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STUDY OF PROBLEM AREAS ON OPTICAL COMMUNICATIONS

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TABLE OF CONTENTS

ABSTRACT

The objectives of this study of problem areas in optical communications concern the theoretical limits for operability and for rate of transmission of information of communication channels employing light or infrared radiation, aerospace to earth or earth to aerospace.

The first section of the main body of the report deals with the nature of different processes of attenuation encountered in the path of the radiation and presents a first preliminary analysis for the quantitative evaluation of scattering and absorption.

The second section discusses the quantum-mechanical modifications of the mathematical theory of communication, which have been presented in the recent literature and which necessarily serve as a basis for any quantitative analysis of communication in the optical and infrared range. According to the directives, the first type of channel taken up for detailed study is a channel operating with a signal level so low at the receiver that a fraction of a photon per sample is received on the average. It employs a binary photon counter as a detector. Strengths and weaknesses of this type of channel are discussed.

One of the critical components for such a channel is an optical filter to reduce the amount of incident background radiation reaching the detector. The third section surveys known filter principles and possible means for tuning such a filter.

INTRODUCTION: DEFINITION OF PROBLEM AREAS

In the last few years considerable attention has been directed to the possible use of optical frequencies for communication. The continuous pressure towards higher and higher frequencies has traditionally been based in the past on the fact that bandwidth is one of the important factors for the capacity of a communication channel ; for constant relative bandwidth the absolute bandwidth grows directly with the midband channel frequency. Another motivating circumstance is the growing antenna gain for constant aperture; the approximate coherency made possible by the laser has extended the range of this advantage into the optical frequency band.

Every communication channel involves flow of energy from a transmitting system to a receiving system; at optical frequencies the minimum 'grain size' of this flow is so large that classical field theory and network theory fail to give an adequate description of the properties of a communication channel. For this purpose it is necessary to develop theoretical models based on quantum mechanics.

The most fundamental problem area may consequently be considered as a quantum-mechanical formulation of the statistical theory of communication and a survey of pertinent quantitative conclusions within a specified range of parameters. The primary target in this area is to find a theoretical upper bound for the rate of transmission of information under various specified conditions.

Other problem areas obviously will be concerned with the realization and implementation of channels capable of approaching this upper bound. These areas are distinguished from each other according to the different operational transformations of the signal processes in a communication channel: encoding, modulation, antenna optics, attenuation, superposition of background noise, filtering, detection and decoding. In several of these secondary problem areas device technology and the 'state of the art' play a dominant part rather than fundamental limitations derived from generally accepted physical and mathematical theory.

The first objective of this study is to determine ultimate limitations in a few of these problem areas. i.e., limitations predicted by theory rather than those set by present technology or state of the art.

The second objective is to formulate recommendations on how to approach these ultimate limits most quickly and efficiently.

The applicational environment motivating this study is communication from distant space vehicles to earth and vice versa. This fact gives the appropriate perspective: the transmission is subject to scattering and absorption in the earth's atmosphere; furthermore, power, weight, and complexity are limited by the fact that one terminal is located in a space vehicle. From hypothetical orbits and ground-terminal locations the ranges of such system parameters as distances, cloud cover, background radiation from reflected sunlight or of thermal origin, etc., may be estimated.

Other parameter ranges specified for this study are:

Modulation Bandwidth	0 - 1000 Mc/s
Wavelengths	$0.4\,\mu$ - $20\,\mu$ (4000 Å) - 200000 Å)
which are equivalent to	
Frequencies	7.5×10^{14} - 1.5×10^{12}
Quantum Energies	3.1 - 6.2×10^{-3} electron volts

From the very broad problem areas mentioned above, the directives for this study are defined from the following more specific task areas:

Optical Communication and Adverse Meteorological Conditions

"A possibility which exists for future opitcal communications links is the employment of direct aerospace-to-earth or earth-to-aerospace transmission. While receiver and transmitter sites can usually be chosen for ideal visibilities, even here a finite probability exists of temporary poor visibility conditions due to excessive rain, fog, smoke, or haze. Assuming the worst possible conditions which can occur, the possibility of information transmission during these conditions is to be determined. As an example, consider extremely dense fog conditions to exist. The existence of such a fog condition will at best, cause considerable attenuation of the received signal. However, there is also a possibility of almost total attenuation of the signal energy and/or high information degradation due to molecular scattering and absorption. The basic question which needs to be resolved is whether situations exist where it is impossible to achieve satisfactory information transmission. Possibilities which need to be studied for satisfactory transmission should include large power output boosts, operation at frequencies avoiding molecular absorption regions, simultaneous transmission at various different frequencies, and choice of modulation techniques. The analysis is to include effects on the coherence properties of the received signal."

Channel Efficiency and Choice of Detection System in Optical Communication

"A second problem to be considered is a study of the desirability of using binary quantum counters as detectors in the ideal laser communication system. Gordon (Proc. IRE, 50, 1898 (Sept 62)) shows that the binary quantum counter has high information efficiency when the total received power is less than one quantum per sample. Consequently, the basic question here concerns the utility, consistent with the basic tenets of the communication system, operating at information receipt of less than one quantum per sample, where the communication system is subject only to the ultimate theoretical limitations discussed above. Consideration shall also be given to coding techniques allowing minimum information degradation."

Optical Bandpass Filters for Communication Channels

"The third area to be considered is the determination of the theoretical limitations which establish the narrowness and the transmissivity of bandpass filters, tunable and untunable, within the spectrum specified. Interference filters are of special interest. The limitations to be considered are only those of the filters themselves and not those due to external system considerations such as Doppler. Recommendations for future and unusual filter research which may approach the established limitations are also of interest."

This first Interim Engineering Report is the result of the work done during the period April 1 to July 1, 1964. This initial stage of the study has necessarily been one of surveying, criticizing and evaluating such previously published work that has a bearing on the problem areas to be investigated. Only in the last few years have quantum physics and information theory met face to face and found it mutually advantageous to explore overlapping zones of interest, despite the fact that some of the fundamental ideas of information theory trace their origins many years back to the work of physicists such as Boltzmann and Szilard. Because of the frontier nature of the area of quantum information theory much of the work reported in the literature is of a tentative nature and must be carefully and critically evaluated before it can be accepted and used as a solid foundation for further work.

1. Optical Communication under Adverse Meteorological Conditions

1.1 Introduction

The first problem area concerns the possibility of maintaining communication in the optical or infrared range aerospace-to-earth or earth-to-aerospace under regular as well as extreme meteorological conditions. This section will give a preliminary discussion of attenuation of light waves in the atmosphere.

Attenuation is most commonly described by the extinction coefficient γ where $I = I_o e^{-\gamma L}$.

