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INVESTIGATION OF PHYSICAL MODEL FOR SATELLITE DETECTION

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<u>OBJECT</u>

Formulate by theoretical analysis, the nature of the disturbances, the existence of passive RF radiation, and the nature of the radiating mechanism resulting from a satellite passing through the ionosphere.

1. PURPOSE

The purpose of the work being performed under the present modification of the basic contract is to indicate whether or not the emission of passive microwave radiation at measurable intensities above the normal atmospheric noise background, results from the passage of a satellite through the ionosphere by theoretically analyzing the nature of the disturbances, the existence of passive radiofrequency radiation, and the nature of the radiating mechanism.

2. ABSTRACT

Work on two types of plasma problems was undertaken, namely, theoretical investigations of several mechanisms which could produce passive microwave radiation from the flow field in the wake of a satellite and a basic plasma study concerning photon transport properties of a non-relativistic plasma.

The IBM 704 computer program for prediction of tracking parameters as functions of time-of-day for satellite passages over any given site has been checked and is ready for use. An interpolation sub-routine has been devised in order that the outputs may be presented at desired time intervals. This sub-routine has been checked and is ready for use with the satellite look-angle program.

An experimental program for measuring the electron and ion densities and temperatures and for detecting possible microwave emissions in the vicinity of a vehicle moving through the ionosphere at satellite velocities, by installing a

Langmuir probe and a microwave radiometer receiver aboard a re-entry vehicle, was initiated and then cancelled during the reporting period.

3. TECHNICAL WORK

3.1 Theoretical Work

3.1.1 General

As indicated in the Semi-Annual Report (2764-30-P), the theoretical work has been divided among a number of problems involving the electromagnetic characteristics of plasmas associated with bodies moving at high speed within the ionosphere. In addition, the IBM 704 general look-angle program for satellite observations from a prescribed site has been improved and thoroughly checked.

3.1.2 Microwave Emission From the Wake of a Satellite

Several approaches to the problem of finding a physical mechanism which would be capable of generating electromagnetic emission at microwave frequencies within the flow field comprising the wake of a satellite have been considered. Professor K. M. Siegel has expressed a belief that such emissions will occur at frequencies well above the local plasma frequencies existing within the wake and that the search for a mechanism to produce electromagnetic emission should be restricted to physical models which are capable of producing such microwave radiation. Such radiations would easily propagate through the surrounding plasma and might be detectable at the surface of the earth.

To date two different models have been proposed which appear to have a mechanism for generation of high frequency microwave radiation. Neither of these models have been checked or refined to the point where any positive statements concerning results can be presented at this time. Therefore, the discussion will be confined primarily to a brief description of the approach used in each case.

Dr. K-M Chen has developed a theory based upon the fact that an electron layer will always be formed at a sharp boundary between a plasma and a rarefied region such as that generated in the wake of a body moving through the ionosphere. Due to the high speed of the electrons relative to the ions and molecules, the time scale of transport phenomena in the electron layer is much smaller than that of similar phenomena in the plasma. Therefore, any oscillations within the electron layer resulting from perturbations will have a higher frequency than the plasma frequency characterizing the plasma region. Since this phenomena would occur within the cavity generated by the motion of the satellite at speeds greater than that of the ionic and molecular constituents of the medium, the emission resulting from these electron layer oscillations will propagate through the external plasma and be detectable by external sensors. Preliminary results of this work are given in Appendix A.

The second approach assumes that the wake cavity is axially symmetric and therefore can be represented by a model in which the number density distribution of the ions is a function of radial distance only and is stationary in time. The distribution function for the electrons is assumed to be a separable function of radial distance and electron velocity, the former having a functional relationship identical to that of the ion distribution and the latter being Gaussian.

If the spatial distribution is disturbed, internal electric currents and time varying charge distributions will be generated and these, in turn, will give rise to an electromagnetic field which will propagate in a radial direction. The radiation spectrum emitted by such a model is believed to be continuous, however, the spectrum has maxima at certain resonant frequencies which are well above the plasma frequency of the plasma region surrounding the cavity.

3.1.3 Wake of a Non-Spherical Satellite

The work of Dr. W. Sawchuk on the problem of determining the variation in wake ionization contours due to changes in angle of attack of a prolate spheroid moving within the ionosphere has been progressing satisfactorily. Some trouble was experienced in locating errors in the program, which is quite complex, and in attempting to simplify the program, after it had been checked out, in order that the machine-time requirement might be reduced considerably. This work has been completed and the simplified program appears to be in good order.

Since this program is actually a combination of two independent programs, one for electrons and one for ions, the checking process involves running each program separately, checking these results, and then coupling the two programs together in such a manner that the proper interactions are accounted for in the final computer program. The first two phases, namely, running and examining the electron and ion programs independently have been completed. Preliminary computer solutions, using the combined programs, are being run at the present time. It is anticipated that a final solution should be forthcoming in the near future.

3.1.4 Electron Drift Velocities in the Plasma Sheath and Wake

Professor C. L. Dolph has completed one phase of his theoretical study in which a macroscopic approach to the problem of radio emission at frequencies above the plasma frequency was used. He considered the drift coupling between longitudinal plasma waves and transverse electromagnetic waves.

