Relation of Scattering Problems to Radiation from Simple Shapes

by K. M. Siegel and A. L. Maffett

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The University of Michigan
Engineering Research Institute
Willow Run Laboratories
Willow Run Airport
Ypsilanti, Michigan

1. INTRODUCTION

For many years the research personnel in the radiation theory field have seldom mixed technically with the people in the scattering field. At most meetings, papers on scattering are given in sessions parallel to those in radiation theory; and even a desire to mix with people in the "other field" was met with considerable frustration. The "radiation field" is ostensibly a much broader field, since it applies to many peaceful pursuits, such as listening to a baseball game. The scattering field has had as its recent primary influence the determination of the scattering properties of targets. The word targets by its very nature is an unfriendly word. The substitution of "objects" for "targets" even sounds ominous to some people. Scatter from the ionosphere and troposphere has broadened the base of scattering theory as the techniques of the "scattering field" are then applied to explaining or predicting propagation effects.

We now realize that the methods of scattering theory are directly applicable to radiation theory. The practical importance of the reciprocity principle is becoming better understood; indeed, any exact scattering answer which determines the exact current on an object, is by the reciprocity theorem, an exact solution in radiation theory. The approximation methods of scattering theory, such as geometric optics, physical optics, and creeping wave type approximations, are directly applicable.

This paper deals primarily with geometric optics type approximations to radiation patterns from simple shapes. We, of course, use the older building blocks of Sommerfeld (Ref. 1), Carslaw (Ref. 2), MacDonald (Ref. 3), Mie (Ref. 4), as well as the new constructions of Bailin and Silver (Ref. 5) for the cone; Bailin (Ref. 6) and Wait (Ref. 7) for the cylinder, Karr (Ref. 8) for the sphere, and Hatcher and Leitner (Ref. 9) and Myers (Ref. 10) for the prolate spheroid.

We hope this paper encourages the interrelationship between radiation and scattering fields and helps meld the research into one larger field.

2. A METHOD FOR CALCULATING APPROXIMATE FAR FIELDS PRODUCED BY SLOT RADIATORS ON SIMPLE SHAPES

Where exact solutions exist for any of the problems mentioned above it is in general true that the nature of the functions and expressions involved in these exact solutions makes numerical results difficult to find. For these situations and for situations where exact solutions do not exist it is necessary to have a good approximation technique.

For the last seven years, techniques for determining the approximate scattering patterns of many simple and complicated shapes have been the subject of considerable effort at The University of Michigan. These techniques have, in the main, been based on geometric and physical optics methods of approximation and can be extended from scattering problems to slot radiation problems. As they pertain to scattering problems, these methods have produced results which have been shown to be in good agreement with exact results for certain bodies (as long as one is on the small wavelength side of the resonance region of the scatterer - Ref. 11), to be in good agreement with experiments for the infinite cone (Ref. 12), and to produce even exact results for the paraboloid in certain situations (Ref. 13).

The optics methods are employed for the scattering problems under the assumptions that, in addition to the fact that the body in question is perfectly conducting and has no sharp points or edges, the incident energy is in the form of a plane wave, and the scattering surface dimensions are large in comparison to a wavelength. In spite of the elimination of bodies with point discontinuities by these assumptions, the method has been applied to determine the nose-on backscattering radar cross-section of a semi-infinite cone and has yielded results which are within experimental accuracies (Ref. 12), and which are precisely the same as the first order results obtained from the exact solution for large and small cone angles.

As mentioned above these optics techniques can be applied, with similar assumptions, to the problem of calculating approximate fields for radiating slots on simple shapes. The surface dimensions of the body must still be large in comparison to a wavelength; and the incident energy in the form of a plane wave for the scattering problem is replaced by radiated energy from a magnetic dipole on the surface, for the slot problem. The surface is still assumed to be perfectly conducting but may have sharp edges.

With these assumptions, the electric field $\vec{E}$ produced by a radiating slot on a body takes the form (with voltage V_o across the slot)

$$\vec{E} \approx \frac{V_o}{2\pi} \operatorname{curl} \int_{\substack{\text{visible portion} \\ \text{of slot}}} \left(\frac{e^{ikR}}{R} \, d\vec{l} \right) , \qquad (1)$$

where R is the distance between field point and integration point along the slot and $d\vec{l}$ is an infinitesimal of length in the direction of the magnetic dipole. In particular, when the body containing the slot becomes an infinite perfectly conducting plane, the expression (1) becomes the exact solution for the boundary value problem of a radiating slot on such a body.

The expression (1) for the electric field $\vec{E}$ will depend on the body under consideration only in the sense that it will depend on the position and orientation of the slot on that body; that is, the optics method typified by Equation (1) will not yield any information concerning possible diffraction effects due to the body itself. Thus the form of Equation (1) will be similar for all bodies to be considered here; indeed, when an approximation of the form (1) is applied to an arbitrary convex body of revolution having a circumferential slot in a plane normal to the axis of revolution, the problem reduces to that for a cone, tangent to the body of revolution at the slot, with a slot at the circle of tangency. For this reason, only the problem of the cone will receive detailed attention.

In the remaining sections the bodies enumerated at the beginning of the paper will be discussed, and calculations from the optics approximation will be compared to calculations obtained from exact methods for certain special situations by some of the authors mentioned in the Introduction.

3. WEDGE

The infinite wedge will be discussed first because solutions from the literature for this problem are most familiar to the reader, and thus it is most instructive to use the wedge to exhibit techniques, even though it is handled in a manner somewhat different from that indicated above. Oberhettinger (Refs. 14, 15) has given expressions for the Green's function due to a line source parallel to the edge of a wedge; if the line source is moved onto the wedge, the resulting Green's function becomes the Green's function due to a uniformly excited slot parallel to the edge of the wedge. The far field which may be obtained from a suitable approximation of the integral representation of Green's function will consist of an optics term (exactly the term that would result by using the method discussed in the previous section) and a diffraction term due to the edge of the wedge (Ref. 16).

In the case when the wedge opens into a plane the exact values of the angular component of the far electric field can easily be found. These exact values are plotted, along with the result of the approximate method (using both optics and diffraction terms), in Figure 1 for several values of ka, where $k = 2\pi/\lambda$ (λ meaning wavelength) and a = distance from edge of wedge to slot. It is interesting to note that the optics term alone would give just the constant value 1 towards which both exact and approximate results spiral for large ka.