Extinction may be attributed to two physically distinct phenomena: absorption and scattering out of the beam. One writes (see, for example, Section 2 of Ref. 1)[1] when α_{abs}, is the attenuation coefficient due to atomic and molecular absorption,

$$\gamma = \alpha_{abs} + \alpha_{sc} \tag{1.1}$$

where α_{sc} is derived from a "scattering coefficient" $\mu(\theta)$ by the relation

$$\alpha_{sc} = 2\int_0^{\pi} \mu(\theta) \sin\theta \, d\theta \tag{1.2}$$

Fundamentally, $\mu(\theta)$ relates the intensity $dI(\theta)$ scattered at angle θ by a medium of thickness $d\ell$ on which is incident the intensity I_o

$$dI(\theta) = I_o \, d\ell \, \mu(\theta) \tag{1.3}$$

$\mu(\theta)$ may be obtained experimentally by use of the defining equation (1.3) or theoretically by solving the electromagnetic scattering problem. This solution

(1)Reference numbers refer to the Bibliography, page 47.

is best known for spheres, which have been treated both in the so-called Rayleigh limits where the sphere is very small compared to the wavelength and for other ranges of r/λ. For a discussion one might see Reference 3.

Now in order to discuss propagation through meteorological conditions, we must be able to characterize them in terms of size distribution and electrical properties of the constituent particles, or else have presumably typical experimental results from which, if necessary, extrapolation to conditions of interest is possible. Let us survey the situation as it seems to us after a moderately intensive search of the literature.

1.2 Rain

A survey is in progress to find size distribution of rain droplets and electrical parameters, which will vary with the concentration of dissolved substances. The lack of information in the literature may hamper us seriously. With this information, scattering could be treated theoretically.

Pure absorption is likely to be less simple, since in our opinion extrapolation from measurements performed on water vapor may be unsound (because of the perturbing effect of neighboring molecules, as in a crystal, the bulk properties of liquid water may differ from those measured in a dispersed vapor phase. Theoretical treatment of this difference would be of a complexity unwarranted under this contract.).

It appears that a renewed search for discussions of propagation through rain will be necessary.

1.3 Fogs

These are discussed in Reference 1 on the basis of some earlier experimental results, which may be summarized in the following statements.

The scattering may be described for all fogs studied by a universal function $F(\theta)$, which is tabulated by Spencer in the reference. In terms of this function, the scattering coefficient is defined by

$$\mu(\theta) = \alpha F(\theta), \tag{1.4}$$

where the attenuation coefficient α merely entered as a scale factor distinguishing the scattering by various fogs. α was found to vary between 5.4×10^{-3}/meter and 2.44/meter for the fogs studied.

No statement is made of the wavelength region for which these conclusions are valid, and indeed a theoretical investigation of the problem would have to be performed before extrapolating to other wavelengths. It seems to be in order to go back to the records of the experimenter, and to contact some of them who are at the U of M about unpublished results, which are likely otherwise to be difficult to obtain.

1.4 Haze

By this term we mean an aerosol of solid particles suspended in the atmosphere.

A size distribution model commonly employed is that proposed by Ref. 4. Experiments on infrared attenuation reported in Ref. 2 seem to fit the predictions based on this model, and furthermore a direct count of the particles confirmed the distribution. We could, in the absence of contradicting evidence, regard it as correct and write

$$n(r) = Cr^{-P}, \quad r_1 > r > r_o \tag{1.5}$$

$$= 0 \quad \text{otherwise}$$

$$\text{with } r_o = 20 \text{ or } 50\mu$$

$$r_1 \sim 0.1\,\mu$$

and P is really a function of r rather than a constant, but roughly $2 \leqslant P \leqslant 4.5$, with the larger values typifying haze and $P = 2$ for water fogs at 100 percent relative humidity. C will depend on the mass of aerosol present, and no information on values has been obtained.

With the distribution (1.5), one has (Ref. 2) for the attenuation by scattering in traversing path length L, $e^{-\sigma_{sc} L}$, the values given by

$$\sigma_{sc} = C\pi\lambda^{3-P} \tag{1.6}$$

As long as the condition is satisfied that

$$r_1 >> \lambda . \tag{1.7}$$

Otherwise a more complicated dependence of σ on λ results. We might investigate the approximate form valid near the limit.

The experiments, with L = 200 yards, indicate values of σ L consistently near unity for $\lambda = 1\mu$ and decreasing to about 0.2 as λ increases to 10μ.

1.5 Work Proposed for the Next Period

The general discussion begun above will be completed by including such topics as absorption in clouds and in the clear atmosphere.

New data will be collected to make it possible to present specific values for representative transmission paths.

2. Efficiency and Choice of Detection System in Optical Communications

2.1 Quantum Mechanics and Communication Theory: General Discussion

In classical communication theory the communication channel with given bandwidth and average signal power at the receiver input terminals, perturbed by white Gaussian noise of given power, serves as a standard case, with which channels of different properties may be compared and approximately evaluated. A significant prediction of this theory is that the maximum rate of transmission of information depends on the ratio of signal power to noise power but not on the absolute magnitude of these powers. This result is reasonable only if power, or energy in any sample time interval, can be subdivided into infinitely small measurable increments.

On the other hand, quantum mechanics states that exchange of energy between physical systems, such as a "transmitter" and a "receiver" takes place in finite discrete increments or quanta. The classical communication theory consequently is founded on the tacit assumption that the signal energy and the noise energy per sample time interval are always very large compared to the energy quantum at the channel frequency. As far as thermal radiation noise is concerned, the classical condition is

$$kT >> hf \qquad (2.1)$$

where k is Boltzmann's constant, T the absolute temperature, h Planck's constant and f the channel frequency. At room temperature $T = 290^{\circ}$ equality occurs at a frequency of $6 \cdot 10^{12}$ cycles per second, which falls in the infrared range, close to the low end at the frequency interval considered in this study.

Simple models of a communication channel that permit introduction of the quantum-mechanical postulates may be constructed in several different ways. A long lossless transmission line or waveguide may temporarily be made into a loop by connecting the two ends directly together (periodic boundary conditions); it is then a resonator with two sets of normal modes, representing waves travelling in opposite directions. For the fundamental pair of modes the line is exactly one wavelength long.

The equilibrium distribution of photons over the normal modes can be calculated from statistical mechanics. This is the maximum-entropy distribution (Einstein-Bose) for given temperature or energy. The transformation from a resonator to a communication channel is easily performed by separating the ends and connecting them to matched terminations.

It may seem that the above system's a highly specialized ideal case, from which no general conclusions may be drawn. However, the equilibrium concept permits such generalization. Under equilibrium (maximum entropy) conditions at a given temperature all systems seen from their terminals or boundaries, must be in a certain sense equivalent, so that no net average transfer of energy takes place, which would contradict the equilibrium.

For this reason the maximum entropy of a wave train of given bandwidth and energy on such a transmission line may be used as a standard of comparison for the rate of information carried by an electromagnetic wave. This is Gordon's "wave capacity" (5)

$$C_w = \Delta f \left\{ \log \left(1 + \frac{S}{\Delta f \cdot hf}\right) + \frac{S}{\Delta f hf} \log \left(1 + \frac{\Delta f \cdot hf}{S}\right) \right\} \qquad (2.2)$$

if background noise is negligible. Here Δf stands for the bandwidth, f for the midband frequency and S the average power. Subtracting the equivocation due to independent background radiation of average power N, Gordon obtains the more general expression

$$C_W = \Delta f \left\{ \log\left(1+\frac{S}{N+\Delta f \cdot hf}\right) + \frac{S+N}{\Delta f\, hf} \log\left(1+\frac{\Delta f \cdot hf}{S+N}\right) - \frac{N}{\Delta f\, hf} \log\left(1+\frac{\Delta f\, hf}{N}\right) \right\} \quad (2.3)$$

Evidently these expressions depend on $S/\Delta f \cdot hf$ as well as on S/N. Shannon's classical channel capacity is the limit of (2.3) for $N >> \Delta f \cdot hf$.

