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DOPPLER RADIATION STUDY

Interim Report No. 1

1 July - 1 October 1968

By C-M Chu

Contract N62269-68-C-0715

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Contract With: Naval Air Development Center
Johnsville, Warminster, Pennsylvania 18974

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FOREWORD

This report (1969-1-Q) was prepared by the University of Michigan Radiation Laboratory, Department of Electrical Engineering. The report was written under Contract N62269-68-C-0715 "Doppler Radiation Study" and covers the period 1 July - 1 October 1968. The research was carried out under the direction of Professor Ralph E. Hiatt, Head of the Radiation Laboratory, and the Principal Investigator was Professor Chiao-Min Chu. The sponsor of this research is the U.S. Naval Air Development Center, Johnsville, Pennsylvania and the Technical Monitor was Mr. Edward Rickner

ABSTRACT

In order to arrive at a model for the bistatic cross-section of a rough surface, which is adequate for the estimation of the reflected radiation from a doppler radar, a survey of the literature on the experimental work and theoretical models, concerning rough surface scattering, has been carried out. As a result of this survey, the dominant characteristics of radiation reflected from rough surfaces, and the basic approaches in arriving at various theoretical models are summarized.

It is suggested that for the present work, a theoretical model based on the statistics of the slope of the surface, incorporating the effect of wind is probably adequate. An outline of the approach in arriving at the expressions for the bistatic cross-section based on the slope characteristics is given.

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INTRODUCTION

This is the first quarterly report on Contract N6229-68-C-0715, "Doppler Radiation Study," and covers the period 1 July through 1 October 1968.

Contract N6229-68-C-0715 is a continuation of Contract N62269-67-C-0545, in the study of the characteristics of the radiation from airborne doppler navigational radar systems. The conclusions derived from the numerical computations of the radiation from a doppler antenna such as AN/ADN-153 seem to indicate that the diffusely reflected radiation from the ground is appreciable and should be investigated more thoroughly. Since our previous calculations were based on the simple model of a Lambert surface, the principal effort of this contracting period is to investigate the realistic model for the reflective properties of a rough surface (particularly of the sea surface) that are appropriate in the calculation of the diffusely reflected radiation.

An extensive literature survey on the theoretical and experimental results concerning the reflection from rough surfaces was carried out. It has been found that various phenomenological and/or statistical models have been proposed to characterize the reflective properties of rough surfaces. Some of these models have been experimentally checked under specific experimental conditions, such as a given frequency, fixed angle of incidence or reflection, or both, and therefore it is difficult to appraise their validity in other situations. Some of these models are theoretically plausible and allow the "scaling" of the reflective properties in wavelength and/or some characterisitic roughness of the surface, but have not been checked experimentally.

In Chapter II of this report, the experimental results on the basic characteristics of the reflected radiation from rough surfaces (particularly the sea surface), which are reported in the open literature available, are summarized. In Chapter III, various theoretical and phenomenological models concerning the reflection of waves from rough surfaces are summarized, and in some cases extended.

It is hoped that a detailed correlation study between the experimental results and various proposed theoretical models will enable us to choose, or propose a practical model for the reflective properties of a rough surface (particularly the sea), which is appropriate for the present work. It is also expected that the proposed model may involve some numerical constants that can be determined experimentally, and suggest some simple, scaled experiments for their determination. This phase of the work has been started in this research period and shall be the primary objective for the next research period.

II

REFLECTIVE PROPERTIES OF ROUGH SURFACES

2.1 Introduction

The study of the diffused reflection from a rough surface has been conducted experimentally and theoretically during the past few decades in connection with radar echoes from rough surfaces. An extensive survey of the theoretical and experimental results has been given in the book by Bechmann and Spizzichino (1963). Since then, additional theoretical and experimental investigations have been carried out by various investigators in the attempt to clarify the effect of polarization, angle of incidence and surface roughness, on the magnitude and angular distribution of the power scattered by a rough surface. It is the object of this section to summarize the results of these studies, including those summarized by Bechmann and Spizzichino, in order to choose a realistic model of surface reflection for the study of doppler radiation.

For the present, it appears that the rough terrain model is the most appropriate one in order to investigate the reflected electromagnetic field. The discussion which follows is based on this model.

2.2 The Scattering Cross-Section

The average quantity which is used to characterize the reflective properties of a rough surface is the bistatic cross-section $\sigma(\hat{\Omega}_1, \hat{\Omega}_2)$, per unit area. It is conventionally defined as 4π times the ratio of the average power scattered per unit area of the surface, per unit solid angle in the direction $\hat{\Omega}_2$, to the incident power from direction $\hat{\Omega}_1$ per unit area of the surface. As illustrated in Fig. 2-1, the mean level of the scattering surface may be taken as the z-plane, the incident direction $\hat{\Omega}_1$ may be expressed in terms of two angles θ_1 and ϕ_1 by

$$\hat{\Omega}_1 = -\left[\hat{x} \sin \theta_1 \cos \phi_1 + \hat{y} \sin \theta_1 \sin \phi_1 + \hat{z} \cos \theta_1\right] \quad . \qquad (2.1)$$

Direction $\hat{\Omega}_2$ may be expressed in terms of two angles θ_2, ϕ_2 by

$$\hat{\Omega}_2 = (\hat{x}\sin\theta_2\cos\phi_2 + \hat{y}\sin\theta_2\sin\phi_2 + \hat{z}\cos\theta_2) \quad . \tag{2.2}$$

Then, for an incident power P_1 in the direction $\hat{\Omega}_1$, the scattered power intercepted by an area dA_2 normal to $\hat{\Omega}_2$ and at a distance r_2 from dA is given by

$$P_2 = \frac{P_1\, dA \cos\theta_1}{4\pi\, r_2^2}\, dA_2\, \sigma(\hat{\Omega}_1, \hat{\Omega}_2) \quad . \tag{2.3}$$

In general, the bistatic cross-section $\sigma(\hat{\Omega}_1, \hat{\Omega}_2)$ depends on the polarization of incident and reflected radiation. To fix the direction of polarization, we shall define the direction of the horizontal polarization of the incident radiation by

$$\hat{e}_{h_1} = \frac{\hat{z} \times \hat{\Omega}_1}{|\hat{z} \times \hat{\Omega}_1|} \tag{2.4}$$

and the direction of the vertical polarization by

$$\hat{e}_{v_1} = \hat{\Omega}_1 \times \hat{e}_{h_1} \quad . \tag{2.5}$$

Similarly, for the scattered radiation in the direction $\hat{\Omega}_2$, the directions of horizontal and vertical polarization are defined by

$$\hat{e}_{h_2} = \frac{\hat{z} \times \hat{\Omega}_2}{|\hat{z} \times \hat{\Omega}_2|} \tag{2.6}$$

and

$$\hat{e}_{v_2} = \hat{\Omega}_2 \times \hat{e}_{h_2} \tag{2.7}$$

respectively. These directions are illustrated in Fig. 2-1. If $\hat{\Omega}_2$ is the specularly reflecting direction, so that

$$\hat{\Omega}_2 = \hat{\Omega}_1 - 2\hat{z}(\hat{z} \cdot \hat{\Omega}_1) \tag{2.8}$$