4. CONE

The case of a cone, $\theta = \theta_o$, where θ is the usual spherical polar variable, with a uniformly excited circumferential slot at distance a from its tip will be illustrated in some detail. Let (r, θ, ϕ) and (a, θ_o, β) designate field and integration points (spherical coordinate system), respectively. Then Equation (1) takes the form

$$\vec{E} \approx \frac{V_o}{2\pi} \operatorname{curl} \int_{\substack{\text{VISIBLE} \\ \text{PORTION} \\ \text{OF SLOT}}} \frac{e^{ikR}}{R} (\hat{j} \cos\beta - \hat{i} \sin\beta)\, a \sin\theta_o \, d\beta \quad , \qquad (2)$$

where $\hat{i}$ and $\hat{j}$ are unit vectors in the x and y directions, respectively, V_o is

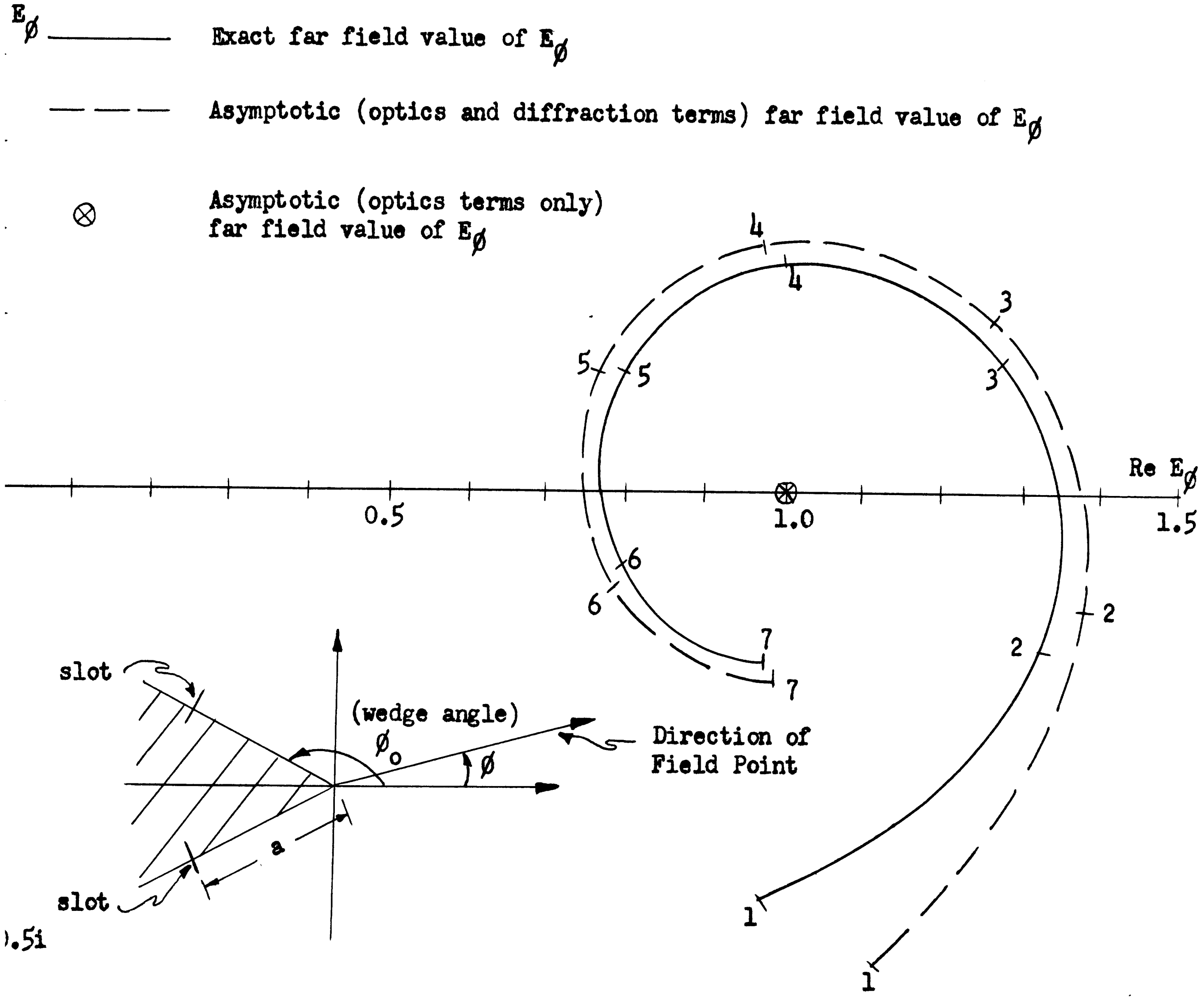


Figure 1

Normalized Field Component, E_ϕ vs ka for Two Uniformly Excited Slots Parallel to the Edge of an Infinite Wedge for the Case When the Wedge Closes into a Plane ($\phi_o = \pi$, $\phi = \frac{\pi}{2}$)
(The numbers on the curves indicate ka values)

a voltage across the slot, and

$$R^2 = r^2 + a^2 - 2\,a\,r\left[\cos\theta\cos\theta_o + \sin\theta\sin\theta_o\cos(\phi-\beta)\right] .$$

Then for $|\theta| < \pi - \theta_o$, the region where the entire slot can be seen, and for $r \gg a$, Equation (2) becomes

$$\vec{E} \approx ka\,V_o\,\frac{e^{ikr}}{r}\sin\theta_o\,e^{-ika\cos\theta\cos\theta_o}J_1(ka\sin\theta\sin\theta_o) \cdot \left[\cos\phi\,\hat{r}\times\hat{j} - \sin\phi\,\hat{r}\times\hat{i}\right] \quad (3)$$

$$= -ka\,V_o\frac{e^{ikr}}{r}\sin\theta_o\,e^{-ika\cos\theta\cos\theta_o}\,J_1(ka\sin\theta\sin\theta_o)\,\hat{\theta},$$

$$|\theta| < \pi - \theta_o \quad ,$$

where $\hat{\theta}$ is a unit vector in the positive θ direction. When $\theta > \pi - \theta_o$, the integration of (2) is accomplished by the method of stationary phase; the error thus incurred is comparable to the error of the integral itself. Then for large ka

$$\vec{E} \approx V_o\,\frac{e^{ikr}}{r}\sqrt{\frac{ka\sin\theta_o}{2\pi\sin\theta}}\,e^{-ika\cos(\theta_o-\theta)-\frac{\pi i}{4}}\,\hat{\theta}, \quad \theta > \pi - \theta_o \; . \quad (4)$$

Bailin and Silver (Ref. 5) have calculated from an exact series

$$\left(ika\,V_o\sqrt{\frac{\pi}{2ka}}\,\frac{e^{ikr}}{r}\right)^{-1} E_\theta \equiv BE_\theta \quad (5)$$

for four points when $\theta_o = 165^o$ and $ka = 50\pi$. A comparison between their results and the results from Equation (4) is given in Table 1.

TABLE 1

$\theta_0 = 165^\circ$, ka = 50

θ	60°	75°	90°	105°
BE_θ from Ref. 4	.182 + .087i	-.102 -.110i	.088 +.141i	.108 +.171i
BE_θ from Eqn. 4	.144 + .098i	-.117 -.117i	.092 + .134i	.117 +.117i