When this classical limit does not hold, the significances of these upper bounds for the rate of transmission of information as applied to actual communication channels is not obvious. Since the emission and absorption of radiation according to quantum mechanics is not completely deterministic but governed by chance within the restrictions of certain transition probabilities, it is doubtful whether a full amount of predetermined information equal to C_W in (2.2) can be coded into an electromagnetic communication channel.

At large signal levels and noise levels the power $\Delta f \cdot hf$ is loosely analogous to a "limit of resolution," such as Shannon's "fidelity criterion" for a white Gaussian variable. It appears in the first term of (2.3) as the power of an additional independent noise source. However, whenever the two later terms are appreciable, the relationships are far more complicated.

For future reference some of the steps leading to the expression (2.2) will be given here. Let λ_o be the length of the transmission line and c the wave velocity. The frequency of the n-th normal mode travelling in one direction (say left to right) is then

$$f_n = n \cdot f_o = n \cdot \frac{c}{\lambda_o} \tag{2.4}$$

In the frequency band $f_c \pm f_m$ there are N_t such modes

$$N_t = \frac{2 f_m}{f_o} = \frac{2 f_m \cdot \lambda_o}{c} \tag{2.5}$$

During equilibrium at T^oK photons are distributed over these modes according to the Einstein-Bose distribution function, which is a maximum-entropy distribution. If the zero-point energy is omitted, the expected value of the number of photons in a normal mode of frequency f is

$$E(m) = \overline{m} = \left[\exp\left(\frac{hf}{kT}\right) - 1\right]^{-1} \tag{2.6}$$

and the probability of exactly m photons

$$P(m) = \left[\frac{1}{1+\overline{m}}\left(\frac{\overline{m}}{1+\overline{m}}\right)\right]^{m} \tag{2.7}$$

The state of the line as far as the wave train travelling from left to right is concerned can be specified either by the numbers of photons in each mode or by N_t amplitude samples taken in the time

$$\Delta t = \frac{\lambda_o}{c}$$

required for all the right-directed energy to leave the line when the right-hand matched terminal is inserted instead of the periodic condition. The equivalent rate of samples per second

$$N = \frac{N_t}{\Delta t} = \frac{2 f_m \cdot \lambda_o}{c} \cdot \frac{c}{\lambda_o} = 2 f_m \tag{2.8}$$

= the bandwidth.

The entropy per mode or per sample is obtained from (2.7)

$$H = - \sum_{m=0}^{\infty} P(m) \log P(m) = \log (1+\bar{m}) + \bar{m} \ \log(1+\frac{1}{\bar{m}}) \tag{2.9}$$

Introducing the average power

$$S = 2f_m \cdot \bar{m} = \Delta f \cdot \bar{m} \tag{2.10}$$

and multiplying by $N = \Delta f$ gives the result (2.2).

It should be noted that the ordinary procedure of sampling in the time domain takes samples of either the electric or the magnetic field (voltage or current) at the terminal at a rate of twice the bandwidth. In the above analysis, on the other hand, the original information of energy per mode is translated to samples of the Poynting vector at the rate of only Δf per second. The lack of phase information reduces the number of specified data by a factor of two.

No contribution from the phase uncertainty has been accounted for in the calculation of the wave entropy given by (2.2) and (2.3) consequently at least a large part of the fundamental uncertainty introduced by the randonness of quantum-mechanical transitions has been omitted from these expressions.

2.2 Quantum Formalism for the Electromagnetic Field in an Optical Channel

The contemplation of communication channels at frequencies in the infrared or optical range in which quantum effects are expected to be of some consequence, inevitably poses the requirement of a quantum-mechanical description of the signal. Equally inevitable is the precise specification of the source as well as of the detection process. From the quantum theoretical standpoint, the former corresponds to the preparation of the system (signal) and the latter to the measurement of some dynamical variable of it. This specification is necessary because of the fact that the uncertainty in any measurement depends strongly upon the preparation and detection.

Following the literature on the subject[(5)-(9)] we consider a signal consisting of a single transverse field mode passing through a linear transmission medium which may be vacuum. Classically, the signal is described by its time Fourier components each of which obeys a harmonic-oscillator equation. These components shall be referred to as the modes of the signal (field) hereafter.[+] The number of these modes depends on the time of observation and the bandwidth. Thus the description of the signal is equivalent to the description of an assembly of harmonic oscillators.

The quantum mechanical description of the signal is effected by describing the oscillators quantum mechanically. Let Q be the position operator and P the momentum operator of the harmonic oscillator. The frequency ω is assumed to be nearly the same for all oscillators. This means that we are considering a narrowband system. The hamiltonian of the harmonic oscillator is

$$H = \frac{1}{2} (P^2 + \omega^2 Q^2). \tag{2.11}$$

P and Q obey the commutation relation

$$\left[Q, P\right] = i\hbar. \tag{2.12}$$

One of these operators corresponds to the electric field and the other to the magnetic field operator.

If we introduce the operators α and α^t defined by

$$\alpha \equiv \frac{1}{\sqrt{2\hbar\omega}} (P - i\omega Q), \tag{2.13a}$$

and

$$\alpha^t \equiv \frac{1}{\sqrt{2\hbar\omega}} (P + i\omega Q), \tag{2.13b}$$

[+] Note the difference between these modes and the spatial field modes. Since we have restricted the system to one spatial mode the term mode will not be used in the sense of spatial modes in the Sequel.

the hamiltonian becomes

$$H = \hbar\omega\,(\alpha^{t}\alpha + \frac{1}{2}). \tag{2.14}$$

α and α^{t} obey the commutation relation

$$\left[\alpha, \alpha^{t}\right] = 1\,. \tag{2.15}$$

We also have the relationships

$$P = \sqrt{\frac{\hbar\omega}{2}}\,(\alpha^{t} + \alpha), \tag{2.16a}$$

$$Q = i\sqrt{\frac{\hbar}{2\omega}}\,(\alpha^{t} - \alpha)\,. \tag{2.16b}$$

The state of the assembly of the oscillators (and hence the state of the signal) is known if the density operator D of the assembly is known. Then the outcome of a measurement of an observable is represented by a probability distribution. For example, if R is a hermitian operator and $\{/r>\}$ is the set of its eigenfunctions, where r denotes the eigenvalue of $/r>$ i.e.

$$R/\,r> = r/r>\,, \tag{2.17}$$

then the probability for the result of a measurement of R to be r is

$$<r/\,D\,/\,r> \tag{2.18}$$

For the sake of notational simplicity we shall use the symbol D_{rr} rather than $<r\,/\,D\,/\,r>$. Also we shall assume that all eigenfunctions $/\,r>$ have been normalized to unity.