This approach to the complex problem of describing mechanisms by which measurable amounts of energy, at microwave frequencies, can be emitted from plasmas associated with satellite flow fields in the ionosphere indicates that resonance will occur at higher harmonics of the plasma frequency. Since the amount of energy that can be present in a resonance frequency band decreases as the harmonic frequency increases, it appears that a model based on such a mechanism is experimentally impractical and therefore must be considered to be outside the realm of possible mechanisms to produce measurable high frequency microwave emission.

3.1.5 Emission at the Onset of Re-entry

Between the free molecule flow regime of satellite flight and the hypersonic flow regime where strong shockwaves dominate due to the increase in density of the ambient medium, there exists a transition region where diffuse shockwaves can be found. Dr. C. Mason has been attempting to formulate a model which would contain some physical mechanism which would be compatible with actual flow field conditions and also would be a source of partially coherent electromagnetic emission.

Since flight through the more dense regions of the ionosphere implies a propagation of a shockwave through partially ionized air, current efforts have been directed towards an attempt to derive the plasma jump conditions (analogous to shockwave relations for non-ionized gases) for a strong shockwave in a partially ionized medium, similar to that found in the ionosphere. If the strong shockwave problem can be solved, then by relaxing the discontinuity condition of such a thin shockwave the electromagnetic characteristics of a diffuse shockwave may be revealed.

3.1.6 Extension of Photon Transport Theory

In developing a momentum-configuration space transport equation for photons, Professor R. K. Osborn and Messrs. E. Klevans and E. Ozizmir have been attempting to take into account general dispersive phenomena by including collective particle effects of the medium. A study of modifications of the transition probabilities for recombination, cyclotron radiation, de-excitation radiation, and Bremsstrahlung is currently underway. Cerenkov radiation from an electron in a fully ionized plasma having an external magnetic field will be considered also.

3.1.7 Satellite Look-Angle Computer Program

To provide a means of checking proposed theories of microwave emission from the wake of a satellite, it was proposed that USASRDL use existing receiving antenna facilities as detection devices. In order to do this, suitable tracking data had to be computed for each satellite to be observed. In addition, it was thought that the emitted frequencies would be dependent upon the altitude of the satellite. Therefore a general look-angle program to be used in conjunction with an IBM 704 computer was modified to provide the following quantities, as functions of time-of-day, for observations from any given site: elevation, azimuth, slant range, altitude above earth, elevation of sun, and illumination. This information can be furnished in either a punched card or a print-out form.

In order to make the format of the computer output compatible with that required for input to the antenna programmer, an interpolation sub-routine has been appended to the computer program. A choice of read-out interval is now available by appropriate adjustment of the sub-routine.

Several sets of data, each containing a number of passages of several satellites over the observation site, have been supplied to USASRDL. These data consisted primarily of azimuth and elevation angles for observation of the Tiros satellites which transmit at 108 MC/s. By using the tracking feed on the receiver antenna it was hoped that a comparison between actual and computed look-angle data could be made. Due to various mechanical and electrical difficulties at the site, no actual look-angle data has been obtained.

3.2 Experimental Work

3.2.1 Piggyback Flight Tests

At the request of AVCO Corporation, the Radiation Laboratory agreed to participate in two Titan Mark IV, Mod. 4 re-entry vehicle based flight tests. The purposes of the tests were to investigate some ionization characteristics of the wake cavity by means of a Langmuir probe and to detect emissions from the wake by means of a microwave radiometer. The duration of each test was intended to span the interval between re-entry vehicle separation and telemetry blackout.

After approximately two months of accelerated procurement of essential components and material, rapid completion of final design of flight equipment, and initial construction of components, the flight tests were cancelled because of the imposing of additional flight test requirements which could not be met within the time and financial schedules of the program. Upon cancellation of the program all work pertaining to it was abandoned.

4. CONCLUSIONS

4.1 Theoretical Work

Substantial progress has been made on most plasma problems. It is anticipated that a number of technical reports will be prepared and published in the near future. These reports will indicate, in part, transport and high frequency emission properties of plasmas. The satellite look-angle program is completely modified to furnish data in a form which is compatible with USASRDL requirements. This program has been thoroughly checked and is ready for use at any time.

4.2 Experimental Work

Due to the unfortunate cancellation of the Piggyback experiments during the initial stages of equipment fabrication, only one conclusion can be drawn. The data, that these experiments would have provided, could have greatly aided the work on satellite wake emission by providing a better picture of the microwave emissions at the source. For a relatively small amount of financing, some unique and useful data could have been obtained had not apparently unrelated factors resulted in a decision to indefinitely suspend the tests.

5. PROGRAM FOR NEXT INTERVAL

It is anticipated that during the remainder of the present contract, at least preliminary results will be forthcoming from all plasma studies currently underway. These results will be presented either as individual technical reports or included in a subsequent periodical report depending upon the completeness of the work. When possible, areas of work where additional or extensions of effort are needed will be indicated.

