = -\frac{i}{9}(k_o b)^3\;\frac{k_2 b\, k_2 d}{1-\frac{1}{3}k_2 b\, k_2 d}\,, \qquad (36)$$

$$B_1 = -\frac{2i}{3}(k_o b)^3\;\frac{k_1^2-k_o^2+2k_2^2\frac{d}{b}}{k_1^2+2k_o^2+2k_2^2\frac{d}{b}}\,. \qquad (37)$$

As the layer thickness $d \to 0$,

$$A_1 \to 0 \quad ,$$

$$B_1 \to -\frac{2i}{3}(k_o b)^3 \frac{N^2 - 1}{N^2 + 2} \tag{38}$$

where $N = \sqrt{\frac{\epsilon_1}{\epsilon_o}}$ is the refractive index of the dielectric, in agreement with the known results for a dielectric sphere (see, for example, Stratton, 1941; p. 572). If, on the other hand, $k_2 \to \infty$ with $d \neq 0$,

$$A_1 \to \frac{i}{3}(k_o b)^3 \quad ,$$

$$B_1 \to -\frac{2i}{3}(k_o b)^3 \quad , \tag{39}$$

and we recover the results for a perfectly conducting sphere.

Our particular interest is in the nature of the transition between these two extremes. Taking first A_1, we observe that if

$$|k_2 b \; k_2 d| << 3$$

A_1 is characteristic of a dielectric sphere, whereas if

$$|k_2 b \; k_2 d| >> 3$$

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the perfectly conducting value is obtained. In consequence, the transition between the two is centered on

$$|k_2 b \; k_2 d| \simeq 3 . \tag{40}$$

For B_1 the transition is a little harder to specify because of the occurence of

k_o and k_1 in the expression (37), but to an adequate degree of accuracy it can be assumed centered on

$$\left|\frac{k_2}{k_1}\right|^2 \frac{d}{b} \simeq \frac{1}{2} \quad . \tag{41}$$

Since, for a metal, $|k_2|^2 = 2/\delta^2$ where δ is the penetration (or skin) depth, the transitional values of d/δ are

$$\begin{aligned} A_1 &: \quad \frac{d}{\delta} = \frac{3}{2}\frac{\delta}{b} \, , \\ B_1 &: \quad \frac{d}{\delta} = \frac{1}{4}\frac{\epsilon_1}{\epsilon_o} k_o \delta k_o b \quad . \end{aligned} \tag{42}$$

Both are extremely small. As an example, consider the case $\epsilon_1 = 4\epsilon_o$ and $k_o b = 0.2$. For a typical metal at microwave frequencies, $k_o \delta = 10^{-5}$ (a value achieved by pure drawn gold at a frequency 37 MHz), giving

$$A_1 \; : \quad \frac{d}{\delta} = 7.5 \times 10^{-5}$$

$$B_1 \; : \quad \frac{d}{\delta} = 0.2 \times 10^{-5}$$

As d is decreased, the coefficient A_1 first departs from its value for a metallic sphere and, in our example, goes over completely to its value for a dielectric sphere before the transition for B_1 occurs. This last is centered on 0.2×10^{-5} and is not completed until d/δ is (say) a factor 5 smaller still. To appreciate how thin such layers are, we remark that for gold at a frequency of 37 MHz, $\delta = 1.3 \times 10^{-5}$m. Thus, for B_1, the transition is centered on a layer thickness d of only 0.26 A^o ($1\,A^o = 10^{-8}$ cm.), and since this is comparable (or less than) the dimensions of a molecule, Maxwell's theory cannot be presumed to hold. Certainly it is difficult to conceive of such a "uniform" layer being deposited.

Changing ${}^{\epsilon_1}/_{\epsilon_0}$, the conductivity of the metal used or the frequency of illumination does not drastically affect these conclusions. As b is increased, the transitional values for A_1 and B_1 approach one another and though this would, in the above example, violate the criterion for Rayleigh scattering, the first order modes will continue to be significant contributors to the scattering for all $k_o b \leq 2$. Though they will not be the only contributors for $k_o b \gg 0.2$, this argument suggests that even for $k_o b$ as large as 2 the transition between the metallic and dielectric sphere behaviors will still occur for layer thickness in the 0.1 to 10 A^o range. A simple requirement for substantial penetration of energy through the layer, e.g. $d < \delta$, is therefore irrelevant to the retention of the dielectric sphere behavior.

Computer Program

For the specific problem of concern to us

$$\mu_1 = \mu_2 = \mu_o \quad ,$$

$$\epsilon_1 \text{ is real}$$

and

$$\epsilon_2 = \epsilon_o + i\epsilon_2'' \tag{43}$$

with

$$\epsilon_2'' = \frac{\sigma}{\omega} \tag{44}$$

where σ is the conductivity (in mhos/m.) of the layer material. For a metal,

σ is of order 10^7 and the corresponding penetration depth is

$$\delta = \frac{1}{k_o} \sqrt{\frac{2\epsilon_o}{\epsilon_2''}} , \tag{45}$$

which is of order 10^{-5} m. at 50 MHz. The complex propagation constant in the metal layer is then

$$k_2 = k_o \sqrt{1 + i \frac{2}{(k_o \delta)^2}} \tag{46}$$

and whilst this is the formula used to compute k_2 in the program, an approximation which is adequate for all practical purposes is

$$k_2 \approx \frac{1+i}{\delta} . \tag{47}$$

For the above choice of permeabilities, the expressions (19) and (20) for the modal impedance factors become

$$\Gamma_n^{(1)} = \frac{k_o}{k_2} \frac{\psi_n(k_1 a) \cos k_2 d + {}^{k_1}/_{k_2}\, \psi_n'(k_1 a) \sin k_2 d}{\psi_n(k_1 a) \sin k_2 d - {}^{k_1}/_{k_2}\, \psi_n'(k_1 a) \cos k_2 d} \tag{48}$$

$$\Gamma_n^{(2)} = \frac{k_o}{k_2} \frac{\psi_n'(k_1 a) \cos k_2 d - {}^{k_1}/_{k_2}\, \psi_n(k_1 a) \sin k_2 d}{\psi_n'(k_1 a) \sin k_2 d + {}^{k_1}/_{k_2}\, \psi_n(k_1 a) \cos k_2 d} \tag{49}$$

compared with which the surface impedance factor is

$$\Gamma = \frac{k_o}{k_2} \cot k_2 d . \tag{50}$$

The computer program is written in Fortran IV and uses in a straightforward manner the expressions (48) and (49) in association with eqs. (2) through (5). Double precision is employed throughout to ensure that the wide variations in some of the factors will not introduce significant errors. The modified Bessel and Hankel functions $\psi_n(z)$ and $\xi_n(z)$ are calculated via backward and forward recursion schemes respectively using a Radiation Laboratory subroutine developed for functions with complex arguments. In eqs. (2) and (3), the infinite series for $S(0)$ and $S(\pi)$ are truncated at the 15th term. Thus, $n \leq 15$ and though this limits the spheres that can be handled to those for which $k_1 b \lesssim 6$, the limitation can be easily overcome by increasing the array sizes and NMAX.

The input data specifies the choice of parameters and also selects from several options on the composition of the output. The propagation constant k_o (KO) is measured in $m.^{-1}$ and specifies the frequency of the incident illumination via $k_o = \omega/c$ where ω is the circular frequency ($= 2\pi f$) and c is the velocity of light in free space. The sphere is specified by its outer radius b (BL) and the layer thickness d (D1); the difference is the radius a (AL) of the inner. All dimensions are in meters. The dielectric constant of the metal coating is defined in eq. (43) and is therefore specified completely by the ratio ϵ_2''/ϵ_o (E2OEO). The dielectric constant of the dielectric sphere is likewise specified by the ratio ϵ_1/ϵ_o (E1OEO).

The values assigned to I1, I2 determine the form of the output. The non-optional part of this includes:

(i) all the input parameters

(ii) the penetration depth δ in the metal (see eq. 45)

(iii) the propagation constants

and (iv) the far field amplitudes $S(0)$, $S(\pi)$ in real and imaginary parts, plus normalised cross sections $\frac{\sigma(0)}{\pi a^2}$, $\frac{\sigma(\pi)}{\pi a^2}$ and the phases arg $S(0)$, arg $S(\pi)$ in degrees, for the coated sphere and an uncoated one (quantities denoted by an asterisk) of the same size.

If I1 $\neq$ 0 (or blank), $\Gamma_n^{(1)}$ and $\Gamma_n^{(2)}$ are printed for both the coated and uncoated spheres, along with the surface impedance factor Γ (see eq. 50). This is useful for seeing how the individual modes are differentially affected.

If I2 $\neq$ 0 (or blank), the coefficients A_n and B_n are printed for both the coated and uncoated spheres. This enables the individual modal contributions to be examined, as well as the adequacy of the truncation at the 15 th term.

Input data:

card 1	k_o	KO	(F 12.5)
card 2	b	BL	(F 20.12)
card 3	d	D1	(F 20.12)
card 4	ϵ_1/ϵ_o	E1OEO	(F 20.12)
card 5	ϵ_2''/ϵ_o	E2OEO	(F 20.5)
card 6	I1, I2		(2I2)

A program listing is included in the Appendix along with the output from a typical run.

Computations

Some of the data that have been obtained for the normalised back and forward scattering cross sections $\frac{\sigma(0)}{\pi a^2}$ and $\frac{\sigma(\pi)}{\pi a^2}$ are shown in Figs. 2 through 7. In all cases the coating material is pure drawn gold whose microwave conductivity is 4.1×10^7 mhos/m. (Stratton, 1941; p. 605). The frequency has been taken to be 47.7465 MHz for which $k_o = 1\ \text{m.}^{-1}$ and $\delta = 1.136 \times 10^{-5}$ m., and the curves are drawn from the sampled values $k_o b = 0.2(0.2)2.0$ for a variety of coating thick-

[redacted]

nesses.

In Figs. 2 through 5, $\epsilon_1 = 4\epsilon_0$. Results have been obtained for coating thicknesses from 10^{-5}m. ($\approx 0.8803\ \delta$) down, but it is not until d has been reduced to about 10^{-9}m. (≈ 10 A^o) that any significant departures from the metallic sphere behavior are observed. The curves in Figs. 2 through 5 are for d = 10, 5, 1, 0.75, 0.5, 0.25 and 0.1 A^o and in each Figure the curves for a metallic and the dielectric (uncoated) sphere are included for comparison. As regards the backscattering cross section, the first thickness of coating to display any indication of the deep minimum* at $k_o b \simeq 1.36$ is 0.25 A^o, and it is necessary to go down to 0.1 A^o to reproduce the minimum with any accuracy. Even then there are still large differences from the curve for the uncoated sphere for $k_o b > 1.5$. In the forward direction the curves are more regular with no pronounced minima, but even so the curve for the thinnest coating (d = 0.1 A^o) still departs noticably from the dielectric one for $k_o b$ in the vicinity of 1.5.

The effect of a change in the permittivity is illustrated in Figs. 6 and 7 where we have taken $\epsilon_1 = 3.5\ \epsilon_0$ and reproduced only the curves for the single coating thickness d = 0.5 A^o. The departures from the dielectric sphere behavior are quite significant.

The obvious conclusion that can be drawn is that if we are to retain the dielectric sphere behavior in the presence of a metallic coating, the coating thickness must be extremely small ($\sim 10^{-6}\delta$). For small spheres in the Rayleigh region this same conclusion was previously arrived at analytically, and it is not hard to see why it is also valid for larger spheres.

