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By: C-M Chu and S-K Cho

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FOREWORD

This report (1969-2-Q) was prepared by The University of Michigan Radiation Laboratory, Department of Electrical Engineering. The report was written under Contract N62269-68-C-0715 "Doppler Radiation Study" and covers the period 1 October 1968 - 1 January 1969. The research was carried out under the direction of Professor Ralph E. Hiatt, Head of the Radiation Laboratory, and the Principal Investigator was Professor Chiao-Min Chu. The sponsor of this research is the U. S. Naval Air Development Center, Johnsville, Pa., and the Technical Monitor is Mr. Edward Rickner.

ABSTRACT

Approximate formulae for the bistatic cross section of a homogeneous, stationary surface are derived. The formulae are based on physical optics. These formulae give the estimates of the aspect and polarization dependence of the cross section based on a single function of the incident and reflected direction. This results from an integral involving the correlation function of surface height.

It is suggested that a study of this "universal" function based on some realistic choice of correlation function of surface height, should be carried out for a reasonable estimate of the aspect dependence of the scattering from a rough surface.

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I

INTRODUCTION

The investigation of the scattering properties of rough surfaces, particularly that of the sea is continued in this period. Based on the Kirchhoff integral formula and tangent plane approximations, integral formulae for the scattering matrices were given in the last quarterly report (1969-1-Q, Dec. 68). In terms of these scattering matrices, the conventionally used bistatic cross section σ_{hh}, σ_{hv}, etc may be considered as the statistical average of some functional forms of the scattering matrix. The primary concern in estimating the cross sections, therefore, is the choice of a reasonable statistical description of the rough surface.

After an extensive survey of the experimental and theoretical works concerning the spectrum of the sea surface, we limit ourselves to rough surfaces that are described statistically as staionary and homogeneous, corresponding to that of an aged open sea.

By assuming a height correlation function $H(\tau_x, \tau_y)$ for the rough surface, it is shown in Appendices A and B that the integral involved in calculating the average bistatic cross sections can be expressed in relatively simple form. In Chapter III, the explicit forms of the various components of the bistatic cross section showing the aspect and polarization dependence of such rough surfaces are given.

All these formal results depend on a "universal" function F which may be expressed as an integral.of the height correlation of the surface. A realistic choice of the correlation function is somewhat uncertain. It is suggested that some simple models based on either measured results on the angular variation of the sea spectrum, or conventional anisotropic Gaussian correlation might be used. This phase of the study shall be continued in the next research period.

In order to check the theoretical formulae derived, and/or to obtain the constants that might be involved in the formulae, an experimental system has been designed and constructed to study the bistatic scattering cross section of surfaces including water surfaces. Preliminary measurements involving flat surfaces of finite and infinite conductivity have been carried out to test the transmitting, receiving and the recording system. It is felt that this system functions very well. Experimental work involving the reflection from rough surfaces of known characteristics (corrugated surface), ground surface and agitated salt water surface are currently under consideration.

II
THE SCATTERING MATRIX

The expressions for the components of the scattering matrix, specifying the scattering properties of a rough surface, derived from the Kirchhoff integral formula and tangent plane approximation for the boundary conditions, have been given in the First Interim Report (1969-1-Q, December 1968). The essential results, expressed in forms that are more convenient for later use, are summarized in this section.

Referring to Fig. 2-1, a plane wave incident from direction $\hat{\Omega}_1$ is scattered by a rough surface. We are interested in finding the field scattered in the direction $\hat{\Omega}_2$. To be specific, let the rough surface be described by the function z(x, y) of which only some partial statistic behavior is known. The incident direction is given by

$$\hat{\Omega}_1 = -\left[\hat{x}\sin\theta_1\cos\phi_1 + \hat{y}\sin\theta_1\sin\phi_1 + \hat{z}\cos\theta_1\right] \quad . \tag{2.1}$$

The directions of polarization associated with the incident field are given by the following: i) the direction of horizontal polarization

$$\hat{e}_{h_1} = \frac{\hat{z}\times\hat{\Omega}_1}{|\hat{z}\times\hat{\Omega}_1|} = \hat{x}\sin\phi_1 - \hat{y}\cos\phi_1 \tag{2.2}$$

and, ii) the direction of vertical polarization

$$\hat{e}_{v_1} = \hat{\Omega}_1 \times \hat{e}_{h_1} = -\hat{x}\cos\theta_1\cos\phi_1 - \hat{y}\cos\theta_1\sin\phi_1 + \hat{z}\sin\theta_1 \quad . \tag{2.3}$$

Similarly, any direction of reflection is given by

$$\hat{\Omega}_2 = \hat{x}\sin\theta_2\cos\phi_2 + \hat{y}\sin\theta_2\sin\phi_2 + \hat{z}\cos\theta_2 \quad . \tag{2.4}$$

The directions of polarization associated with the reflected field are, i) direction of horizontal polarization,

