Topics in Electrodynamics of Moving Media

Final Report

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By

C. T. Tai, V. Pyati and R. M. Kalafus

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National Aeronautics and Space Administration
Langley Research Center
Langley Station
Hampton, Virginia 23365

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INTRODUCTION

In preparing the final report, we have given some technical outline of those works which have not yet appeared in a technical report or been submitted for publication. Work performed on the project that has been published is indicated in the references. The latter includes a detailed study of the problem of a small antenna over a moving dielectric medium (Section 2), and a paper concerning the present views on the theory of electrodynamics of moving media (Section 3). In Section 4 are discussed some of the complexities introduced into the theory by extending it to conducting media. Several applications are presented in Section 5 which involve the Cerenkov radiation effect. In Section 6, earlier work on a drifting, magneto-ionic plasma is extended to finding the characteristic indices of refraction for some configurations of plasma parameters. The last section deals with the formulation of the time-dependent Green's function for moving, lossless media.

II

RADIATION DUE TO AN OSCILLATING DIPOLE OVER A LOSSLESS SEMI-INFINITE MOVING DIELECTRIC MEDIUM

A thesis bearing the above title was completed by Pyati (1966) during this period, and copies of this were distributed to NASA on March 25, 1966. An oral paper describing this work was presented at the Washington URSI meeting, April 19 - 21, 1966. It is expected that a written paper will soon be submitted for publication in a technical journal. The main contribution resulting from this study is the exact determination of the effects of the motion on the radiation pattern of a Hertzian antenna.

III

DEFINITE AND INDEFINITE FORMS OF MAXWELL-MINKOWSKI EQUATIONS

The basic equations which govern the field vectors in a moving medium are now well understood. These equations may be presented in various forms but they are all equivalent. All of them can be derived from the original theory of Minkowski. A paper concerning the present views on electrodynamics of moving media was presented at the International Convention of the Institute of Electrical and Electronics Engineers (Tai, 1966). Twenty copies of this paper were sent to the Langley Research Center. It is hoped that this work has clarified some of the recent controversies since the publication of the book by Fano, Chu and Adler (1960).

IV

ELECTRODYNAMICS OF MOVING, CONDUCTING MEDIA

4.1 Ohm's Law

Upon the introduction of finite conductivity into the study of isotropic moving media, one is immediately beset by the existence of two apparently different forms of Ohm's law:

$$\overline{J}_c^{(1)} = \gamma\sigma' (\overline{E} + \overline{v} \times \overline{B}) \qquad \text{(Weyl, 1922)}$$

and

$$\overline{J}_c^{(2)} = \frac{\sigma'}{\gamma} \overline{\overline{\gamma}} \cdot \overline{\overline{\gamma}} \cdot (\overline{E} + \overline{v} \times \overline{B}) \qquad \text{(Sommerfeld, 1952, and Cullwick, 1959)}$$

where $\overline{J}_c$ is the conduction current density

σ' the rest-frame conductivity of the medium

$\overline{v} = v\hat{z}$ is the velocity of the medium

$\gamma = (1 - \beta^2)^{-1/2}$

$$\overline{\overline{\gamma}} = \begin{bmatrix} \gamma & 0 & 0 \\ 0 & \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\beta = v/c$

Schlomka (1950) claimed that $J_c^{(2)}$ is correct on the basis of an electron-theoretic model. His crucial step, however, involves attaching a physical meaning to the relative velocity of two bodies as seen by a third observer. This is dubious and unsatisfying, and warranted further investigation. A study of the derivations showed that the decomposition of current density into conduction and convection terms is not unique. An expression was derived for the Joule heat loss per unit volume per unit time,

$$\left(\frac{\sigma'}{\gamma}\left[\bar{\bar{\gamma}}\cdot(\bar{E}+\bar{v}\times\bar{B})\right]^2\right)$$

that is independent of the decomposition, using elementary relativistic thermodynamics. Since both field and energy terms are uniquely specified, either decomposition may be used, and the choice of Ohm's law is immaterial.