Given a plane wave in free space incident normally on a slab of material

---

* This minimum is undoubtedly a consequence of the single-bounce (glory ray) contribution; see, for example, Peters, 1969.

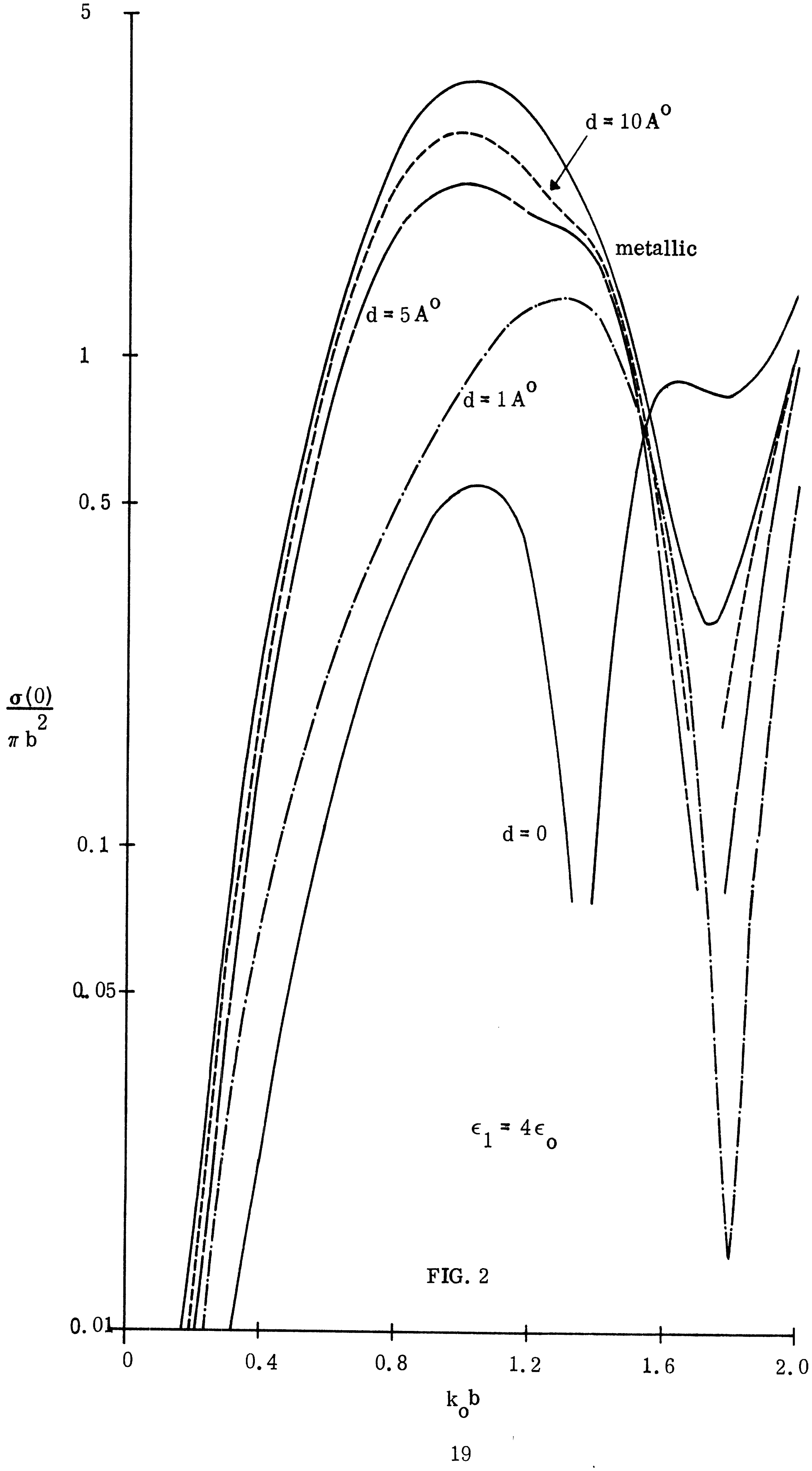
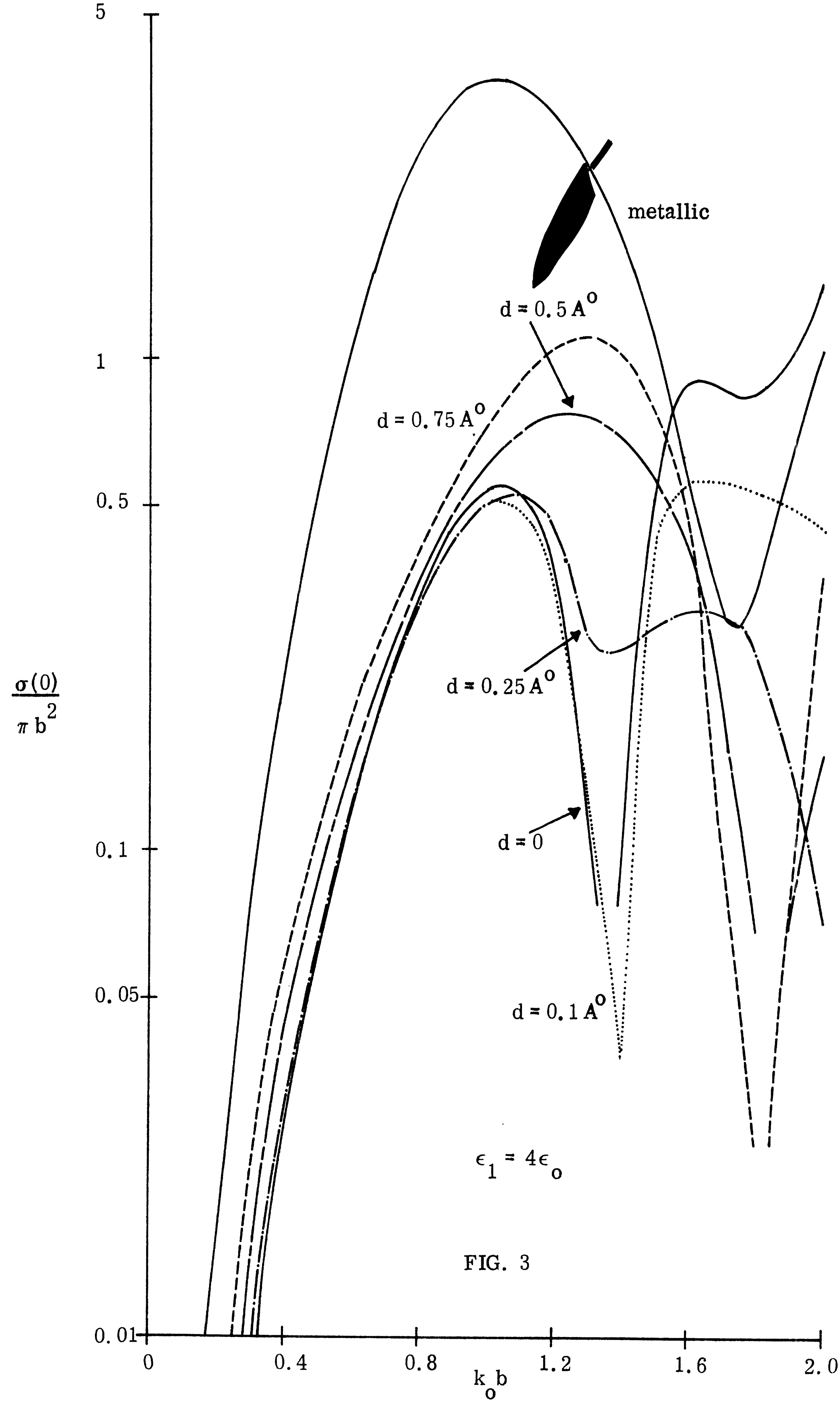
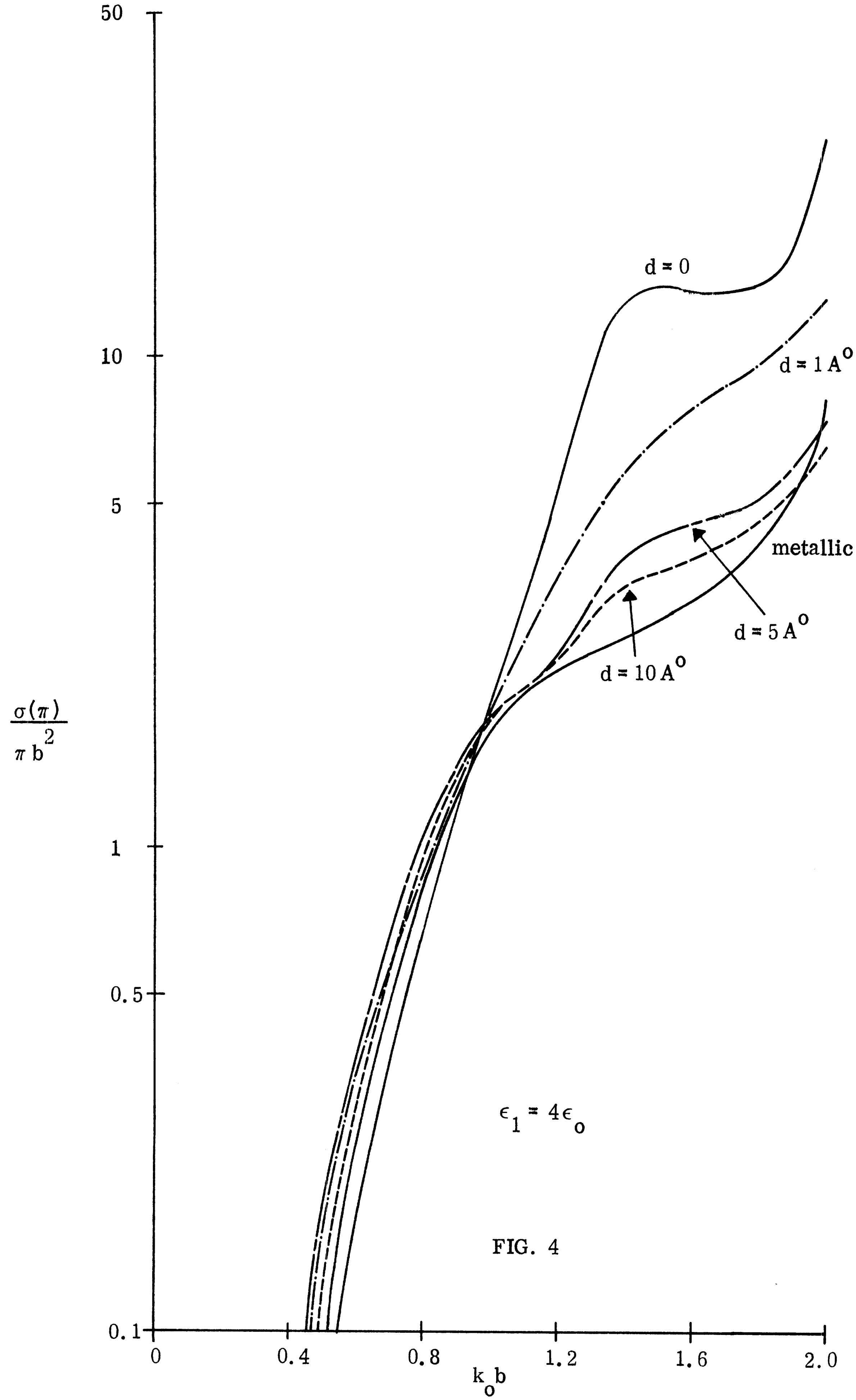