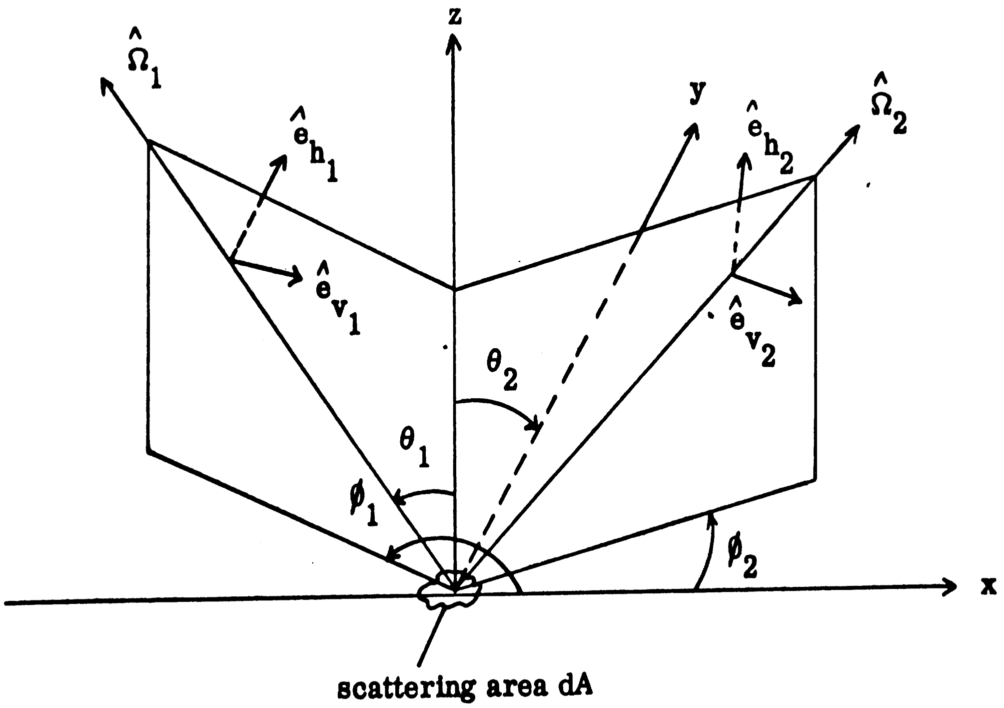

FIG. 2-1: DIRECTIONS OF INCIDENT AND REFLECTED WAVES AND THE DIRECTIONS OF POLARIZATION.

then

$$\hat{e}_{h_2} = \hat{e}_{h_1} \tag{2.9}$$

and

$$\hat{e}_{v_2} = - \hat{e}_{v_1} \quad . \tag{2.10}$$

On the other hand, if $\hat{\Omega}_2$ is the backscattering direction, so that

$$\hat{\Omega}_2 = - \hat{\Omega}_1 \quad , \tag{2.11}$$

then

$$\hat{e}_{h_2} = - \hat{e}_{h_1} \tag{2.12}$$

and

$$\hat{e}_{v_2} = \hat{e}_{v_1} \quad . \tag{2.13}$$

To stress the polarization dependence of the scattering cross-section, we may consider four types of scattering cross-section σ_{AB}, where A and B may stand for h or v. For example, σ_{hv} is the scattering cross-section corresponding to horizontally polarized scattered radiation when the incident wave is vertically polarized.

Experimental data concerning the bistatic cross-section are very scarce. In the work of Hunter and Senior (1966), Pidgeon (1966) and others, where the bistatic cross-section is measured, the incidence angle θ_1 is limited only to nearly 90^o degrees and the reflection direction is either nearly specular or nearly in the backscattering direction. The detailed information on the directional distribution of the scattered power from a rough surface in the microwave range has not been so far reported in experimental work.

On the other hand, experimental work on the backscattering cross-section has been reported extensively. In the subsequent sections of this chapter, we shall therefore confine our attention to the backscattering cross-section. In the backscattering direction, since

$$\begin{aligned} \theta_1 &= \theta_2 \triangleq \theta \\ \phi_1 &= \phi_2 \triangleq \phi \quad , \end{aligned} \tag{2.14}$$

we may represent

$$\sigma(\hat{\Omega}_1, \hat{\Omega}_2) \triangleq \sigma(\theta, \phi) \quad . \tag{2.15}$$

2.3 Experimental Determination of $\sigma(\theta, \phi)$

In the experimental work, the backscattering cross-section is generally correlated with the backscattered power. For a c.w. system, it can be easily seen that the backscattered power is

$$P_r = \frac{P_t G_t^2 \lambda^2}{(4\pi)^3 R^2} \iint d\theta \, d\phi \cos^2\theta \, f^4(\theta, \phi) \, \sigma(\theta, \phi) \tag{2.16}$$

where

P_t = transmitted power

G_t = gain of the transmitting antenna

λ = operating wavelength

R = distance from the transmitter to the scattering area

and

$f(\theta, \phi)$ = the field pattern of the transmitting antenna.

Therefore, for a wide beam transmitting antenna, only some spatial average of the scattering cross-section is measured. For a narrow beam, we assume

$$f(\theta, \phi) = 1 \quad \begin{cases} \theta = \theta_o \pm \frac{1}{2} \Theta \\ \phi = \phi_o \pm \frac{1}{2} \Phi \end{cases} \tag{2.17}$$

$$= 0 \qquad \text{otherwise,}$$

where Φ, and Θ are the angular beam width of the transmitting antenna, and θ_o, ϕ_o is the direction of the major lobe of the transmitting beam, then

$$P_r \cong \frac{P_t G_t^2 \lambda^2}{(4\pi)^3 R^2} \cos^2\theta_o \, \sigma(\theta_o, \phi_o) \quad . \tag{2.18}$$

Alternately, the backscattering cross-section can be determined from the average power returned by using wide beam transmitter and short pulsed signal. In that case, the returned power corresponding to each pulse spreads in time, and may be identified for any interval of time as the scattered power from each portion of the surface, where θ_o and ϕ_o may be different. Such a scheme has been used by Edison et al (1960). For a transmitter with its major lobe incident on the surface, the relation $\sigma(\theta, \phi)$ may be expressed in the following form

$$\frac{1}{2\pi}\int_0^{2\pi} \sigma(\theta_o, \phi_o)\, d\phi_o = \frac{P_r(t)}{\dfrac{c\lambda^2}{64\pi^2}\displaystyle\int_t^{t+\tau} \frac{P_t(d-t)\, G^2(t)\, dt}{(h+\frac{ct}{2})^3}} \tag{2.19}$$

where

h = height of the antenna

t = time measured from the starting of received pulse

c = velocity of light

d = time delay from the starting of the transmitted pulse and the [illegible] received pulse

τ = pulse width

and

$$\cos\theta_o = \frac{h}{h + (\frac{ct}{2})} \quad .$$

If one assumes that the scattering surface is isotropic, so that $\sigma(\theta_o, \phi_o)$ is independent of ϕ_o, then

$$\sigma(\theta_o, \phi_o) = \frac{P_t(r)}{\frac{c\lambda^2}{64\pi^2}\int_t^{t+\tau} \frac{P_t(d-t)G^2(t)dt}{(h+\frac{ct}{2})^3}} \tag{2.20}$$

which is the formula used by Edison, et al. The limitation here, besides the assumption of isotropic surface, is that the result is correct only for a small range of θ.

Most experimental results reported in the literature, based their calculation of $\sigma(\theta, \phi)$ from correlation formulas such as (2.18) or (2.20) or some variations of these equations. Thus, the results of experimental data should be understood as some spatial average, and therefore independent of ϕ_o. It is felt that for rough surfaces such as the sea, the effect of the wind should be illustrated in the spatial variation of $\sigma(\theta, \phi)$ with ϕ. But such an explicit relation for such dependence does not seem to be available in the literature.

2.4 Survey of Experimental Results

The dominant characteristics of the back scattering cross-section from rough surfaces have been reported by experimental investigators, in numerous articles. A brief summary of these dominant features is outlined in this section. Since there exist numerous publications which discuss different aspects of this problem, only some representative ones are referenced here.

2.4.1 Aspect Variation

Wiltse, et al (1957) reported that at 9.6, 24, 38, and 48.7 GHz, for a moderate sea, $\sigma(\theta, \phi)$ is a decreasing function of θ, with the slope being greatest at $\theta \cong 30^o$. The change of $\sigma(\theta, \phi)$ from $\theta = 0^o$ to $\theta = 30^o$, even for a calm sea, may vary by a factor as large as 5000.

The results of Grant and Yaplee (1957), at wavelengths of 8.6, 1.25, and 3.2 cm of various types of terrain seem to indicate similar variation.

Edison, et al (1960) reported that the measured average values of $\sigma(\theta, \phi)$ at 3.8 GHz and 415 MHz over terrains, forests, deserts, etc., in the range of θ from 0^o to 25^o seem to correlate fairly well with the relation

$$\sigma(\theta, \phi) = \frac{\theta\, a^2 \csc^2\theta}{4\pi\xi^2} \exp\left\{-\frac{a^2}{4\xi^2}\tan^2\theta\right\} \qquad (2.21)$$

where

ξ = the mean square height deviation of the rough surface

and

a = the horizontal correlation distance of height variation.