The expectation value of this probability distribution is

$$\mathrm{Tr}\,(DR) = \sum_r D_{rr} r \qquad (2.19)$$

The density operator has the property

$$\mathrm{Tr}\,D = \sum_r D_{rr} = 1, \qquad (2.20)$$

and this holds true for all representations $/r\rangle$ since the trace is independent of the representation used.

Let now $/s\rangle$ be a second representation consisting of the eigenfunctions of an operator S. It is assumed that both representations are separately complete for the description of the assembly. Then we have the following general relationship

$$D_{s's} = \sum_{rr'} \langle s'/r'\rangle D_{r'r} \langle r/s\rangle \qquad (2.21)$$

where it has been tacitly assumed that R has a discrete spectrum. If its spectrum is continuous the summation is replaced by integration.

Observables which are of interest to us in connection with the harmonic oscillator are the energy $(\hbar\omega\,\alpha^t\alpha + \frac{\hbar\omega}{2})$, and the fields P and Q. The spectrum of $\alpha^t\alpha$ is discrete while P and Q have continuous spectra. The set of eigenfunctions of any of these operators is complete for the description of the harmonic oscillator. Let $\{/n\rangle\}$, $\{/p\rangle\}$ and

$\{|q>\}$ be these sets respectively.

The density operator which describes the signal at the receiver is determined by the source of the signal, and by the transmission medium. The detection process determines the representation $|s>$ in which we would like to calculate the diagonal matrix elements of D. If for example, the detection is based on a measurement of P then the probability distribution that we would like to know is D_{pp}.

Let us assume now that we have a signal with a fixed average energy per observation interval, which we call $\mathcal{E}$. Then we shall have

$$\sum_{n=0}^{\infty} n D_{nn} = M, \tag{2.22a}$$

and

$$\mathcal{E} = \hbar\omega M, \tag{2.22b}$$

where

$$\alpha^{t}\alpha | n > = n|n>. \tag{2.22c}$$

M is interpreted as the average number of photons arriving at the receiver per observation interval.

Now we consider a detection scheme which consists in measuring the energy of each mode. One measurement per mode is assumed. Clearly we may interpret this measurement as measuring the number of photons because there is a one to one correspondence between these two specifications. The outcome of a measurement of the number of photons in one mode is given by D_{nn} which is a first-order probability distribution over the non-negative integers. This distribution has a variance σ_n^2 associated with it given by

$$\sigma_n^2 = \sum_0^\infty n^2 D_{nn} - M^2 \tag{2.23}$$

where the subscript (n) indicates the distribution to which it refers. In general the variance of D_{ss} shall be denoted by σ_s^2. The variance σ_n^2 is a measure of the uncertainty inherently associated with the measurement of the number of photons in the mode.

The following point, about which the literature is rather confusing, should be emphasized. The hamiltonian of the harmonic oscillator could be taken either as $\hbar\omega(\alpha^t\alpha + \frac{1}{2})$ or simply as $\hbar\omega\alpha^t\alpha$. Since in most instances we deal with differences in energy the choice is immaterial. If the term $\frac{\hbar\omega}{2}$ is included the average number of photons is $(M + \frac{1}{2})$. Otherwise it is M. In both cases however the variance σ_n^2 is as given by Eq. (2.23), as a simple algebraic calculation shows.

Suppose now that the detection is based on a field measurement where again one measurement per mode is understood. We may assume, without loss of generality that the measured variable is Q. Then we are interested in the probability distribution D_{qq} which is given in terms of D_{nn} by the following equation:

$$D_{qq} = \sum_{m,n=0}^{\infty} D_{mn} \left(\frac{\omega}{\hbar\pi}\right)^{\frac{1}{2}} \frac{2^{-\frac{m+n}{2}}}{m!\,n!} e^{-\frac{\omega}{\hbar} q^2} H_m\left(\sqrt{\frac{\omega}{\hbar}}\, q\right) H_n\left(\sqrt{\frac{\omega}{\hbar}}\, q\right) \qquad (2.24)$$

This is obtained from Eq. (2.21) by using the explicit form of $/n\rangle$ in terms of q, as given for example in Ref. 10. $H_n(\psi)$ are the Hermite polynomials. The variable q varies continuously from $-\infty$ to $+\infty$. If all D_{mn} are known D_{qq} can be calculated although not always in closed form. There are nevertheless several conclusions that can be drawn without knowing D_{qq} explicitly.

Let us consider the variance σ_q^2 of this distribution. We have

$$\sigma_q^2 = \frac{\omega}{\hbar} \left[\int_{-\infty}^{+\infty} q^2 D_{qq}\, dq - \left(\int_{-\infty}^{+\infty} q\, D_{qq}\, dq \right)^2 \right] \qquad (2.25)$$

where $\omega/\hbar$ is introduced in order to make α_q^2 dimensionless. This variance can be calculated without explicit knowledge of D_{qq}. Indeed note that

$$\int_{-\infty}^{+\infty} q^2 D_{qq}\, dq = \mathrm{Tr}(DQ^2) \equiv \langle Q^2 \rangle \qquad (2.26a)$$

and

$$\int_{-\infty}^{+\infty} q\, D_{qq}\, dq = \mathrm{Tr}(DQ/ \equiv \langle Q \rangle \qquad (2.26b)$$

where $\langle\ \rangle$ simply means the expectation value of the corresponding operator and is introduced for brevity. For D's that satisfy certain conditions, which are presumably satisfied in the case of communication signals, we have

$$\langle \omega^2 Q^2 \rangle = \langle P^2 \rangle = \langle \hbar\omega\,(\alpha^t \alpha + \frac{1}{2}) \rangle$$

Since the traces are independent of the representation and since from Eq. (12a) we know, $\langle \hbar\omega(\alpha^t\alpha + \frac{1}{2} \rangle = \hbar\omega\,(M + \frac{1}{2})$, we conclude that

$$\frac{\omega}{\hbar} \int_{-\infty}^{+\infty} q^2\, D_{qq}\, dq = \langle \frac{\omega}{\hbar}\, Q^2 \rangle = M + \frac{1}{2} \qquad (2.27)$$

For future reference we note that

$$\langle Q \rangle = \mathrm{Tr}(DQ) = i\sqrt{\frac{\hbar}{2\omega}} \sum_{m=0}^{\infty} \sqrt{m+1}\left(D_{m,\,m+1} - D_{m+1,\,m}\right) \qquad (2.28)$$

which is obtained by using Eq. (6b) and the fact that $\alpha^t / m\rangle = \sqrt{m+1}/ m+1\rangle$ and $\alpha / m\rangle = \sqrt{m}/ m-1\rangle$. For the moment, however, we shall simply use the notation $\langle Q \rangle$, keeping in mind that $\langle Q \rangle$ is the average value of the field to be measured. Thus Eq. (15) becomes

$$\sigma_q^2 = (M + \frac{1}{2}) - \frac{\omega}{\hbar} \quad \langle Q \rangle^2 \qquad (2.29)$$

Again it is to be noted that the presence of the term $\frac{1}{2}$ is dependent of whether it is included in the hamiltonian or not.

This variance gives us a measure of the uncertainty associated with a measurement of the field.

