Inverse Scattering Investigation

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I

INTRODUCTION

The problem in question consists of determining means of solving the inverse scattering problem where the transmitted field is given, and the received fields are measured and this data used to discover the nature of the target. First, the set or sets of necessary and sufficient measurements required to specify the size and shape of the target are to be determined, followed by a study of the degradation of the size and shape determinations due to a reduction in the number of necessary and sufficient conditions.

II

REVIEW OF INVERSE SCATTERING TECHNIQUES

There seems to be a variety of problems that are named "The inverse scattering problem" and what follows is certainly not a complete account of the existing literature on the subject.

Connected with the inverse scattering problems is the question of to what extent knowledge of the far-field from sources of finite extension determines the distribution of the same sources. The relationship has been investigated by Müller (1954, 1956) for the scalar and vector case respectively. The results are that the far-field determines the radius of the smallest sphere such that the far-field can be generated by sources all of which are inside the sphere. Furthermore, if the far-field vanishes the total field is identically zero outside every region, such that it contains all sources. These results also follow from an expansion theorem given by Wilcox (1956). The statements are also true in two dimensions where an expansion theorem is due to Karp (1961). It should be noted that the smallest sphere mentioned above does not necessarily determine the extension of the real sources. For instance, if the sources are distributed over a certain volume in such a manner that the far-field can be expanded in a finite number of surface harmonics, an identical far-field can be obtained from a number of mulipoles, i.e. from sources inside an infinitesimal sphere around the origin.

Some acoustic and electromagnetic scattering problems can be formulated in terms of the Schrödinger equation. A group of one-dimensional problems has been treated by Moses and deRidder (1963), and a three-dimensional scalar problem by Kay (1962). The physical problem considered by Kay is to find the variation of electron density in a weakly ionized gas from a knowledge of the scattering amplitude resulting from the incidence of a plane electromagnetic wave. However, his results seem to be applicable to scalar scattering by a plane wave from an arbitrary isotropic body. Instead of the ordinary time-independent Schrödinger equation, Kay considers the integro-differential equation

$$\Delta u + k^2 u - \int v(\underline{x}, \underline{x}') u(\underline{x}', \underline{k}) d\underline{x}' = 0 \qquad (1)$$

which takes the form of the ordinary Schrödinger equation, if v is a distribution of the form $v(\underline{x}, \underline{x}') = V(\underline{x})\,\delta(\underline{x}-\underline{x}')$. He required knowledge of the scattering amplitude over a hemisphere from a wave incoming from the same half-space in an arbitrary direction for all values of k to determine the function $v(\underline{x}, \underline{x}')$. The question of uniqueness and existence of $v(\underline{x}, \underline{x}')$ under any general condition is not touched upon. Instead, a particular condition on the solution $u(\underline{x}, \underline{k})$ is introduced which leads to a unique $v(\underline{x}, \underline{x}')$.

An extensive bibliography, to the date of publication, of the quantum mechanics inverse scattering problem is given by Faddeyev (1963).

A three-dimensional scalar problem is also treated by Petrina (1960). The scattering body is there assumed to be homogeneous and isotropic so the Helmholtz equation with a wave number k_1 is satisfied inside the body and the same equation with a wave number k_0 is satisfied outside the body. Petrina gives the following relation between the scattering amplitude and the shape of the scattering body.

$$\left. \frac{\partial f(k_0, k_1, \underline{\tau})}{\partial (k_0^2)} \right|_{k_0^2 = k_1^2} = -\frac{1}{4\pi} \int_B e^{i \underline{\tau} \cdot \underline{y}} \, d\underline{y} \quad . \qquad (2)$$

The integration is to be performed over the volume of the scattering body and

$$\underline{\tau} = \underline{k_0} - \frac{k_0}{|\underline{x}|} \underline{x} \ ,$$

where $\underline{k_0}$ is the wave vector of the incident plane wave and $\underline{x}$ is in the direction of observation. The integral on the right hand side of Eq. (2) can be considered as the Fourier transform of a function which takes the value 1 inside the scatterer and vanishes outside. Thus, knowledge of the left hand side for all $\underline{\tau}$ determines the shape of the scatterer. However, this means determination of the behavior of the scattering amplitude when the wave constant of the surrounding medium is changed, which is not measurable in a physical situation.