Finally Renau and Collinson (1965) performed back scattering measurements for the angle of incidence varying from 0^o to 89^o when a laser beam is incident upon manufactured rough surfaces whose rms surface slope is denoted by h/ℓ. They determined that at 0^o angle of incidence the cross-section varies as the square of h/ℓ and that it is independent of wavelength. As the angle of incidence increases the cross section increases with h/ℓ and h/λ with an upper limit closely agreeing with what Lambert's scattering law would predict. In this case h represents the the rms surface height and h/ℓ is the rms surface slope.

2.4.2 Frequency Dependence

Goldstein (1946) reported that $\sigma(\theta, \phi)$ varies with the zeroth power of λ for a rough sea and varies as λ^{-4} for a calm sea. Katzin (1957), based on the results of measurements at 3.2, 9.1, and 24 cm wavelengths indicates that $\sigma(\theta, \phi)$ varies as λ^{-1} for a sea surface.

Grant, et al (1957) reported that for various types of terrain, σ decreases with λ.

Wiltse, et al (1957) reported that $\sigma_{VV}(\theta, \phi)$ seems to be relatively independent of frequency, especially for small values of θ.

Boring, et al (1957) made an extensive experimental study of sea returns at frequencies of 6.3 GHz and 35 GHz. Their results indicated that $\sigma(\theta, \phi)$ for any polarization at 35 GHz is about 3 to 4 dB higher than that of the corresponding value at 6.3 GHz. Roughly, there is a $1/\lambda$ dependence for σ_{VV}, σ_{HH} and $1/\sqrt{\lambda}$ dependence for σ_{VH} or σ_{HV}.

Long (1965), made a further analysis of the Boring results, and reported that σ_{HH} and σ_{VV} are independent of frequency at low frequencies, and have $1/\lambda$ dependence for larger frequencies. Moreover, the ratios σ_{HH}/σ_{HV} and σ_{VV}/σ_{HV} are relatively insensitive to frequency. Chia, et al (1965) report the reflection coefficient of forest, desert and water terrain, for measurements taken at 9 MHz from an altitude of 1000 km. They also present tables of data taken by other groups for various terrains at several frequencies and from different heights above the earth. Finally Renau and Collinson (1965) determined that the cross section differs for the two polarizations. At grazing incidence, the cross-section is larger for the parallel polarization.

2.4.3 Polarization Effects

Wiltse, et al (1957) reported that for a moderate sea, $\sigma_{HH}(\theta, \phi)$ is usually less than σ_{VV} by several dB for large values of θ; while for small θ, σ_{HH} and σ_{VV} are nearly the same. $\sigma_{VH}(\sigma_{HV})$ is on the average 12 dB less than σ_{VV} for large θ and 20 dB less at $\theta = 0$.

The results reported by Boring, et al (1957) seem to imply that for a calm sea, $\sigma_{VV} \gg \sigma_{HH}$, but as the wind speed increases, σ_{HH} approaches closer to σ_{VV}. In some cases, σ_{HH} may exceed σ_{VV} for very rough sea conditions.

Hunter, et al (1966), reported that at 9.375 GHz, with right circularly polarized transmitter and horizontal and vertically polarized receivers, $\sigma_{RV} < \sigma_{RH}$ for cross wind and $\sigma_{RV} > \sigma_{RH}$ for upwind or downwind. σ_{RV}, the cross-section for right circularly polarized transmission and vertically polarized receivers, is easily shown to be equal to

$$\sigma_{RV} = \frac{\sigma_{VV} + \sigma_{VH}}{2} \quad .$$

2.4.4 Effect of Wind and Sea State

Wiltse, et al (1957) reported that for $\theta > 20^{\circ}$, rough sea results in large values of $\sigma(\theta, \phi)$, but for near vertical incidence, the trend is usually reversed.

The work of Boring, et al (1957) reported the results of a statistical analysis of a large number of experiments of sea return at frequencies of 6.3 GHz and 35 GHz. It is found that the variation of $\sigma(\theta, \phi)$ may be empirically represented by the expression.

$$\sigma(\theta, \phi) = C_1 W^{C_2} (90^o - \theta)^{C_3} 10^{\left[C_4 H + C_5 \cos \phi\right]}$$

where

H is the height of waves in feet

ϕ is so chosen that $\phi = 0$ is the upwind direction

W is the wind speed in knots. This expression was obtained from a statistical analysis of a large number of experimental results. The constants C_1, C_2, C_3, C_4 and C_5 differ for different states of polarization and for the two frequencies measured. From their data, it seems that in most cases, σ_{AB} increases with wind speed.

Clarke and Hendry (1964) studied the power reflected in the specular direction, when the incident wave is horizontally polarized and at an angle of incidence of 45^o, for various states of roughness of the reflecting surface.

Beard (1967) examined the forward scattered field for artificially created water waves by various methods. The water samples varied from distilled water to ocean water and the frequency used was 100 GHz. His results indicate that in any theoretical approach for the study of scattering from rough surfaces the sphericity of the wave fronts must be taken into account.

Finally, Renau and Callinson (1965) report that for a constant rms height h the cross section increases with higher frequency and large angles of incidence.

III

THEORETICAL MODELS FOR ROUGH SURFACE

3.1 Introduction

In order to explain the various aspects of the return from a rough surface, several theoretical models have been proposed to account for the dominant features of the reflected signal. Due to the lack of detailed information on the structure of the rough surface, and the dielectric properties of the surface, only the average properties based on some statistical description of the surface can be described. Basically, if one specifies a rough surface by some random function

$$z = z(x, y) \tag{3.1}$$

with zero mean value, and a postulated statistical property, then together with the dielectric constant of the surface, the formal solution of the scattered field from the surface may be obtained approximately, and a statistical average of the return signal can be taken. Most of the theoretical work is based on a gaussian surface. Recently (Valenzuela, 1967) the use of the actual wave spectrum of the sea was attempted. It is felt that, introducing the actual statistical description of the sea surface, including the effect of the wind, one should obtain more accurate prediction of the scattered radiation for the sea surface.

Mathematically, the theoretical model for rough surface scattering may be classified into three different classes. These are:

(a) The phenomenological model. By postulation the surface is taken as an ensemble of distributed spheres, facets and half planes. A review of this phenomenological model has been given by Beckmann, et al (1963).

(b) The use of series expansion. In this approach, the incident and scattered fields are expanded in series and boundary conditions are used in determining the coefficients of expansion. Due to the complicated procedure in determining the coefficients, only the first and second order approximate solutions have been reported so far. It is doubtful that any attempt to obtain the more detailed, higher order solutions is feasible. A brief description of the results from this approach is given in Section 3.2.

(c) The use of the Kirchhoff approximation. The use of the Kirchhoff integral representation of the scattered field offers a convenient means of finding the approximate expressions for the scattered field from a rough surface. This has been used by various investigators in obtaining theoretical models for rough surface scattering. In the high frequency limit, this offers some basis for the phenomenological models introduced. It is felt that the extension of Kirchhoff's formulation, incorporating the dielectric properties of the surface and some realistic statistical description of the surface, may lead to a useful model appropriate for the present work. In the remainder of this chapter, the Kirchhoff approximation and possible extensions are discussed in some detail.

3.2 The Method of the Series Expansion

The method of approximate solutions, for the fields scattered by a slightly rough surface, by a series expansion has been introduced by Rice (1951), used by Peake (1959) and recently extended by Valenzuela (1967). The results of these investigations are summarized in this section.