5
1
0.5
0.1
0.05
0.01
$\frac{\sigma(0)}{\pi b^2}$
d = 10 A°
metallic
d = 5 A°
d = 1 A°
d = 0
$\epsilon_1 = 4\epsilon_o$
0
0.4
0.8
1.2
1.6
2.0
$k_o b$

FIG. 2



5
1
0.5
0.1
0.05
0.01
0
0.4
0.8
1.2
1.6
2.0
$k_o b$
$\frac{\sigma(0)}{\pi b^2}$
metallic
d = 0.5 A°
d = 0.75 A°
d = 0.25 A°
d = 0
d = 0.1 A°
$\epsilon_1 = 4\epsilon_o$

FIG. 3



50
10
5
1
0.5
0.1
$\frac{\sigma(\pi)}{\pi b^2}$
0
0.4
0.8
1.2
1.6
2.0
$k_o b$
d = 0
d = 1 A°
metallic
d = 5 A°
d = 10 A°
$\epsilon_1 = 4\epsilon_o$

FIG. 4

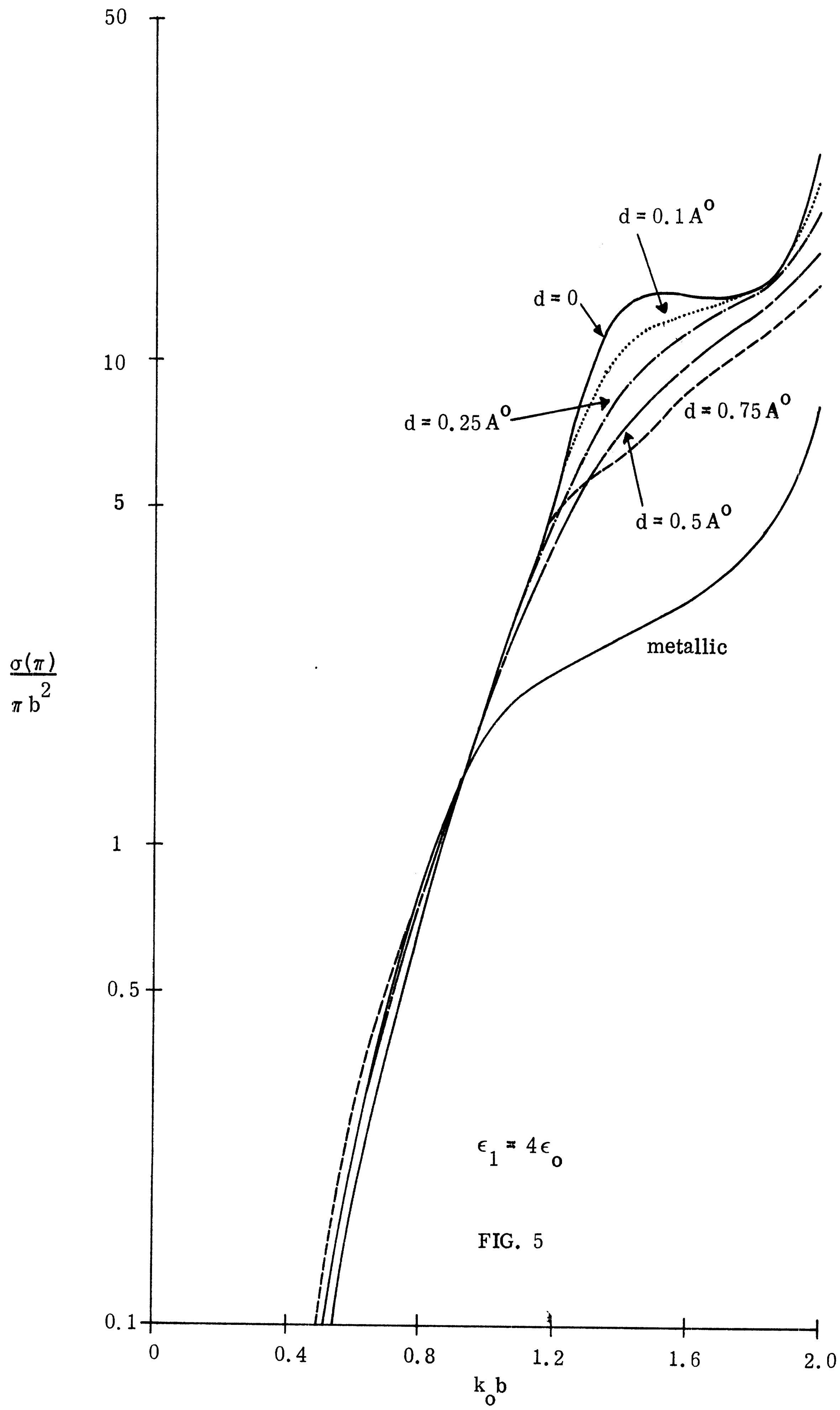

50
10
5
1
0.5
0.1
$\frac{\sigma(\pi)}{\pi b^2}$
d = 0.1 A°
d = 0
d = 0.25 A°
d = 0.75 A°
d = 0.5 A°
metallic
$\epsilon_1 = 4\epsilon_o$
0
0.4
0.8
1.2
1.6
2.0
$k_o b$

FIG. 5

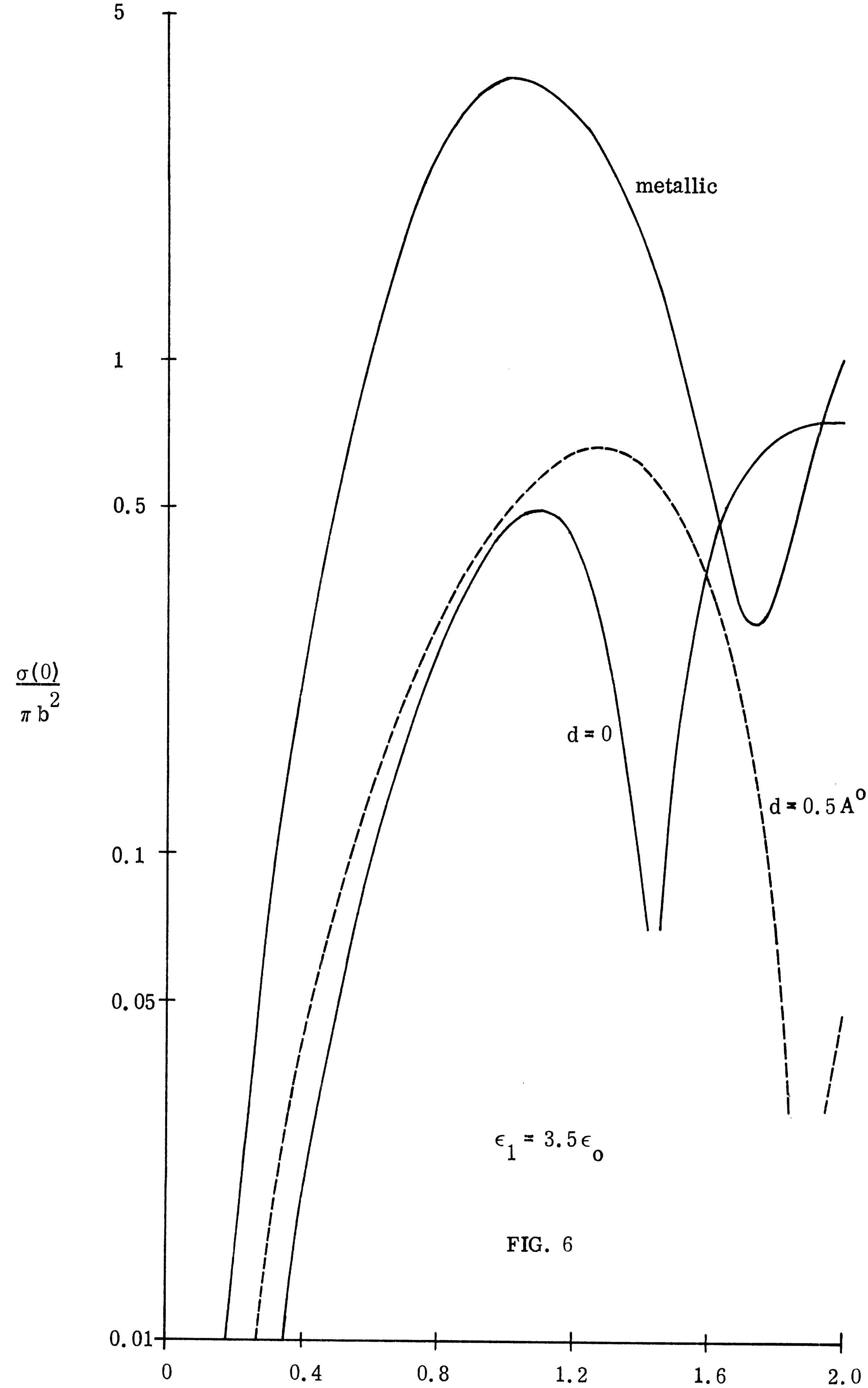

5
1
0.5
0.1
0.05
0.01
0
0.4
0.8
1.2
1.6
2.0
metallic
d = 0
d = 0.5 A°
$\frac{\sigma(0)}{\pi b^2}$
$k_o b$
$\epsilon_1 = 3.5\epsilon_o$