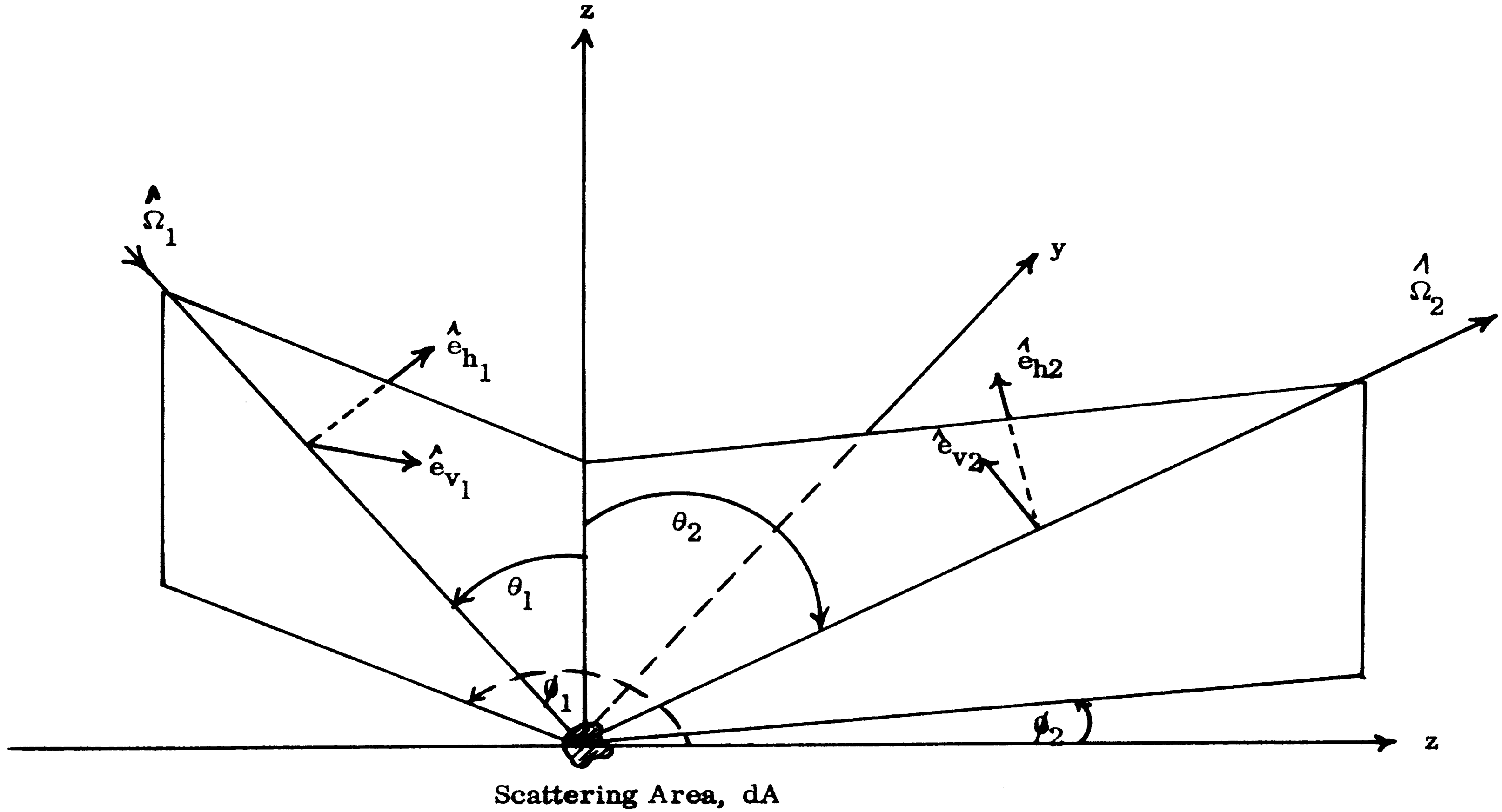

FIG. 2-1: DIFFRACTIONS OF INCIDENT AND REFLECTED WAVES AND THE DIRECTIONS OF POLARIZATION.

$$\hat{e}_{h_2} = \frac{\hat{z} \times \hat{\Omega}_2}{|\hat{z} \times \hat{\Omega}_2|} = -\hat{x}\sin\phi_2 + \hat{y}\cos\phi_2 \tag{2.5}$$

and, ii) vertical polarization,

$$\hat{e}_{v_2} = \hat{\Omega}_2 \times \hat{e}_{h_2} = -\hat{x}\cos\theta_2\cos\phi_2 - \hat{y}\cos\theta_2\sin\phi_2 + \hat{z}\sin\theta_2 \ . \tag{2.6}$$

If the incident field is represented in terms of polarized components by

$$\underline{E}_1 = \left[\hat{e}_{h_1}E_{h_1} + \hat{e}_{v_1}E_{v_1}\right] e^{ik\hat{\Omega}_1 \cdot \underline{r}} \ , \tag{2.7}$$

the far zone reflected field, in the direction $\hat{\Omega}_2$, is also resolved into the two directions of polarization, and is expressed as

$$\underline{E}_2 = \left[\hat{e}_{h_2} E_{h_2} + \hat{e}_{v_2} E_{v_2}\right] \frac{e^{ikr}}{r} \ . \tag{2.8}$$

The relation between the components of the scattered field and those of the incident field are related formally by a scattering matrix. This relation is given by

$$\begin{bmatrix} E_{h_2} \\ E_{v_2} \end{bmatrix} = \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \begin{bmatrix} E_{h_i} \\ E_{v_1} \end{bmatrix} \ . \tag{2.9}$$

By using Kirchhoff's integral formulation and the tangent plane approximation for the boundary conditions, the elements of the scattering matrix may be approximately represented by the integrals;

$$S_{\ell m} = \frac{i\omega\mu_0}{4\pi\eta} \int dx' \int dy' \, e^{-ik(q_x x' + q_y y' + q_z z')} \, g_{\ell m} \ , \tag{2.10}$$

where

$$q_x = \sin\theta_1\cos\phi_1 + \sin\theta_2\cos\phi_2 \ , \tag{2.11}$$

$$q_y = \sin\theta_1\sin\phi_1 + \sin\theta_2\sin\phi_2 \ , \tag{2.12}$$

and

$$q_z = \cos\theta_1 + \cos\theta_2 \quad . \qquad (2.13)$$

The functions $g_{\ell m}$, in the case of finite index of refraction (N), are complicated functions involving the conventionally used reflection coefficients,

$$R_\perp = \frac{\cos\gamma - \sqrt{N^2 - \sin^2\gamma}}{\cos\gamma + \sqrt{N^2 - \sin^2\gamma}} \qquad (2.14)$$

and

$$R_{/\!/} = \frac{N^2\cos\gamma - \sqrt{N^2 - \sin^2\gamma}}{N^2\cos\gamma + \sqrt{N^2 - \sin^2\gamma}} \qquad (2.15)$$

where γ is the angle of incident wave with the local unit normal and varies with x, y, according to the relation

$$\cos\gamma = \frac{-\sin\theta_1\cos\phi_1\frac{\partial z}{\partial x} - \sin\theta_1\sin\phi_1\frac{\partial z}{\partial y} + \cos\theta_1}{\left[1 + (\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2\right]^{1/2}} \qquad (2.16)$$

Anticipating that, for most cases, when $N \rightarrow \infty$, $R_\perp \cong -1$, and $R_{/\!/} \cong +1$, one may express $g_{\ell m}$ in the following form.

$$g_{\ell m} = -(1+R_\perp)P_{\ell m} - (1-R_{/\!/})Q_{\ell m} + 2\,G_{\ell m} \quad .$$

The function, P, Q, and G depend on the direction of incidence, reflection and the slope of the surface. For highly conducting surfaces, the dominant contribution to the scattering matrix comes from the factors $G_{\ell m}$, and the other terms may be treated as a pertubbation. By straightforward algebraic manipulation the explicit expressions of the dominant coefficients $G_{\ell m}$, in terms of $\theta_1, \phi_1, \theta_2, \phi_2$, and the slope at the surface $z_z = \partial z/\partial x$ and $z_y = \partial z/\partial y$ are given below:

$$G_{hh} = z_x\sin\theta_1\cos\phi_2 + z_y\sin\theta_1\sin\phi_2 - \cos\theta_1\cos(\phi_2 - \phi_1) \qquad (2.17)$$

$$G_{hv} = \sin(\phi_2 - \phi_1) \qquad (2.18)$$

$$G_{vh} = z_x(\cos\theta_1\sin\theta_2\sin\phi_1 - \cos\theta_2\sin\theta_1\sin\phi_2) + z_y(\sin\theta_1\cos\theta_2\cos\phi_2 - \sin\theta_2\cos\theta_1\cos\phi_1)$$
$$+\cos\theta_1\cos\theta_2\sin(\phi_2-\phi_1) \qquad (2.19)$$

$$G_{vv} = -z_x\cos\phi_1\sin\theta_2 - z_y\sin\phi_1\sin\theta_2 + \cos\theta_2\cos(\phi_2-\phi_1) \quad . \qquad (2.20)$$