The satellite look-angle program will be used to furnish appropriate tracking data to USASRDL upon request. Depending upon the receiving frequency capabilities of the USASRDL equipment, the Radiation Laboratory will suggest frequencies to be observed based upon the latest theoretical estimates available at the Laboratory.

APPENDIX A

High Frequency Oscillation Induced in the Electron Sheath Which Formed on a Sharp Plasma Boundary

by Kun-Mu Chen

Theoretical prediction of the phenomena which may be observed at a sharp plasma boundary after it is suddenly created is presented. It is found that an electron sheath will be formed on the sharp plasma boundary as the steady state condition. In this electron sheath a small perturbation may cause a high frequency oscillation. The frequency of the oscillation is found to be much higher than the ambient plasma frequency. At the altitude of 500Km, this type of oscillation gives a frequency in the order of KMC. Therefore, the propagation of this oscillation through the ionosphere is possible and the detection on the ground is hopeful.

Introduction

In the study of the passive radiation caused by a space vehicle in the ionosphere, an oscillation with a frequency much higher than the corresponding plasma frequency is expected to exist in the neighborhood of a space vehicle. It is hoped that this high frequency oscillation can propagate through the ionosphere with a very small attenuation so that detection on the ground can be fruitful. When a space vehicle travels in the ionosphere a vacuum cavity is created immediately behind the vehicle. There a sharp boundary is created in a plasma medium. If the size of the vehicle is large, a hollow cylinder is formed at the tail of the vehicle.

The phenomena which may take place at the sharp boundary of this hollow cylinder can be investigated by studying a one-dimensional geometry case. In this study, we aim to find what happens at a sharp plane plasma boundary. We imagine the whole space was originally filled with a uniform plasma and a half space of the plasma is suddenly swept away at $t = 0$. The question asked is what will happen after $t = 0$, particularly, to conditions existing at the boundary for small t. This is based on the belief that if there is any electron oscillation existing it must take place at the boundary and in a transient period which ends before the massive positive ions make substantial movements.

The geometry for the analysis is shown in Figure 1. At $t = 0$, the half space $x \geq 0$ is filled with a uniform plasma and another half space $x \leq 0$ is vacuum. The positive ions are assumed to remain stationary for small t and the electrons are assumed to move freely except for the action of coulomb forces. Physically we can see that electrons stream out from the plasma background with some initial velocity and gradually slow down as they climb the potential hill. They will turn around at a point where the kinetic energy is balanced by the potential energy, gradually accelerate to their initial speed at the boundary, and re-enter the plasma background. If the positive ions are assumed to be stationary, the electron flow field, at the plasma-vacuum interface, will reach a steady state configuration when the electrons leaving the plasma are equal in number to those re-entering the plasma. Thus an electron sheath will be formed on the boundary. Analytically this electron sheath can be determined.

Due to the nature of the electron density distribution and the potential distribution in this electron sheath, a high frequency oscillation is possible if once a small perturbation is introduced in this electron sheath. Theoretically, the existence of an electron oscillation in this electron sheath can be predicted. Its frequency is found to be much higher than the plasma frequency of the plasma background. At an altitude of 500 KM, this type of electron oscillation has a frequency in the KMC range. The plasma frequency at this altitude is in the neighborhood of 10Mc. Therefore, the propagation of this oscillation will not be greatly attenuated.

The scheme of the analysis is as follows: The steady state configuration of the electron sheath is obtained first. This includes showing that the sheath thickness is of the order of a Debye length and finding the potential and density distributions in this electron sheath. The non-steady state solution is sought by assuming that a small perturbation is introduced inside the sheath. Theoretically it can be shown that an electron oscillation of high frequency can exist in this electron sheath. The frequency and the damping coefficient of the oscillation are determined. This type of oscillation may be a hybrid transverse and logitudinal wave in nature. The transverse part of the wave will radiate directly. The longitudinal part of the wave will induce another transverse oscillation in the ambient plasma if there exists a temperature or density gradient and a magnetic field. This analysis was done by Tidman.+

1. Basic Equations and Assumptions

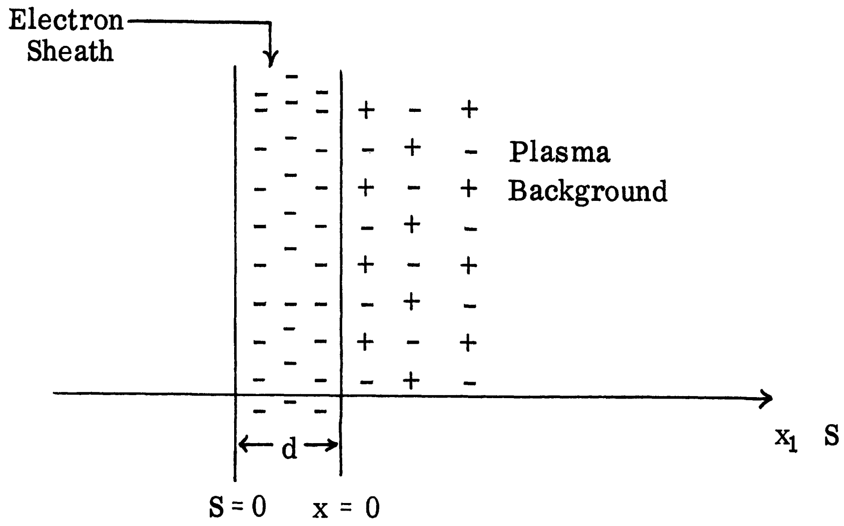

Figure 1. Geometry for Analysis

In Figure 1, the plasma background occupies a half space $x \leq 0$ and an electron sheath of thickness d is formed on the boundary. It is assumed that at t = o the half space $x \geq 0$ was filled with a uniform plasma and the half space $x \leq 0$ was void. After t = o, the electrons stream out from the plasma background, while the positive ions are assumed to remain stationary, and then are pulled back into the plasma, thereby generating an electron sheath at the boundary as a steady state condition.