FIG. 6

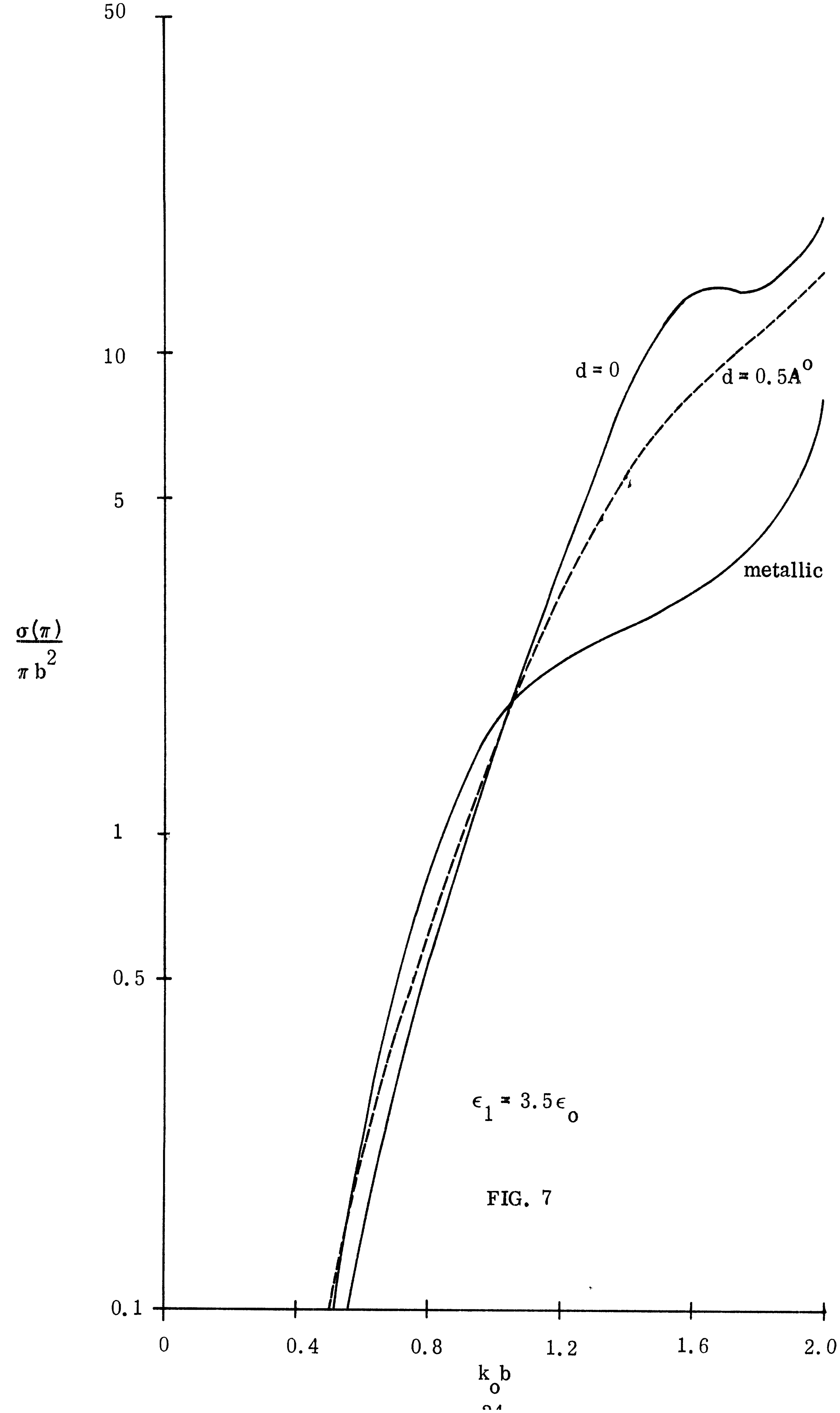

50
10
5
1
0.5
0.1
$\frac{\sigma(\pi)}{\pi b^2}$
d = 0
d = 0.5A°
metallic
$\epsilon_1 = 3.5\epsilon_o$
0
0.4
0.8
1.2
1.6
2.0
$k_o b$

FIG. 7

having $\epsilon = \epsilon_2$, $\mu = \mu_2$ and thickness d, and backed by infinitely extended material having $\epsilon = \epsilon_1$, $\mu = \mu_1$, the magnitude of the transmission coefficient is

$$T = \frac{2}{\left| \left(1 + \frac{Z_1}{Z_o}\right) \cos k_2 d - i \left(\frac{Z_2}{Z_o} + \frac{Z_1}{Z_2}\right) \sin k_2 d \right|} . \qquad (51)$$

If the slab material is metallic and $k_2 d \ll 1$, eq. (51) reduces to

$$T \simeq 2 \left\{ 1 + \sqrt{\frac{\epsilon_o}{\epsilon_1}} \left(1 + 2 \frac{d/\delta}{k_o \delta}\right) \right\}^{-1}$$

where we have put $\mu_1 = \mu_2 = \mu_o$, and for this to exceed (say) 50 percent, the requirement is

$$\frac{d}{\delta} < \frac{k_o \delta}{2} \left(3 \sqrt{\frac{\epsilon_1}{\epsilon_o}} - 1\right). \qquad (52)$$

The right hand side is of order 10^{-5} at 50 MHz, which again implies a layer thickness comparable to 1 $\mathrm{\mathring{A}}$. In other words, the extreme mismatch at the surface of the metal for all except the most infinitesimal thicknesses prevents any penetration through.

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Peters, L., Jr. (1969), Modified geometrical optics for dielectric scattering: Short course "Application of optical methods to microwave problems," The Ohio State University.

Scharfman, H. (1954), Scattering from dielectric coated spheres in the region of the first resonance, J. Appl. Phys. 25, 1352-1356.

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Appendix

Computer Program

```
0001       COMPLEX*16 ABES(2,15),ADBES(2,15),AHANK(2,15),ADHANK(2,15),
     C     THE A---- ARRAYS JUST STORE THE CORRESPONDING
     C     SPHERICAL BESSEL FUNCTIONS.
          1OUT(4,15)
0002       COMPLEX*16 BES(17),DBES(17),HANK(17),DHANK(17)
0003       COMPLEX*16 MULT,A,B,C,D,X,KB,KOK1,KOK2,S1,S2,S3,S4,KD
0004       COMPLEX*16 AI,BI,CI,DI
0005       REAL*8 A0B,K0,K0B,AL,BL,E10E0,E20E0,D1,DELT,T
0006       REAL*8 SIG1,SIG2,SIG3,SIG4,DREAL,DIMAG
0007       REAL*8 P1,P2,P3,P4,R1,R2,R3,R4
0008       MULT(A,B,C,D)=A*B-C*D
     C     TAKES THE REAL PART.
0009       DREAL(A)=(A+DCONJG(A))/2.D0
     C     TAKES THE IMAGINARY PART.
0010       DIMAG(A)=(0.0,-1.0)*(A-DCONJG(A))/2.D0
0011       PIE=3.1415962
     C     TO INCREASE THE NUMBER OF MODES TO BE CALCULATED
     C     NMAX MUST BE INCREASED AS WELL AS ALL ARRAYS.
0012       NMAX=16
     C     THE INPUT IS READ IN AS FOLLOWS;K0/BL/D1/E10E0/E20E0/I1,I2
0013       READ(5,1) K0
     C     K0 IS THE FREE-SPACE PROPAGATION CONSTANT.
0014     1 FORMAT(F12.5)
0015       READ(5,1) BL
     C     BL IS THE RADIUS OF THE OUTER SPHERE.
0016       READ(5,61) D1
0017       READ(5,61) E10E0
     C     E10E0 IS THE RELATIVE DIELECTRIC CONSTANT OF
     C     THE INNER SPHERE.
0018    61 FORMAT(F20.12)
0019       READ(5,62) E20E0
     C     E20E0 IS THE IMAGINARY PART OF THE DIELECTRIC
     C     CONSTANT OF THE METAL COATING.
0020    62 FORMAT(F20.5)
0021       READ(5,40) I1,I2
0022    40 FORMAT(2I1)
```

```
      AL=BL-D1
      WRITE(6,2) AL
    2 FORMAT('1','THE RADIUS OF THE INNER SPHERE IS',
     1F20.12,2X,'METERS')
      WRITE(6,3) BL
    3 FORMAT(' ','THE RADIUS OF THE OUTER SPHERE IS',
     1F20.8,2X,'METERS')
      WRITE(6,4) D1
    4 FORMAT(' ','THE THICKNESS OF THE COATING IS',
     1D20.8,2X,'METERS')
C     PENETRATION DEPTH CALCULATIONS.
      DELT=DSQRT(2.0D0/(KO*KO*E20E0))
      WRITE(6,5) DELT
    5 FORMAT(' ','THE PENETRATION DEPTH IS',D15.7,2X,'METERS')
      T=D1/DELT
      WRITE(6,6) T
    6 FORMAT(' ','THE COATING THICKNESS IS ',D15.7,2X,'TIMES
     1THE PENETRATION DEPTH')
      KOB=KO*BL
      WRITE(6,7) KOB
    7 FORMAT(' ','KOB IS ',F15.7)
      KB=DCMPLX(KOB,0.DC)
      AOB=AL/BL
      T=1.0000/DSQRT(E10E0)
C     THE RELATIVE PROPAGATION CONSTANTS ARE DETERMINED.
      KOK1=DCMPLX(T,0.D0)
      KOK2=DCMPLX(1.0D0,E20E0)
      KOK2=1.0000D/CDSQRT(KOK2)
      X=KB
      WRITE(6,71) E20E0
   71 FORMAT(' ','E2/E0 IS',2X,D15.7)
      WRITE(6,72) E10E0
   72 FORMAT(' ','E1/E0 IS',2X,D15.7)
      WRITE(6,73) KOK1
   73 FORMAT(' ','KO/K1 IS',2(2X,D15.7))
      WRITE(6,74) KOK2
   74 FORMAT(' ','KO/K2 IS',2X,D15.7,2X,D15.7)
```

```
      CALL CBES(BES,DBES,HANK,DHANK,X,NMAX)
C     THE SPHERICAL BESSEL FUNCTIONS FOR ARGUMENT KOB
C     ARE CALCULATED.
      DO 8 I=2,NMAX
      ABES(1,I)=BES(I)
      ADBES(1,I)=DBES(I)
      AHANK(1,I)=HANK(I)
    8 ADHANK(1,I)=DHANK(I)
C     THE SPHERICAL BESSEL FUNCTIONS FOR ARGUMENT K1A
C     ARE CALCULATED.
      X=KB*AOB/KOK1
      CALL CBES(BES,DBES,HANK,DHANK,X,NMAX)
      DO 11 I=2,NMAX
      ABES(4,I)=BES(I)
      ADBES(4,I)=DBES(I)
      AHANK(4,I)=HANK(I)
   11 ADHANK(4,I)=DHANK(I)
C     THE SPHERICAL BESSEL FUNCTIONS FOR ARGUMENT K1B
C     ARE CALCULATED.
      X=X/AOB
      CALL CBES(BES,DBES,HANK,DHANK,X,NMAX)
      A=DCMPLX(1.0D0,E20E0)
C     K2D IS CALCULATED.
      A=D1*D1*A
      B=CDSQRT(A)
      KD=KO*B
      WRITE(6,101) KD
  101 FORMAT(' ','K2D IS ',D20.12,2X,D20.12)
      A=CDCOS(KD)
      B=-CDSIN(KD)/(KB*AOB/KOK2)
      C=(KB/KOK2)*CDSIN(KD)
      D=CDCOS(KD)/AOB
      AI=A
      BI=B*(KOK1/KOK2)*(KOK1/KOK2)
      CI=KOK2*KOK2*C
      DI=KOK1*KOK1*D
```

```
      DO 12 J=2,NMAX
      AHANK(4,J)=MULT(ABES(4,J),A,ADBES(4,J),B)
      AHANK(4,J)=AHANK(4,J)/(MULT(ABES(4,J),C,ADBES(4,J),D))
C     PROPORTIONAL TO GAMMA-1.
      ADHANK(4,J)=MULT(ABES(4,J),AI,ADBES(4,J),BI)
      ADHANK(4,J)=ADHANK(4,J)/(MULT(ABES(4,J),CI,ADBES(4,J),DI))
C     PROPORTIONAL TO GAMMA-2.
      OUT(1,J)=ABES(1,J)+ADBES(1,J)*AHANK(4,J)
      OUT(1,J)=OUT(1,J)/(AHANK(1,J)+ADHANK(1,J)*AHANK(4,J))
C     AN=OUT(1,N).
      OUT(2,J)=ABES(1,J)+ADBES(1,J)*ADHANK(4,J)
      OUT(2,J)=OUT(2,J)/(AHANK(1,J)+ADHANK(1,J)*ADHANK(4,J))
C     BN=OUT(2,N).
      HANK(J)=-BES(J)/DBES(J)
C     *-ED QUANTITIES REFER TO THE DIELECTRIC SPHERE.
C     PROPORTIONAL TO GAMMA-1*.
      DHANK(J)=HANK(J)/(KOK1*KOK1)
C     PROPORTIONAL TO GAMMA-2*.
      OUT(3,J)=ABES(1,J)+ADBES(1,J)*HANK(J)
      OUT(3,J)=OUT(3,J)/(AHANK(1,J)+ADHANK(1,J)*HANK(J))
C     AN*=OUT(3,N).
      OUT(4,J)=ABES(1,J)+ADBES(1,J)*DHANK(J)
      OUT(4,J)=OUT(4,J)/(AHANK(1,J)+ADHANK(1,J)*DHANK(J))
C     BN*=OUT(4,N).
   12 CONTINUE
C     OPTIONAL OUTPUT RELATED TO I1.
      IF(I1.EQ.0) GO TO 16
      WRITE(6,13)
   13 FORMAT(' ',15X,'ORDER',23X,'GAMMA-1',23X,'GAMMA-2')
      DO 15 K=2,NMAX
      L=K-1
      AHANK(4,K)=KB*AHANK(4,K)
      ADHANK(4,K)=-1.0D0/(KB*ADHANK(4,K))
      WRITE(6,14) L,AHANK(4,K),ADHANK(4,K)
   14 FORMAT(' ',15X,I5,4(5X,D15.7))
   15 CONTINUE
```

```
      WRITE(6,30)
   30 FORMAT(' ',15X,'ORDER',22X,'GAMMA-1*',22X,'GAMMA-2*')
      DO 31 K=2,NMAX
      L=K-1
      HANK(K)=KB*HANK(K)
      DHANK(K)=-1.0D0/(KB*DHANK(K))
      WRITE(6,32) L,HANK(K),DHANK(K)
   32 FORMAT(' ',15X,I5,4(5X,D15.7))
   31 CONTINUE
      IF(D1.EQ.0.0) GO TO 16
      X=KOK2*CDCOS(KD)/CDSIN(KD)
      WRITE(6,83) X
   83 FORMAT(' ','THE IMPEDANCE OF THE SURFACE LAYER IS',
     12X,D15.7,2X,D15.7)
   16 S1=0.00000
      S2=0.00000
      S3=0.00000
      S4=0.00000
      DO 17 M=2,NMAX
      K=M-1
      H=K
      S1=((-1)**K)*(H+.5000)*(OUT(1,M)-OUT(2,M))+S1
      S2=(H+.5000)*(OUT(1,M)+OUT(2,M))+S2
      S3=((-1)**K)*(H+.5000)*(OUT(3,M)-OUT(4,M))+S3
      S4=(H+.5000)*(OUT(3,M)+OUT(4,M))+S4
   17 CONTINUE
      S1=(0.0,1.0)*S1
      S2=(0.0,1.0)*S2
      S3=(0.0,1.0)*S3
      S4=(0.0,1.0)*S4
      WRITE(6,41) S1
   41 FORMAT(' ','S(O) IS',D20.8,5X,D20.8)
      WRITE(6,42) S2
   42 FORMAT(' ','S(PIF) IS',D20.8,5X,D20.8)
      WRITE(6,43) S3
   43 FORMAT(' ','S(O)* IS',D20.8,5X,D20.8)
      WRITE(6,44) S4
   44 FORMAT(' ','S(PIF)* IS',D20.8,2X,D20.8)
```

```
      SIG1=S1*DCONJG(S1)/(KOB/2.000)**2
      P1=DREAL(S1)
      R1=DIMAG(S1)
      P1=DATAN2(R1,P1)
      P1=P1*180.0/PIE
      SIG2=S2*DCONJG(S2)/(KOB/2.000)**2
      P2=DREAL(S2)
      R2=DIMAG(S2)
      P2=DATAN2(R2,P2)
      P2=P2*180.0/PIE
      SIG3=S3*DCONJG(S3)/(KOB/2.000)**2
      P3=DREAL(S3)
      R3=DIMAG(S3)
      P3=DATAN2(R3,P3)
      P3=P3*180.0/PIE
      SIG4=S4*DCONJG(S4)/(KOB/2.000)**2
      P4=DREAL(S4)
      R4=DIMAG(S4)
      P4=DATAN2(R4,P4)
      P4=P4*180.0/PIE
C     N IS THE NORMALIZATION FACTOR
      WRITE(6,45) SIG1,P1
   45 FORMAT(' ','SIGMA(0)/N IS ',F15.7
     1,2X,',PHI IS',F15.7,2X,'DEGREES')
      WRITE(6,46) SIG2,P2
   46 FORMAT(' ','SIGMA(PIE)/N IS',F15.7
     1,2X,',PHI IS ',F15.7,2X,'DEGREES')
      WRITE(6,47) SIG3,P3
   47 FORMAT(' ','SIGMA(0)*/N IS',F15.7
     1,2X,',PHI IS ',F15.7,2X,'DEGREES')
      WRITE(6,48) SIG4,P4
   48 FORMAT(' ','SIGMA(PIE)*/N IS',F15.7
     1,2X,',PHI IS',F15.7,2X,'DEGREES')
```

```
C     OPTIONAL OUTPUT RELATED TO I2.
      IF(I2.EQ.0) GO TO 100
      WRITE(6,25)
   25 FORMAT(' ',10X,'SPHERE WITH LAYER',26X,'DIELECTRIC SPHERE')
      DO 103 JJ=2,NMAX
      WRITE(6,102) OUT(1,JJ),OUT(2,JJ),OUT(3,JJ),OUT(4,JJ)
  102 FORMAT(' ',8D13.5)
  103 CONTINUE
  100 CONTINUE
      STOP
      END
```