4.2 Characterization of Charge Densities

In extending the work of Tai (1965a) on point sources in isotropic, moving media to conducting media, the characterization of charge density requires careful interpretation. Denoting quantities measured in the rest system of the medium by primes, the total current density $\bar{J}'$ is made up of convection and conduction terms in general:

$$\bar{J}' = \rho_s' \,\bar{u}' + \sigma' \bar{E}'$$

where ρ_s' is the density of the charges introduced into the medium with velocity $\bar{u}'$. The quantity ρ_s' cannot be considered to be the total charge density, for this leads to inconsistencies; there must be an additional partial charge density caused by the presence of the source charge density ρ_s'. Furthermore, the current density viewed in the stationary (unprimed) system has two convection currents, one due to the motion of the source charge $(\rho_s \,\bar{u})$, and one due to the motion of the medium $(\gamma \rho_r' \,\bar{v})$.

$$\bar{J} = \bar{J}_c^{(1)} + \gamma \rho_r' \,\bar{v} + \rho_s \,\bar{u}$$

where $\rho_r' = \rho' - \rho_s'$, and ρ' is the total charge density. With this interpretation, the introduction of source charges has no effect on Ohm's law, as would be required. The charge density $\gamma\rho_r'$ is determined completely by ρ_s, and in the special case where ρ_s is a point charge

$$\rho_s = q\,\delta(x)\,\delta(y)\,\delta(z)$$

then

$$\gamma\rho_r' = \begin{cases} 0\,, & z < 0 \\ -\dfrac{\gamma\sigma'}{\epsilon' v}\, q\,\delta(x)\,\delta(y)\exp\left[-\dfrac{\sigma' z}{\epsilon'\gamma v}\right], & z > 0 \end{cases}$$

The scalar potential ϕ for the time-independent case (measured in the stationary system) must satisfy the following differential equation:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{1}{a}\frac{\partial^2\phi}{\partial z^2} - \sigma'\mu'\gamma v\,\frac{\partial\phi}{\partial z} =$$

$$-\frac{q}{\epsilon' a}\,\delta(x)\,\delta(y)\,\delta(z) + \frac{q\gamma\sigma'}{\epsilon'^2 v}\,\delta(x)\,\delta(y)\exp\left[-\frac{\sigma' z}{\epsilon'\gamma v}\right]\cdot\begin{Bmatrix} 0, & z<0 \\ 1, & z>0\end{Bmatrix}$$

where

$$a = \left[\gamma^2(1 - n^2\beta^2)\right]^{-1}$$

$$n^2 = \mu'\epsilon'/\mu_o\epsilon_o$$

and ϵ' and μ' are the permittivity and permeability, measured in the rest-frame of the medium.

A partial solution has been found in closed form for the scalar potential, that is, a solution found by ignoring the second term on the right of the equation. By Fourier transform analysis, the partial scalar potential function ϕ_1 can be shown to be:

$$\phi_1 = \left\{ \begin{array}{ll} 0\,, & |a|\,z < r \\ \dfrac{-q\; e^{-bz}\; \cosh\left[b\left(z^2 - \frac{r^2}{|a|}\right)^{1/2}\right]}{2\pi\epsilon |a| \;\left(z^2 - \frac{r^2}{|a|}\right)^{1/2}}\,, & |a|\,z > r \end{array} \right\}$$

where $\quad b = \dfrac{\sigma' \mu' \gamma v}{2|a|}$,

and it is assumed that $n\beta > 1$. The surface described by $|a|\,z = r$ is a cone, and ϕ_1 exhibits an exponential decay for large z . In the rest system of the medium, the radiation properties are determined by the vector potential A' . This in turn can be shown to depend only on the partial solution ϕ_1 , so that it is unnecessary to evaluate the total scalar potential ϕ . The results are identical with those of section 5.3 .

V

CERENKOV RADIATION TOPICS

5.1 Unbounded, Lossless Media

The well-known Cerenkov effect, arising from charged particles travelling with velocities exceeding the speed of light in the medium, has many interesting properties. The assumption of a point charge as a source results in infinite energy densities in the calculation of the fields, while the assumption of a dispersionless medium results in infinite energy. For a point charge, the shock wave has the form of a cone (see Fig. 1); fields are zero ahead of the cone, and behind the cone behave as

$$\frac{1}{\sqrt{(z' + v\,t')^2 - (n^2 \beta^2 - 1)\, r'^2}}$$

in the rest system of the medium, or

$$\frac{1}{\sqrt{|a|\, z^2 - r^2}}$$

in the rest system of the charge.