+Tidman, D. A., Phys. Rev., 117, No. 2, 336 (January 15, 1960).

In the analysis, the unperturbed velocity distribution of electrons is assumed to be mono-energetic in x-direction. i.e.

$$f_{eo}(\vec{v}, \vec{x}) \sim \delta(V - V_s)$$

and

$$1/2\ m\ V_s^2 = 1/2\ KT, \text{ or } V_s = \sqrt{\frac{KT}{m}}$$

where m, T, are the electron mass and temperature.

The basic equations to be used are

$$\frac{\partial^2 \phi}{\partial x^2} - \mu_o \epsilon_o \frac{\partial^2 \phi}{\partial t^2} - \mu_o \sigma \frac{\partial \phi}{\partial t} = - \frac{e}{\epsilon_o}(n_+ - n_-) \tag{1}$$

$$\frac{1}{2}\ m\ V_s^2 = \frac{1}{2}\ m\ V^2 - c\phi \tag{2}$$

$$\frac{\partial n_-}{\partial t} + \frac{\partial}{\partial x}(n_- V) = 0 \tag{3}$$

Equation (1) is the wave equation for the scalar potential, equation (2) is the law of energy conservation and equation (3) is the equation of continuity. n_+ and n_- are the number densities of positive ions and electrons. V is the x component of electron velocity and ϕ is the scalar potential. e is the magnitude of electron charge. μ_o and ϵ_o are the permeability and the permittivity of the free space.

The boundary conditions to be used are as follows:

ϕ is continuous at x = 0 and $\hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \eta$ at x = 0.

Using equations (1) to (3) with some boundary conditions the whole system can be analyzed. As a matter of procedure, the steady state solution of the electron sheath will be determined first and then the non-steady state solution of the electron sheath will be obtained from linearized basic equations.

2. Steady State Solution of Electron Sheath

Based on the assumption that the electrons have a unique velocity, V_s, in the x direction, electrons streaming out from the plasma background will turn back at some point. A steady state condition will be reached when the leaving electron stream is balanced by the returning electron stream. Thus an electron sheath will be formed and its thickness is assumed to be d.

For the convenience of analysis, the following changes of variables are made.

$$e\psi = e\phi + 1/2\ m\ V_s^2 \qquad (4)$$

$$s = x + d \qquad (5)$$

After these changes of variables, equations (1) to (3) become

$$\frac{\partial^2 \psi}{\partial s^2} - \mu_o \epsilon_o \frac{\partial^2 \psi}{\partial t^2} \mu_o \sigma \frac{\partial \psi}{\partial t} = \frac{-e}{\epsilon_o} (n_+ - n_-), \qquad (1a)$$

$$e\psi = 1/2\ m\ V^2, \qquad (2a)$$

and
$$\frac{\partial n_-}{\partial t} + \frac{\partial}{\partial s}(n_- V) = 0. \tag{3a}$$

A steady state solution (in this case a static solution) for the electron sheath can be obtained in the following manner. Equations (1a) to (3a) simplify to

$$\frac{\partial^2 \psi}{\partial s^2} = -\frac{e}{\epsilon_o}(n_+ - n_-), \tag{6}$$

$$V = \sqrt{\frac{2e\psi}{m}}, \tag{7}$$

and
$$\frac{\partial}{\partial s}(n_- V) = 0. \tag{8}$$

From equation (8),

$$n_- = \frac{n_o V_s}{V} \tag{9}$$

where n_o and V_s are the density and the x-component of velocity of electrons at $s = \infty$.

The substitution of equations (7) and (9) in equation (6) leads to

$$\frac{\partial^2 \psi}{\partial s^2} = -\frac{e}{\epsilon_o}\left(n_+ - \sqrt{\frac{m}{2e}}\; n_o V_s \psi^{-1/2}\right). \tag{10}$$

The scalar potential ψ is determined from equation (10) in two regions, namely, $0 \leq s \leq d$ and $s \geq d$.

In the region where $0 \leq s \leq d$, or inside of the electron sheath, n_+ is zero so that equation (10) becomes

$$\frac{\partial^2 \psi}{\partial s^2} = \frac{e}{\epsilon_o} \sqrt{\frac{m}{2e}}\; n_o V_s \psi^{-1/2} . \tag{11}$$

The appropriate solution for ψ is

$$\psi = \left(\frac{9}{4} \frac{e}{\epsilon_o} \sqrt{\frac{m}{2e}}\; n_o V_s\right)^{2/3} s^{4/3}. \tag{12}$$

Equation (12) is valid for $0 \leq s \leq d$.