Comparing σ_n^2 to σ_q^2 we draw the trivial conclusion that they are different in general. The implications of this statement however are less trivial, since it shows that the uncertainty which is encountered in a measurement inherently depends upon what is the quantity to be measured. Consequently the effective noise in a communication channel is going to depend strongly on the detection scheme. Thus a system which is theoretically noiseless may turn out to have noise if the receiver is changed. Again this effect has nothing to do with imperfections of the instrument or with external sources of noise. It will exist in vacuum as well.

At this point we can discuss a general model for an infrared or optical communication channel from the standpoint of quantum noise. The model is proposed in Ref. 6. Here we discuss it from a slightly different angle and in fact on the basis of Eq. (19). Suppose that the energy arriving at the receiver consists of a well defined signal $\langle Q \rangle$ plus noise energy which has been added by external sources. Since the total energy arriving at the receiver is $\hbar\omega M$, the noise energy will necessarily be $\hbar\omega M - \omega^2 \langle Q \rangle^2$ which we will call $h\omega N$ i.e.

$$\hbar\omega M - \omega^2 \langle Q \rangle^2 = \hbar\omega N \tag{2.30}$$

Then we have

$$\hbar\omega\sigma_q^2 = \hbar\omega N + \frac{\hbar\omega}{2} \tag{2.31}$$

It is conceivable however that the noise from external sources (including a distant transmitter) be zero. At least it is theoretically permissible to assume so. Then we would have $N = 0$. The term $\frac{\hbar\omega}{2}$ however, will never vanish and we may say that this is the minimum quantum noise in a receiver which measures the field.

From Eq. (2.23) it is seen that in order to make any statement about the noise associated with an energy measuring receiver a more detailed knowledge of D is needed. In fact one needs to know D_{nn} for all n.

In reference 6 the same conclusion is drawn after a specific form for D is postulated. Here it appears that the results are more general, the only restriction being the model for the channel. These restrictions however are of a rather general nature. The basic restriction perhaps is the assumption that there is a well defined signal with a known average $\langle Q \rangle$ and negligible variance. However, if the signal has been attenuated over a long transmission path this assumption is certainly questionable. The application of the formalism discussed above to extremely weak signals will be further investigated.

2.3 Binary Channel with Photon-Counter Detection

It has been pointed out above that within the larger part of the frequency range considered here in general

$$kT << hf \tag{2.1}$$

so that thermal background radiation plays a minor part in limiting the optimum rate of transmission of information. The decisive parameter is then the ratio of the received signal power to $\Delta f \cdot hf$, or the number of photons per sample at the receiver. For an optical communication channel operating with a small fraction of a photon per sample and with negligible background radiation it is possible to devise and analyze a very simple theoretical model, which sheds some light on the operation of an optical communication channel at minimum power levels.

The optical system is used as a binary channel; the basic unit of time is taken as the time between independent samples, i.e., the reciprocal of the bandwidth. A binary channel is illustrated by the diagram in Fig. 2.1., if the output is always yes or no, pulse or no pulse;

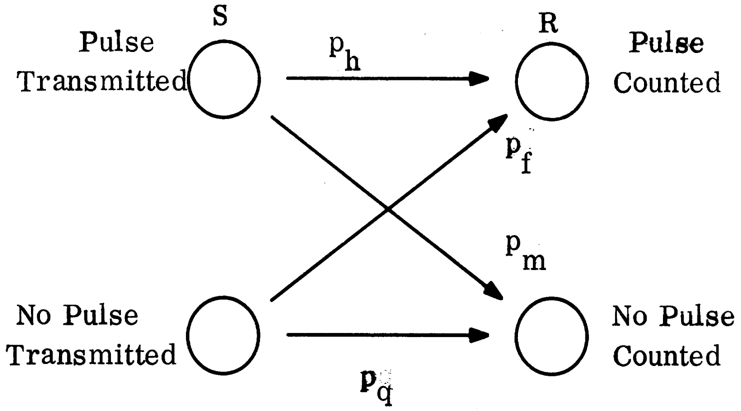

FIG. 2.1.

i.e., the third alternative "don't know" or erasure is not included. The detector or counter is here incorporated in the channel and the transition probabilities between transmitted pulses and received counts include the statistics of the detector as well as the statistics of the propagation. The counter may be considered to be a phototube or photo multiplier.

If background radiation and detector dark current are negligible, the false-alarm probability p_f is zero and its complementary probability p_q is unity. When a pulse is sent, the number of photons counted can be assumed to have a Poisson distribution; this is the maximum-entropy minimum-bias assumption. The probability of r counted photons is then

$$P(r) = \frac{\bar{r}^r}{r!} \cdot e^{-\bar{r}} \qquad (2.32)$$

In the binary channel the only two alternatives of interest are

$$p_m = P(o) = e^{-\bar{r}} = 1-r \qquad (2.33)$$

$$p_n = P(r \geqslant 1) = 1 - e^{-\bar{r}} = r \qquad (2.34)$$

where $\bar{r}$ is the expected value of the number of counted photons per pulse.

The probability Q of a transmitted pulse is also the duty cycle of the transmitter. For given average power then the pulse power is inversely proportional to Q; so is the expected value $\bar{r}$ of the number of photons counted.

The rate of transmission of information per sample may be written

$$\begin{aligned} H(s) + H(R) - H(S,R) &= -Q \log Q - (1-Q)\log(1-Q) \\ &\quad -Q\,p \log Q\,p - (1-Qp)\log(1-Qp) \\ &\quad + Qp \log Qp + Q(1-p)\log Q(1-p) \\ &\quad + (1-Q)\log(1-Q) \qquad (2.35) \\ &= 1 \log(1-Qp) - Qp \log \frac{Q(1-p)}{1-Qp} + Q \log(1-p) \end{aligned}$$

since the probability matrix of (S, R) is

$$\begin{vmatrix} P(1,1) & P(0,1) \\ P(1,0) & P(0,0) \end{vmatrix} = \begin{vmatrix} Qp & 0 \\ Q(1-p) & 1-Q \end{vmatrix} \qquad (2.36)$$

The optimum duty cycle Q is found by maximizing (2.35) under the constraint of given energy averaged over all samples, pulse or no pulse

$$\bar{m} = Q \cdot \bar{r} = \text{const.} \qquad (2.37)$$

A transcendental relation between Q_{opt} and $\bar{m}$ or between the corresponding $\bar{r}$ and $\bar{m}$ may be obtained by a variational procedure. Gordon[5] has given a graph of a numerical solution of this equation and discussed the conclusions to be drawn from the analysis of the binary optical channel.

The graph is most easily obtained from the above-mentioned relation written in the following form:

$$\bar{m} = \bar{r}\left[\exp\left\{\frac{\bar{r}(\bar{r}+1)}{e^{\bar{r}} - \bar{r} - 1}\right\} + 1 - e^{-\bar{r}}\right]^{-1} \tag{2.38}$$

It may be worthwhile to illustrate the results by a few numerical examples.