For a line charge moving perpendicular to its axis with velocity v , while the wedge angle α is the same as before (see Fig. 1), there is no wake. That is, the fields are confined to a small region around the shock wave, and vanish in front and behind. This was worked out for the case of lossless, non-dispersive media.

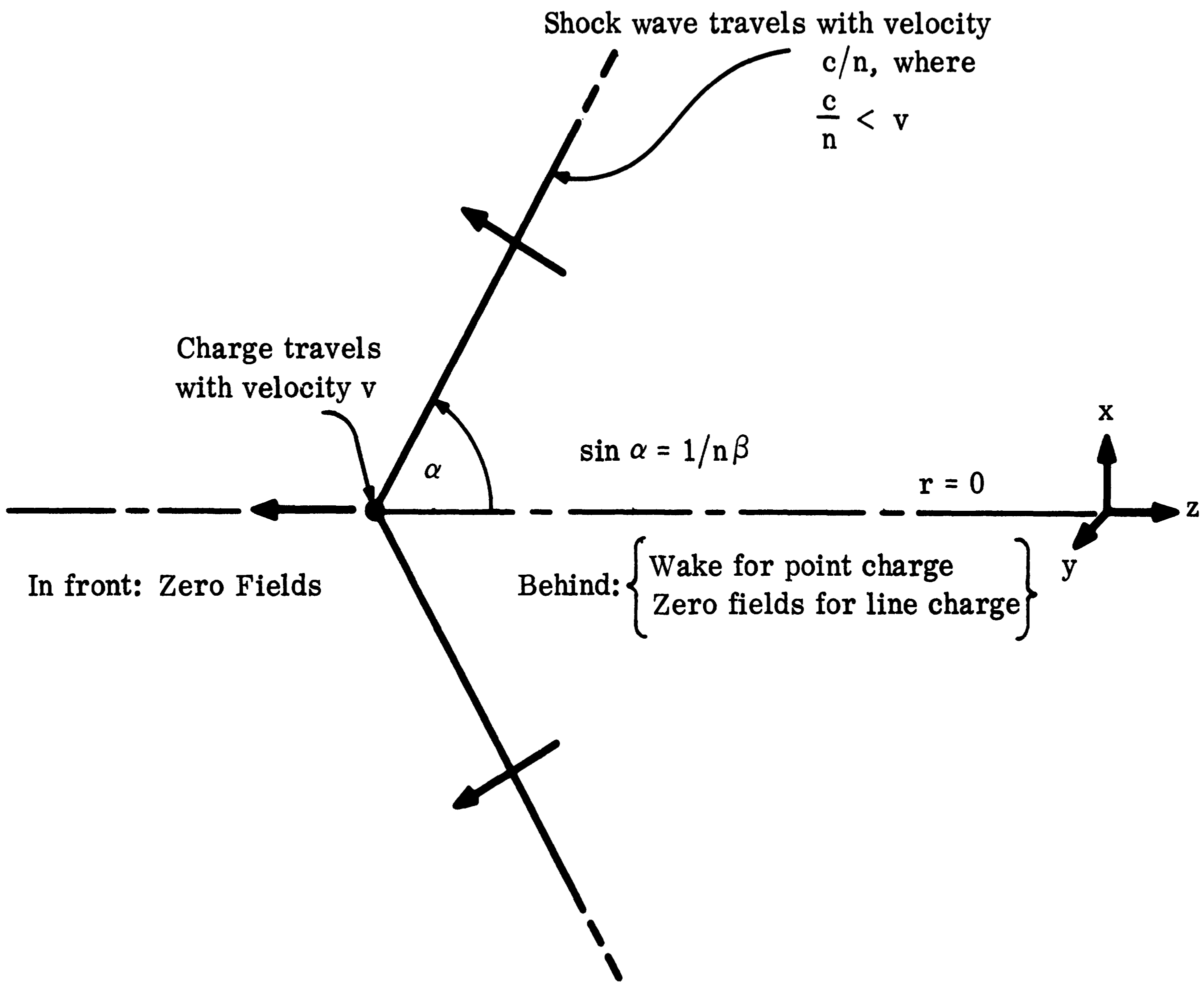

FIG. 1: CERENKOV SHOCK WAVE GEOMETRY

5.2 Bounded Media

The introduction of perfectly conducting boundaries causes some interesting effects. An observer in the rest system of the medium finds the radiation spectrum discrete, instead of continuous. It is immaterial whether or not the boundary is moving or stationary if it is perfectly conducting.

For the case of a point charge moving along the axis of a perfectly conducting cylinder of radius "b" having an appropriate medium ($n\,\beta > 1$), the solution has three regions, indicated in Fig. 2. In regions I and II, the solution is the same as for the unbounded case; in region III the fields have the behavior

$$\sum_{m=1}^{\infty} \sin\left[\frac{(z' + vt')\, j_m}{b(n^2\beta^2 - 1)^{1/2}}\right] \frac{J_o(\frac{r'}{b} j_m)\, N_o(j_m)}{J_1(j_m)}$$

where $r' = \sqrt{x'^2 + y'^2}$, and j_m is the m-th zero of $J_o(x)$, the Bessel function. Work on the energy distribution was found to have been discussed in detail by Russian authors (Bolotovskii, 1961).

For the corresponding two-dimensional case, that of a line charge moving between two parallel conducting plates, the fields are still restricted to the shock wave front which forms the zig-zag pattern shown in Fig. 3. The frequency spectrum seen by an observer in the rest-frame of the medium is again discrete, the frequencies excited depending on the eigen-values of the parallel-plate geometry.