[redacted]

```
      C      CBES CALCULATES THE SPHERICAL BESSEL FUNCTIONS
      C      UP TO ORDER NMAX WITH ARGUMENT X.
0001         SUBROUTINE CBES(BES,DBES,HANK,DHANK,X,NMAX)
0002         COMPLEX*16 BES(17),DBES(17),HANK(17),DHANK(17),I
0003         COMPLEX*16 X,JO,J(90),P,Y(75)
      C      INDEX IS ONE GREATER THAN THE ORDER.
      C      BES(N+1)=JN(X),DBES(N+1)=(XJN(X))',
      C      HANK(N+1)=HN(X),DHANK(N+1)=(XHN(X))'
0004         I=(0.00,1.00)
0005         NN=NMAX+16
0006         NO=NN-1
0007         NP=NN-2
0008         NO=NMAX+1
0009         NR=NMAX+2
0010         J(NN)=(0.00,0.00)
0011         J(NO)=(1.00,1.00)
0012         JO=CDSIN(X)/X
0013         Y(1)=-CDCOS(X)/X
0014         Y(2)=Y(1)/X-JO
0015         DO 10 M=1,NP
0016         N=NO-M
0017      10 J(N)=(2*(N-1)+3)*J(N+1)/X-J(N+2)
0018         DO 12 N=3,NR
0019      12 Y(N)=((2*(N-1)-1)/X)*Y(N-1)-Y(N-2)
0020         P=JO/J(1)
0021         BES(1)=J(1)*P
0022         HANK(1)=BES(1)+I*Y(1)
0023         DO 20 L=1,NO
0024         K=L+1
0025         BES(K)=J(K)*P
0026         HANK(K)=BES(K)+I*Y(K)
0027         DBES(L)=L*BES(L)-X*BES(K)
0028         DHANK(L)=L*HANK(L)-X*HANK(K)
0029      20 CONTINUE
0030         RETURN
0031         END
```

Sample Output

```
THE RADIUS OF THE INNER SPHERE IS        0.99999999000   METERS
THE RADIUS OF THE OUTER SPHERE IS            1.00000000   METERS
THE THICKNESS OF THE COATING IS      0.10000000D-08  METERS
THE PENETRATION DEPTH IS  0.1135924D-04  METERS
THE COATING THICKNESS IS   0.8803408D-04  TIMES           THE PENETRATION DEPTH
K0B IS        1.0000000
E2/E0 IS     0.1550000D 11
F1/E0 IS     0.4000000D 01
K0/K1 IS     0.5000000D 00     0.0
K0/K2 IS     0.5679618D-05   -0.5679618D-05
K2D IS   0.8803408431110D-04     0.8803408430550D-04
```

ORDER	GAMMA-1		GAMMA-2	
1	-0.4508173D-02	-0.6419956D-01	0.1448251D-01	-0.6108235D-01
2	-0.9709307D-02	-0.6302025D-01	0.6891510D-02	-0.6377139D-01
3	-0.1399083D-01	-0.6132416D-01	0.4683221D-02	-0.6417436D-01
4	-0.1767906D-01	-0.5924016D-01	0.3588164D-02	-0.6431595D-01
5	-0.2085921D-01	-0.5586449D-01	0.2922230D-02	-0.6438350D-01
6	-0.2356682D-01	-0.5428505D-01	0.2470619D-02	-0.6442138D-01
7	-0.2582985D-01	-0.5158166D-01	0.2142686D-02	-0.6444489D-01
8	-0.2767948D-01	-0.4382396D-01	0.1893065D-02	-0.6446053D-01
9	-0.2915161D-01	-0.4606989D-01	0.1696364D-02	-0.6447149D-01
10	-0.3028546D-01	-0.4336542D-01	0.1537189D-02	-0.6447948D-01
11	-0.3112156D-01	-0.4074520D-01	0.1405639D-02	-0.6448549D-01
12	-0.3169974D-01	-0.3823380D-01	0.1295035D-02	-0.6449012D-01
13	-0.3205776D-01	-0.3584728D-01	0.1200708D-02	-0.6449377D-01
14	-0.3223036D-01	-0.3359483D-01	0.1119287D-02	-0.6449670D-01
15	-0.3224869D-01	-0.3149023D-01	0.1048277D-02	-0.6449909D-01
ORDER	GAMMA-1*		GAMMA-2*	
1	-0.9187552D 00	0.0	0.2721073D 00	0.0
2	-0.4187552D 00	0.0	0.5970075D 00	0.0
3	-0.2827840D 00	0.0	0.8840670D 00	0.0
4	-0.2161349D 00	0.0	0.1156417D 01	0.0
5	-0.1758781D 00	0.0	0.1421439D 01	0.0
6	-0.1486099D 00	0.0	0.1682257D 01	0.0
7	-0.1288373D 00	0.0	0.1940431D 01	0.0
8	-0.1138003D 00	0.0	0.2196831D 01	0.0
9	-0.1019584D 00	0.0	0.2451980D 01	0.0
10	-0.9237994D-01	0.0	0.2706215D 01	0.0
11	-0.8446630D-01	0.0	0.2959760D 01	0.0
12	-0.7781442D-01	0.0	0.3212772D 01	0.0
13	-0.7214252D-01	0.0	0.3465363D 01	0.0
14	-0.6724742D-01	0.0	0.3717615D 01	0.0
15	-0.6297378D-01	0.0	0.3969591D 01	0.0