Ex. 1. A binary channel operating under the conditions: $\bar{m} = 0.06$, $Q_{opt} = 0.06$, $\bar{r} = 1.0$, which satisfy (16) and (17), has a rate of transmission of 0.176 binary units per sample. The maximum entropy of the transmitted signal above is 0.328 bits per sample. The difference, 0.152 bits, constitutes the equivocation introduced by the Poisson detection process. The rate of transmission is 53 % of the wave entropy per sample obtained from (2.2) and (2.10)

$$C_W / \Delta f = \log(\bar{m} + 1) + \bar{m} \log(1 + \frac{1}{\bar{m}}) \tag{2.39}$$

The entropy of the received signal is found to be 70 % of the wave entropy.

Ex. 2. Reducing $\bar{m}$ to 0.0032 gives a $Q_{opt} = 0.0064$ and $\bar{r} = 0.5$. The corresponding channel rate is approximately 0.0238 of a binary unit per sample, which is 76.5 % of the wave entropy per sample. The entropy of the received signal is 81 % of the wave entropy per sample. In this case the maximum entropy of the transmitted signal is 0.056 of a binary unit per sample, indicating an equivocation of 0.032 bit per sample.

The trends indicated by these numerical examples are monotone. The rate of transmission of information decreases of course when the average energy per sample is reduced; at the same time the rate approaches the upper bound estimated by (2.2). This result has been stated in the form that the "efficiency of a binary channel with quantum-counter detection approaches 100 °/o as the number of photons per sample is reduced. This statement may be useful in comparing the quantum counter with other detectors in this range of energy per sample. But it should not be interperted as suggesting that this "high-efficiency" operation is desirable or even practical.

As the energy per sample is reduced and the "efficiency" grows, the rate of transmission of information falls off very rapidly. The ratio of equivocation per sample to the entropy of independent samples per sample grows to values 50 °/o and higher. Very highly redundant codes have to be used to eliminate transmission errors, and the coding-decoding problems become very difficult, requiring elaborate computers.

It is really not surprising that a high "efficiency" in relation to the upper bound in (2.2) can be obtained, since also the latter expression is based on the statistics of a quantum count. The difference is of course that the binary counter does not distinguish between the reception of one, two, three--- etc. photons. Clearly then the "efficiency" can be expected to be high under operating conditions when the probabilities of the simultaneous arrival of two and more photons are small compared to the probabilities of zero and one. In the first example above the probabilities p(0), p(1), p(2)--- are 0.368, 0.368, 0.184, 0.061 . . . . . in case of a tranmitted pulse, while the higher efficiency of the second example correlates with the more rapid convergence of the sequence 0.606, 0.303, 0.025, 0.003-----

As a binary communication channel this system has some peculiar features of its own. The strong asymmetry introduced by $p_f = 0$ means of course that the conventional coding theory developed for symmetric channels does not apply and that the solution of the coding problem involves exploring some new territory.

It should be realized, of course, that the false-count probability p_f is in practical cases not exactly zero. The background temperature differs somewhat from zero, and the dark-current of a phototube is not completely absent. Under conditions where Q_{opt} is very small, it may consequently be advisable to investigate how small p_f really is. When p_f cannot be neglected, the analysis of optimum operating conditions presented above has to be modified considerably.

In order to realize the advantages that the binary counter channel promises to offer the following requirements must be met at any selected operating wavelength:

1. Photon counters must be available with

    a) high quantum efficiency

    b) fast operation (10^{-9} sec. or better)

    c) negligible false-count probability (dark current)

2. If background radiation exists means must be available to reduce the fraction reaching the counter, such as

    a) antenna optics (space filters)

    b) optical frequency filters

3. A coding-decoding system must be developed that permits error-free operation when the channel equivocation per independent sample is very high.

None of these requirements is at present satisfied over the whole wavelength range specified in the directives for this study. There is consequently a considerable amount of work to be done, first in investigating to what extent these requirements can be met, from a theoretical point of view, second in recommending the direction of further research, whenever the theoretical answer is affirmative.

Even if the first answer is encouraging, a critical comparison with other channel principles can hardly be omitted. Such a comparison, however, will have to be postponed till a later report.

The performance limits of photon counter cathodes fall outside the special problem areas specified for this study and will only be discussed briefly in the future. Optical frequency filters, on the other hand, constitute such a problem area, and a general survey and preliminary discussion is included in the last section of this report. The encoding-decoding problem belongs to the problem areas of the communication theory of the binary photon-counter channel, which is the subject of the present section. We shall come back to this problem in a future report.

3. Optical Band Pass Filters for Communication Channels

3.1 Survey and a Preliminary Evaluation of Known Filter Principles

The third problem area selected for this study is motivated by the importance of minimizing by optical means the background radiation incident on a quantum-counter detector in a communication system, since in this detection scheme no opportunity of intermediate-frequency frequency discrimination is offered.

Filtering of optical and infrared radiation in the wavelength interval 0.4 to 20μ is generally accomplished by the use of interference or absorption type filters. There are however, other kinds of filters available for special applications. Optical filters can be classified according to the physical mechanism involved in their operation as: (1) selective absorption; (2) selective reflection; (3) scattering; (4) interference; (5) polarization; and

(6) refraction[13]. We will briefly discuss some filters now being used, their characteristics and the possibility of changing their center wavelengths or of tuning them.

Glass color filters and gelatin filters are commercially available in the 0.2 to 1.0 micron range. These filters operate on the selective absorption principle and their pass band characteristics are generally not very sharp. They can be useful as bandpass filters in conjunction with some other narrow-band filter to eliminate multiple pass bands of the latter. Tuning of such filters is not possible.

Transmission interference filters[13], in the simplest form a Fabry-Perot interferometer, are available commercially with transmission peaks of 30 % and a width of 150 Å in the range 3900 Å to 7800 Å. When the semi-reflecting metal films of the Fabry-Perot interferometer are replaced with quarter-wavelength stacks of alternately high and low indexes of refraction, narrower pass bands and higher peak transmissions can be obtained, while the sidebands are decreased. In this way peak transmission of 73% and pass bands of 70 Å are typically achieved, although with some sacrifice in the peak transmission, pass bands as narrow as 20 Å are possible. At these frequencies, a wavelength pass band of 20 Å corresponds to a frequency pass band of about 10^{12} cps.[14] Interference filters in the 1.4 to 4.5 micron range are available with pass bands of about

1/50 of the center frequency, or about 10^{13} cps. The interference filter can also be made as a reflection, rather than a transmission filter. Hadley and Dennison[33] discuss both thoroughly. The interference type filter is feasible to tune; this possibility will be discussed below.