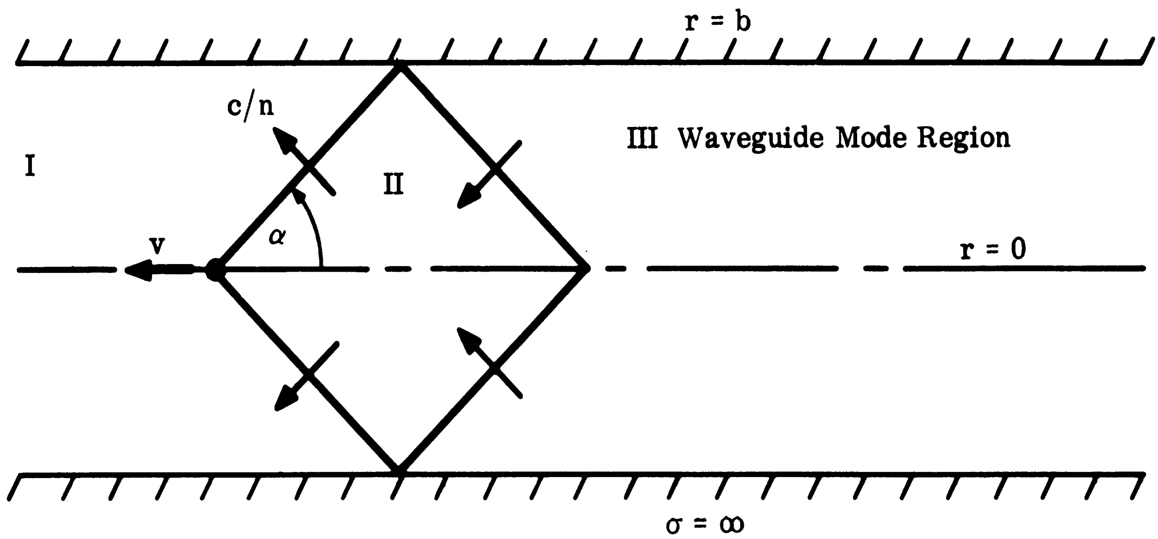

FIG. 2: SHOCK WAVE GEOMETRY, CYLINDRICAL BOUNDARY

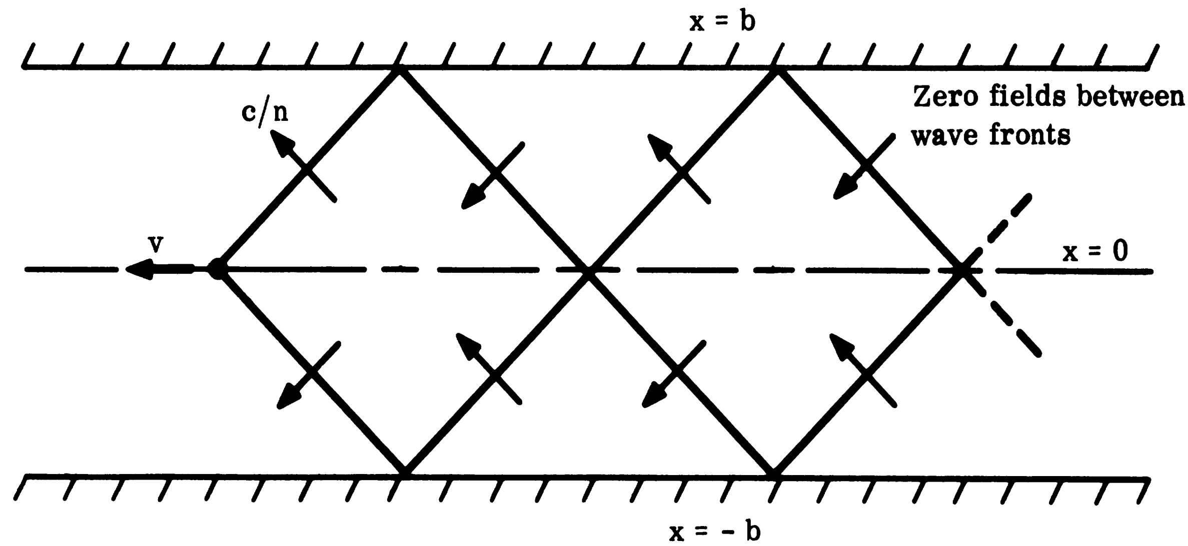

FIG. 3: SHOCK WAVE GEOMETRY, PARALLEL PLATE BOUNDARY

5.3 Unbounded Conducting Media

The case of a point charge moving in a conducting medium was considered in the program, tying in with the general study of conductive, moving media. As one would expect, the introduction of conductivity in the medium does not change the shape of the shock wave. The fields ahead of the shock wave are zero, and behind are attenuated in comparison with the lossless case. The behavior of the vector potential becomes:

$$\frac{e^{-p(z'+vt')}\cosh\left[p\sqrt{(z'+vt')^2-(n^2\beta^2-1)\,r'^2}\right]}{\sqrt{(z'+vt')^2-(n^2\beta^2-1)\,r'^2}}$$

where $p = \dfrac{\sigma' \mu' v}{2\sqrt{n^2\beta^2-1}}$

Note that when the conductivity σ' vanishes, p vanishes, and the behavior becomes that of the lossless case above. The exponentially decaying factor is the type of behavior to be expected from the introduction of losses.

5.4 Dispersion