In the region where $s \geq d$, or inside of the plasma background, n_+ can be assumed to be n_o. Equation (10) then becomes

$$\frac{\partial^2 \psi}{\partial s^2} = \frac{e}{\epsilon_o} \sqrt{\frac{m}{2e}}\; n_o V_s \psi^{-1/2} - \frac{e}{\epsilon_o} n_o \tag{13}$$

The solution to equation (13) can be written as

$$\psi = \left(\frac{9}{4} \frac{e}{\epsilon_o} \sqrt{\frac{m}{2e}}\; n_o V_s\right)^{2/3} s^{4/3} - \frac{1}{2} \frac{e}{\epsilon_o} n_o s^2 + c_1 s + c_2 \tag{14}$$

Equation (14) is valid for $s \geq d$. c_1 and c_2 are arbitrary constants to be determined by the boundary conditions at $s = d$.

From the boundary condition, ψ is continuous at $s = d$, equation (15) is obtained.

$$-\frac{1}{2}\frac{e}{\epsilon_o} n_o d^2 + c_1 d + c_2 = 0 \qquad (15)$$

From the boundary condition, $\hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \eta$, i.e. the discontinuity of the electric displacement is proportional to the surface charge density, another equation can be obtained, namely,

$$-\frac{e}{\epsilon_o} n_o d + c_1 = \frac{e}{\epsilon_o} n_o \delta, \qquad (15a)$$

where δ represents the apparent depth or thickness of the surface charge layer. At this step it is assumed that the dielectric constants in regions $0 \leq s \leq d$ and $s \geq d$ are the same because we are dealing with a wave whose frequency is much higher than the corresponding plasma frequency. In equation (15a) $n_o e \delta$ is assumed to be the surface charge density. It is reasonable to assume that δ is very small or even zero as long as the accumulation of charge on the boundary surface is small or zero.

After c_1 and c_2 are found from equations (15) and (15a), ψ is explicitly obtained as

$$\psi = \left(\frac{9}{4}\frac{e}{\epsilon_o}\sqrt{\frac{m}{2e}}\, n_o V_s\right)^{2/3} s^{4/3} \qquad \text{for } 0 \leq s \leq d. \qquad (16)$$

$$\psi = \left(\frac{9}{4}\frac{e}{\epsilon_o}\sqrt{\frac{m}{2e}}\, n_o V_s\right)^{2/3} s^{4/3} - \frac{1}{2}\frac{e}{\epsilon_o} n_o (s-d)^2 + \frac{e}{\epsilon_o} n_o \delta (s-d) \qquad (17)$$

for $s \geq d$.

The electron density n_- can be obtained by substituting equations (16) and (17) in equations (7) and (9). The results are

$$n_- = \left(\frac{4}{9}\frac{\epsilon_o}{e}\right)^{1/3} \left(\sqrt{\frac{m}{2e}}\, n_o V_s\right)^{2/3} s^{-2/3} \qquad \text{for } 0 \leq s \leq d \qquad (18)$$

and
$$n_- = \sqrt{\frac{m}{2e}}\, n_o V_s \left[\frac{9}{4}\frac{e}{\epsilon_o}\sqrt{\frac{m}{2e}}\, n_o V_s\, s^{4/3} - \frac{1}{2}\frac{e}{\epsilon_o} n_o (s-d)^2\right]^{-1/2}$$

$$\text{for } s \geq d. \qquad (19)$$

It should be noted that δ is assumed to be zero and that equations (17) and (19) are probably invalid at $s \rightarrow \infty$. However, as long as we are only interested in the electron sheath they are adequate for the further development of this analysis.

3. Non-Steady State Solution of Electron Sheath

The steady state solution of the electron sheath is determined in the previous section. Now we will attempt to find what will happen if a small perturbation is introduced inside the electron sheath. It can be shown that a small perturbation inside the electron sheath will cause a high frequency oscillation. The frequency and the damping coefficient of the oscillation are obtained explicitly in the following analysis.

Expressions for the total values of ψ, n_-, and V, including perturbations, can be written as

$$\psi'(s,t) = \psi(s) + \psi_1\,(s,t), \tag{20}$$

$$n'_-(s,t) = n_-\,(s) + n_1\,(s,t)\,, \tag{21}$$

and

$$V'(s,t) = V(s) + V_1\,(s,t)\,. \tag{22}$$

where $\psi(s)$, $n_-(s)$ and $V(s)$ are the steady state solutions while $\psi_1(s,t)$, $n_1(s,t)$ and $V_1(s,t)$ are the perturbation terms. We also assume that

$$\psi_1 \ll \psi, \quad n_1 \ll n_-, \quad \text{and} \quad V_1 \ll V\,.$$

The substitution of equations (20) to (22) into equations (1a) to (3a) leads to the following set of linearized equations;

$$\frac{\partial^2 \psi_1}{\partial s^2} - \mu_o\,\epsilon_o \frac{\partial^2 \psi_1}{\partial t^2} - \mu_o\,\sigma \frac{\partial \psi_1}{\partial t} = \frac{e}{\epsilon_o}\, n_1\,, \tag{23}$$

$$V_1 = \frac{1}{2} \sqrt{\frac{2e\psi}{m}}\; \frac{1}{\psi}\; \psi_1\,, \tag{24}$$

and

$$\frac{\partial n_1}{\partial t} + \frac{\partial}{\partial s}\,(V n_1) + \frac{\partial}{\partial s}\,(n_- V_1) = 0\,. \tag{25}$$