Another filter which is useful in the visible and infrared range is the Christiansen filter.[13], [15], [16] In this filter a powdered dielectric is suspended in an appropriate fluid. The two components are chosen so that their refractive-index vs. wavelength curves have different slopes and cross near a desired wavelength. At the point where the refractive indices are the same, the radiation passing through the filter is not deflected. On either side of this point, however, the particles of dielectric scatter the radiation, thereby attenuating it. Pass bands of 200 Å in the range 4000 to 8000 Å have been obtained. The width of the pass band is determined by the relative slopes of the two refractive index curves. The fact that the index of refraction of a liquid changes more rapidly with temperature than that of a solid has been used to make a tunable band-pass filter.[13]

Clever use of the birefringence of crystal quartz plates has been applied in the design of the birefringent filter.[13] It is also called the interference polarization filter or Lyot–Öhman filter. Quartz plates are cut with their optical axes parallel to their large faces, and these plates are mounted between polarizes whose axes are oriented at 45°

with respect to the quartz optical axes. The incident radiation is converted to linearly polarized light at the first polarizer, and thereafter, only that part of the light whose axis of polarization is parallel to the polarizer axis will pass. If each succeeding quartz plate is twice as thick as that which precedes it, then in a given wavelength interval, it will have twice as many maxima and minima. The transmission of the entire assembly of plates and polarizers is the product of the transmission of the individual sections, and there are now sharp transmission peaks separated by λ_o / N where λ_o is the lowest-order wavelength for maximum transmission, and N is the order of the maxima. Filters using 6 quartz plates have been made with a pass band of 4.1 Å at 6563 Å[13]. Some commercial filters are available with pass bands as narrow as 1 Å. These filters have a potential for tuning.

There is another type of interference filter called the frustrated total-reflection filter which has been used in the vicinity of 5000 Å.[14] Its operation is based on the fact that the reflection is not total when radiation is incident on a boundary between high and low index of refraction materials at an angle greater than the critical angle, if the low index of refraction material is thin enough. The filter is constructed of two flat triangular pieces of glass joined along their hypotenuses but separated by two thin layers of low refractive index and a high index of refraction spacer in between. The wavelength of maximum transmission is determined by the optical thickness of the spacer, the angle of reflection in it and the phase change on reflection while the width of the pass band depends on the thickness of the low refractive index layer. Bandwidths of 120 Å to 30 Å has been achieved with peak transmissions of 93 % to 12 %.

Two other filters which are restricted primarily to the infrared range are commonly used. They are the semiconductor filter and the residual ray plate or restrahlen filter.[13],[15],[16] The semiconductor filter is a low-pass filter which absorbs wavelengths shorter than that associated with its energy gap between the valence and conduction bands. The cut-off characteristics are quite sharp, but because impurity atoms can introduce electron energy levels between the valence and conduction bands, the impurity content must be kept low if absorption beyond the cut-off wavelength is to be minimized. The semi-conductor filter has a cut-off wavelength in the range of 1 to 8 microns and can be used with interference filters to obtain a rather narrow pass band and to eliminate the side bands of the interference filter. Since the width of the energy gap of the semiconductor is temperature-dependent, the cut-off wavelength may exhibit a temperature-dependent shift.

The residual-ray plate filter depends for its operation on the ionic structure of a crystal. When radiation impinges on the atoms in a crystal, the electric field tends to set the ions into motion. A resonance will occur which is related to the restoring-force constant of the crystal and the masses of the two ions. The crystal will reemit a band of wavelengths about this natural frequency. The principal restrictions on the use of the residual-ray plate filter is that there is only a limited number of center wavelengths available. In addition, they are limited generally to applications at wavelengths greater than 15 microns.

One other choice that is commonly used to filter light or disperse it is the prism or grating spectroscope. Such a device is essentially one which converts a frequency spectrum into a spatial spectrum by means of a dispersive element. The resolving power of a rock-salt prism spectroscope is about 200 at 5 microns and 400 at 14 microns which is equivalent to a pass band on the order of 300 A at 10 microns wavelength.[17] The grating instrument in the wavelength range 3 to 25 microns has no better resolution than a well designed prism spectroscope, since the theoretical resolving power of the grating is seldom realized in infrared work.[18] This is due to the fact that the limit is set not by the grating, but by the slit width which must be employed for adequate illumination. The present practical limit of the resolving power of the grating is 8000 from 1.5 to 7 microns, or a pass band on the order of 6 Å at 5 microns wavelength. At 20 microns wavelength the pass band would be about 160 Å. The spectroscope may be of some interest in designing a tunable filter.

In the application in which we are interested, the ideal filter bandwidth is on the order of 10^9 cps, the expected modulation bandwidth. The corresponding passbands at 0.4 microns and 20 microns wavelength are 5.3×10^{-3} Å and 13 Å respectively. If we recall the passband widths quoted above for present commerical filters, we observe that these desired bandwidths are 1 to 2 orders of magnitude smaller than that is available with the filters discussed. It appears that the best chance for obtaining the desired bandwidth occurs at the longer wavelengths of the interval 0.4 to 20 microns.

When the question of tuning a filter is considered, of these filters mentioned above, two kinds seem easily adaptible for tuning, the interference and the Christiansen filter. The spectroscope may also have some potential in this area, as mentioned previously. Tuning of the interference filter is feasible by electrooptic, piezoelectric and perhaps also magneto-optic and magneto-strictive means, while the Christiansen filter is, in principle at least, capable of electro-optic tuning. A quite extensive amount of work has been published in recent years in this connection concerning the modulation of light by application of electric or magnetic fields to optic materials. We mention briefly here the common electro-and magneto-optic effects and present some examples where they have been used.

The electro-optic effect in a crystal is a change in the refractive index with the strength of the applied electric field. If the change is linearly related to the field strength, it is known as the Pockel's effect. A coherent-light phase modulator using the Pockel's effect was described by Peters,[19] who used a slow-wave microwave structure to provide a modulating electric field in a crystal of ammonium dihydrogen phosphate. The experimental evidence suggests that a modulation bandwidth several octaves greater than 1 Gc could be obtained. Kaminow[20] carried out a somewhat similar experiment, using a crystal of potassium dihydrogen phosphate in a cylindrical cavity and a modulation frequency of 0.25 Gc. Gil'varg and Kolesov[21] reported on the use of these two crystals in a high-speed shutter and Niblack and Wolf[22] report on a polarization modulation-demodulation system utilizing a Pockel's cell for the modulation of a continuous-wave helium-neon gas laser as the light source.

When the change in the refractive index is proportional to the square of the applied electric field, it is known as the Kerr electro-optic effect, which should be distinguished from the Kerr magneto-optic effect which will be mentioned later. The Kerr electro-optic effect is characteristic of a crystal with a center of symmetry[23]; if the crystal has no center of symmetry, then the electro-optic effect may be linear. Holshouser, et. al[24] constructed a Kerr-cell microwave light modulator which used a re-entrant microwave cavity to provide the modulation field and which achieved up to 80 per cent modulation at 3 Gc.

The magneto-optic effect may also be classified as linear or quadratic. The linear magneto-optic effect is called the Faraday effect; when the effect is proportioned to the square of the magnetic field it is known as the Cotton-Mouton effect. Schmidt et. al[25] have used the enhanced Faraday effect near the absorption line of sodium vapor to modulate the plane of polarization of light traveling through the vapor. Pulse modulation was carried out up to 698 Mc with no evidence of reduction in frequency response. Fork and Bradley[26] reported on the Faraday rotation in mercury vapor near an optical resonance. Since only light in the vicinity of the resonance experiences appreciable rotation, they suggest that by using a pair of crossed polarizers with the cell a narrow-band modulated filter, having a pass band of about 1500 Mc, can be realized.

Another magneto-optic effect which may be of interest is the Kerr magneto-optic effect. This term refers to a change in the polarization or intensity of light reflected from the surface of a magnetized medium. When we speak of the Kerr effect hereafter, we will mean the Kerr electro-optic effect, unless we specify otherwise.