The coefficients $P_{\ell m}$ and $Q_{\ell m}$, which have less effect on the scattering matrix for surfaces of large index of refraction, may be expressed as

$$P_{hh} = -Q_{vv} = \frac{AC}{\Delta}$$
$$P_{hv} = Q_{vh} = \frac{AD}{\Delta}$$
$$P_{vh} = Q_{hv} = \frac{BC}{\Delta}$$
$$P_{vv} = -Q_{hh} = \frac{BD}{\Delta}$$

where

$$A = z_x^2\left[\sin\theta_1\cos\theta_2\sin\phi_1\sin\phi_2 - \cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_1\cos\phi_1\cos\phi_2\right]$$
$$+z_y^2\left[\sin\theta_1\cos\theta_2\cos\phi_1\cos\phi_2 - \cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_1\sin\phi_1\sin\phi_2\right]$$
$$-z_xz_y\left[\sin\theta_1\cos\theta_2\sin(\phi_2+\phi_1) + \sin\theta_1\cos\theta_1\sin(\phi_2+\phi_1)\right]$$
$$-z_x\left[\sin\theta_1\sin\theta_2\cos\phi_1 - \cos\theta_1\cos\theta_2\cos\phi_2 + \cos^2\theta_1\cos\phi_2 + \sin^2\theta_1\cos\phi_1\cos(\phi_2-\phi_1)\right]$$
$$-z_y\left[\sin\theta_1\sin\theta_2\sin\phi_1 - \cos\theta_1\cos\theta_2\sin\phi_2 + \cos^2\theta_1\sin\phi_2 + \sin^2\theta_1\sin\phi_1\cos(\phi_2-\phi_1)\right]$$
$$+\sin\theta_1\cos(\phi_2-\phi_1)\left[\cos\theta_1+\cos\theta_2\right] \quad .$$

$$B = z_x^2\sin\theta_1\left[\sin\phi_1\cos\phi_2 + \cos\phi_1(\cos\theta_1\cos\theta_2\sin\phi_2 + \sin\theta_1\sin\theta_2\sin\phi_1)\right]$$
$$-z_y^2\sin\theta_1\left[\cos\phi_1\sin\phi_2 + \sin\phi_1(\cos\theta_1\cos\theta_2\cos\phi_2 + \sin\theta_1\sin\theta_2\cos\phi_1)\right]$$
$$-z_xz_y\sin\theta_1\left[\cos(\phi_2+\phi_1) + \cos\theta_1\cos\theta_2\cos(\phi_2+\phi_1) + \sin\theta_1\sin\theta_2\cos2\phi_1\right.$$
$$\left. -\sin\theta_1(1+\cos\theta_1\cos\theta_2)\sin(\phi_2-\phi_1)\right]$$
$$+z_x\left[\sin^2\theta_1\cos\theta_2\cos\phi_1\sin(\phi_2-\phi_1) + \cos\theta_1(\sin\phi_2 + \cos\theta_1\cos\theta_2\sin\phi_2 + \sin\theta_1\sin\theta_2\sin\phi_1)\right]$$
$$+z_y\left[\sin^2\theta_1\cos\theta_2\sin\phi_1\sin(\phi_2-\phi_1) + \cos\theta_1(\cos\phi_2 + \cos\theta_1\cos\theta_2\cos\phi_2 + \sin\theta_1\sin\theta_2\cos\phi_1)\right]$$

$$C = -z_x \cos\theta_1 \cos\phi_1 - z_y \cos\theta_1 \sin\phi_1 - \sin\theta_1$$

$$D = -z_x \sin\phi_1 + z_y \cos\phi_1$$

$$\Delta = \left[1+z_x^2+z_y^2\right]^{3/2} \left[z_x^2(1-\sin^2\theta_1\cos^2\phi_1) + z_y^2(1-\sin^2\theta_1\sin^2\phi_1) - 2z_xz_y\sin^2\theta_1\sin\phi_1\cos\phi_1 + 2z_x\sin\theta_1\cos\theta_1\cos\phi_1 + 2z_y\sin\theta_1\cos\theta_1\sin\phi_1 + \sin^2\theta_1\right].$$

III

THE SCATTERING CROSS SECTION

In terms of the scattering matrix introduced in the previous chapter, the conventional scattering cross section can easily be deduced. For example, if the incident field is horizontally polarized,

$$\underline{E}_1 = \hat{e}_{h_1} E_{1_h} e^{ik\hat{\Omega}_1 \cdot \underline{r}} , \tag{3.1}$$

then the component of the scattered field polarized in the horizontal direction is given by

$$\underline{E}_{2_h} = \left[\hat{e}_{h_2} S_{hh} E_{h_1}\right] \frac{e^{ikr}}{r} . \tag{3.2}$$

If the scattering area is denoted by A, then the scattering cross section is easily shown to be

$$\sigma_{hh}(\hat{\Omega}_1, \hat{\Omega}_2) = 4\pi \cos\theta_1 \frac{S_{hh} S^*_{hh}}{A} . \tag{3.3}$$

In terms of the integral representation of S_{hh}, we have

$$S_{hh} S^*_{hh} = \frac{k^2}{16\pi^2} \int dx \int_A dy \int dx' \int_A dy' \, g_{hh}(x,y) g^*_{hh}(x',y') \left\{ e^{-ikq_x(x-x')} e^{-ikq_y(y-y')} e^{-ikq_z(z-z')} \right\} \tag{3.4}$$

where for simplicity we denote

$$z' \stackrel{\Delta}{=} z(x', y') \tag{3.5}$$

In most rough surfaces, only some partial statistical knowledge of the surfaces is known, so that the exact evaluation of the integrals in (3.4) is not feasible. From the statistical properties of the surface, however, one may find the expected value, or statistical averages of these expressions.

Denote $x' = x + \tau_x$ and $y' = y + \tau_y$. The expected value of Eq. (3.4) may be expressed as

$$\langle S_{hh} S_{hh}^{*} \rangle = \frac{k^2}{16\pi^2} \int dx \int_A dy \int d\tau_x \int d\tau_y \, e^{ik(q_x \tau_x + q_y \tau_y)}$$
$$\langle g_{hh}(x,y) g_{hh}^{*}(x+\tau_x, y+\tau_y) e^{ikq_z(z'-z)} \rangle . \quad (3.6)$$

For a homogeneous random surface, for which the statistical properties are invariant under the translation of coordinates (i.e. the scattering area is uniform), we infer that

$$\langle g_{hh}(x,y) g_{hh}^{*}(x+\tau_x, y+\tau_y) e^{ikq_z(z'-z)} \rangle \stackrel{\Delta}{=} K_{hh}(\tau_x, \tau_y) \quad (3.7)$$

which is independent of the coordinates, x, y. With this homogeneity assumption, Eq. (3.6) may be reduced approximately to

$$\langle S_{hh} S_{hh}^{*} \rangle = \frac{k^2}{16\pi^2} A \int d\tau_x \int d\tau_y \, e^{ikq_x \tau_x} e^{ikq_y \tau_y} K_{hh}(\tau_x, \tau_y) \quad (3.8)$$

Thus, we have

$$\sigma_{hh}(\hat{\Omega}_2, \hat{\Omega}_1) = \frac{k^2}{4\pi} \cos\theta_1 \int d\tau_x \int d\tau_y \, e^{ikq_x \tau_x} e^{ikq_y \tau_y} K_{hh}(\tau_x, \tau_y) . \quad (3.9)$$