The surface described by the random function

$$z = z\,(x, y) \tag{3.1}$$

is assumed to have the following statistical properties:

(a) The expected value of z is zero, i.e.,

$$\langle z \rangle = 0 \quad . \tag{3.2}$$

(b) The expected value of z^2 is given by

$$\langle z^2 \rangle \triangleq \bar{z}^2 \quad . \tag{3.3}$$

(c) The correlation function for the random function z is given by

$$< z(x, y),\ z(x+\xi\ ,\ y+n) > \triangleq \overline{z}^2 \rho(\xi, n) \quad , \qquad (3.4)$$

and

(d) The surface is slightly rough, i.e.,

$$\left|\frac{\partial z}{\partial x}\right| , \quad \left|\frac{\partial z}{\partial y}\right| << 1 \ . \qquad (3.5)$$

Based on these assumptions, the first order results of Peake (1959) may be generalized to the following expressions for the bistatic cross section:

$$\sigma_{HH}(\theta_1, \phi_1, \theta_2, \phi_2)$$

$$= 4\pi k^4 \cos\theta_1 \cos\theta_2 \cos^2(\phi_2 - \phi_1) \left| \frac{(\epsilon - 1)}{(\cos\theta_1 + \sqrt{\epsilon - \sin\theta_1})^2} \right|^2 X$$

$$X\ w\left[k(\sin\theta_1 \cos\phi_1 + \sin\theta_2 \cos\phi_2),\ k(\sin\theta_1 \sin\phi_1 + \sin\theta_2 \cos\phi_2)\right]$$

(3.6)

and

$$\sigma_{VV}(\theta_1, \phi_1, \theta_2, \phi_2)$$

$$= 4\pi k^2 \cos\theta_1 \cos\theta_2 \cos^2(\phi_2 - \phi_1) \left| \frac{(\epsilon - 1)\,\epsilon(1 + \sin^2\theta_1) - \sin^2\gamma}{\left[\epsilon\cos\theta_1 + \sqrt{\epsilon - \sin^2\theta_1}\right]^2} \right|^2 X$$

$$X\ w\left[k(\sin\theta_1 \cos\phi_1 + \sin\theta_2 \cos\phi_2),\ k(\sin\theta_1 \cos\phi_1 + \sin\theta_2 \cos\phi_2)\right]$$

(3.7)

where ϵ is the relative complex dielectric constant of the scattering surface, and $w(\alpha, \beta)$ is the Fourier transform of $z^2 \rho(\xi, n)$.

For back scattering and an isotropic rough surface, i.e.,

$$\rho(\xi, n) \equiv \rho(\sqrt{\xi^2+n^2}, 0) \quad , \tag{3.8}$$

the above reduces to the back scattering cross-section given by Peake (1959).

A second order solution, for a perfectly conducting rough surface has been given by Valenzuela. The results were applied to cross polarization effect of sea returns. In terms of the sea spectrum $A^2(\nu)$, Valenzuela obtained the following result for the scattering cross-section:

$$\sigma_{VH}(\theta) = \sigma_{HV}(0) \cong \frac{\pi^2}{2} k^2 \cos^4\theta \; T_{VH} \quad \times$$

$$\times \int_0^\infty \frac{\nu^5 A^2(\nu - k\sin\theta) A^2(\nu^2 + k\sin\theta)}{\left|\epsilon\sqrt{k^2-\nu^2} + \sqrt{\epsilon k^2 - \nu^2}\right|^2} \, d\nu \quad . \tag{3.8}$$

Using the Neumann spectrum for a fully developed sea, Valenzuela presented the value of σ_{VH} for several values of wind speed for $\lambda = 24$ cm. Application of this results for different ranges of wavelength, involves lengthy calculations so that whether this model agrees with the experimental features of wavelength dependence cannot be assessed at the present time.

3.3 Kirchhoff's Integral Formulation

A mathematical formulation of electromagnetic scattering, which gives approximate but useful results, is the vector extension of Kirchhoff's integral formula. The Kirchhoff's integral formula can be deduced from the physical concept of induced sources as illustrated in Fig. 3-1. When an electromagnetic wave is incident on this surface, the scattered radiation may be interpreted as the re-radiation due to the surface electric and magnetic currents.

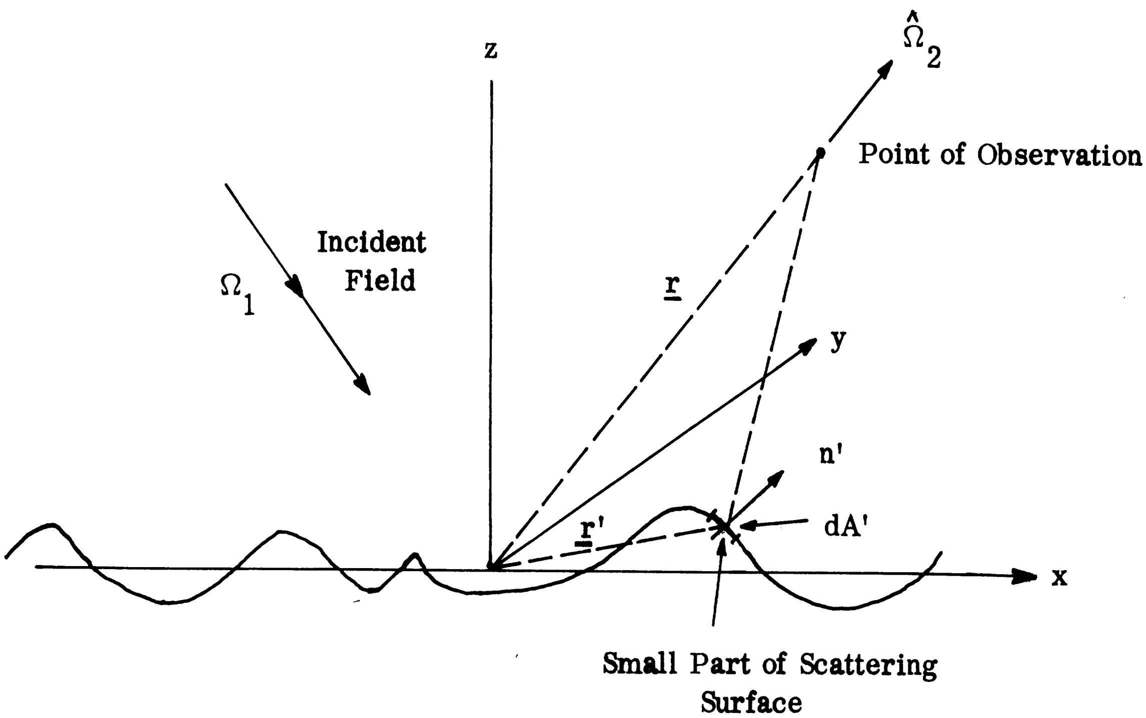

FIG. 3-1: CONFIGURATION FOR THE APPLICATION OF KIRCHHOFF'S INTEGRAL FORMULA

Consider for example an elementary area dA', with normal n', located at r'. If the total (incident and scattered) field at this surface is given by $\underline{E}_s(\underline{r}')$ and $\underline{H}_s(\underline{r}')$, then the surface electric and magnetic currents per unit area at $\underline{r}'$ are given by

$$\underline{J}_e = \hat{n}' \times \underline{H}_s(\underline{r}') \quad , \tag{3.10}$$

$$\underline{J}_m = \hat{n}' \times \underline{E}_s(\underline{r}') \tag{3.11}$$

respectively. The electric field due to these induced sources, observed at any point $\underline{r}$, is given by

$$d\underline{E}(\underline{r}) = \underbrace{(\hat{n}' \times \underline{E}_s(\underline{r}')) \times \nabla' G(\underline{r},\underline{r}')}_{\text{due to magnetic current}} + \underbrace{i\omega\mu_0(\hat{n}' \times \underline{H}(\underline{r}') \cdot \left[G(\underline{r},\underline{r}') + \frac{1}{k^2}\nabla'\nabla' G(\underline{r},\underline{r}')\right]}_{\text{due to electric current}} \tag{3.13}$$

where

$$G = \frac{e^{ik|\underline{r} - \underline{r}'|}}{4\pi|\underline{r} - \underline{r}'|} \tag{3.13}$$

and

$$\nabla' = \hat{x}\frac{\partial}{\partial x'} + \hat{y}\frac{\partial}{\partial y'} + \hat{z}\frac{\partial}{\partial z'} \quad . \tag{3.14}$$

Thus, if the tangential field components $\hat{n} \times \underline{E}_s$ and $\hat{n} \times \underline{H}_s$ on the surface are known at all parts of surface, the scattered fields may be obtained by adding the contributions from each part of the surface, i.e.,

$$\underline{E}_2(\underline{r}) = \int_{\text{Surface}} dA \left\{ (\hat{n}' \times \underline{E}_s(\underline{r}')) \times \nabla' G(\underline{r},\underline{r}') + i\omega\mu_0(\hat{n} \times \underline{H}(\underline{r}')) \cdot \left[G(\underline{r},\underline{r}') + \nabla'\nabla' G(\underline{r},\underline{r}')\right]\right\}. \tag{3.15}$$

Similarly the scattered magnetic field is given by

$$\underline{H}_2(\underline{r}) = \int_{Surface} dA \left\{ (\hat{n}' x \underline{H}_s(\underline{r}')) x \nabla' G(\underline{r}, \underline{r}') - i\omega\epsilon_o (\hat{n}' x \underline{E}_s(\underline{r}')) \cdot \left[G(\underline{r}, \underline{r}') + \frac{1}{k^2} \nabla' \nabla' G(\underline{r}, \underline{r}') \right] \right\} . \tag{3.16}$$

Equations (3.13) and (3.16), are exact, provided that the exact surface fields are used in the integration.