The piezoelectric effect is also of interest in this context. It is exhibited by some crystals in which a stress is set up when an electric field is applied; conversely, a stress produces an electric field. This effect has been used by Hauser et al[27] in designing a stressed-plate shutter. Seraphin et al[28] employed the piezoelectric effect in designing a Fabry-Perot type light modulator using a quartz spacer, the thickness of which was changed piezoelectrically. Another interesting application of this effect was made by Astheimer et al[29], who designed a frustrated total internal reflection infrared modulator. Its principle of operation is similar to that of the frustrated total-reflection filter which was discussed previously. In this case, however, there is only air separating the two halves of the device, and the separation is controlled piezoelectrically. Since the transmission falls off as $\sinh^{-2} \frac{d}{\lambda}$ where d is the separation and λ the wavelength, we see that decreasing λ or increasing d decreases the transmission. It can thus act as a low-pass filter whose cut-off frequency can be altered by changing d. It could possibly be used with an interference filter to make a tunable band-pass filter.

The magnetostrictive effect, that is distortion produced in certain materials when subjected to a magnetic field, has also been used in this area. Bennet and Kindlemann[30] report on the use of magnetostriction for making angular and separation adjustments on the end plates of a helium-neon maser. The device was constructed with the eventual use in mind of achieving negative-feedback frequency stabilization.

As discussed above, the electro-optic effect involves a change of the index of refraction with applied electric field. If a Christiansen filter were to be made using an electro-optic fluid, then the cross over point of the refraction indexes of the two materials composing the filter could be shifted in frequency, providing a tuning mechanism for the filter. From a practical standpoint however, the relative slopes required for the dispersion relations of the two materials in order to obtain a sufficiently narrow pass band may be unrealistic considering the properties of materials presently used for such filters, particularly since one of them must be an electro-optic material. In addition, the tuning range that can be achieved may be too limited to make such a filter of interest. These questions will be considered more fully later.

The various interference filters such as the polarization, frustrated internal reflection and reflection and transmission filters are tunable in principle, though the techniques of accomplishing the tuning will be different. For the present we discuss briefly the transmission-type filter.

The transmissivity of the interference transmission filter for normal incidence is given by,

$$T = \frac{1}{\left[1+\frac{A}{T}\right]^{2}} \cdot \frac{1}{\left[1 + \frac{4R}{\left[1-R\right]^{2}} \sin^{2}\frac{\delta}{2}\right]} \qquad (3.1)$$

where R, T and A are the spectral reflectivity, transmissivity and absorptivity of a single reflection layer of the fitler and

$$\delta = \frac{4\pi nd}{\lambda} + 2\,\phi \qquad (3.2)$$

In this formula, n is the refractive index of the medium between the two layers, d is the layer separation, λ is the free-space wavelength and d is the phase shift for reflection from one layer.

We observe that T is a maximum whenever $\sin\frac{\delta}{2} = 0$, i.e., when

$$\frac{2\pi nd}{\lambda} + \phi = N\pi\,,\ N = 1,\ 2,\ 3,\ \ldots \qquad (3.3)$$

N is referred to as the order of the interference fringe. We see that the wavelength of maximum transmission for a given order of interference N is determined by n, d and ϕ. Thus the filter should be tunable by adjusting any one or more of these 3 parameters. If for the moment we neglect changes in ϕ and consider only n and d variable, and if we wish to tune the filter over the interval λ_1 to λ_2 where $\lambda_2 = \Delta \lambda_1$, then

$$\lambda_1 (\Delta - 1) = \frac{2}{N} (n_2 d_2 - n_1 d_1) \tag{3.4}$$

n_1, d_1 and n_2, d_2 are the index of refraction and the separation at the ends of the tuning band. We can rearrange this expression to get

$$n_2 d_2 = \Delta n_1 d_1 = \frac{\Delta N \lambda_1}{2} \tag{3.5}$$

So we see that the product of the index of refraction and the separation must be changed in proportion to the desired change in center wavelength λ_1 of the pass band. In addition, the absolute change in the product of nd is proportional to N, so that the change required is a minimum for a given λ when $N = 1$.

To get an idea of the order to magnitude of the quantities involved, suppose we consider piezoelectrically tuning an air filled Fabry-Perot interferometer over an octave bandwidth from 5 to 10 microns wavelength. Then $\lambda_1 = 5$ microns, $n = 1$, and $N = 1$, $d_2 = 5$ microns, $d_1 = 2.5$ microns. If barium titanate,, which has the largest piezoelectric constant known, 3×10^{-4} microns per volt,[29] were to be used to accomplish the 2.5 micron change in separation then a voltage of almost 10,000 volts would be required. This large voltage can be decreased by placing slabs of piezoelectric in series mechanically, but in parallel electrically. And since the relative displacement of the plates of the filter could be reversed by changing the direction of the electric field, the 10,000 volt requirement could be decreased to 5,000 volts. The requirements for tuning the simple Fabry-Perot interferometer do not seem unreasonable then.

The frustrated internal reflection filter and the Christiansen filter together suggest another possible filter configuration that would be tunable. We will refer to this configuration, which has the geometry of the Wernicke prism[32], as the Wernicke filter. A cross-section of this filter is shown in Fig. 3.1.

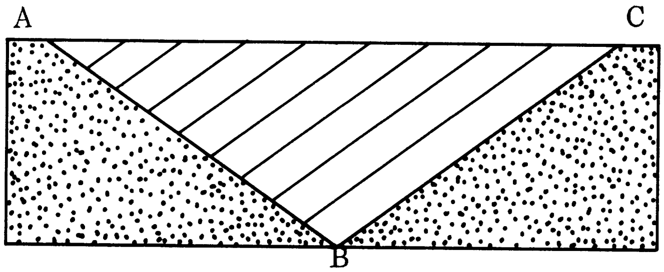

FIG. 3.1. Sketch of a Wernicke Filter.

If the two materials of which this filter is composed have refractive index relations vs wavelength with different slopes, which cross over at some wavelength, then at this particular wavelength, the incident radiation will pass through the filter with no deflection. At wavelengths longer or shorter than this however, reflections will occur at interfaces AB and CB. For wavelengths where the indexes of refraction are sufficiently different, total internal reflection will take place at these interfaces, but on opposite sides of the center wavelength. If now, one or the other of the materials is electrooptic in nature, the center wavelength could be changed. The consideration mentioned above with respect to the Christiansen filter apply here also. There is an additional feature here, however, in that the radiation emerging from the Wernicke filter would be spatially dispersed on either side of the center wavelength, providing a potential additional filtering mechanism.

We have reviewed here some common methods used for filtering in the optical and near infrared regions of the spectrum. Some possible mechanisms by which a filter might be tuned were discussed, and some examples of the application of these mechanisms in the modulation of light and infrared radiation were presented. Some possible schemes for constructing a tunable filter were considered.

3.2 Proposed Work for the Next Period

In the next quarter we intend to investigate some specific filter designs more thoroughly, particularly concerning theoretical limitations on their bandpass characteristics and tunability.

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