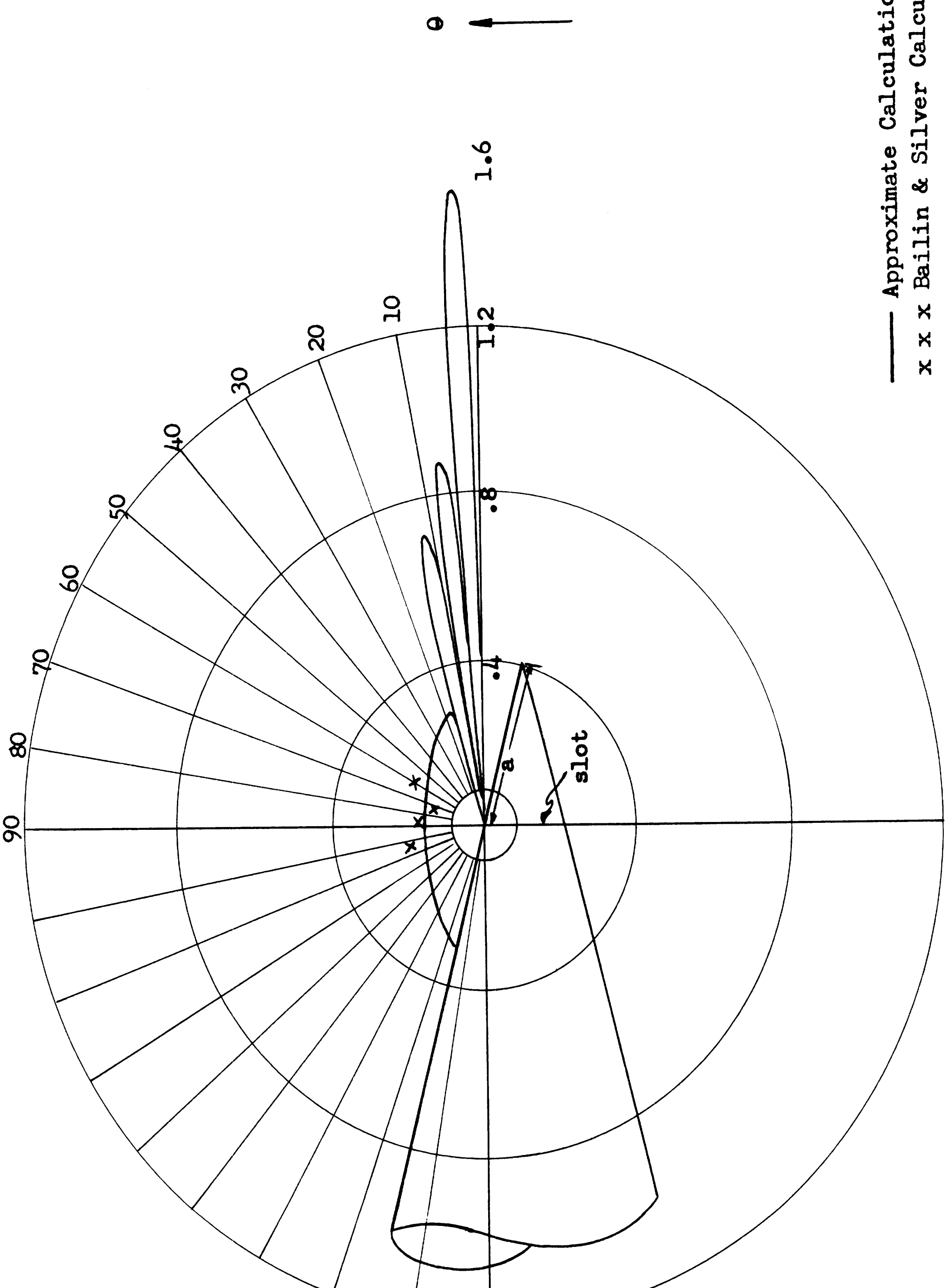
Figure 2 gives a graphical picture, $|BE_\theta|$ versus θ, of Table 1.

For other than uniform excitation it is necessary only to supply the specified excitation in the integrand of (2). Figures 3 and 4 give contour plots of field intensities for excitations cos ϕ and cos 2ϕ, respectively.

5. CYLINDER

The approximation method (1) has been employed to obtain the far field components E_θ and E_ϕ for both axial and circumferential half-wavelength slots on an infinite circular cylinder. Bailin (Ref. 6) has made calculations for these fields using a method (adapted from expressions due to Silver and Saunders (Ref. 17)) more accurate than the method of this paper. Bailin's results are compared with the results obtained from (1) both magnitude-wise and phase-wise in Tables 2 through 7.

Wait (Ref. 7) has obtained results for the same type of slots on an elliptic cylinder. Figure 5 gives a comparison of his and our results for a short (compared to a wavelength) circumferential slot on an elliptic cylinder for the limiting case when the cylinder becomes a ribbon.



——— Approximate Calculation
x x x Bailin & Silver Calculation

Figure 2
Normalized Field Intensity, $|BE_\phi|$, vs θ
For a Uniformly Excited Circumferential Slot
On a 30° Cone With ka = 50π

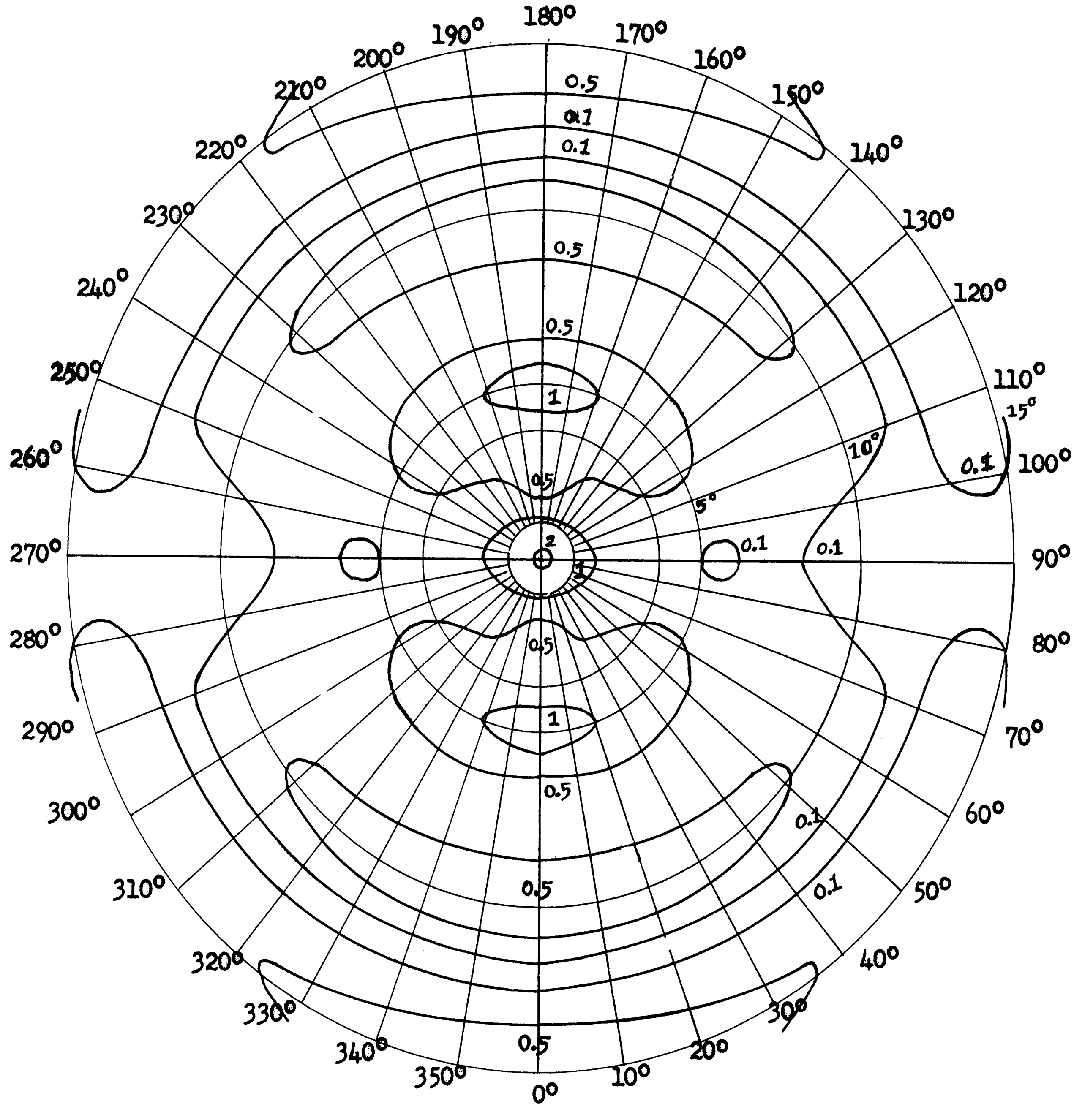
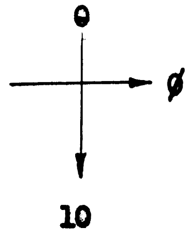