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THE IMPEDANCE OF THE SURFACE LAYER IS   -0.3291710D-09    -0.6451613D-01
S(0) IS          0.77373104D 00                0.34765660D 00
S(PIE) IS          0.34327150D 00                0.57947234D 00
S(0)* IS         0.32238805D 00                0.17324210D 00
S(PIE)* IS          0.67997936D 00             0.19920757D 00
SIGMA(0)/N IS            2.8780993   ,PHI IS      24.1955448  DEGREES
SIGMA(PIE)/N IS            1.8144940   ,PHI IS      59.3580184  DEGREES
SIGMA(0)*/N IS           0.5357875   ,PHI IS      28.2522821  DEGREES
SIGMA(PIE)*/N IS            2.0082223   ,PHI IS      16.3285203  DEGREES
        SPHERE WITH LAYER                                   DIELECTRIC SPHERE
  0.72211D-01  0.19240D 00  0.30082D 00 -0.39702D 00   0.81419D-02 -0.89865D-01   0.12414D 00 -0.32974D 00
  0.48846D-02  0.15854D-01  0.27560D-02 -0.29972D-01   0.40229D-05 -0.20057D-02   0.30793D-03 -0.17545D-01
  0.19988D-03  0.45954D-03  0.30354D-04 -0.75295D-03   0.90584D-09 -0.30097D-04   0.19643D-06 -0.44321D-03
  0.38961D-05  0.68240D-05  0.33937D-06 -0.11385D-04   0.89211D-13 -0.29868D-06   0.44996D-10 -0.67079D-05
  0.44079D-07  0.62546D-07  0.26951D-08 -0.11208D-06   0.43070D-17 -0.20753D-08   0.43668D-14 -0.66081D-07
  0.32734D-09  0.39037D-09  0.15573D-10 -0.77109D-09   0.11279D-21 -0.10620D-10   0.20704D-18 -0.45501D-09
  0.17235D-11  0.17777D-11  0.68123D-13 -0.39094D-11   0.17336D-26 -0.41636D-13   0.53316D-23 -0.23090D-11
  0.67879D-14  0.61635D-14  0.23304D-15 -0.15201D-13   0.16635D-31 -0.12898D-15   0.80762D-28 -0.89868D-14
  0.20795D-16  0.16863D-16  0.64003D-18 -0.46754D-16   0.10469D-36 -0.32356D-18   0.76533D-33 -0.27665D-16
  0.51065D-19  0.37434D-19  0.14416D-20 -0.11656D-18   0.44975D-42 -0.67064D-21   0.47647D-38 -0.69027D-19
  0.10290D-21  0.68814D-22  0.27106D-23 -0.24030D-21   0.13634D-47 -0.11677D-23   0.20279D-43 -0.14241D-21
  0.17342D-24  0.10663D-24  0.43178D-26 -0.41642D-24   0.29993D-53 -0.17319D-26   0.60980D-49 -0.24694D-24
  0.24828D-27  0.14127D-27  0.59012D-29 -0.61505D-27   0.49029D-59 -0.22142D-29   0.13319D-54 -0.36495D-27
  0.30589D-30  0.16197D-30  0.69950D-32 -0.78346D-30   0.60791D-65 -0.24656D-32   0.21635D-60 -0.46514D-30
  0.32794D-33  0.16239D-33  0.72593D-35 -0.86951D-33   0.58202D-71 -0.24125D-35   0.26676D-66 -0.51649D-33

 STOP      0
EXECUTION TERMINATED
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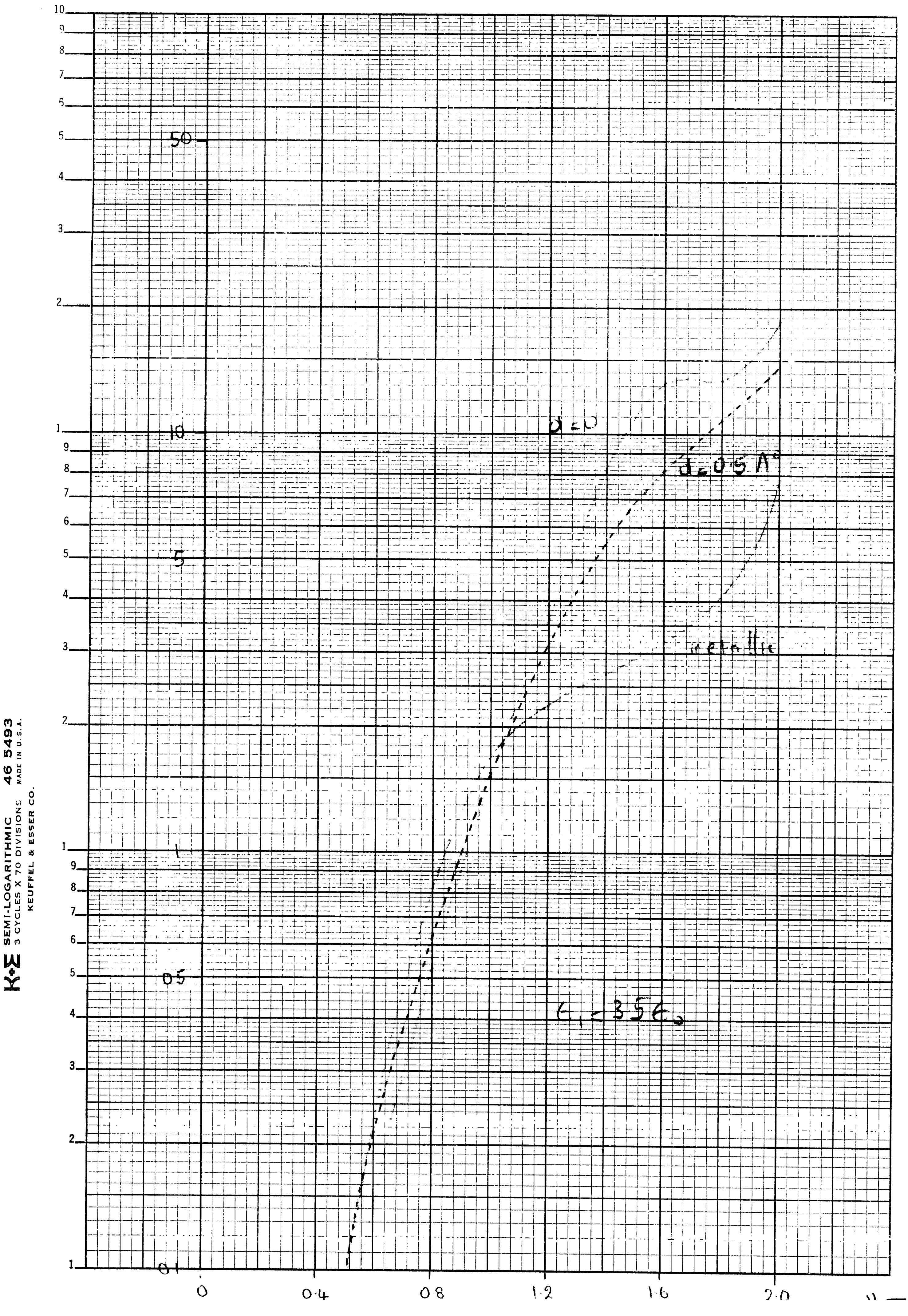
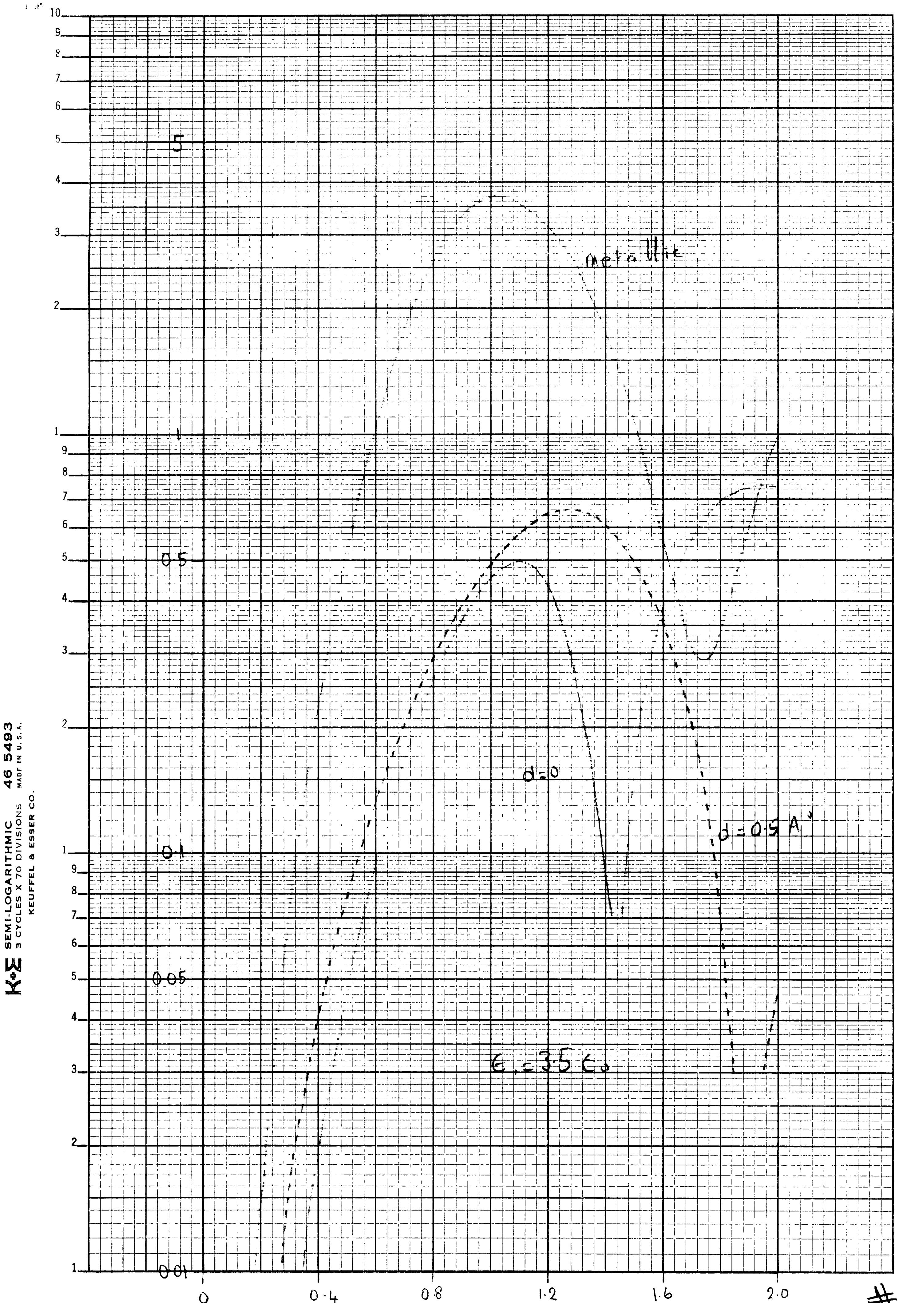
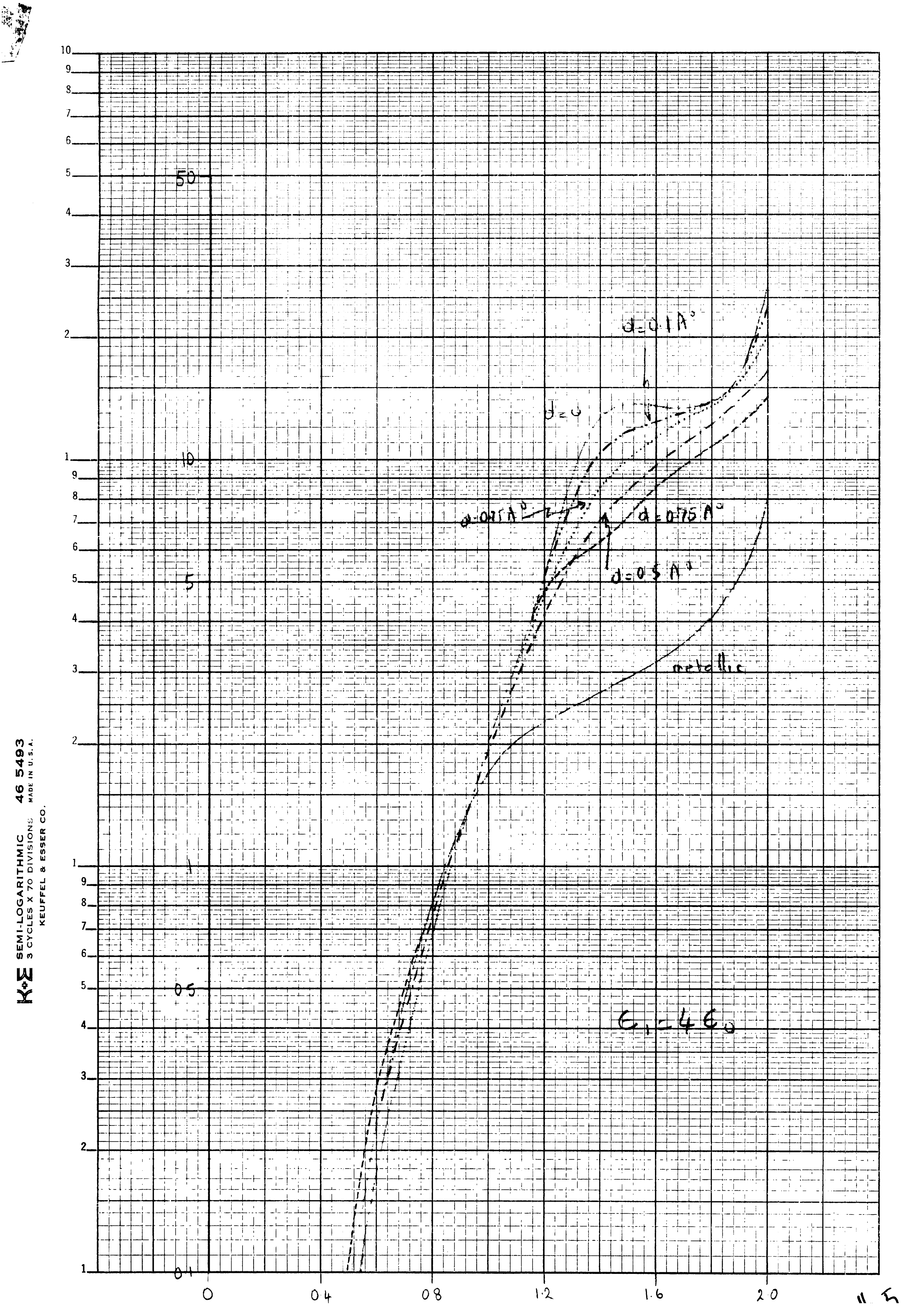