Some results for two-dimensional acoustically soft or hard obstacles is given by Karp (1961). He forms determinants whose elements are f_{ij} where $f_{ij}=f(\theta_i, \theta_j)$ is the scattering amplitude at an angle of observation θ_i for an angle of incidence θ_j of the plane incoming wave. Necessary and sufficient conditions for a point to be on the surface of the scatterer is thus derived for the special case that $\det(f_{ij})$ vanishes, where $\theta_i=\theta_1, \theta_2, \ldots \theta_n$ are n different angles. Furthermore, it is shown that if $f(\theta, \theta_0)$ only depends on the difference $\theta-\theta_0$, the scatterer must be a circle.

The inverse scattering problem in geometrical optics has been investigated by Keller (1959). If the scattering amplitude and reflection coefficient are known, explicit formulas determining the illuminated part of the surface can be obtained for two dimensional problems. In the three-dimensional case the bistatic radar cross section is proportional to the reflection coefficient and the product of the principal radii of curvature R_1, R_2 at the point of reflection. The problem of determining a surface when its Gaussian curvature, $G= 1/(R_1R_2)$, for all directions of the normal to the surface is given, is known as Minkowski's problem. It has a unique solution for any sufficiently smooth convex body (c.f. Nirenberg (1953)). If the differential scattering cross section is known for two different incident waves coming from opposite directions and the reflection coefficient is also known, the Gaussian curvature is determined everywhere and the inverse problem has a unique solution. It also follows that the problem of determining the shape of the scatterer from knowledge of the backscattering cross section in all directions has a unique solution in the geometrical optics formulation for a smooth convex body.

The geometric optics method no longer applies of the scattering body has any section where one principal radius of curvature is infinite. For a body of revolution where this is the case or where the radius of the cross section varies slowly along the axis of revolution an approximate method due to Blasberg is described in Altman et al (1964). Using the physical optics approximation the backscattered far field is shown to be proportional to the Fourier transform of the function $r(x)e^{-ikr(x)}$ at the point $k\sin\theta$, where $r(x)$ is the radius of the cross section as a function of a coordinate x along the axis of revolution. The relation is valid for small values of θ where $\pi/2-\theta$ is the angle between the direction of propagation of the incident plane wave and the axis of revolution of the body. Consequently, if a substantial part of the backscattering is confined to small angles, the inverse Fourier transform of the scattered far field with respect to $d=k\sin\theta$ integrated over $\theta=-\pi/2$ to $\theta=\pi/2$ will be a function which is close to $r(x)e^{-ikr(x)}$ for x values inside the body and close to zero for points outside. According to Brindley (1965) the Blasberg approximation has been successfully used to determine the shape of objects from empiric data.

Another theory of high frequency scattering is employed by Freedman (1963). There the incoming wave consists of a modulated pulse and the scattered field in an arbitrary direction in the lit region is a superposition of pulses of the same form. Each discontinuity in

$$\frac{d^nA(R)}{dR^n} \quad (n=0, 1, 2, \ldots)$$

where A(R) is the projection towards the transmitter of those parts of the scatterer which are within distance R, is assumed to generate a component towards the scattered signal. The magnitude of each scattering component is proportional to the size of its generating discontinuity. A more sophisticated treatment of the impulse response from a finite object is given by Kennaugh and Moffatt (1965).

III

THE EQUIVALENT SOURCES OF THE SCATTERED FIELD AND THEIR RELATIONSHIP TO THE SCATTERING BODY

The scattered field can be thought of as arising from an equivalent set of sources located in the interior or on the surface of the scattering body. Their precise location is not determined as yet. The question then arises as to how much can be inferred about the possible equivalent source distributions if the complete radiation pattern of the scattered field is known (i.e. both phase and amplitude) for a fixed frequency.

The far-zone scattered field will be represented in the form

$$\underline{E} \underset{r\to\infty}{\sim} e^{ikr}/r \; \underline{E}_0(\theta, \phi) \tag{3}$$

where (r, θ, ϕ) is a spherical polar coordinate system, with the origin in the vicinity of, or in the interior of, the body. Using the notation that $\hat{\underline{i}}_r, \hat{\underline{i}}_\theta, \hat{\underline{i}}_\phi$ are unit vectors associated with the coordinate system, then

$$\underline{E}_0 = \hat{\underline{i}}_\theta \, E_0^2 + \hat{\underline{i}}_\phi \, E_0^3 \quad . \tag{4}$$