After making a Laplace transformation, equations (23) to (25) become

$$\frac{\partial^2 \Phi_1}{\partial s^2} - \mu_o\,\epsilon_o \left[p^2\,\Phi_1 - p\psi_1(0) - \dot{\psi}(0)\right] - \mu_o\sigma \left[p\,\Phi_1 - \psi(0)\right] = \frac{e}{\epsilon_o}\, N_1, \tag{26}$$

$$\overline{V}_1 = \frac{1}{2} \sqrt{\frac{2e}{m\psi}}\; \Phi_1\,, \tag{27}$$

and $$pN_1 - n_1(0) + \left(\frac{\partial V}{\partial s}\right) N_1 + \left(\frac{\partial n_-}{\partial s}\right) \bar{V}_1 + V\left(\frac{\partial N_1}{\partial s}\right) + n_-\left(\frac{\partial \bar{V}_1}{\partial s}\right) = 0, \tag{28}$$

where Φ_1, N_1, $\bar{V}_1$, are the Laplace transformed forms of ψ_1, n_1, and V_1. $\dot{\psi}_1(0)$ and $\dot{n}_1(0)$ are the first time derivatives of ψ_1 and n_1 at $t = 0$.

Before performing the Fourier transformation of equations (26) to (28), it is necessary and convenient to assign to the coefficients of perturbed quantities fixed values determined by evaluation at $s = d/2$. Thus after Fourier transformation, equations (26) to (28) lead to

$$-k^2 \underset{\sim}{\Phi}_1 - \mu_o \epsilon_o \left(p^2 + \frac{\sigma}{\epsilon_o} p\right) \underset{\sim}{\Phi}_1 = \frac{e}{\epsilon_o} \underset{\sim}{N}_1 - F_1, \tag{29}$$

$$\underset{\sim}{\bar{V}}_1 = \sqrt{\frac{e}{2m}} \frac{1}{\sqrt{\bar{\psi}}} \underset{\sim}{\Phi}_1, \tag{30}$$

and $$\left(p + \left(\frac{\partial \bar{V}}{\partial s}\right) - ik\bar{V}\right) \underset{\sim}{N}_1 = \underset{\sim}{\dot{n}}_1(0) - \left[\left(\frac{\partial \bar{n}_-}{\partial s}\right) - ik\,\bar{n}_-\right] \underset{\sim}{\bar{V}}_1, \tag{31}$$

where $\bar{V}$, $\left(\frac{\partial \bar{V}}{\partial s}\right)$, $\bar{n}_-$, $\left(\frac{\partial \bar{n}_-}{\partial s}\right)$, $\bar{\psi}$, are the values at $s = d/2$. $\underset{\sim}{\Phi}_1$ is the Fourier transform of Φ_1 and etc. From equations (30) and (31) we have

$$\underset{\sim}{N}_1 = \frac{\underset{\sim}{\dot{n}}_1(0) - \left[\left(\frac{\partial \bar{n}_-}{\partial s}\right) - ik\,\bar{n}_-\right]\sqrt{\frac{e}{2m}} \frac{1}{\sqrt{\bar{\psi}}} \underset{\sim}{\Phi}_1}{p + \left(\frac{\partial \bar{V}}{\partial s}\right) - ik\,\bar{V}} = \frac{\underset{\sim}{\dot{n}}_1(0) + \left[\frac{n_o V_s}{4\bar{\psi}^2}\left(\frac{\partial \bar{\psi}}{\partial s}\right) + ik\,\frac{n_o V_s}{2\bar{\psi}}\right]\underset{\sim}{\Phi}_1}{p + \sqrt{\frac{e}{2m}} \frac{1}{\sqrt{\bar{\psi}}}\left(\frac{\partial \bar{\psi}}{\partial s}\right) - ik\sqrt{\frac{2e\bar{\psi}}{m}}} \tag{32}$$

The substitution of equation (32) into equation (29) gives

$$\underset{\sim}{\Phi_1} = \frac{F}{k^2 + \mu_o \epsilon_o p^2 + \mu_o \sigma p + \dfrac{\dfrac{e}{\epsilon_o}\left[\dfrac{n_o V_s}{4\bar{\psi}^2}\left(\dfrac{\partial \bar{\psi}}{\partial s}\right) + ik \dfrac{n_o V_s}{2\bar{\psi}}\right]}{p + \sqrt{\dfrac{e}{2m}}\dfrac{1}{\sqrt{\bar{\psi}}}\left(\dfrac{\partial \bar{\psi}}{\partial s}\right) - ik\sqrt{\dfrac{2e\bar{\psi}}{m}}}} \tag{33}$$

with

$$F = \frac{-\dfrac{e}{\epsilon_o}\displaystyle\int_{-\infty}^{\infty} \dot{n}_1(0)\, e^{iks}\, ds}{p + \sqrt{\dfrac{e}{2m}}\dfrac{1}{\sqrt{\bar{\psi}}}\left(\dfrac{\partial \bar{\psi}}{\partial s}\right) - ik\sqrt{\dfrac{2e\bar{\psi}}{m}}} + \mu_o \epsilon_o \int_{-\infty}^{\infty} \left[(p\psi_1(0) + \dot{\psi}(0) + \frac{\sigma}{\epsilon_o}\psi(0)\right] e^{iks}\, ds\ .$$

The second term of the above expression is F_1 in equation (29).