In general, therefore, for a rough surface that may be statistically represented by a stationary random variable, the scattering cross sections may be estimated by evaluating the expected values

$$K_{\ell m}(\tau_x, \tau_y) = \langle g_{\ell m}(x,y) g_{\ell m}^{*}(x+\tau_x, y+\tau_y) e^{ikq_z(z'-z)} \rangle \quad (3.10)$$

The scattering cross section conventionally used can then be evaluated by the expression

$$\sigma_{\ell m}\ (\hat{\Omega}_2, \hat{\Omega}_1) = \frac{k^2}{4\pi} \cos\theta_1 \int d\tau_x \int d\tau_y e^{ikq_x \tau_x} e^{ikq_y \tau_y} K_{\ell m}(\tau_x, \tau_y) \quad . \tag{3.11}$$

Due to the complicated integrals involved in the procedures in calculating the cross section and the uncertainties in the statistical description of a rough surface such as that of the sea, estimation of $\sigma_{\ell m}$ for a rough surface presents an extremely difficult task. In the present case, some simple models for the rough surface were chosen, and approximate formulas for evaluating the scattering cross sections based on these models are derived.

IV

ASPECT DEPENDENCE OF CROSS SECTION

In order to evaluate the statistical averages of the scattering cross sections given in Eq. (3.3) of the last section, some partial statistical knowledge of the surface must be known. Due to the complexity of physical processes involved in causing the disturbance over a surface such as the sea, a single analytical description for the rough surface is of course impossible. Kinsman (1965) gives an extensive review of the theoretical and experimental work on the wind waves. For a fully aroused sea, it seems that the following two statements concerning the sea waves might be approximately true:

i) The surface is spatially homogeneous and temporally stationary.

ii) The surface height is approximately Gaussian-distributed, although the Gram-Charlier distribution fits the experimental data better ; the difference between these two distributions are very small.

iii) Due to surface wind, the sea spectrum is anisotrppic; the directional dependence of the sea spectrum has been measured experimentally.

Based on these approximate statements, we shall assume the rough surface to be anisotropic, stationary, homogeneous random surface. The surface is statistically specified by a correlation function

$$H(\tau_x, \tau_y) = \left\langle z(x, y) z(x' - \tau_x, y' - \tau_y) \right\rangle = H(\tau, \alpha) \qquad (4.1)$$

where

$$\tau_x = \tau \cos \alpha \qquad (4.2)$$

$$\tau_y = \tau \sin \alpha \ . \qquad (4.3)$$

If the x-direction is chosen as the direction of the wind, some of these directional spectra have been experimentally determined. As illustrated in Appendix A and B for such a model the statistical average of the scattering cross sections may be evaluated. In this section, the analytical expressions for the scattering cross sections, knowing the function $H(\tau_x, \tau_y)$, is developed.

To develop the formal expressions for the scattering cross section, we shall follow the approximate procedure developed in Appendix B.

First the coefficients $g_{\ell m}$ defined in Chapter II are expanded into series forms to the first order approximation in z_x and z_y:

$$g_{\ell m} = a_{\ell m} + b_{\ell m} z_x + c_{\ell m} z_y \quad . \tag{4.4}$$

Explicitly, we have

$$a_{hh} = -2\cos\theta_1 \cos(\phi_2-\phi_1) + (1+R_\perp)(\cos\theta_1+\cos\theta_2)\cos(\phi_2-\phi_1)$$

$$\begin{aligned} b_{hh} = 2\sin\theta_1\cos\phi_2 &- \frac{(1+R_\perp)}{\sin\theta_1}\Big[(1+\cos\theta_1\cos\theta_2)\cos\phi_1\cos(\phi_2-\phi_1) \\ &\quad +\cos\phi_1\sin\theta_1\sin\theta_2 - \cos\theta_1(\cos\theta_1+\cos\theta_2)\cos\phi_2\Big] \\ &+ \frac{(1-R_{//})}{\sin\theta_1}\Big[(1+\cos\theta_1\cos\theta_2)\sin\phi_1\sin(\phi_2-\phi_1)\Big] \end{aligned}$$

$$\begin{aligned} c_{hh} = 2\cos\theta_1\sin\phi_2 &- \frac{(1+R_\perp)}{\sin\theta_1}\Big[(1+\cos\theta_1\cos\theta_2)\sin\phi_1\cos(\phi_2-\phi_1) \\ &\quad +\sin\phi_1\sin\theta_1\sin\theta_2 - \sin\phi_2\cos\theta_1(\cos\theta_1+\cos\theta_2)\Big] \\ &- \frac{(1-R_{//})}{\sin\theta_1}(1+\cos\theta_1\cos\theta_2)\cos\phi_1\sin(\phi_2-\phi_1) \end{aligned}$$

$$a_{hv} = 2\sin(\phi_2-\phi_1) - (1-R_{//})(1+\cos\theta_1\cos\theta_2)\sin(\phi_2-\phi_1)$$

$$\begin{aligned} b_{hv} = \frac{(1+R_\perp)}{\sin\theta_1}(\cos\theta_1+\cos\theta_2)\sin\phi_1\cos(\phi_2-\phi_1) &+ \frac{(1-R_{//})}{\sin\theta_1}\Big[(\cos\theta_1+\cos\theta_2)\cos\phi_1\sin(\phi_2-\phi_1) \\ &-\cos\theta_1(1+\cos\theta_1\cos\theta_2)\sin\phi_2 - \sin\theta_1\cos\theta_1\sin\theta_2\sin\phi_2\Big] \end{aligned}$$

$$\begin{aligned} C_{hv} = -\frac{(1+R_\perp)}{\sin\theta_1}(\cos\theta_1+\cos\theta_2)\cos\phi_1\cos(\phi_2-\phi_1) &+ \frac{(1-R_{//})}{\sin\theta_1}\Big[(\cos\theta_1+\cos\theta_2)\sin\phi_1\sin(\phi_2-\phi_1) \\ &+\cos\theta_1(1+\cos\theta_1\cos\theta_2)\cos\phi_2 + \sin\theta_1\cos\theta_1\sin\theta_2\cos\phi_1\Big] \end{aligned}$$

$$a_{vh} = 2\cos\theta_1\cos\theta_2\sin(\phi_2-\phi_1)-(1+R_\perp)(1+\cos\theta_1\cos\theta_2)\sin(\phi_2-\phi_1)$$

$$b_{vh} = 2\left[\cos\theta_1\sin\theta_2\sin\phi_1-\sin\theta_1\cos\theta_2\sin\phi_2\right]$$
$$+\frac{(1+R_\perp)}{\sin\theta_1}\Big[(\cos\theta_1+\cos\theta_2)\cos\phi_1\sin(\phi_2-\phi_1)-\cos\theta_1(1+\cos\theta_1\cos\theta_2)\sin\phi_2$$
$$-\sin\theta_1\cos\theta_1\sin\theta_2\sin\phi_1\Big]+\frac{(1-R_{//})}{\sin\theta_1}(\cos\theta_1+\cos\theta_2)\sin\phi_1\cos(\phi_2-\phi_2)$$