In most practical cases, the point of observation is high above the ground, so that the far zone approximation may be introduced. Referring to Fig. 3-1, any point of observation may be represented by

$$\underline{r} = \hat{\Omega}_2 \underline{r} \quad , \tag{3.17}$$

and the approximation

$$|\underline{r} - \underline{r}'| \cong (\underline{r} - \underline{r}') \cdot \hat{\Omega}_2 \qquad (r >> r') \quad , \tag{3.18}$$

is adequate. Then, Eq. (3.15), and Eq. (3.16) may be reduced to

$$\underline{E}_2(\underline{r}) = \frac{i\omega\mu_o}{\eta} \frac{e^{ikr}}{\pi} \int_{Surface} dA \left\{ \hat{\Omega}_2 x (\hat{n}' x \underline{E}_s(\underline{r}')) - \eta \Omega_2 x \left[\Omega_2 x (\hat{n}' x \underline{H}_s(\underline{r}')) \right] \right\} e^{-ik\hat{\Omega}_2 \cdot \underline{r}'} \tag{3.19}$$

and

$$\underline{H}_2(\underline{r}) = \frac{i\omega\mu_o}{\eta} \frac{e^{ikr}}{4\pi r} \int_{Surface} dA \left\{ \frac{1}{\eta} \hat{\Omega}_2 x \left[\hat{\Omega}_2 x (\hat{n}' x \underline{E}_s(\underline{r}')) \right] + \hat{\Omega}_2 x (\hat{n}' x \underline{H}_s(\underline{r}')) \right\} e^{-ik\hat{\Omega}_2 \cdot \underline{r}'} . \tag{3.20}$$

Equation (3.19) and Eq. (3.20), are the fundamental relations used in the approximate calculations of the scattered field, [for example, see Hoffman 1955, Aksenov, 1961].

For plane wave incidence, the incident fields are given by:

$$\underline{E}_1(\underline{r}') = \underline{E}_1(0)\, e^{ik\hat{\Omega}_1 \cdot \underline{r}'} \tag{3.21}$$

$$\underline{H}_1(\underline{r}') = \frac{1}{\eta}\hat{\Omega}_1 \times \underline{E}_1(0)\, e^{ik\hat{\Omega}_1 \cdot \underline{r}'} \tag{3.22}$$

then, if $\underline{E}_s$ and $\underline{H}_s$ are interpreted as the surface fields due to an incident field with the electric and magnetic field respectively given by $\underline{E}_1(0)$ and $\frac{1}{\eta}\hat{\Omega}_1 \times \underline{E}_1(0)$ at each part of the surface, we may have

$$\underline{E}_2(\underline{r}) = \frac{i\omega\mu_o}{\eta}\frac{e^{ikr}}{4\pi r}\int_{\text{Surface}} dA \left\{ \hat{\Omega}_2 \times (\hat{n}' \times \underline{E}_s(\underline{r}')) - \eta\hat{\Omega}_2 \times \left[\hat{\Omega}_2 \times (\hat{n}' \times \underline{H}_s(\underline{r}')) \right] \right\} e^{ik(\hat{\Omega}_1 - \hat{\Omega}_2)\cdot \underline{r}'} . \tag{3.23}$$

Similar equations, can be obtained for $\underline{H}_2(\underline{r})$.

In practice, the integral given by Eq. (3.23) cannot be carried out exactly, even if we know the shape of the surface, due to the difficulties in obtaining $\underline{E}_s$ and $\underline{H}_s$. A commonly used approximation is the so-called local tangent plane approximation. Assume that the local radius of curvature of the scattering surface is much larger than the wave length, so that locally, the reflected fields at any point may be considered the same as those reflected by a plane tangent to the reflecting surface at that point. With this approximation, it may be easily shown that, (Aksenov, 1961)

$$\hat{n}' \times \underline{E}_s = (1+R_\perp)(\underline{E}_1(0)\cdot \hat{t})\,\hat{n}' \times \hat{t} + (1-R_{||})(\underline{E}_1(0)\cdot \hat{t}')\,\hat{n} \times \hat{t}' \tag{3.24}$$

$$\eta\hat{n}' \times \underline{H}_s = (1-R_\perp)(\underline{E}_1(0)\cdot \hat{t})\,\hat{n}' \times \hat{t}' - (1+R_{||})(\underline{E}_1(0)\cdot \hat{t}')\,\hat{n} \times \hat{t} \tag{3.25}$$

where

$$\hat{t} = \frac{\hat{\Omega}_1 \times \hat{n}'}{\left|\hat{\Omega}_1 \times \hat{n}'\right|} \tag{3.26}$$

and

$$\hat{t}' = \hat{\Omega}_1 \times \hat{t} \quad . \tag{3.27}$$

Also

$$R_{\perp} = \frac{\cos\gamma - \sqrt{N^2 - \sin^2\gamma}}{\cos\gamma + \sqrt{N^2 - \sin^2\gamma}} \quad , \tag{3.28}$$

$$R_{||} = \frac{N^2\cos\gamma - \sqrt{N^2 - \sin^2\gamma}}{N^2\cos\gamma + \sqrt{N^2 - \sin^2\gamma}} \quad , \tag{3.29}$$

$$\eta = \sqrt{\frac{\mu_o}{\epsilon_o}} \quad , \tag{3.30}$$

$$\cos\gamma = -\hat{\Omega}_1 \cdot \hat{n}$$

and

N = index of refraction of the surface.

In particular, if the reflecting surface is perfectly conducting, then,

$$R_{\perp} = -1 \tag{3.31}$$

and

$$R_{||} = +1 \quad . \tag{3.32}$$

For this case, we have,

$$\hat{n}' \times \underline{E}_s = 0 \quad , \tag{3.33}$$

and

$$\eta\, \hat{n}' \times \underline{H}_s = 2\, \hat{n}' \times \underline{H}_1(0) \tag{3.34}$$

which are the approximate relations used by most investigators to simplify computations.

By introducing Eqs. (3.24) and (3.25) into Eq. (3.23) an approximate relation results between the incident electric field $\underline{E}_1$ and the scattered electric field $\underline{E}_2$. This result is given by

$$\underline{E}_2(r, \hat{\Omega}_2) = \frac{i\omega\mu_o}{\eta} \frac{e^{ikr}}{r}$$

$$\hat{\Omega}_2 \times \int dA' \left[(1+R_\perp)(\hat{n}' \times \hat{t})(\hat{E}_1 \cdot \hat{t}) + (1-R_{||})(\hat{n}' \times \hat{t}')(\underline{E}_1 \cdot \hat{t}')\right] e^{ik(\hat{\Omega}_1 - \hat{\Omega}_2)\cdot \underline{r}'} \tag{3.35}$$

$$-\hat{\Omega}_2 \times \hat{\Omega}_2 \times \int dA' \left[(1-R_\perp)(\hat{n}' \times \hat{t})(\underline{E}_1 \cdot \hat{t}) - (1+R_{||})(\hat{n}' \times \hat{t})(\underline{E}_1 \cdot \hat{t}')\right] e^{ik(\hat{\Omega}_1 - \hat{\Omega}_2)\cdot \underline{r}'} \quad . \tag{3.36}$$

In order to clarify the polarization effect on the scattered field, let us, by using the directions of polarization defined by Eq. (2.4) through Eq. (2.5), resolve the incident and scattered fields into the vertically and horizontally polarized components as indicated below:

$$\underline{E}_1 = \left[\hat{e}_{h1} E_{h1} + \hat{e}_{v1} E_{v1}\right] e^{ik\hat{\Omega}_1 \cdot \underline{r}} \tag{3.37}$$

and

$$\underline{E}_2 = \left[\hat{e}_{h2} E_{h2} + \hat{e}_{v2} E_{v2}\right] \frac{e^{ikr}}{r} \quad . \tag{3.38}$$