Figure 3
Contour Plot of Radiation Pattern From a 30° Cone
With Circumferential Slot Having cos Ø Excitation (ka = 50π)



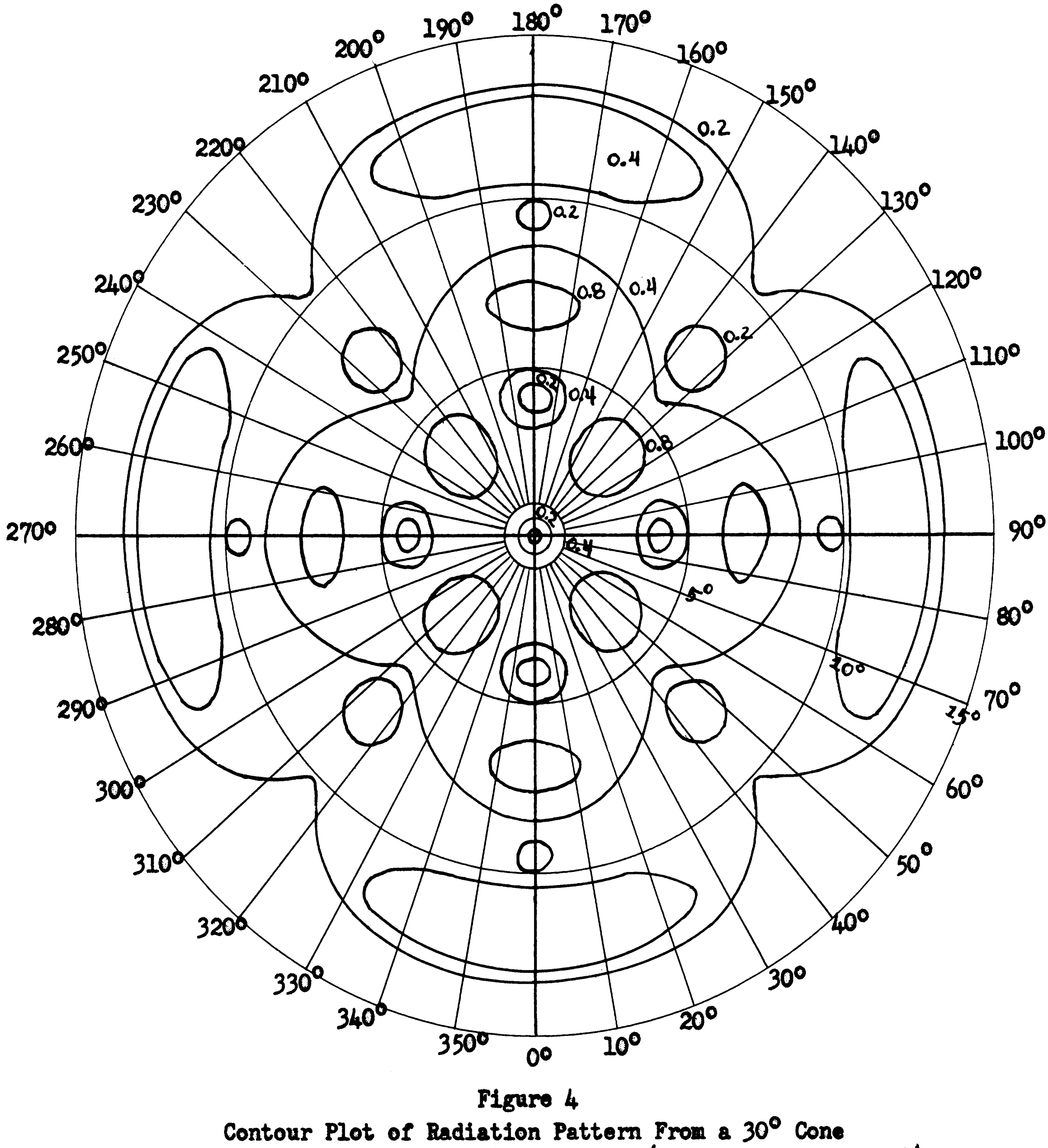


Figure 4
Contour Plot of Radiation Pattern From a 30° Cone
With Circumferential Slot Having cos 2ϕ Excitation ka = 50π

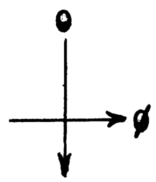


TABLE 2

ϕ° \ θ°	90°		69°.6		55°.6		39°.6		30°	
	Calc	Bailin	Calc	Bailin	Calc	Bailin	Calc	Bailin	Calc	Bailin
0°	1.0		1.0		1.0		1.0		1.0	
15°	.951	.952	.953	.954	.956	.959	.960	.967	.961	.975
30°	.816	.822	.823	.828	.832	.841	.846	.860	.854	.876
45°	.628	.643	.638	.653	.653	.675	.680	.705	.687	.731
60°	.419	.454	.428	.467	.444	.493	.466	.530		

Normalized Field Component, $\dfrac{|E_\theta(\theta,\phi)|}{|E_\theta(\frac{\pi}{2},0)|}$, for a

Half-Wavelength Circumferential Slot on an Infinite

Cylinder with ka = 12

TABLE 3

ϕ° \ θ°	90°		69°.6		55°.6		39°.6		30°	
	C	B	C	B	C	B	C	B	C	B
0°	0°		43°.1	40°.2	120°	126°.3	250°	247°.2	344°	345°.3
15°	23°.5	23°.4	65°.25	62°.2	139°.6	145°.5	262°.1	262°.4	356°.1	357°.3
30°	95°.7	92°.2	130°.1	127°.2	197°.1	202°	304°.4	307°.1	390°.3	392°.3
45°	202°.6	202°.3	233°.5	231°	287°.3	292°.4	379°	378°.8		
60°	347°.6	347°.1	375°.3	367°.6	406°.9	411°.7	470°.2	473°.6		

Phase of $E_\theta(\theta,\phi)$ Relative to $E_\theta(\frac{\pi}{2}, 0)$ for a

Half-Wavelength Circumferential Slot on an Infinite

Cylinder with ka = 12

TABLE 4

θ_0	80°.9		69°.6		55°.6		39°.6		30°	
ϕ^o	C	B	C	B	C	B	C	B	C	B
5°	.0138	.015	.0303	.029	.0492	.049	.0670	.056	.0753	.072
15°	.0403	.045	.0891	.084	.145	.144	.199	.191	.223	.214
30°	.0747	.083	.166	.155	.271	.269	.376	.358	.427	.403
45°	.0996	.109	.222	.203	.369	.355	.520	.479	.595	.541
60°	.115	.120	.258	.224	.434	.396			.717	.629

Normalized Field Component, $\frac{|E_\phi(\theta, \phi)|}{|E_\theta(\frac{\pi}{2}, 0)|}$, for a