When dispersion is considered, a convenient quantity to utilize is the amount of radiated energy passing through a small cylinder of length dz'. In symbols, we are interested in the quantity

$$\frac{dW'}{dz'} = 2\pi r' \int_{-\infty}^{\infty} S_r' \, dt'$$

where S_r' is the Poynting vector component in the radial direction, from the point of view of the medium rest system. The expression for the lossless case is, in M.K.S. units (cf. Tamm (1939))

$$\frac{dW'}{dz'} = \frac{q^2}{8\pi} \int_{n\beta>1} \omega \mu' \left(1 - \frac{1}{n^2 \beta^2}\right) d\omega$$

where the integration is over only those (positive) frequencies for which $n\beta > 1$. For conducting media one would not expect this quantity to be independent of radius, because of losses. Indeed, for frequencies high enough that $\omega >> \frac{v}{r' \sqrt{n^2 \beta^2 - 1}}$ and $\omega >> \frac{\sigma' n^2 \beta^2}{\epsilon'}$, the integrand is modified only by the exponential factor

$$\exp\left[\frac{-\sigma' \mu' v r'}{\sqrt{n^2 \beta^2 - 1}}\right] .$$

The modifying factor is still tractable when the second condition on ω is removed, but there has not yet been found a convenient factor good for all frequencies.