$$c_{vh} = 2\left[\sin\theta_1\cos\theta_2\cos\phi_2-\cos\theta_1\sin\theta_2\cos\phi_1\right]+\frac{(1+R_\perp)}{\sin\theta_1}\Big[(\cos\theta_1+\cos\theta_2)\sin\phi_1\sin(\phi_2-\phi_1)$$
$$+\cos\theta_1(1+\cos\theta_1\cos\theta_2)\cos\phi_2+\sin\theta_1\cos\theta_1\sin\theta_2\cos\phi_1\Big]$$
$$-\frac{(1-R_{//})}{\sin\theta_1}(\cos\theta_1+\cos\theta_2)\cos\phi_1\cos(\phi_2-\phi_1)$$

$$a_{vv} = 2\cos\theta_1\cos(\phi_2-\phi_1)-(1-R_{//})(\cos\theta_1+\cos\theta_2)\cos(\phi_2-\phi_1)$$

$$b_{vv} = -2\sin\theta_2\cos\phi_1-\frac{(1+R_\perp)}{\sin\theta_1}(1+\cos\theta_1\cos\theta_2)\sin\phi_1\sin(\phi_2-\phi_1)+\frac{(1-R_{//})}{\sin\theta_1}\Big[(1+\cos\theta_1\cos\theta_2)$$
$$\cos\phi_1\cos(\phi_2-\phi_1)+\sin\theta_1\sin\theta_2\cos\phi_2-\cos\theta_1(\cos\theta_1+\cos\theta_2)\cos\phi_2\Big]$$

$$c_{vv} = -2\sin\theta_2\sin\phi_1+\frac{(1+R_\perp)}{\sin\theta_1}(1+\cos\theta_1\cos\theta_2)\cos\phi_1\sin(\phi_2-\phi_1)$$
$$+(1-R_{//})\Big[(1+\cos\theta_1\cos\theta_2)\sin\phi_1\cos(\phi_2-\phi_1)+\sin\theta_1\sin\theta_2\sin\phi_1-$$
$$-\cos\theta_1(\cos\theta_1+\cos\theta_2)\sin\phi\Big].$$

Here, the approximate $R_{//}$ and $R_\perp$ are evaluated through $\cos\gamma = \cos\theta_1$. Then, following Appendix B, we choose correlation function $H(\tau_x, \tau_y)$ and compute the integral

$$F(q_x, q_y, q_z) \stackrel{\Delta}{=} \int_{-\infty}^{\infty} d\tau_x \int_{-\infty}^{\infty} d\tau_y \exp\left\{-k^2 q_z^2 \left[H(0,0) - H(\tau_x, \tau_y)\right] - ik\left[q_x \tau_x + q_y \tau_y\right]\right\} \tag{4.5}$$

and then the integrals involving the evaluation of the cross sections may be expressed in terms of F. From Eq. (3.11) we immediately deduce

$$\sigma_{\ell m}(\hat{\Omega}_2, \hat{\Omega}_1) = \frac{4\pi k^2 \cos\theta_1}{q_z^2} F(q_x q_y, q_z) \left[a_{\ell m} q_z - b_{\ell m} q_x - c_{\ell m} q_y\right]^2 \quad . \tag{4.6}$$

It is evident that from any analysis carried out here the choice of the correlation function $H(\tau_x, \tau_y)$ and subsequent evaluation of $F(q_x, q_y, q_z)$ completely determines the aspect dependence of the average value of the scattering cross sections.

The choice of some reasonable expressions for $H(\tau_x, \tau_y)$ seems to be the major problem confronting us at present, coupled by some approximate method of carrying out the integration for $F(q_x, q_y, q_z)$. We expect that, after careful study of the experimental data as mentioned in Kinsman (1965), some reasonable functional form of $H(\tau_x, \tau_y)$ may be deduced. If this proves to be impossible, then, perhaps, following the commonly used technique in dealing with random variables, assuming scale lengths ℓ_x and ℓ_y we will postulate

$$H(\tau_x, \tau_y) = H(0,0) \exp\left[-\frac{\tau_x^2}{\ell_x^2} - \frac{\tau_y^2}{\ell_y^2}\right] \quad . \tag{4.7}$$

This phase of work shall be continued in the next research period.

V

CONCLUSIONS AND RECOMMENDATIONS

A first order theory for the bistatic scattering cross section of a rough surface incorporating the effect of the dielectric constant of the surface, has been developed. The results depend on the choice of a model of surface height correlation. Due to the approximations involved in the derivation, and uncertainty in the choice of a reasonable model of the height correlation, the usefullness of the results derived cannot be certain. However, it extends the present theories of rough surface scattering which are mainly limited to backscattering, to arbitrary direction, and within the same degree of approximation. Therefore, it seems to be worthwhile to choose some reasonable form of height correlation and investigate the implications of the aspect variations of scattering cross section as predicted by these first order formula. If possible, the results of the theory should be checked against the limited experimental data available in open literature.

An experimental system for the measurement of bistatic cross section that is designed in conjunction with this work should also prove to be useful in asserting the validity of the theoretical results and suggest possible modifications.

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APPENDIX A

STATISTICAL DESCRIPTION OF ROUGH SURFACE

In carrying out the statistical average of $S_{\ell m}S^*_{\ell m}$ to obtain $\sigma_{\ell m}$, statistical properties of the surface are required. Descriptions of a random surface and the approximate distribution functions involving the surface height and slopes are given in this appendix.

We consider the surface height given by

$$z = z(x, y) \tag{A.1}$$

as a random function. By proper choice of the reference height, we let the average value of z be zero, i.e.

$$\langle z(x, y)\rangle = 0 \tag{A.2}$$

A statistical description of this random function is the correlation of the height between two positions on the surface defined as

$$H(x, y; x', y') = \langle z(x, y),\ z'(x', y')\rangle \quad . \tag{A.3}$$

For a homogeneous (spatial) surface, the correlation function should be invariant with respect to any coordinate translation. Thus, if we denote

$$x' = x-\tau_x \tag{A.4}$$

$$y' = y-\tau_y \ , \tag{A.5}$$

then

$$H(x, y; x', y') = \langle z, z'\rangle = H(\tau_x, \tau_y) \tag{A.6}$$

where, for simplicity, we denote

$$z' = z(x', y') = z(x-\tau_x, y-\tau_y) \ . \tag{A.7}$$

If we let

$$\tau_x = \tau \cos\theta \tag{A.8}$$

$$\tau_y = \tau \sin\theta \ , \tag{A.9}$$

then

$$H(\tau_x, \tau_y) = H(\tau, \theta) \ . \tag{A.10}$$

In practical measurements involving sea surfaces, some knowledge of the function $H(\tau, \theta)$ is known.