Half-Wavelength Circumferential Slot on an Infinite

Cylinder with ka = 12

TABLE 5

θ^o	80°.9		69°.6		55°.6		39°.6		30°	
ϕ^o	C	B	C	B	C	B	C	B	C	B
5°	185°.8	188°.7	221°.7	220°.4	301°.5	304°.5	427°.4	423°.2	522°.9	521°.4
15°	210°	212°	243°	239°	317°.7	321°.7	440°.7	436°.7	534°.8	531°.1
30°	277°.8	278°.6	307°.7	303°.4	374°.9	377°.1	482°.7	480°.7	569°	563°.9
45°	385°.8	384°.6	410°.5	405°	464°.7	465°.4	557°	549°.6	625°.2	618°.6
60°	526°.2	523°.2	551°.3	537°.6	583°	580°.4	647°.2	640°.7	695°.4	689°

Phase of $E_\phi(\theta, \phi)$ Relative to $E_\theta\left(\frac{\pi}{2}, 0\right)$ for a

Half-Wavelength Circumferential Slot on an Infinite

Cylinder with ka = 12

TABLE 6

θ°	90°		69°.6		55°.6		39°.6		30°	
ϕ°	C	B	C	B	C	B	C	B	C	B
0°	1.0	1.0	.9112	.918	.7654	.753	.553	.554	.418	.414
15°		.999		.914		.753		.553		.411
30°		.991		.907		.744		.546		.405
45°		.969		.889		.726		.532		.394
60°		.920		.845		.690		.498		.367

Normalized Field Component, $\frac{|E_\phi(\theta,\phi)|}{|E_\phi(\frac{\pi}{2},0)|}$, for an Axial Half-Wavelength Slot on an Infinite Cylinder with ka = 12. (The calculated value of $\frac{|E_\phi(\theta,\phi)|}{|E_\phi(\frac{\pi}{2},0)|}$ is independent of ϕ for large values of r.)

TABLE 7

θ°	90°		69°.6		55°.6		39°.6		30°	
ϕ°	C	B	C	B	C	B	C	B	C	B
0°	0°	0°	43°.1	39°.9	120°	125°.5	250°	245°.2	344°	341°.8
15°	23°.4	23°.3	65°.1	61°.9	139°.5	144°.5	262°	260°.1	356°	353°.2
30°	95°	91°.4	129°.5	125°.9	196°.5	200°	304°	303°.4	390°	387°.2
45°	201°	199°.6	232°	227°.7	286°	288°	378°	372°.8	446°	440°.5
60°	344°	340°.7	372°	360°.6	404°	403°.3	468°	462°.8	516°	511°.3

Phase of E_ϕ Relative to $E_\phi(\frac{\pi}{2}, 0)$ for an Axial Half-Wavelength Slot on an Infinite Cylinder with ka = 12.

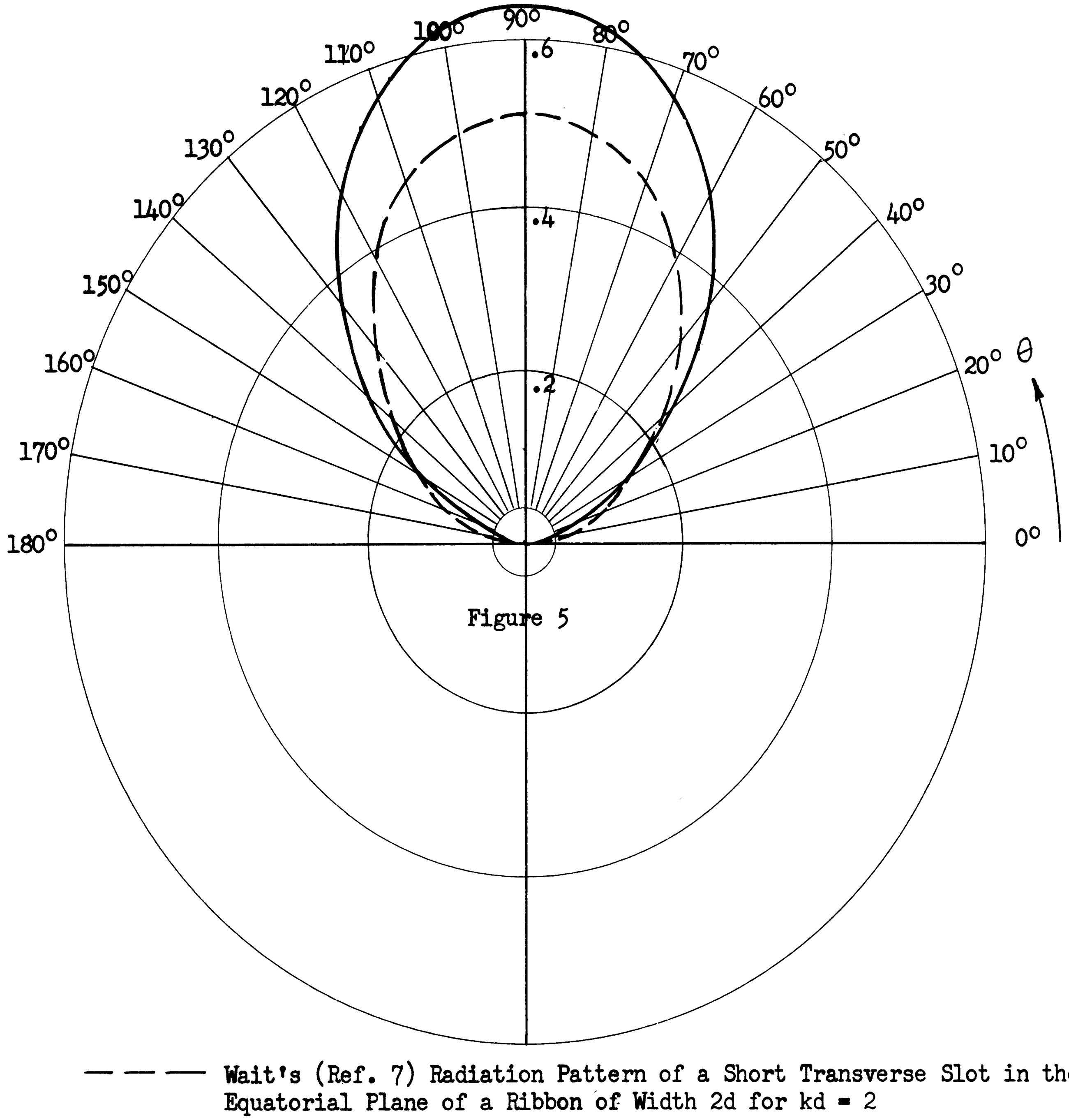


Figure 5

— — — Wait's (Ref. 7) Radiation Pattern of a Short Transverse Slot in the Equatorial Plane of a Ribbon of Width 2d for kd = 2

——— Radiation Pattern (Eqn. 1) of a Short Transverse Slot in the Equatorial Plane for an Elliptic Cylinder in Limit as Cylinder Flattens into a Plane (because of the optics approximation this corresponds to a pattern of a slot in an infinite plane)

(plot is proportional to $r^2 |\vec{E}|^2$)

6. PROLATE SPHEROID AND PARABOLOID

Hatcher and Leitner (Ref. 9) have considered the radiation problem of a prolate spheroidal conductor excited by an electric dipole at its tip. They have considered spheroids of varying thicknesses, with wavelengths approximately π, $\pi/2$, and $\pi/3$ times their major axes, and have obtained patterns giving the interesting results that most of the radiation is directed back of the equatorial plane (i.e., between $\theta = 90^\circ$ and 180°) for the ka considered by them (a being one-half the interfocal length of the spheroid). Geometric optics methods of course would not apply to this case!