The dispersion equation is obtained by letting the denominator of equation (33) equal to zero, namely,

$$\mu_o \epsilon_o p^2 + \mu_o \sigma p + \frac{\dfrac{e}{\epsilon_o}\left[\dfrac{n_o V_s}{4\bar{\psi}^2}\left(\dfrac{\partial \bar{\psi}}{\partial s}\right) + ik \dfrac{n_o V_s}{2\bar{\psi}}\right]}{p + \sqrt{\dfrac{e}{2m}}\dfrac{1}{\sqrt{\bar{\psi}}}\left(\dfrac{\partial \bar{\psi}}{\partial s}\right) - ik\sqrt{\dfrac{2e\bar{\psi}}{m}}} + k^2 = 0. \tag{34}$$

To determine the nature of the electron oscillations in the electron sheath the roots of p are to be determined from equation (34). To determine the center frequency of the oscillations in the dispersion relation we set $k = 0$. In case $k \neq 0$, p can be solved as a function of k.

In the limit of k = 0, equation (34) becomes

$$p^3 + (A + C)p^2 + AC\,p + B = 0\,, \tag{35}$$

where

$$A = \sqrt{\frac{e}{2m}}\;\frac{1}{\sqrt{\bar{\psi}}}\;\left(\frac{\overline{\partial \psi}}{\partial s}\right) = \sqrt{n_o}\;\times 10^2\,,$$

$$B = \frac{e}{\mu\,\epsilon_o^{\,2}}\;\frac{n_o\,V_s}{4\,\bar{\psi}^2}\;\left(\frac{\overline{\partial\psi}}{\partial s}\right) = (n_o^{3/2}\,T^{-1}) \times 7.5 \times 10^{15}\,,$$

and

$$C = \frac{\sigma}{\epsilon_o}\;.$$

The substitution of $p = -\alpha + i\omega$ in equation (35) gives two equations

$$(A + C - 3\alpha)\,\omega^2 + AC\alpha - (A+C)\alpha^2 + \alpha^3 = B \tag{36}$$

and

$$\omega^2 + 2(A+C)\,\alpha - 3\alpha^2 = AC\,, \tag{37}$$

or alternatively,

$$\alpha^3 - (A+C)\alpha^2 + \frac{1}{4}\,(A^2+3AC+C^2)\alpha + \frac{1}{8}\,(B - A^2C - AC^2) = 0 \tag{38}$$

and

$$\omega^2 = AC + 3\alpha^2 - 2(A + C)\alpha\;. \tag{39}$$

α and ω can be completely determined from equations (38) and (39) if the constants A, B, and C are known. Unfortunately C is too difficult to be determined accurately because the possible loss mechanisms are quite involved. We are only able to solve equations (38) and (39) for two extreme cases, namely, the hot and the cold plasmas.

I. The Hot Plasma

For the hot plasma, the inequality of

$$(A^2 + 3AC + C^2)^{1/2} >> (B - A^2C - AC^2)^{1/3}$$

is assumed to be valid. In this case

$$\omega^2 \doteq \frac{(A+C)(B - A^2C - AC)^2}{A^2 + 3AC + C^2} + AC$$

and

$$\alpha \doteq \frac{B - A^2 C - AC^2}{-2(A^2 + 3AC + C^2)} \quad .$$

As the limit of zero loss, $C \to 0$, ω and α can be expressed as

$$\omega = \sqrt{\frac{B}{A}} = \frac{2}{\sqrt{\mu_0 \epsilon_0}} \sqrt{\frac{e^2 n_0}{\epsilon_0 KT}} \tag{40}$$

and

$$\alpha = - \frac{B}{2A^2} \quad . \tag{41}$$

This gives an oscillation frequency of

$$\boxed{f = \frac{3}{\pi} \frac{1}{h} \times 10^8 \qquad c/sec} \tag{42}$$

where $h = \sqrt{\frac{\epsilon_0 KT}{e^2 n_0}}$ = debye length of the ambient plasma (in meters).

The damping coefficient, α, turns out to be a small negative value in the limit of zero loss. This implies that if there is no loss at all the oscillation may have a growing wave nature.

However, if we consider the loss mechanisms, the oscillation is more likely to be a damped wave instead of a growing wave. We are encouraged in one respect, if this type of electron oscillation exists it probably will not damp out too fast or we may even expect a large amplitude oscillation.