From Eq. (3.36), it is seen that the components of the incident field and the scattered field may be related by a matrix:

$$\begin{bmatrix} E_{h2} \\ E_{v2} \end{bmatrix} = \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \begin{bmatrix} E_{h1} \\ E_{v1} \end{bmatrix} \tag{3.39}$$

where each component of the scattering matrix S_{ab} may be expressed by:

$$\begin{aligned} S_{ab} = \frac{i\omega\mu_o}{4\pi\eta} \int dA' \, e^{ik(\hat{\Omega}_1 - \hat{\Omega}_2)\cdot \underline{r}'} \Big\{ & - (1+R_\perp) \quad (\hat{e}_{b1}\cdot\hat{t}) \quad (\hat{\Omega}_2 x \hat{e}_{a2})\cdot(\hat{n}' x \hat{t}) \\ & - (1-R_{||}) \quad (\hat{e}_{b1}\cdot\hat{t}) \quad (\hat{\Omega}_2 x \hat{e}_{a2})\cdot(\hat{n}' x \hat{t}') \\ & + (1-R_\perp) \quad (\hat{e}_{b1}\cdot\hat{t}) \quad \hat{e}_{a2}\cdot(\hat{n}' x \hat{t}') \\ & - (1+R_{||}) \quad (\hat{e}_{b1}\cdot\hat{t}') \quad \hat{e}_{a2}\cdot(\hat{n}' x \hat{t}) \Big\} \quad . \end{aligned} \tag{3.40}$$

The relations between the elements of the scattering matrix and the scattering cross-section may be easily deduced from the definition of the scattering cross section. For example, for an incident wave with polarization direction a, (h or v), the power scattered by a surface of A, per unit solid angle with polarization b is given by

$$\frac{dP_2}{d\Omega} = |S_{ba}|^2 \, P_1 \quad .$$

Thus, if the surface A is relatively uniform, the scattering cross-section per unit area is given by

$$\sigma_{ba} = \frac{\langle S_{ba}^2 \rangle}{A \cos \theta_1} \tag{3.41}$$

where < > enclosing any quantity indicates the statistical average of that quantity.

In general, the evaluation of S_{ab} and the subsequent deduction of σ_{ab} is complicated, and depends on the postulation of the statistics of the surface. Most theoretically models in the literature may be interpreted as the results of carrying out this integration and averaging process approximately. Some of these approximate procedures and subsequent results are discussed in the next few sections.

3.4 Model of Specular Reflecting Regions

The first order approximation of the Kirchhoff integral, is obtained by means of a stationary phase integration and yields the geometric optics result for the reflected field. Although this is a generally known fact in classical field theory for a long time [MacDonald , 1912], specific application of this concept to rough surface scattering was not used until 1966 when Kodis applied it with the scatterer being an infinitely conducting rough surface. Extension of the results of Kodis to rough surfaces with a finite index of refraction, and some qualitative results of this approach are investigated in this section.

For the approximate evaluation of the integral (3.40), by means of a stationary phase integration, we assume that the exponential factor $e^{ik(\hat{\Omega}_1 - \hat{\Omega}_2)\cdot \underline{r}'}$, due to its rapid oscillatory behavior as $\underline{r}'$ varies, is the dominating factor, and the rest of the integrand is slowly varying. Thus, the contribution to the value of the integral is primarily due to regions in the neighborhood of some $\underline{r}'$, such that

$$(\hat{\Omega}_1 - \hat{\Omega}_2)\cdot \underline{r}' = \text{constant} \quad . \tag{3.42}$$

There may be several values of $\underline{r}'$, say $\underline{r}_1, \underline{r}_2 \ldots \underline{r}_j \ldots$ which satisfy this condition. These points are known as stationary points or specularly reflecting points, because at these points the geometry is such that waves may be considered as specularly reflecting from the direction $\hat{\Omega}_1$ to $\hat{\Omega}_2$. To illustrate this, let us denote

$$\underline{q} \triangleq (\hat{\Omega}_1 - \hat{\Omega}_2) = \hat{x}\, q_x + \hat{y} q_y + \hat{z} q_z \tag{3.43}$$

and any point on the rough surface by

$$\underline{r}' = \hat{x}\, x' + \hat{y}\, y' + \hat{z}\, z'\,(x', y') \tag{3.44}$$

where

$$z' = z'\,(x', y')$$

is the equation for the rough surface. Then, Eq. (3.42) means that

$$q_x x' + q_y y' + q_z z'\,(x', y') = \text{constant}$$

at a stationary point. Therefore,

$$q_x + q_z \frac{\partial z'}{\partial x'} = 0 \tag{3.45}$$

and

$$q_y + q_z \frac{\partial z'}{\partial x'} = 0 \tag{3.46}$$

at a stationary point. For a physical interpretation of the stationary points which satisfy these equations, we note that the normal to the surface is given by

$$n' = \frac{\hat{x} \frac{\partial z'}{\partial x'} + \hat{y} \frac{\partial z'}{\partial y'} - \hat{z}}{\sqrt{(\frac{\partial z'}{\partial x'})^2 + (\frac{\partial z'}{\partial y'})^2 + 1}} \quad . \tag{3.47}$$

Thus, it is easy to see that the condition satisfied by a stationary point may be represented by the vector relation

$$\hat{n}' \times \underline{q} = \hat{n}' \times (\hat{\Omega}_1 - \hat{\Omega}_2) = 0 \quad . \tag{3.48}$$

As illustrated in Fig. 3-2, at any stationary point $\underline{r}'_j$, Eq. (3.49) indicates that $\hat{n}'$ bisects the angle between $\hat{\Omega}_1$ and $\hat{\Omega}_2$ as indicated, and waves are reflected from $\hat{\Omega}_1$ to $\hat{\Omega}_2$ specularly at $\underline{r}'_j$, where $j = 1, 2, 3, \ldots$.

Thus, with the geometrical optics approximation, the first order scattered field from an incident direction $\hat{\Omega}_1$ to any direction $\hat{\Omega}_2$ may be considered as the diffraction field due to regions near all the stationary points of the surface. This mathematical observation indicates that the phenomenological models consider the reflecting surface as a random distribution of hemispheres, paraboloids, or other bumps on the surface and they may be considered as the same class of approximation as those for specular reflecting regions. Equivalent qualitative results should be obtained if the statistical descriptions of the surface are the same.

To carry out the mathematical approximation of geometric optics further, the direction of incidence $\hat{\Omega}_1$ is fixed and the reflected field is observed at a fixed direction $\hat{\Omega}_2$, then, at any reflection point $\underline{r}'_j$, the vectors $\hat{n}'$, $\hat{t}$, $\hat{t}'$ are fixed (independent of the position $\underline{r}'_j$), hence may be taken out of the integral sign in Eq. (3.40). The results are,

$$\begin{aligned} S_{ab} \doteq \frac{i\omega\mu_o}{4\pi\eta} F(\hat{\Omega}_1, \hat{\Omega}_2, k) \frac{1}{\hat{n}' \cdot \hat{z}} \Big[& - (1+R_\perp)(\hat{e}_{b1} \cdot \hat{t})(\hat{\Omega}_2 \times \hat{e}_{a2}) \cdot (\hat{n}' \times \hat{t}) \\ & - (1-R_{||})(\hat{e}_{b1} \cdot \hat{t}')(\hat{\Omega}_2 \times \hat{e}_{a2}) \cdot (\hat{n}' \times \hat{t}') \\ & + (1-R_\perp)(\hat{e}_{b1} \cdot \hat{t})\, \hat{e}_{a2} \cdot (\hat{n}' \times \hat{t}') \\ & - (1+R_{||})(\hat{e}_{b1} \cdot \hat{t}')\, \hat{e}_{a2} \cdot (\hat{n}' \times \hat{t}) \Big] \end{aligned} \tag{3.49}$$