Myers (Ref. 10) has obtained patterns for slots on prolate spheroids of approximate length to width ratios of 10/1, 22/1, and 316/1 for ka = 1,2,3, where a is as shown above. Here the limitations of the approximate method are quite evident for the following reasons: For the slot excitation chosen by Myers the far field intensity, in the region where the slot is completely visible, is nearly zero; thus, although this is the region in which our method produces the most reliable result, no decent comparisons can be made. In the region where only part of the slot is visible, it is necessary to assume $ka \gg 1$ to accomplish the integration of (1); thus again no comparisons can be made because of the small magnitude of Myers' ka.

It is, however, possible to obtain patterns for a spheroid with circumferential slot for sufficiently large ka by the simple device, as indicated in Section 2, of investigating the equivalent cone problem. To fix ideas, consider the spheroid

$$\frac{z^2}{p^2} + \frac{x^2 + y^2}{q^2} = 1, \qquad p > q \tag{7}$$

with circumferential slot in the plane, $z = z_o$, $0 < z_o \leq p$, perpendicular to the focal axis. Then to obtain the cone which would, by our method, produce the same pattern as the ellipsoid (7), it is only necessary to choose for the cone parameters θ_o and a (see Section 4) the values

$$\theta_o = \arctan \frac{-q\, z_o}{p\sqrt{p^2 - z_o^2}}$$

$$a = \frac{1}{pz_o}\sqrt{(p^2 - z_o^2)(p^4 - c^2 z_o^2)}\quad , \quad c^2 = p^2 - q^2 \quad . \tag{8}$$

The same remarks hold for the paraboloid of revolution

$$x^2 + y^2 = -4\, pz$$

with circumferential slot in the plane $z = z_o$, $z_o < 0$. Here it will be necessary to choose

$$\theta_o = \arctan \frac{1}{2z_o}$$

$$a = -4pz_o\sqrt{1 - 4z_o^2}$$

in order to obtain the same pattern that would be obtained for the cone $\theta = \theta_o$ with a as specified.

In this way Figures 2, 3, and 4 can also be interpreted as patterns for certain spheroids and paraboloids with suitably located circumferential slots.

The paraboloid serves also to illustrate nicely the statement made in the Introduction that "any exact scattering answer which determines the exact current on an object is, by the reciprocity theorem, an exact solution in radiation theory." It has already been pointed out (Section 2) that

Schensted (Ref. 13) has shown the geometric optics answer to be the exact scattering answer for a plane wave incident along the axis of a paraboloid. Thus, the expectation is that the far field produced by a magnetic dipole located arbitrarily on the surface of a paraboloid would be given exactly in the direction of the axis of the paraboloid; and this expectation has been confirmed. The expression for the magnetic far field in the direction of the axis of the paraboloid due to a magnetic dipole on the surface of the paraboloid is just twice the free space expression for the magnetic dipole. (i.e., $\vec{H} \approx - \frac{2e^{ik(z-z_o)}}{z-z_o} \hat{\phi}$ for a magnetic dipole oriented in the $\hat{\phi}$-direction, where z is distance to field point parallel to the axis of the paraboloid, z_o is corresponding distance to the dipole.) Even so, this result might surprise some, because optics approximations are expected, from their derivations, to give poor results for the field of a dipole located in a region of high curvature.

In this same vein it is worth noting that similar reasoning can be applied to the cone. We recall that the bistatic radar cross-section is very closely approximated by the physical optics formulation for illumination along the axis of the cone. This suggests that the use of the physical optics field in the reciprocity theorem will give a higher order approximation for the field in the direction of the cone axis due to a slot excitation of the cone.

In the case of a circumferential slot on either the paraboloid or the cone we note that only the cosine excitation will produce a non-vanishing contribution along the axis of symmetry. Using the above method for the paraboloid will, as indicated, give the exact field along the axis of symmetry for the cosine excited circumferential slot. Although the reciprocity theorem is not obeyed by the physical optics field we note that Felsen (Ref. 18) has demonstrated that for plane wave illumination along the cone axis the physical optics field agrees with the first order small cone angle approximation to the exact scattered field outside the region of specular reflection $\theta < (2\theta_0 - \pi)$. Recently Felsen (Ref. 19) has indicated that his results can be continued past the specular reflection region and, hence, to the surface of the cone, where the agreement with physical optics also obtains. Thus we may use the physical optics field on the surface of the cone. We expect to have reciprocity obeyed to first order in the small cone angle approximation and to first order in $(ka)^{-1}$. In this way we obtain the field on the axis of symmetry produced by the slot on the cone to the same order of approximation.

7. SPHERE

Far field radiation patterns have been obtained by Karr (Ref. 8) for ka = 1, 1.5, 2, and 3, where a is the sphere radius, for various positions of a circumferential slot on the sphere. In Figures 6, 7, and 8 comparisons are made, for the slot located at $\theta = 45^{\circ}$, between Karr's and our results for ka = 1.5, 2, and 3, respectively. The comparisons are made only in the region where the slot is completely visible from the field point since ka is too small to reasonably apply our methods outside that region.

In order to evaluate our method outside this region of complete visibility we have employed Karr's results to compute several points for ka = 15. Figure 9 gives a comparison of these points with the curve calculated from