II. The Cold Plasma

For the cold plasma the inequality of

$$(A^2 + 3AC + C^2)^{1/2} << (B - A^2C - AC^2)^{1/3}$$

is assumed to be valid. In this case α and ω can be determined in the limit of $C = 0$ as follows:

$$\alpha_1 = -\frac{B^{1/3}}{2} \qquad \omega_1 = -\frac{\sqrt{3}}{2} B^{1/3}$$

$$\alpha_2 = \left(\frac{1}{4} + i\frac{\sqrt{3}}{4}\right) B^{1/3} \quad \text{and} \quad \omega_2 = \left(\frac{\sqrt{3}}{4} + i\frac{3}{4}\right) B^{1/3}$$

$$\alpha_3 = \left(\frac{1}{4} - i\frac{\sqrt{3}}{4}\right) B^{1/3} \qquad \omega_3 = \left(\frac{\sqrt{3}}{4} - i\frac{3}{4}\right) B^{1/3}.$$

These results imply two types of wave. The first type of wave is with

$$\boxed{f = \frac{\sqrt{3}}{4\pi} B^{1/3} = 0.265 \frac{(C_o^2 V_s)^{1/3}}{h} \quad c/sec,} \tag{43}$$

where

$$h = \sqrt{\frac{\epsilon_o KT}{e^2 n_o}} = \text{debye length of the ambient plasma (in meters)},$$

$$C_o = \text{velocity of light} = 3 \times 10^8 \text{ m/sec},$$

and

$$V_s = \sqrt{\frac{KT}{m}} = \text{thermal velocity of electron (in m/sec)}.$$

This wave has a negative damping coefficient equal to

$$\alpha = -\frac{1}{2} B^{1/3} \tag{44}$$

as the limit of zero loss. Again if the loss mechanisms are considered the wave may be characterized by a damped wave instead of a growing wave.

The second type of wave has a positive damping coefficient equal to

$$\alpha = B^{1/3}$$

and it is anticipated that this wave will damp out very fast. Naturally this type of wave is of no interest to us.

III.

In case $k \neq 0$, equation (34) should be solved more generally and p should be determined as a function of k among the other parameters. If k is small, the oscillation frequency can be corrected to

$$\omega'^2 \doteq \omega^2 + \frac{1}{\mu_o \epsilon_o} k^2 . \qquad (45)$$

The propagation constant, k, can be determined with the help of boundary conditions.

<u>Discussion</u>

The results are summarized as follows:

I. For the hot plasma when the condition of $(A^2+3AC+C^2)^{1/2} >> (B - A^2C - AC^2)^{1/3}$ is satisfied, the oscillation has a frequency

$$f = \frac{3}{\pi} \frac{1}{h} \times 10^8 \quad c/sec. \qquad (42)$$

II. For the cold plasma when the condition of $(A^2+3AC+C^2)^{1/2} << (B - A^2C - AC^2)^{1/3}$ is satisfied, the oscillation frequency is

$$f = 0.265 \quad \frac{(C_o^2 V_s)^{1/3}}{h} \quad c/sec. \qquad (43)$$

with the symbols being defined as

$$A = \sqrt{n_o} \times 10^2 ,$$

$$B = (n_o^{3/2} \times T^{-1}) \times 7.5 \times 10^{15} ,$$

$$C = \sigma / \epsilon_o ,$$

$$h = \sqrt{\frac{\epsilon_o KT}{e^2 n_o}} ,$$

$$V_s = \sqrt{\frac{KT}{m}} ,$$

and

$$C_o = 3 \times 10^8 .$$

All units are in MKS system.

It is interesting to note that when the temperature of the cold plasma is increased, V_s approaches C_o and the oscillation frequency of the cold plasma tends toward that of the hot plasma. In both the hot and the cold plasmas the oscillation seems to have a negative damping coefficient in the absence of any loss mechanism. This fact may be useful because we can be more certain that the damping effect of the oscillation will not be too great after considering all the loss mechanisms.

To show the significance of the results a typical example is considered. We assume this type of electron oscillation to take place in the wake of a satellite which is moving at an altitude of 500KM. The frequency of oscillation can be obtained as follows:

Assuming $n_o = 10^{12}$ $1/m^3$, $T = 2,000^oK$, if the plasma is classified as a cold plasma, the frequency can be calculated from equation (43) to be

$$\underline{f = 2.14 \text{ KMC.}}$$

If the plasma is considered to be a hot plasma, the frequency can be calculated from equation (42) to be

$$\underline{f = 30.5 \text{ KMC.}}$$

Note that the ambient plasma frequency in this region is f_p = 9 Mc.

Since the temperature and the electron density of the ionosphere are not well specified and due to the approximate feature of the analysis we may conclude that the frequencies calculated from equations (43) and (42) may be considered to be lower and upper bounds of the oscillation frequency.

Fortunately the frequency range included in the lower and upper limits is not too wide, therefore, the theoretical prediction can be of value to experimental investigation.

The type of electron oscillation proposed in this study should be checked by some experiments. If the theoretical prediction agrees with the experimental data reasonably well, we may conclude that this type of electron oscillation plays an important role in the passive radiation from the satellite.