The mean square value of the surface height, from (A. 6), is given by

$$m_o^2 = \langle z^2 \rangle = H(0,0) \ . \qquad \text{(A. 11)}$$

For most physical problems, $H(\tau_x, \tau_y)$ must be analytical functions of the variables and is evidently even in τ_x and τ_y. Therefore,

$$\left.\frac{\partial H}{\partial \tau_x}\right|_{\substack{\tau_x=0 \\ \tau_y=0}} \triangleq H_x(0,0) = 0 \qquad \text{(A. 12)}$$

and

$$H_y(0,0) = 0 \ . \qquad \text{(A. 13)}$$

Since, for this class of random functions, the operations of differentiation and taking the average commute (Moyal, 1949) the correlation between slopes and heights can be easily obtained. The results are, explicitly,

$$\langle z, z'_x \rangle = -H_x(\tau_x, \tau_y) \qquad \text{(A. 14)}$$

$$\langle z, z_x \rangle = 0 \qquad \text{(A. 15)}$$

$$\langle z_x, z' \rangle = H_x(\tau_x, \tau_y) \qquad \text{(A. 16)}$$

$$\langle z_x, z'_x \rangle = -H_{xx}(\tau_x, \tau_y) \qquad \text{(A. 17)}$$

$$\langle z_x, z_x \rangle = -H_{xx}(0,0) \qquad \text{(A. 18)}$$

$$\langle z, z'_y \rangle = -H_y(\tau_x, \tau_y) \qquad \text{(A. 19)}$$

$$\langle z, z_y \rangle = 0 \qquad \text{(A. 20)}$$

$$\langle z_y, z' \rangle = H_y(\tau_x, \tau_y) \qquad \text{(A. 21)}$$

$$\langle z_y, z'_y \rangle = -H_{yy}(\tau_x, \tau_y) \qquad \text{(A. 22)}$$

and

$$\langle z_y, z_y \rangle = -H_{yy}(0,0) \ . \qquad \text{(A. 23)}$$

In evaluating the average values of the scattering cross sections given in Chapter III, we are interested in obtaining the average of quantities,

$$\left\langle g_{\ell m}(z_x, z_y)\, g^*_{\ell m}(z'_x, z'_y)\, e^{-ikq_z(z-z')} \right\rangle .$$

For evaluation of these quantities, it is necessary to know the higher order correlations such as

$$\langle z_x, z_y; z'_x, z'_y; z, z' \rangle \quad .$$

Knowledge of these higher correlation functions for rough surfaces or sea surfaces is practically unknown and too difficult to measure. By relying heavily on the physical reasoning that a random surface may be considered as a summation of infinitely many small perturbations, we may infer that the random surface is approximately Gaussian so that the joint probability distributions of the quantities z_x, z_y, z'_x, z'_y and $(z-z')$ may be expressed from the correlation of each pair of the variables.

To simplify notations, let us denote

$$u_1 \stackrel{\Delta}{=} z-z', \quad u_2 \stackrel{\Delta}{=} z_x, \quad u_3 \stackrel{\Delta}{=} z_y, \; u_4 \stackrel{\Delta}{=} z'_x, \quad u_5 \stackrel{\Delta}{=} z'_y$$

and the correlation

$$\rho_{ij} \stackrel{\Delta}{=} \langle u_i u_j \rangle .$$

For a random surface of given $H(\tau_x, \tau_y)$, then it is easily seen that

$$\rho_{11} = 2\left[H(0,0) - H(\tau_x, \tau_x)\right]$$
$$\rho_{22} = \rho_{44} = -H_x(0,0)$$
$$\rho_{33} = \rho_{55} = -H_y(0,0)$$
$$\rho_{12} = \rho_{14} = \rho_{21} = \rho_{41} = -H_x(\tau_x, \tau_y)$$
$$\rho_{13} = \rho_{15} = \rho_{31} = \rho_{51} = -H_y(\tau_x, \tau_y)$$
$$\rho_{23} = \rho_{32} = -H_{xy}(0,0)$$
$$\rho_{34} = \rho_{42} = -H_{xx}(\tau_x, \tau_y)$$
$$\rho_{25} = \rho_{52} = \rho_{34} = \rho_{43} = -H_{xy}(\tau_x, \tau_y)$$
$$\rho_{35} = \rho_{53} = -H_{yy}(\tau_x, \tau_y)$$
$$\rho_{45} = \rho_{54} = -H_{yy}(0,0) \quad .$$

In terms of these correlations, the joint probability distribution can be written approximately as

$$f(u_1, u_2, u_3, u_4, u_5) = \frac{1}{\sqrt{(2\pi)^5 |\rho|}} \exp \sum_{i,j}{}' \frac{m_{ij} u_i u_j}{2|\rho|}$$

where $|\rho|$ is the determinant formed by ρ_{ij}, and m_{ij} are the cofactors of ρ_{ij} in the determinant.

Therefore, for the average of any function of the five variables, we may use the probability distribution and evaluate

$$\langle J \rangle = \int_{-\infty}^{\infty} du_1 \int_{-\infty}^{\infty} du_2 \int_{-\infty}^{\infty} du_3 \int_{-\infty}^{\infty} du_4 \int_{-\infty}^{\infty} du_5 J(u_1, u_2, u_3, u_4, u_5) \, f(u_1, u_2, u_3, u_4, u_5) \; .$$

Thus, in principle, using the Gaussian approximation, if we know the correlation function $H(\tau_x, \tau_y)$ of the surface, the scattering matric can be computed. The procedure is, however, extremely complicated and some extra approximations may be introduced in order to obtain manageable results.

APPENDIX B

EVALUATION OF STATISTICAL AVERAGES

Based on the development of the approximate joint probability of $u_1 \triangleq z-z'$, $u_2 = z_x$, $u_3 = z_y$, $u_4 = z'_x$ and $u_5 = z'_y$, the statistical averages involved in the calculations of scattering cross sections can be, in principle, carried out. However, due to the complicated functional forms of $g_{\ell m}$ as shown in Ch. IV, exact evaluations of the cross sections seem to be very impractical. From the expression

$$g_{\ell m} = -(1+R_{\perp})P_{\ell m} -(1-R_{\parallel}) Q_{\ell m} +2 G_{\ell m} ,$$

we see that $G_{\ell m}$, the dominant term,is a linear combination of constant z_x and z_y. To simplify our computation, therefore, we shall approximate the other terms also by a combination of constant term, z_x and z_y. This approximation would give exact results for the case of a perfectly conducting surface and is expected to give good results when the reflective surface if fairly smooth (i. e. z_x, z_y is small).