VI

DRIFTING, MAGNETO-IONIC MEDIUM - FIRST ORDER THEORY

Tai (1965b) applied Minkowski's theory of moving media to the case of a drifting, magneto-ionic plasma, where the drift velocity v is small compared to the velocity of light in vacuum c. This involves an eighth-order set of equations for the index of refraction n, given by

$$\det \bar{\bar{A}} = 0 .$$

where the components of $\bar{\bar{A}}$ are the following:

$$A_{xx} = (n^2 - 1)(1 - n\beta_z - jZ) + (1 - n\beta_z) X$$

$$A_{xy} = -j(n^2 - 1) Y_z$$

$$A_{xz} = (1 - n\beta_z - jZ) n\beta_x - jn\beta_y Y_z - j(1 - n\beta_z) Y_y$$

$$A_{yx} = j(n^2 - 1) Y_z$$

$$A_{yy} = (n^2 - 1)(1 - n\beta_z - jZ) + (1 - n\beta_z) X$$

$$A_{yz} = (1 - n\beta_z - jZ) n\beta_z + jn\beta_x Y_z + j(1 - n\beta_z) Y_x$$

$$A_{zx} = -j(n^2 - 1) Y_y + n\beta_x X$$

$$A_{zy} = j(n^2 - 1) Y_x + n\beta_y X$$

$$A_{zz} = -(1 - n\beta_z)(1 - n\beta_z - jZ) - jn\beta_x Y_y + jn\beta_y Y_x + X$$

where

$\bar{\beta} = \bar{v} / c$

$\bar{v}$ = drift velocity of the medium

$$X = \frac{N q_e^2}{m_e \omega^2 \epsilon_o} = \left(\frac{\omega_p}{\omega}\right)^2$$

ω_p = plasma frequency

$$Z = \frac{\omega_c}{\omega}$$

ω_c = collision frequency

$$\bar{Y} = \frac{\mu_o |q_e| \bar{H}_o}{m_e \omega} = \frac{\bar{\omega}_m}{\omega}$$

ω_m = gyromagnetic frequency .

For the stationary magneto-ionic plasma, there are four possible values of the index of refraction: two "ordinary" waves and two "extraordinary" waves. When the medium moves as well, there are in general eight values, or modes, possible.

Making use of the assumption that $v << c$, the index of refraction was found for a variety of orientations of magnetic field, drift velocity, and direction of propagation. These are presented below for a wave of the form

$$e^{j\omega (t - \frac{n}{c} z)}$$

1. Propagation, drift, magnetic field parallel: $\overline{Y} = Y\hat{z}$, $\overline{\beta} = \beta\hat{z}$

$$n_{1,2} = \pm \sqrt{1 - \frac{X}{1 + Y - jZ}} + \beta \frac{X}{2} \frac{(Y - jZ)}{(1 + Y - jZ)^2}$$

$$n_{3,4} = \pm \sqrt{1 - \frac{X}{1 - Y - jZ}} - \beta \frac{X}{2} \frac{(Y + jZ)}{(1 - Y - jZ)^2}$$

$$n_{5,6} = \frac{1 \pm Y - jZ}{\beta}$$

$$n_{7,8} = \frac{1 \pm \sqrt{X}}{\beta}$$

2. Drift and magnetic field parallel, transverse to propagation:

$\overline{Y} = Y\hat{x}$, $\overline{\beta} = \beta\hat{x}$

$$n_{1,2} = \pm \sqrt{1 - \frac{X}{1 - jZ}}$$

$$n_{3,4} = \pm \sqrt{1 + \frac{X(1 - jZ - X)}{Y^2 - (1 - jZ)(1 - jZ - X)}}$$

(no change from the stationary medium case)