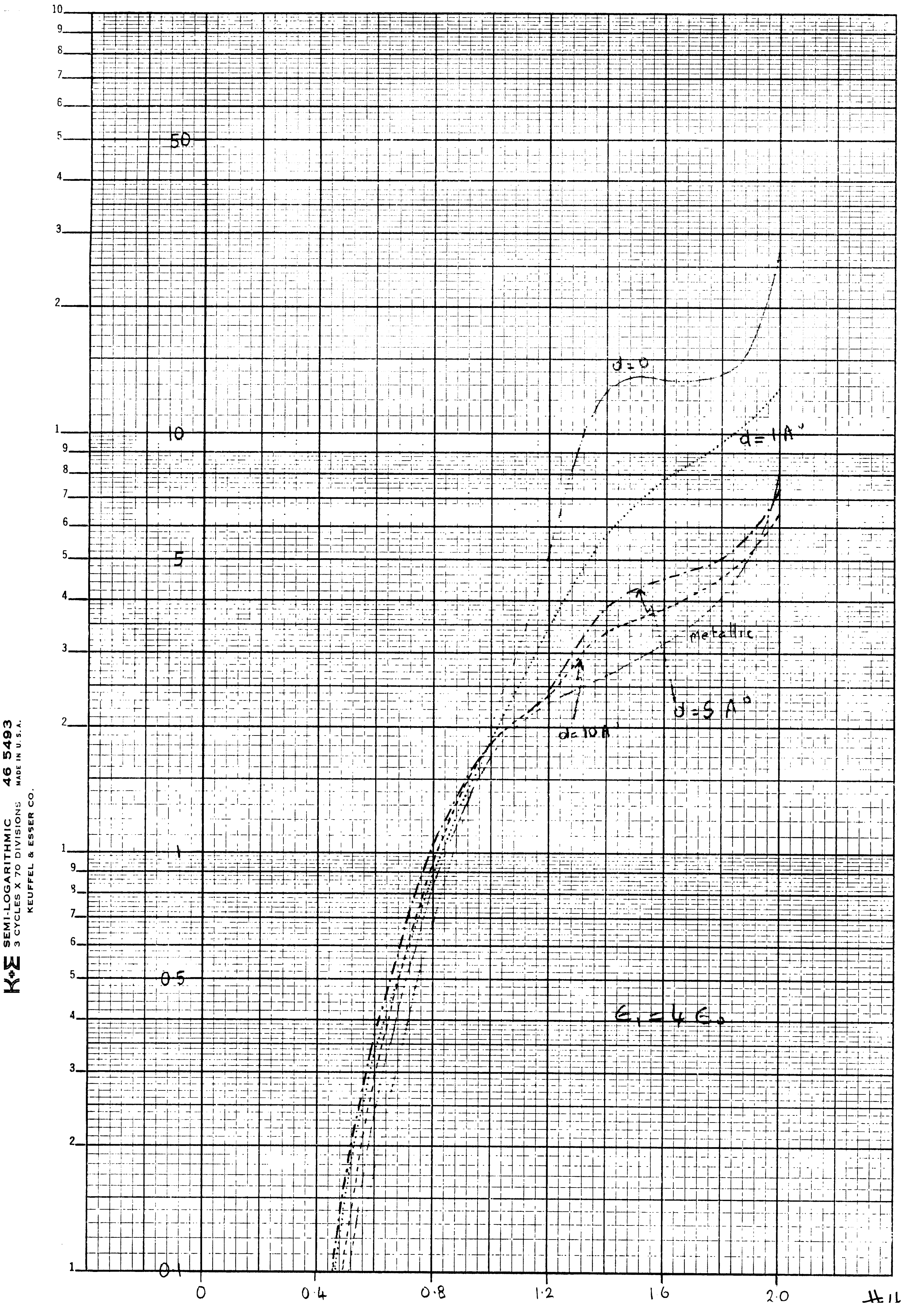
50
10
5
1
0.5
0.1
0
0.4
0.8
1.2
1.6
2.0
d=0
d=0.5 A°
metallic
$\epsilon_1 = 3.5\epsilon_0$
K+E SEMI-LOGARITHMIC 46 5493
3 CYCLES X 70 DIVISIONS MADE IN U.S.A.
KEUFFEL & ESSER CO.



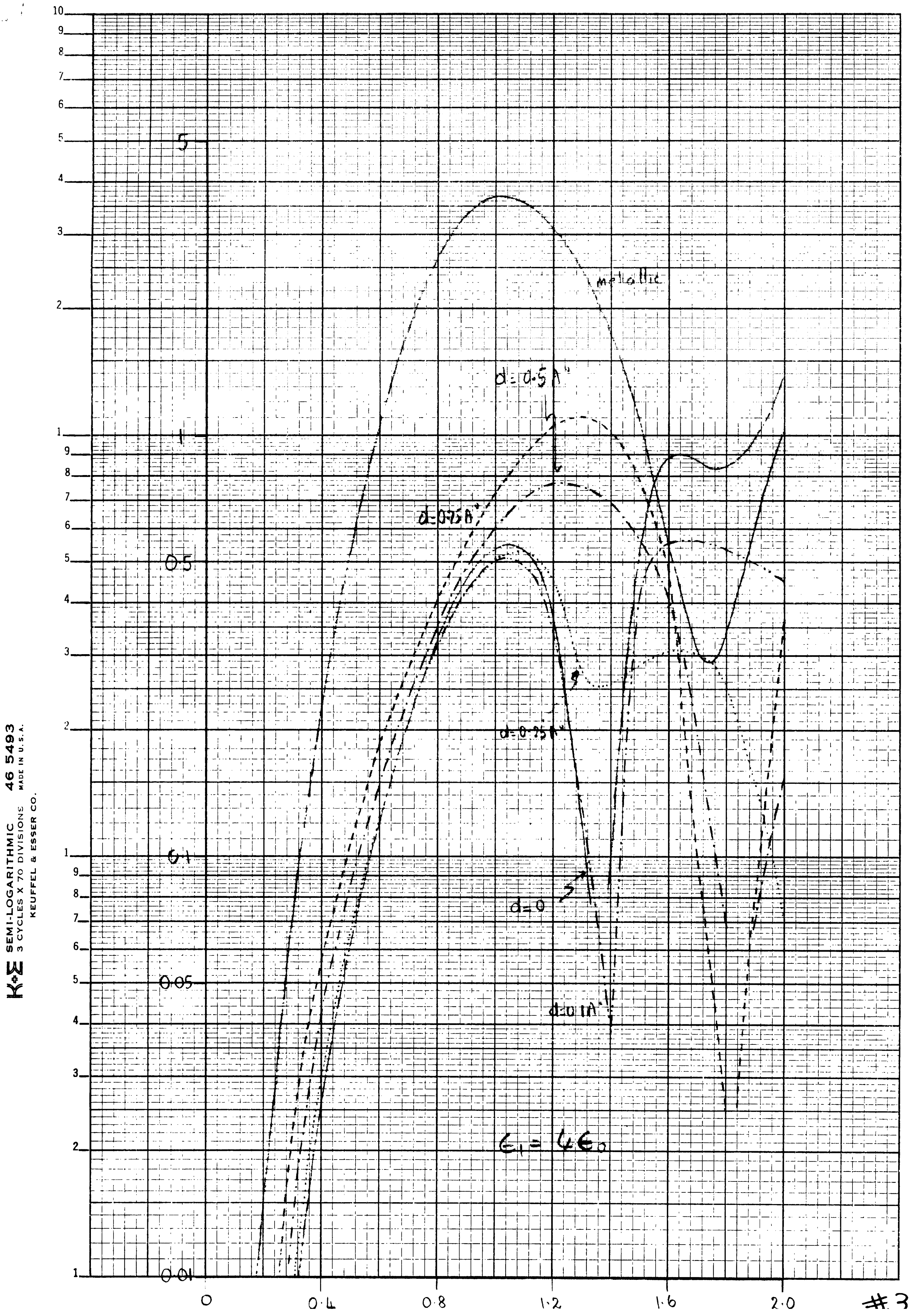
metallic
d=0
d=0.5 A°
ε = 3.5 ε₀
5
1
0.5
0.1
0.05
0.01
0
0.4
0.8
1.2
1.6
2.0
K+E SEMI-LOGARITHMIC 46 5493
3 CYCLES X 70 DIVISIONS MADE IN U.S.A.
KEUFFEL & ESSER CO.

#6


d = 0.1 A°
d = 0
d = 0.25 A°
d = 0.75 A°
d = 0.5 A°
metallic
ε₁ = 4 ε₀
50
10
5
1
0.5
0.1
0
0.4
0.8
1.2
1.6
2.0
K+E SEMI-LOGARITHMIC 46 5493
3 CYCLES X 70 DIVISIONS MADE IN U.S.A.
KEUFFEL & ESSER CO.