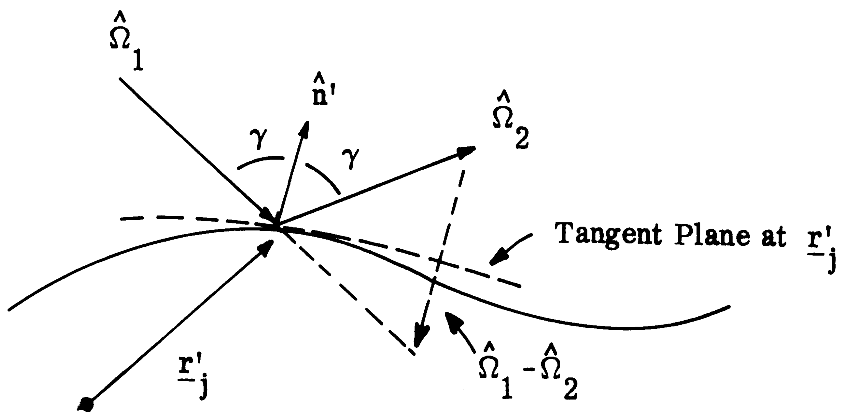

FIG. 3-2: WAVE REFLECTION FROM A SPECULAR (STATIONARY) POINT.

where

$$F(\hat{\Omega}_1, \hat{\Omega}_2, k) = \sum_j \int_{\text{Area Near } \underline{r}'_j} e^{ik(\hat{\Omega}_1 - \hat{\Omega}_1) \cdot \underline{r}'_j} \, dx'dy' \tag{3.50}$$

is the factor denoting the geometrical properties of the surface near each specular point $\underline{r}'_j$.

Explicitly, if

$$\hat{\Omega}_1 = -(\hat{x}\sin\theta_1\cos\phi_1 + \hat{y}\sin\theta_1\sin\phi_1 + \hat{z}\cos\theta_1) \tag{3.51}$$

$$\hat{\Omega}_2 = (\hat{x}\sin\theta_2\cos\phi_2 + \hat{y}\sin\theta_2\sin\phi_2 + \hat{z}\cos\theta_2) \tag{3.52}$$

then, by straight forward algebraic manipulation, we have,

$$S_{hh} = \frac{i\omega\mu_o}{4\pi\eta}\left[C_1 R_\perp - C_2 R_{||}\right] F(\Omega_1, \Omega_2, k) \tag{3.53}$$

$$S_{h\nu} = \frac{i\omega\mu_o}{4\pi\eta}\left[-C_3 R_\perp - C_4 R_{||}\right] F(\Omega_1, \Omega_2, k) \tag{3.54}$$

$$S_{\nu h} = \frac{i\omega\mu_o}{4\pi\eta}\left[C_4 R_\perp + C_3 R_{||}\right] F(\Omega_1, \Omega_2, k) \tag{3.55}$$

$$S_{\nu\nu} = \frac{i\omega\mu_o}{4\pi\eta}\left[-C_2 R_\perp + C_1 R_{||}\right] F(\Omega_1, \Omega_2, k) \; . \tag{3.56}$$

The constants C_1, C_2, C_3, C_4 depend on the incident and reflected directions, and may be expressed as

$$C_1 = \frac{-2PQ}{S} \tag{3.57}$$

$$C_2 = \frac{2\sin\theta_1 \sin\theta_2 \sin^2(\phi_2 - \phi_1)}{S} \tag{3.58}$$

$$C_3 = \frac{2\sin\theta_2 P \sin(\phi_2 - \phi_1)}{S} \tag{3.59}$$

$$C_4 = \frac{-2\sin\theta_1 Q \sin(\phi_2 - \phi_1)}{S} \tag{3.60}$$

and

$$P = \cos\theta_1 \sin\theta_2 - \sin\theta_1 \cos\theta_2 \cos(\phi_2 - \phi_1) \tag{3.61}$$

$$Q = \cos\theta_2 \sin\theta_1 - \sin\theta_2 \cos\theta_1 \cos(\phi_2 - \phi_1) \tag{3.62}$$

$$S = \left[1 - \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 \cos(\phi_2 - \phi_1)\right]\left[\cos\theta_1 + \cos\theta_2\right] . \tag{3.63}$$

The expressions for σ_{ab}, as indicated by the expression (3.41), depend on the expected value of the following relation

$$< F(\hat{\Omega}_1, \hat{\Omega}_2, k) \; F^*(\hat{\Omega}_1, \hat{\Omega}_2, k) >$$

$$= < \sum_j \sum_t \int dx'_j \, dy'_j \int dx'_t \, dy'_t \; e^{ik(\hat{\Omega}_1 - \hat{\Omega}_2)\cdot(r'_j - r'_t)} \; . \tag{3.64}$$

The estimation of the value of the above expression depends greatly on the statistical description of the rough surface. For high frequency scattering, Kodis (1966) showed that an asymptotic result for the integral may be given by

$$< F(\hat{\Omega}_1, \hat{\Omega}_2, k)\, F^*(\hat{\Omega}_1, \hat{\Omega}_2, k) >$$

$$\cong \frac{\pi^2}{k^2 \left[\cos\theta_1 + \cos\theta_2\right]^2} < \bar{R} >< N(\hat{\Omega}_1, \hat{\Omega}_2) > A$$

where

$< \bar{R} >$ is the average gaussian curvature at the reflection point, and

$< N(\hat{\Omega}_1, \hat{\Omega}_2) >$ is the average specular reflecting points per unit area.

Both $< \bar{R} >$ and $< N(\hat{\Omega}_1, \hat{\Omega}_2) >$ depend on the particular statistical model used for the rough surface. By using different postulates, it is shown by Spetner, et al (1960) that the wavelength dependence for the back scattering cross-section may be $1/\lambda^2$, $1/\lambda^4$, $1/\lambda^6$ over different frequency ranges. These models, however, do not seem to agree with the experimental evidence of $1/\lambda$ (or less) dependence on the wavelength.

Recently Barrick (1968) relates the integrals involved in Eq. (3.64) to the probability distribution of the slope of the surface. His results may be expressed as:

$$< F(\hat{\Omega}_1, \hat{\Omega}_2, k)\, F^*(\hat{\Omega}_1, \hat{\Omega}_2, k) >$$

$$\cong \frac{4A\pi^2}{k^2} \frac{1}{(\cos\theta_1 + \cos\theta_2)} P\left[\frac{k(\sin\theta_1 \cos\phi_1 + \sin\theta_2 \cos\phi_2)}{(\cos\theta_1 + \cos\theta_2)} \times \right.$$

$$\left. \times \frac{k(\sin\theta_1 \sin\phi_1 + \sin\theta_2 \sin\phi_2)}{(\cos\theta_1 + \cos\theta_2)}\right]$$

where

$P(u, \nu)$ is the probability densily distribution for the slopes of the surface

$$\frac{\partial z}{\partial x} = u$$

$$\frac{\partial z}{\partial y} = \nu \quad .$$

For back scattering and a Gaussian distribution of slopes,

$$\theta_1 = \theta_2 = \theta \quad ,$$

$$\phi_1 = \phi_2 = \phi \quad ,$$

$$P(u, \nu) = \frac{1}{2\pi R} e^{-\frac{u^2+\nu^2}{2k^2}}$$

and we have,

$$< F(\hat{\Omega}_1, \hat{\Omega}_2, k) \; F^*(\hat{\Omega}_1, \hat{\Omega}_2, k) >$$

$$= \frac{\pi A}{2k^2 \cos^2\theta} \frac{1}{s} e^{-\frac{k^2 \cot^2\theta}{2S}} \quad .$$

This exponential form of variation of back scattering cross-section with θ occurrs in almost all the theoretical models.

It is felt that by using more realistic forms of the slope distribution of the sea surface, such as the slope distribution of the sea surface in the presence of wind measured by Schooley (1962), would yield a more realistic result for the aspect dependence of the scattering cross-section.

It is to be noted that the approximate theoretical model obtained by using the geometric optics approach does not reveal the cross polarization effect in the back scattering direction, as being reported by various experimental works. Refer to Eq. (3.53) through (3.63), for the back scattering direction, one finds that in the limit

$$C_1 = C_3 = C_4 = 0$$

and

$$C_2 = \frac{2}{\cos\theta} \quad .$$

So that this model predicts that

$$\sigma_{HV} = \sigma_{VH} \doteqdot 0 \quad .$$

For a theoretical model which indicates the effect of cross polarization a more accurate integration of the Kirchhoff's integral which corresponds to the physical optics approach should be used. This is outlined in the next section.