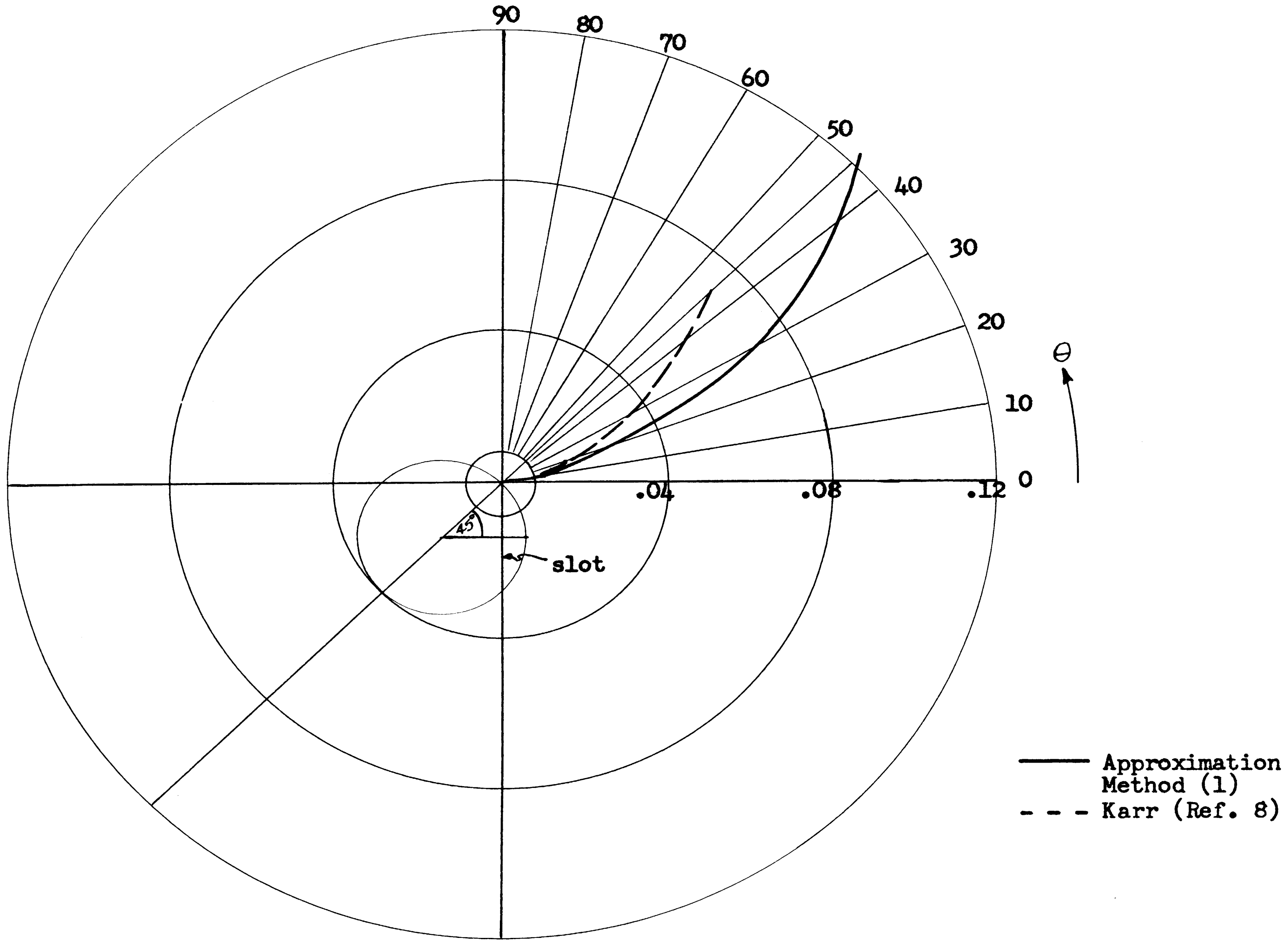


Figure 6

Radiation Pattern For Circumferential Slot (at $\theta = 45^\circ$)

On Sphere in Region Where Entire Slot is Visible for ka = 1.5

(plot is proportional to $r^2 |\vec{E}|^2$)

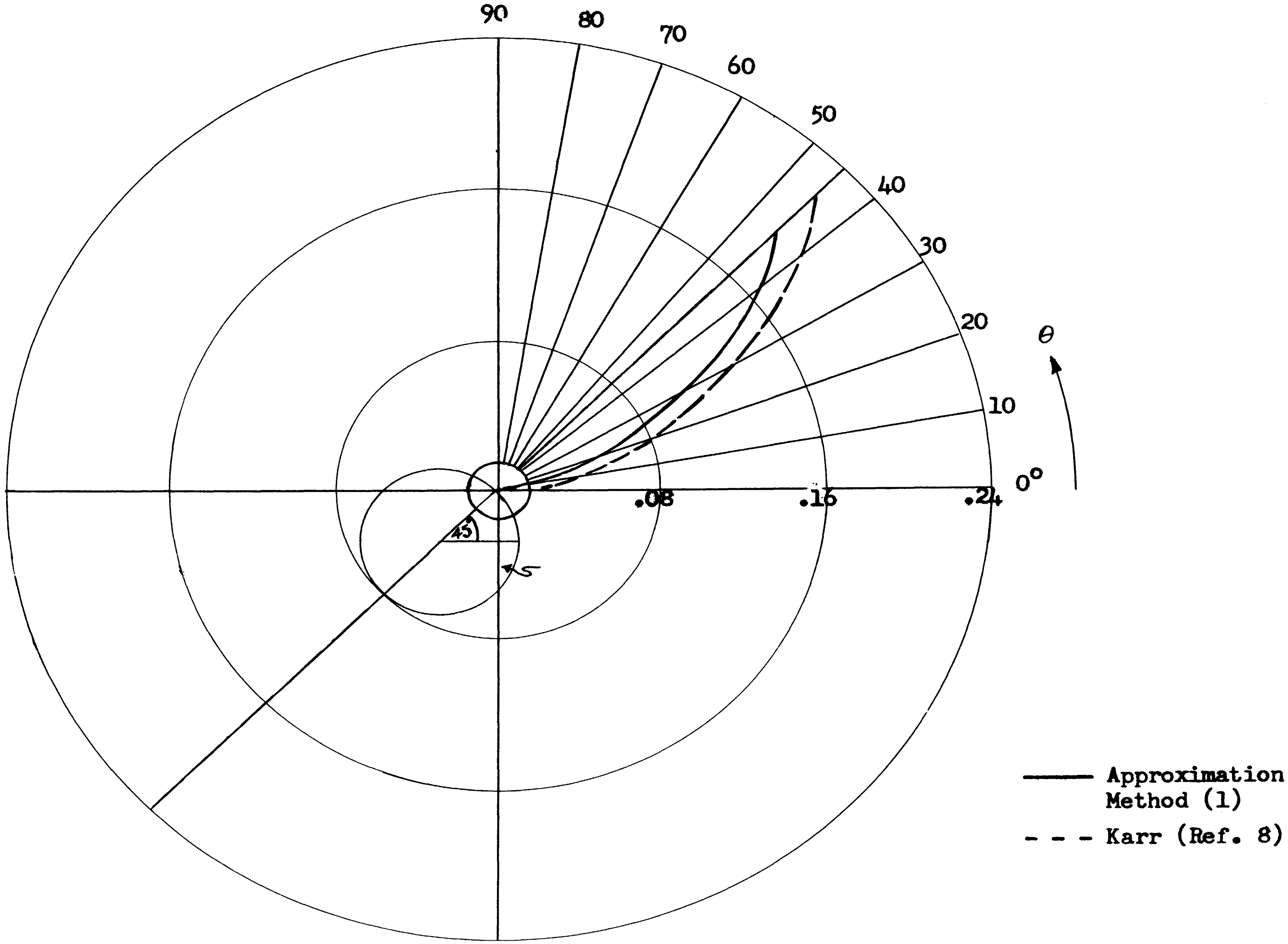


Figure 7
Radiation Pattern For Circumferential Slot (at $\Theta = 45^o$) on Sphere in Region Where Entire Slot is Visible for ka = 2
(plot is proportional to $r^2 |\vec{E}|^2$)

—— Approximation Method (1)
- - - Karr (Ref. 8)