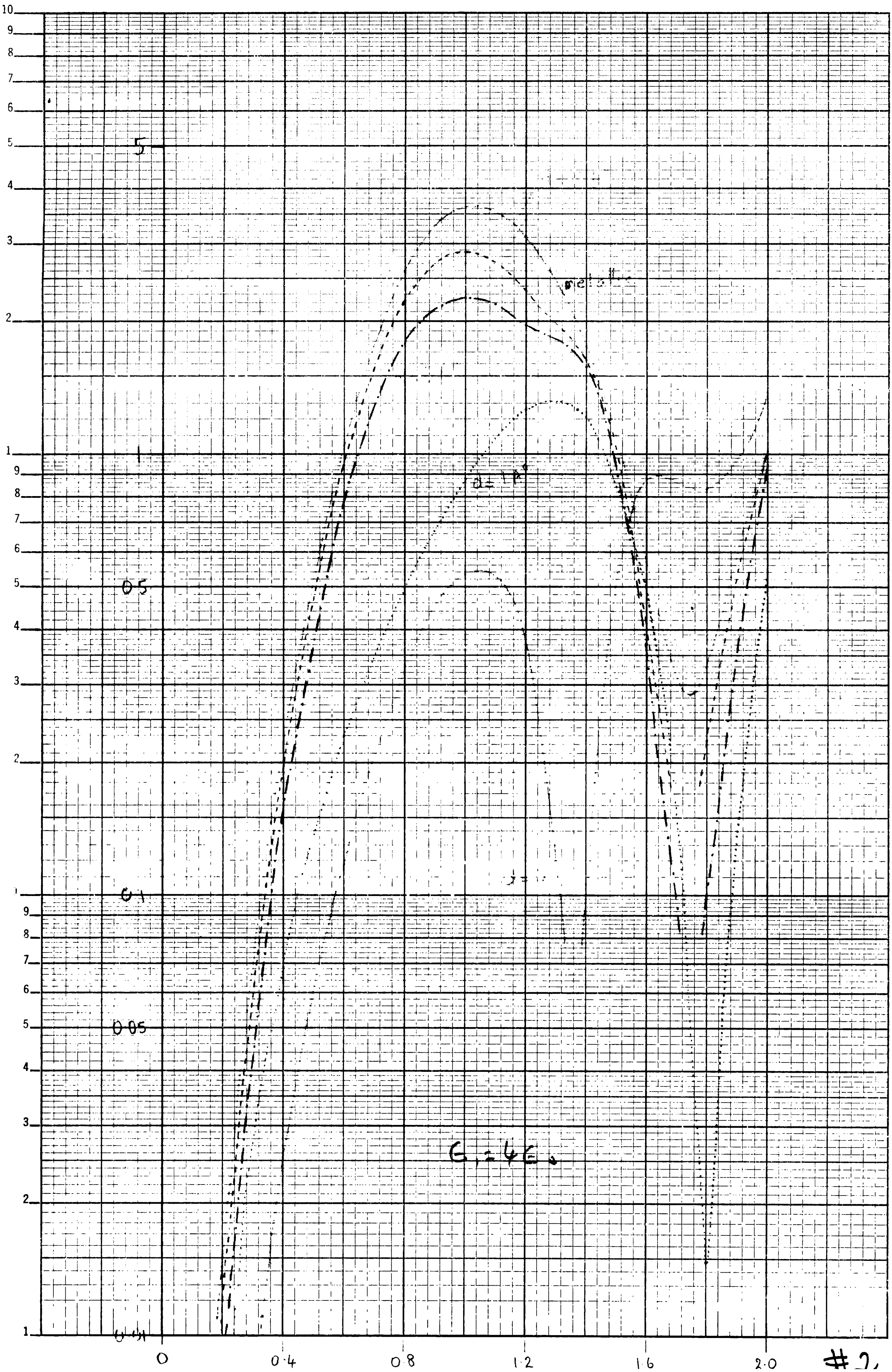


50
10
5
1
0.5
0.1
0
0.4
0.8
1.2
1.6
2.0
d=0
d = 1 A°
Metallic
d = 5 A°
d=10 A°
ε₁ = 4 ε₀
#11
K+E SEMI-LOGARITHMIC 46 5493
3 CYCLES X 70 DIVISIONS MADE IN U.S.A.
KEUFFEL & ESSER CO.



5
1
0.5
0.1
0.05
0.01
0
0.4
0.8
1.2
1.6
2.0
metallic
d = 0.5 Å
d = 0.75 Å
d = 0.25 Å
d = 0
d = 0.1 Å
$\epsilon_1 = 4\epsilon_0$
K+E SEMI-LOGARITHMIC 46 5493
3 CYCLES X 70 DIVISIONS MADE IN U.S.A.
KEUFFEL & ESSER CO.

#3



#2

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THE RADIUS OF  THE INNER SPHERE IS       0.99990000000  METERS
THE RADIUS OF  THE OUTER SPHERE IS            1.00000000  METERS
THE THICKNESS OF THE COATING IS       0.10000000D-03  METERS
THE PENETRATION DEPTH IS  0.1135924D-04  METERS
THE COATING THICKNESS IS    0.88034080 01  TIMES           THE PENETRATION DEPTH
KOB IS          1.0000000
E2/E0 IS     0.1550000D 11
E1/E0 IS     0.4000000D 01
K0/K1 IS     0.5000000D 00     0.0
K0/K2 IS     0.5679618D-05    -0.5679618D-05
K2D IS    0.880340843111D 01     0.880340843055D 01
            ORDER                     GAMMA-1                               GAMMA-2
              1       -0.5679619D-05     -0.5679618D-05     -0.5679619D-05     -0.5679618D-05
              2       -0.5679619D-05     -0.5679618D-05     -0.5679619D-05     -0.5679618D-05
              3       -0.5679619D-05     -0.5679618D-05     -0.5679619D-05     -0.5679618D-05
              4       -0.5679619D-05     -0.5679618D-05     -0.5679619D-05     -0.5679618D-05
              5       -0.5679619D-05     -0.5679618D-05     -0.5679619D-05     -0.5679618D-05
              6       -0.5679619D-05     -0.5679618D-05     -0.5679619D-05     -0.5679618D-05
              7       -0.5679619D-05     -0.5679618D-05     -0.5679619D-05     -0.5679618D-05
              8       -0.5679619D-05     -0.5679618D-05     -0.5679619D-05     -0.5679618D-05
              9       -0.5679619D-05     -0.5679618D-05     -0.5679619D-05     -0.5679618D-05
             10       -0.5679619D-05     -0.5679618D-05     -0.5679619D-05     -0.5679618D-05
             11       -0.5679619D-05     -0.5679618D-05     -0.5679619D-05     -0.5679618D-05
             12       -0.5679619D-05     -0.5679618D-05     -0.5679619D-05     -0.5679618D-05
             13       -0.5679619D-05     -0.5679618D-05     -0.5679619D-05     -0.5679618D-05
             14       -0.5679619D-05     -0.5679618D-05     -0.5679619D-05     -0.5679618D-05
             15       -0.5679619D-05     -0.5679618D-05     -0.5679619D-05     -0.5679618D-05
            ORDER                     GAMMA-1*                              GAMMA-2*
              1       -0.9187552D 00       0.0              0.2721073D 00       0.0
              2       -0.4187552D 00       0.0              0.5970075D 00       0.0
              3       -0.2827840D 00       0.0              0.8840670D 00       0.0
              4       -0.2161349D 00       0.0              0.1156417D 01       0.0
              5       -0.1758781D 00       0.0              0.1421439D 01       0.0
              6       -0.1486099D 00       0.0              0.1682257D 01       0.0
              7       -0.1288373D 00       0.0              0.1940431D 01       0.0
              8       -0.1138003D 00       0.0              0.2196831D 01       0.0
              9       -0.1019584D 00       0.0              0.2451980D 01       0.0
             10       -0.9237994D-01       0.0              0.2706215D 01       0.0
             11       -0.8446630D-01       0.0              0.2959760D 01       0.0
             12       -0.7781442D-01       0.0              0.3212772D 01       0.0
             13       -0.7214252D-01       0.0              0.3465363D 01       0.0
             14       -0.6724742D-01       0.0              0.3717615D 01       0.0
             15       -0.6297878D-01       0.0              0.3969591D 01       0.0
THE IMPEDANCE OF THE SURFACE LAYER IS   -0.5679619D-05   -0.5679618D-05
S(0) IS        0.8796205CD 00            0.3683C756D 00
S(PIE) IS        0.40351678D 00              0.50898104D 00
S(0)* IS       0.32239805D 00              0.17324210D 00
S(PIE)* IS       0.67997936D 00          0.19920757D 00
SIGMA(0)/N IS        3.6375307   ,PHI IS     22.7196171  DEGREES
SIGMA(PIE)/N IS        1.6875500   ,PHI IS      51.5928005  DEGREES
SIGMA(0)*/N IS       0.5357875   ,PHI IS     28.2522821  DEGREES
SIGMA(PIE)*/N IS        2.0082223   ,PHI IS     16.3285203  DEGREES
        SPHERE WITH LAYER                           DIELECTRIC SPHERE
  0.45353D-01  0.20807D 00  0.29193D 00 -0.45465D 00  0.81419D-02 -0.89865D-01  0.12414D 00 -0.32974D 00
  0.29645D-03  0.17202D-01  0.92264D-03 -0.30358D-01  0.40229D-05 -0.20057D-02  0.30793D-03 -0.17545D-01
  0.31333D-06  0.54113D-03  0.57398D-06 -0.75587D-03  0.90584D-09 -0.30097D-04  0.19643D-06 -0.44321D-0
  0.52577D-09  0.89547D-05  0.16020D-09 -0.11409D-04  0.89211D-13 -0.29968D-06  0.44996D-10 -0.67079D-0
  0.56942D-11  0.92607D-07  0.25067D-12 -0.11224D-06  0.43070D-17 -0.20753D-08  0.43668D-14 -0.66081D-0
  0.47969D-13  0.65770D-09  0.13749D-14 -0.77185D-09  0.11279D-21 -0.10620D-10  0.20704D-18 -0.45501D-0
  0.28789D-15  0.34102D-11  0.60080D-17 -0.39123D-11  0.17336D-26 -0.41636D-13  0.53316D-23 -0.23090D-1

  0.12928D-17  0.13484D-13  0.20545D-19 -0.15210D-13  0.16635D-31 -0.12898D-15  0.80762D-28 -0.89868D-1
  0.45088D-20  0.42018D-16  0.56408D-22 -0.46776D-16  0.10469D-36 -0.32356D-18  0.76533D-33 -0.27665D-1
  0.12568D-22  0.10585D-18  0.12703D-24 -0.11661D-18  0.44975D-42 -0.67064D-21  0.47647D-38 -0.69027D-1
  0.28642D-25  0.22009D-21  0.23881D-27 -0.24038D-21  0.13634D-47 -0.11677D-23  0.20279D-43 -0.14241D-2
  0.54368D-28  0.38413D-24  0.38036D-30 -0.41653D-24  0.29993D-53 -0.17319D-26  0.60980D-49 -0.24694D-2
  0.87291D-31  0.57080D-27  0.51979D-33 -0.61519D-27  0.49029D-59 -0.22142D-29  0.13319D-54 -0.36495D-2
  0.12010D-33  0.73088D-30  0.61609D-36 -0.78361D-30  0.60791D-65 -0.24656D-32  0.21635D-60 -0.46514D-3
  0.14317D-36  0.81483D-33  0.63934D-39 -0.86966D-33  0.58202D-71 -0.24125D-35  0.26676D-66 -0.51649D-3

 STOP     0
EXECUTION TERMINATED
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