With this approximation, we may write

$$g_{\ell m} \cong a_{\ell m} + b_{\ell m} u_2 + c_{\ell m} u_3 \tag{B. 1}$$

and

$$g'^{*}_{\ell m} \cong a_{\ell m} + b_{\ell m} u_4 + c_{\ell m} u_5 \tag{B. 2}$$

so that

$$g_{\ell m} g^{*}_{\ell m} \cong a^2_{\ell m} + a_{\ell m} b_{\ell m}(u_2+u_4) + a_{\ell m} c_{\ell m}(u_3+u_5) + b_{\ell m} c_{\ell m}(u_2 u_5 + u_3 u_4) + b^2_{\ell m} (u_2 u_4) + c^2_{\ell m} (u_3 u_5) \quad . \tag{B. 3}$$

The statistical averages to be carried out, therefore, are basically three types. These are evaluated below.

a) For the constant term, we use the distribution

$$f(u_1) = \frac{1}{\sqrt{2\pi\rho_{11}}} \exp\left[-\frac{u_1^2}{2\rho_{11}}\right] \quad . \tag{B.4}$$

This yields

$$\langle e^{-ikq_z u_1}\rangle = \frac{1}{\sqrt{2\pi\rho_{11}}} \int_{-\infty}^{\infty} du_1 \exp\left[-ikq_z u_1 - \frac{u_1^2}{2\rho_{11}}\right] = \exp\left[-\frac{1}{2}k^2 q_z^2 \rho_{11}\right] . \tag{B.5}$$

b) For the terms linear in u_j, we use the distribution

$$f(u_1, u_j) = \frac{1}{2\pi\sqrt{\Delta_j}} \exp\left[-\frac{\rho_{jj}u_1^2 - 2\rho_{1j}u_1 u_j + \rho_{11}u_j^2}{2\Delta_j}\right] \tag{B.6}$$

where

$$\Delta_j = \begin{vmatrix} \rho_{11} & \rho_{1j} \\ \rho_{1j} & \rho_{jj} \end{vmatrix} = \rho_{11}\rho_{jj} - \rho_{1j}^2 \quad . \tag{B.7}$$

Therefore,

$$\langle u_j e^{-ikq_z u_1}\rangle = \frac{1}{2\pi\sqrt{\Delta_j}} \int_{-\infty}^{\infty} u_j du_j \int_{-\infty}^{\infty} du_1$$

$$\exp\left[-ikq_z u_1 - \frac{\rho_{jj}u_1^2 - 2\rho_{1j}u_1 u_j + \rho_{11}u_j^2}{2\Delta j}\right] \tag{B.8}$$

The above integral can be carried out analytically by re-arranging the exponential term such as

$$-ikq_z u_1 - \frac{\rho_{jj}u_1^2 - 2\rho_{1j}u_1 u_j + \rho_{11}u_j^2}{2\Delta_j} = -\frac{1}{2\Delta_j}\left[\sqrt{\rho_{jj}}\,u_1 - \frac{\rho_{1j}u_j - ikq_z\Delta_j}{\sqrt{\rho_{jj}}}\right]^2$$

$$-\frac{1}{2\rho_{jj}}\left[u_j + ik\rho_{1j}q_z\right]^2 - \frac{k^2 q_z^2 \rho_{11}}{2} \tag{B.9}$$

Straightforward successive integration yields

$$\langle u_j e^{-ikq_z u_1} \rangle = -i\rho_{1j} kq_z \exp\left[-\frac{k^2 q_z^2 \rho_{11}}{2}\right] . \qquad \text{(B. 10)}$$

c) for the quadratic terms, we use the distribution

$$f(u_1, u_2, u_3) = \frac{1}{(2\pi)^{3/2}\sqrt{\Delta}} \exp\left[-\frac{\sum_i \sum_j M_{ij} u_i u_j}{2\Delta}\right] \qquad \text{(B. 11)}$$

where

$$\Delta = \begin{vmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{21} & \rho_{22} & \rho_{23} \\ \rho_{31} & \rho_{32} & \rho_{33} \end{vmatrix} \qquad \text{(B. 12)}$$

and M_{ij} is the cofactor of the element ρ_{ij} in Δ. By rearranging terms it is easy to verify that

$$-ikq_z u_1 - \frac{\sum_i \sum_j M_{ij} u_i u_j}{2\Delta} = -\frac{1}{2\Delta}\left[\sqrt{M_{11}}\, u_1 + \frac{M_{12}u_2 + M_{13}u_3 + ikq_z\Delta}{\sqrt{M_{11}}}\right]^2$$

$$-\frac{1}{2M_{11}}\left[\sqrt{\rho_{33}}\, u_2 - \frac{\rho_{23}u_3 + ikq_z M_{12}}{M_{11}}\right]^2$$

$$-\frac{1}{2\rho_{33}}\left[u_3 + ikq_z \rho_{31}\right]^2 - \frac{k^2 q_z^2 \rho_{11}}{2} . \qquad \text{(B. 13)}$$

Using this result, straightforward integration one at a time yields

$$\int_{-\infty}^{\infty} du_1 \int_{-\infty}^{\infty} u_2 du_2 \int_{-\infty}^{\infty} u_3 du_3 f(u_1, u_2, u_3) \exp\left[-ikq_z u_1\right]$$

$$= \left[\rho_{23} + k^2 q_z^2 \rho_{12}\rho_{13}\right] \exp\left[-\frac{k^2 q_z^2 \rho_{11}}{2}\right] .$$

In general, therefore, we have the relation

$$\langle u_i u_j e^{-ikq_z u_1} \rangle = \left[\rho_{ij} - k^2 q_z^2 \rho_{1i}\rho_{1j}\right] \exp\left[-\frac{k^2 q_z^2 \rho_{11}}{2}\right] \quad . \qquad (B.14)$$

With these results,we have the following approximate formula useful in the evaluation of the average values of the scattering cross sections:

$$\langle g_{\ell m} g^*_{\ell m} \rangle = \exp\left[-\frac{1}{2} k^2 q_z^2 \rho_{11}\right] \Big\{ a^2_{\ell m} - ikq_z a_{\ell m} b_{\ell m}\left[\rho_{14}+\rho_{12}\right]$$

$$-ikq_z a_{\ell m} c_{\ell m}\left[\rho_{13}+\rho_{15}\right] + b_{\ell m} c_{\ell m}\left[\rho_{25}+\rho_{34}\right]$$

$$+ b^2_{\ell m}\rho_{24} + c^2_{\ell n}\rho_{35} - k^2 q_z^2\, b_{\ell m} c_{\ell m} (\rho_{12}\rho_{15}+\rho_{13}\rho_{14})$$

$$-k^2 q_z^2 \left[b^2_{\ell m}\rho_{12}\rho_{14} + c^2_{\ell m}\rho_{13}\rho_{15}\right]\Big\} \quad . \qquad (B.15)$$

Using these relations, we can deduce the following relation which is useful in the evaluation of cross sections:

$$\langle g_{\ell m}(z_x, z_y) g^*_{\ell m}(z'_x, z'_y) e^{-ikq(z-z')} \rangle = \exp\left[-\frac{1}{2} k^2 q_z^2 \rho_{11}\right]$$

$$\Big\{ a^2_{\ell m} + 2a_{\ell m} b_{\ell m}(-ikq_z\rho_{12}) + 2a_{\ell m} c_{\ell m}(-ikq_z\rho_{13}) + b^2_{\ell m}(\rho_{24} - k^2 q_z^2 \rho^2_{12})$$

$$+ c^2_{\ell m}(\rho_{35} - k^2 q_z^2 \rho^2_{13}) + 2b_{\ell m} c_{\ell m}(\rho_{25} - k^2_z q^2_z \rho_{12}\rho_{13}) \Big\} \quad . \qquad (B.16)$$

In evaluating the cross sections we must carry out the integral

$$\int_{-\infty}^{\infty} d\tau_x \int_{-\infty}^{\infty} d\tau_y \langle g_{\ell m}(z_x, z_y) g^*_{\ell m}(z'_x, z'_y) e^{-iq(z-z')} \rangle e^{-ikq_x \tau_x} e^{-ikq_y \tau_y} \quad .$$