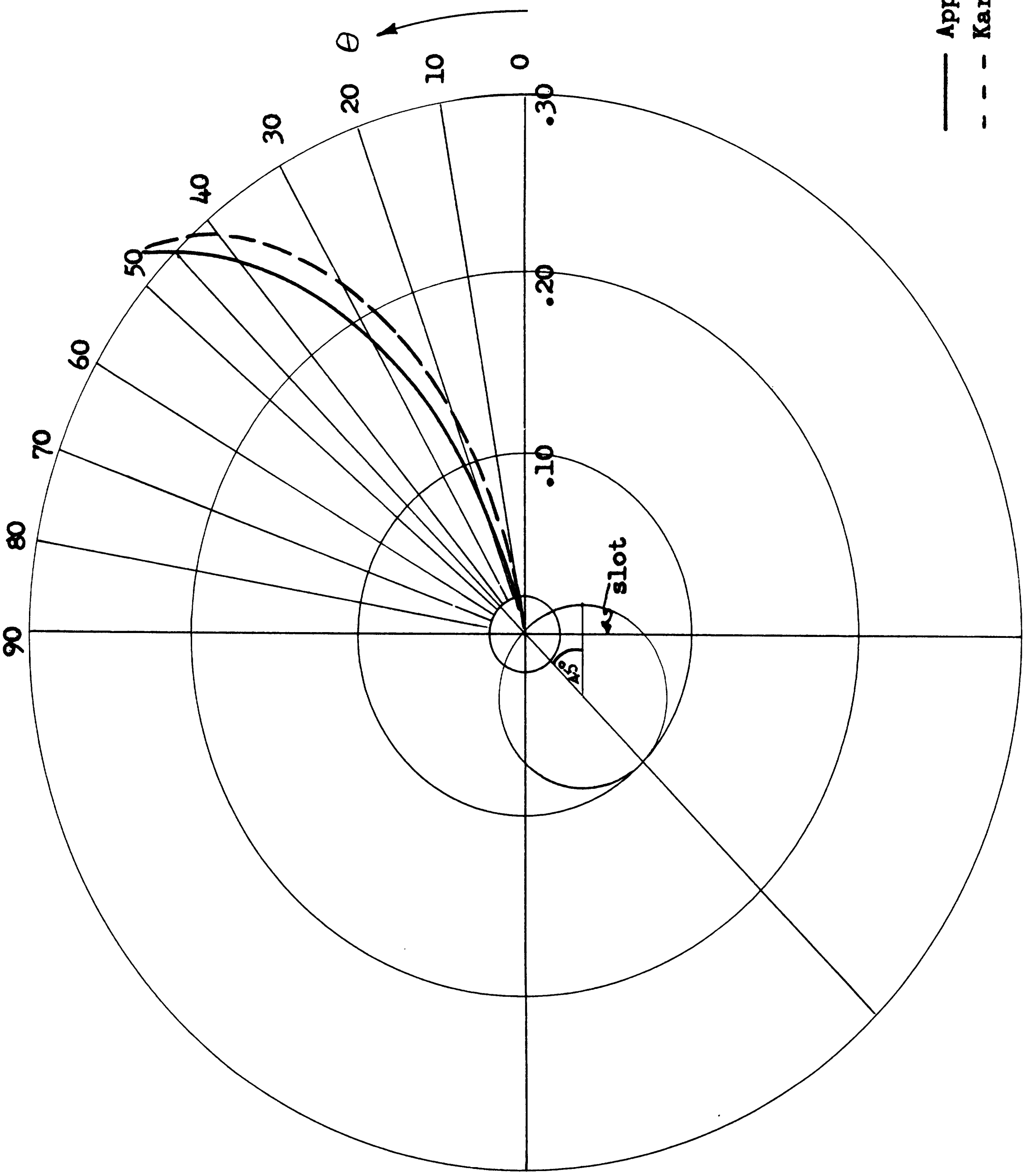


Figure 8

Radiation Pattern For Circumferential Slot (at $\theta = 45^{\circ}$)
On Sphere in Region Where Entire Slot is Visible For ka = 3
(plot is proportional to $r^2|\vec{E}|^2$)

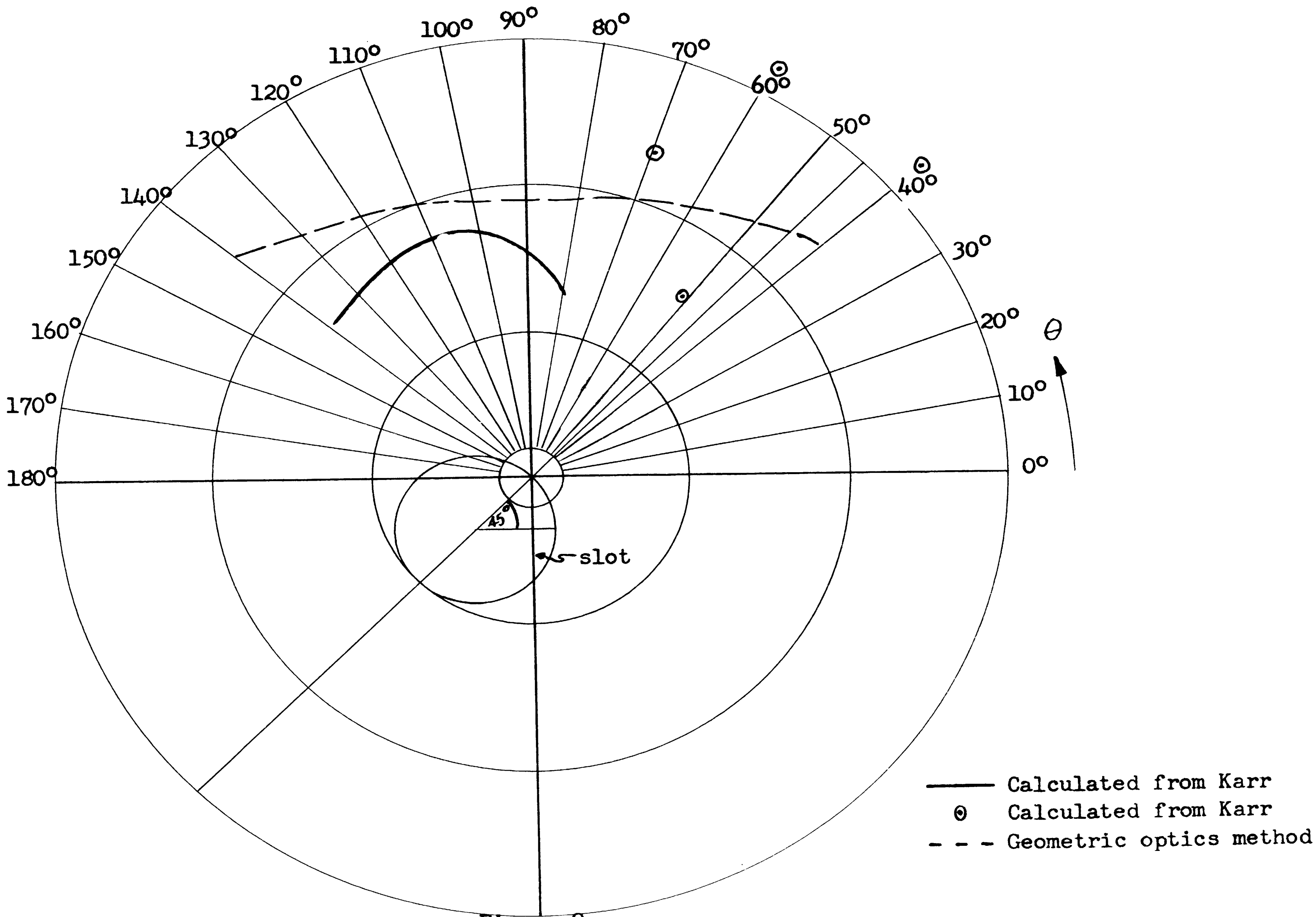


Figure 9
Radiation Pattern of Uniformly Excited Circumferential Slot
On a Sphere at $\Theta_o = 45°$ for ka = 15
(plot is proportional to $r^2|\vec{E}|^2$)

our approximation method for the region where only part of the slot is visible from the field point.

It is of course to be noted from what has been pointed out in Section 6 that our patterns for the slot-on-sphere case given above are exactly those that would be obtained for a cone with $\theta_0 = 135^\circ$.

8. CONCLUSIONS

This paper purports to exhibit the ease with which optics approximations to radiation problems are obtained. With experience one can build up the know-how of when these type approximations are good enough for the design problem at hand. When these approximations yield poor results there are other approximation techniques available in scattering theory which yield better results. We believe that the techniques used in scattering theory calculations should be examined, analyzed, and categorized for use in radiation theory problems.

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