3. Propagation, drift, and magnetic field mutually orthogonal:

$$\overline{Y} = Y\hat{x} \quad , \quad \overline{\beta} = \beta\hat{y}$$

$$n_{1,2} = \pm\sqrt{1 - \frac{X}{1 - jZ}}$$

$$n_{3,4} = \pm\sqrt{1 + \frac{(X(1-jZ+X)}{Y^2 - (1-jZ)(1-jZ+X)}} - \frac{j\beta}{2}\,\frac{XY(1-jZ)(1-jZ-X)}{\left[Y^2 - (1-jZ)(1-jZ-X)\right]^2}$$

$$n_5 = \frac{j}{\beta}\left[\frac{Y}{1-jZ} - \frac{(1-jZ-X)}{Y}\right].$$

The effect of drift is a shift of the order of v / c in the ordinary and extraordinary wave indices, and the appearance of totally new modes. These new modes correspond to very slow waves having velocities on the order of the drift velocity. These are rapidly attenuated for most frequencies and plasma parameters encountered. The extent to which these modes can be excited would be the subject of a boundary-value problem. M. Epstein (1962) reported some initially pessimistic results on this matter for the all-parallel case.

VII

THE TIME-DEPENDENT DYADIC GREEN'S FUNCTION FOR A MOVING ISOTROPIC MEDIUM

The dyadic Green's function for a harmonically oscillating field in a moving isotropic medium was previously derived by means of an operational method (Tai, 1965a). There it was shown that the associated scalar Green's function is, for a particular frequency ω :

Case 1: $n\beta < 1$

$$g_\omega(\overline{R}\,|\,\overline{R}_o) = \frac{a^{1/2}\, e^{-j\omega\left[\frac{n a^{1/2}}{c} R_a - \Omega\xi\right]}}{4\pi R_a}$$

where $R_a = (\rho^2 + a\,\xi^2)^{1/2}$

$$\rho^2 = (x - x_o)^2 + (y - y_o)^2$$

$$\xi = z - z_o$$

$$a = \frac{1-\beta^2}{1-n^2\beta^2}\,, \qquad \Omega = \frac{(n^2-1)}{(1-n^2\beta^2)}\,\frac{\beta}{c}$$

$$\beta = v/c\,, \quad n^2 = \frac{\mu'\epsilon'}{\mu_o\epsilon_o} = \mu'\epsilon' c^2$$

v is the velocity of the medium

Case 2: $n\beta > 1$ (Cerenkov condition)

$$g_\omega(\overline{R}|\overline{R}_o) = \begin{cases} 0, & \xi|a|^{1/2} < \rho \\ \dfrac{|a|^{1/2}\, e^{j\omega\Omega\xi} \cos\left[\dfrac{\omega n}{c}|a|^{1/2} R_a'\right]}{4\pi R_a'}, & \xi|a|^{1/2} > \rho \end{cases}$$

where

$$R_a' = (|a|\,\xi^2 - \rho^2)^{1/2}$$

$$|a| = \frac{1-\beta^2}{n^2\beta^2 - 1}$$

The time-dependent case has recently been treated by Compton (1966). There he derived the following expression for $G_t(\overline{R}|\overline{R}_o)$:

$$G_t(\overline{R}|\overline{R}_o) = -\frac{1}{4\pi}\,\frac{\sqrt{1-(\beta/n)^2}}{1-\beta^2}\,\frac{1}{R_b}\,\delta\left(\tau - \sqrt{\frac{1-(\beta/n)^2}{1-\beta^2}}\,\frac{n}{c}R_b\right)$$

where

$$R_b = \left[\rho^2 + \frac{1-(\beta/n)^2}{1-\beta^2}\left(\xi - \frac{n^2-1}{n^2-\beta^2}\,v\,\tau\right)^2\right]^{1/2}$$

and $\tau = t - t_o$.

It can be shown that the complicated analysis involved in Compton's work can be avoided by taking the Fourier integral of the harmonic solution $\left(g_{\omega}(\bar{R}\,|\,\bar{R}_o)\right)$ with respect to the angular frequency ω. A note comparing these two derivations will be submitted for publication in the near future.

VIII

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