Now, from the following relations in Appendix A,

$\rho_{11}=2\left[H(0,0)-H(\tau_x,\tau_x)\right]$ $\rho_{12}=-H_x(\tau_x,\tau_y)$

$\rho_{13}=-H_y(\tau_x,\tau_y)$ $\rho_{35}=-H_{yy}(\tau_x,\tau_y)$

$\rho_{25}=-H_{xy}(\tau_x,\tau_y)$ $\rho_{24}=-H_{xx}(\tau_x,\tau_y)$

we see that Eq. (B. 16) may be simplified completely. To deduce the simplified relation, let us define a function

$$F(q_x q_y, q_z) \triangleq \int_{-\infty}^{\infty} d\tau_x \int_{-\infty}^{\infty} d\tau_y \exp\left[-\frac{1}{2}k^2 q_z^2 \rho_{11}\right] \exp\left[-ikq_x\tau_x - ikq_y\tau_y\right]. \tag{B. 17}$$

By Fourier inversion we have

$$\exp\left\{-k^2 q_z^2\left[H(0,0)-H(\tau_x,\tau_x)\right]\right\}$$
$$= \frac{k^2}{(2\pi)^2}\int_{-\infty}^{\infty} dq_x \int_{-\infty}^{\infty} dq_y F(q_x, q_y, q_z)\exp\left[+ikq_x\tau_x + ikq_y\tau_y\right]. \tag{B. 18}$$

By differentiating (B. 18) with respect to τ_x we have

$$k^2 q_z^2 H_x(\tau_x,\tau_y)\exp\left\{-k^2 q_z^2\left[H(0,0)-H(\tau_x,\tau_x)\right]\right\}$$
$$= \frac{k^2}{(2\pi)^2}\int_{-\infty}^{\infty} dq_x \int_{-\infty}^{\infty} dq_y \, F(q_x, q_y, q_z)\, ikq_x \exp\left[ikq_x\tau_x + ikq_y\tau_y\right]. \tag{B. 19}$$

Thus, we have

$$-ikq_z\rho_{12}\exp\left\{-\frac{k^2 q_z^2}{2}\rho_{11}\right\}$$
$$= \frac{k^2}{(2\pi)^2}\int_{-\infty}^{\infty} dq_x \int_{-\infty}^{\infty} dq_y \left(-\frac{q_x}{q_z}\right)F(q_x, q_y, q_z)\exp\left[ikq_x\tau_x + ikq_y\tau_y\right]. \tag{B. 20}$$

Fourier inversion of (B.20) yields

$$\int_{-\infty}^{\infty} d\tau_x \int_{-\infty}^{\infty} d\tau_y \, (-ikq_z\rho_{13}) \exp\left\{-\frac{k^2q_z^2}{2}\rho_{11}\right\}$$

$$= -\frac{q_x}{q_z} F(q_x, q_y, q_z) \quad . \qquad \text{(B.21)}$$

Similarly, with τ_y,

$$\int_{-\infty}^{\infty} d\tau_x \int_{-\infty}^{\infty} d\tau_y \, (-ikq_z\rho_{13}) \exp\left[-\frac{k^2q_z^2}{2}\rho_{11}\right] = -\frac{q_y}{q_z} F(q_x, q_y, q_z) \; . \qquad \text{(B.22)}$$

Twice differentiation of the above with respect to τ_x and τ_y yields

$$k^2q_z^2 \begin{Bmatrix} H_{xx}+k^2q_z^2 H_x^2 \\ H_{xy}+k^2q_z^2 H_xH_y \\ H_{yy}+k^2q_z^2 H_y^2 \end{Bmatrix} \exp\left\{-k^2q_z^2\left[H(0,0)-H(\tau_x,\tau_y)\right]\right\}$$

$$= \frac{k^2}{(2\pi)^2} \int_{-\infty}^{\infty} dq_x \int_{-\infty}^{\infty} eq_y F(q_x, q_y, q_z) \exp\left[ikq_x\tau_x + ikq_y\tau_y\right] \begin{Bmatrix} -k^2q_x^2 \\ -k^2q_yq_y \\ -k^2q_y^2 \end{Bmatrix} . \qquad \text{(B.23)}$$

Identifying the respective values of terms, etc., (s.the) corresponding correlation functions, the above relation may be expressed as

$$\begin{Bmatrix} \rho_{24}-k^2q_z^2\rho_{12}^2 \\ \rho_{25}-k^2q_z^2\rho_{12}\rho_{13} \\ \rho_{35}-k^2q_z^2\rho_{13}^2 \end{Bmatrix} \exp\left[-\frac{k^2q_z^2}{2}\rho_{11}\right] = \frac{k^2}{(2\pi)^2}\int_{-\infty}^{\infty} dq_x \int_{-\infty}^{\infty} dq_y F(q_x, q_y, q_z)$$

$$\exp\left[ikq_x\tau_x+ikq_y\tau_y\right] \begin{Bmatrix} q_x^2 \\ q_xq_y \\ q_z^2 \end{Bmatrix} \frac{1}{q_z^2} \quad . \qquad \text{(B. 24)}$$

The Fourier inversion of the above becomes

$$\int_{-\infty}^{\infty} d\tau_x \int_{-\infty}^{\infty} d\tau_y \begin{Bmatrix} \rho_{24}-k^2q_z^2\rho_{12}^2 \\ \rho_{25}-k^2q_z^2\rho_{12}\rho_{13} \\ \rho_{35}-k^2q_z^2\rho_{13}^2 \end{Bmatrix} \exp\left[-\frac{k^2q_z^2}{2}\rho_{11}-ikq_x\tau_x-ikq_y\tau_y\right]$$

$$= \frac{1}{q_z^2} F(q_x, q_y, q_z) \begin{Bmatrix} q_x^2 \\ q_yq_y \\ q_y^2 \end{Bmatrix} \quad . \text{(B. 25)}$$

Substituting these relations in (B. 16) and carrying out the integration, we have a relatively simple relation for the calculation of the scattering cross section.

$$\int_{-\infty}^{\infty} d\tau_x \int_{-\infty}^{\infty} d\tau_y \left\langle g_{\ell m}(z_x, z_y) g^*_{\ell m}(z'_x, z'_y) e^{-iq(z-z')} \right\rangle \exp\left[-ikq_x\tau_x-ikq_y\tau_y\right]$$

$$= \frac{1}{q_z^2} F(q_x, q_y, q_z) \left[a_{\ell m}q_z - b_{\ell m}q_x - c_{\ell m}q_y\right]^2 \quad . \text{(B. 26)}$$

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13. ABSTRACT

Approximate formulae for the bistatic cross section of a homogeneous, stationary surface are derived. The formulae are based on physical optics. These formulae give the estimates of the aspect and polarization dependence of the cross section based on a single function of the incident and reflected direction. This results from an integral involving the correlation function of surface height.

It is suggested that a study of this "universal" function based on some realistic choice of correlation function of surface height should be carried out for a reasonable estimate of the aspect dependence of the scattering from a rough surface.

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