3.5 The Kirchoff Integral and Statistical Average (Physical Optics)

The formal evaluation of the expected value of the bistatic cross-section from Kirchhoff's integral is very complicated for surfaces of finite conductivity. Following the formulation given by Eq. (3.49), we may formally write,

$$S_{\ell m} = \frac{i\omega\mu_o}{4\pi\eta} \int dx' \int dy' \; e^{ik(\hat{\Omega}_1 - \hat{\Omega}_2)\cdot r} \frac{1}{\hat{n}\cdot\hat{z}}$$

$$\Big\{ -(1+R_\perp)\, A_\ell B_m$$

$$-(1-R_{||})\, C_\ell D_m$$

$$+(1-R_\perp)\, M_\ell B_m$$

$$-(1+R_{||})\, N_\ell D_m \Big\} \qquad (3.65)$$

where the function A_ℓ, C_ℓ, etc. are given by

$$A_\ell = (\hat{\Omega}_2 \times \hat{e}_{\ell_2}) \cdot \hat{n} \times \hat{t} \qquad (3.66)$$

$$C_\ell = (\hat{\Omega}_2 \times \hat{e}_{\ell_2}) \cdot \hat{n} \times \hat{t}' \qquad (3.67)$$

$$M_\ell = \hat{e}_{\ell_2} \cdot \hat{n} \times \hat{t}' \qquad (3.68)$$

$$N_\ell = \hat{e}_{\ell_2} \cdot \hat{n} \times \hat{t} \qquad (3.69)$$

$$B_m = \hat{e}_{m_1} \cdot \hat{t} \qquad (3.70)$$

and

$$D_m = \hat{e}_{m_1} \cdot \hat{t}' \quad . \qquad (3.71)$$

Due to the complicated dependence of $R_\perp$ and $R_{||}$ on the slope of the surface through the factor

$$\cos\gamma = -\hat{\Omega}_1 \cdot \hat{n} = \frac{z_x \sin\theta_1 \cos\phi_1 + z_y \sin\theta_1 \sin\phi_1 - \cos\theta_1}{\sqrt{1 + z_x^2 + z_y^2}} \qquad (3.72)$$

the evaluation of the expected value of $< S_{\ell m} \cdot S^*_{\ell m} >$ for the general case has not been investigated.

Approximations for the expressions of $S_{\ell m}$ by assuming that the reflection coefficients $R_\perp$ and $R_{||}$ are nearly constant, or by assuming that the rough surface is infinitely conducting can simplify the mathematical results considerably. For example, if the reflecting surface is infinitely conducting, so that

$$R_\perp = -1$$
$$R_{||} = 1 \quad ,$$

then Eq. (3.65) may be simplified into

$$S_{\ell m} = \frac{i\omega\mu_o}{2\pi\eta} \int dx' \int dy' \; e^{ik(\hat{\Omega}_1 - \hat{\Omega}_2) \cdot \underline{r}'} \; G_{\ell m} \qquad (3.73)$$

where

$$G_{\ell m} = \frac{1}{n \cdot z} \left[M_\ell B_m - N_\ell D_m \right] \quad . \qquad (3.74)$$

Explicitly, if

$$\hat{\Omega}_1 = -\left[\hat{x}\sin\theta_1\cos\phi_1 + \hat{y}\sin\theta_1\sin\phi_1 + \hat{z}\cos\theta_1\right] \qquad (3.75)$$

$$\hat{\Omega}_2 = \left[\hat{x}\sin\theta_2\cos\phi_2 + \hat{y}\sin\theta_2\sin\phi_2 + \hat{z}\cos\theta_2\right] \qquad (3.76)$$

and

$$\hat{n} = \frac{-\hat{x}z_x - \hat{y}z_y + \hat{z}}{\sqrt{1+z_x^2+z_y^2}} \quad , \qquad (3.77)$$

it can be easily shown that,

$$G_{hh} = \hat{e}_{h2} \cdot \hat{n} \times \hat{e}_{v1}$$

$$= \sin(\phi_1 - \phi_2) \qquad (3.78)$$

$$G_{vh} = \hat{e}_{v2} \cdot \hat{n} \times \hat{e}_{v2}$$

$$= \sin\theta_1\cos\theta_2\left[z_x\sin\phi_2 - z_y\cos\phi_2\right] -$$

$$- \sin\theta_1\cos\theta_2\left[z_x\sin\phi_1 - z_y\cos\phi_1\right] -$$

$$- \cos\theta_1\cos\theta_2\sin(\phi_1 - \phi_2) \qquad (3.79)$$

$$G_{hv} = -\hat{e}_{h2} \cdot \hat{n} \times \hat{e}_{h1}$$

$$= \cos\theta_1\cos(\phi_1 - \phi_2) - \sin\theta_1\left[z_x\cos\phi_2 + z_y\sin\phi_2\right] \qquad (3.80)$$

and

$$G_{vv} = -\hat{e}_{v2} \cdot \hat{n} \times \hat{e}_{h1}$$

$$= \cos\theta_2\cos(\phi_1 - \phi_2) - \sin\theta_2\left[z_x\cos\phi_2 + z_y\sin\phi_2\right] \quad . \qquad (3.81)$$

The expected values of $< S_{\ell m} S^*_{\ell m} >$ for the case of the surface height

$$z = z(x, y)$$

is a Gaussian distributed random variable which has been carried out by Hoffman (1955) and extended in detail by Fung (1964). The results are quite complicated, and because the assumption of Gaussian distributed surface appears to be an unrealistic model for the sea surface with wind effects, they are not quoted here.

It is felt that for the case of a sea surface with wind effects, a more realistic approach is to start with the statistical description of the slope of the surface. Experimental values of such distribution have been reported by Schooley (1962). But analytical expressions for such a distribution and the correlation of such distribution with the theoretical spectrum of sea waves, (Wright,1967) has yet to be postulated. If a realistic statistical model of the slope given by the distribution function $F(z_x, z_y)$ can be postulated, the expected values of $<S_{\ell m} S^*_{\ell m}>$ can be carried out, for high frequency scattering by extending the approach given by Barrick (1968). In principle, each of the integrals $S_{\ell m}$ may be represented by the form

$$S_{\ell m} \sim \int dx' \int dy' \, K(z_x, z_y) \, e^{ik(\hat{\Omega}_2 - \hat{\Omega}_1) \cdot \underline{r}'}$$

where K is a function of the slope, different for different polarizations. Thus, if the statistical description of the slope of the surface is known, the expected values of the cross-section may be carried out. The theoretical analysis involved in this approach has started, and it is expected to be finished in the next quarterly report.

IV
CONCLUSIONS AND RECOMMENDATIONS

In order to find a realistic model for the reflection properties of a rough surface to be used in calculating the reflected radiation, a survey of existing theoretical models and experimental results for rough surface scattering has been carried out. It is concluded that a simple, realistic, yet adequate model for the bistatic cross-section incorporating the roughness and wind, for surfaces such as the sea is lacking.

From the review of the basic approaches for arriving at a theoretical model, it is concluded that two basic approaches may be employed to postulate a theoretical model. The use of geometric optics approach yields a simple model for the bistatic cross-section. This model, however, does not agree with the experimental results in the cross polarization effects. The approach of physical optics using a realistic statistical description of the slope of the surface seems to be very promising in arriving at a model for the purpose of this investigation. Derivation of expressions for the expected values of bistatic cross-section for several postulated slope statistics may be carried out by extending the approach of Barrick (currently under study and expected to be completed in the near future). It is planned that a controlled experiment using a water tank to simulate the sea s surface will be carried out during the next research period in order to verify the validity of the theoretical expressions and estimate the parameters (such as mean value of slope, correlation distance, etc.) involved.

LIST OF ILLUSTRATIONS

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13. ABSTRACT

In order to arrive at a model for the bistatic cross-section of a rough surface, which is adequate for the estimation of the reflected radiation from a doppler radar, a survey of the literature on the experimental work and theoretical models, concerning rough surface scattering, has been carried out. As a result of this survey, the dominant characteristics of radiation reflected from rough surfaces, and the basic approaches in arriving at various theoretical models are summarized.

It is suggested that for the present work, a theoretical model based on the statistics of the slope of the surface, incorporating the effect of wind is probably adequate. An outline of the approach in arriving at the expressions for the bistatic cross-section based on the slope characteristics is given.

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