From Wilcox (1956) it is seen that the fields can be uniquely determined in the region outside the smallest sphere enclosing a set of equivalent sources. To be more precise, let r=c by the radius of the smallest sphere (with the center at the origin of the coordinate system) enclosing a set of sources. The $\underline{E}$ can be expressed in the form

$$\underline{E} = e^{ikr}/r \sum_{n=0}^{\infty} \frac{\underline{E}_n(\theta, \phi)}{r^n} \, , \qquad r > c + \epsilon \tag{5}$$

where ϵ is an arbitrary small number greater than zero. The higher order coefficients $\underline{E}_n$ are uniquely determined from the following recurrence relations

$$ikE_1^1 = -r\,\underline{\nabla}\cdot\,\underline{E}_0$$

$$2iknE_{n+1}^1 = n(n-1)E_n^1 + DE_n^1 , \qquad n=1\cdot 2\cdot 3$$

$$2iknE_n^2 = n(n-1)E_{n-1}^2 + DE_{n-1}^2 + D_\theta \underline{E}_{n-1} \qquad n=1\cdot 2\cdot 3 \qquad (6)$$

$$2iknE_n^3 = n(n-1)E_{n-1}^3 + DE_{n-1}^3 + D_\phi \underline{E}_{n-1} \qquad n=1\cdot 2\cdot 3$$

where the operators D, D_θ and D_ϕ are given by the relations

$$Df = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2} ,$$

$$D_\theta \underline{F} = \frac{2\partial F^1}{\partial\theta} - \frac{1}{\sin^2\theta}F^2 - \frac{2\cos\theta}{\sin^2\theta}\frac{\partial F^3}{\partial\phi} , \qquad (7)$$

$$D_\phi \underline{F} = \frac{2}{\sin\theta}\frac{\partial F^1}{\partial\phi} + \frac{2\cos\theta}{\sin^2\theta}\frac{\partial F^2}{\partial\phi} - \frac{1}{\sin^2\theta}F^3 .$$

Thus, complete knowledge of the far field pattern implies the determination of the field outside the smallest sphere enclosing an equivalent source.

The next problem is to relate this result to the scattering body. Consider a body composed, at the present, of arbitrary material. Let the exterior surface of the body be given by S. A plane wave of harmonic time dependence exp(-iωt) will be assumed to be incident upon the body generating a scattered field. The scattered field at a point $\underline{x}$ outside the body can be represented in terms of the total field generated on the surface as follows,

$$\underline{E}(\underline{x}) = -\frac{1}{4\pi}\int_S \left[i\omega\mu_0(\underline{n}x\underline{H})\phi + (\underline{n}x\underline{E})x\nabla'\phi + (\underline{n}\cdot\,\underline{E})\nabla'\phi\right] ds' \qquad (8)$$

where $\underline{n}$ is the unit outward normal to the surface and

$$\phi = e^{ikR}/R \tag{9}$$

with $R = |\underline{x}-\underline{x}'|$, and $\underline{x}'$ being a point of integration on the surface.

Employing the expansion

$$e^{ikR}/R = ik\sum_{n=0}^{\infty} (2n+1)j_n(kr')h_n^{(1)}(kr)P_n(\cos\gamma) \quad , \tag{10}$$

one can derive the following expansion for ϕ in terms of r, where r is the distance from the origin to the point $\underline{x}$,

$$\phi = \frac{e^{ikr}}{r} \sum_{n=0}^{\infty} \frac{A_n}{r^n} \tag{11}$$

where

$$A_n = \frac{1}{(2k)^n(n)!} \sum_{p=0}^{\infty} (2n+2p+1)(-i)^p \frac{\Gamma(2n+p+1)}{\Gamma(p+1)} j_{n+p}(kr')P_{n+p}(\cos\gamma) \tag{12}$$

with

$$\cos\gamma = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\phi-\phi') \tag{13}$$

The angle γ is the angle between the two vectors $\underline{x}$ and $\underline{x}'$.

In particular it can be shown that

$$A_0 = \exp\left[-ikr'\cos\gamma\right] \tag{14}$$

and

$$2iknA_n = \left[n(n-1) + D\right] A_{n-1} \tag{15}$$

where D is the differential operator defined previously Eq. (7). Investigating the behavior of A_n for $n\to\infty$, one can show that the expansion given by Eq.(11), is uniformly convergent for $r>r'$. From the following relationship

$$\underline{\nabla}' \phi = -\underline{\nabla}\phi \tag{16}$$

where the prime indicates differentiation with respect to the source variable $\underline{x}'$, one obtains

$$\underline{\nabla}' \phi = \frac{e^{ikr}}{r} \sum_{n=0}^{\infty} \frac{\underline{B}_n}{r^n} \tag{17}$$

where

$$\underline{B}_n = \hat{\underline{i}}_r \left[-ikA_n + rA_{n-1}\right]$$

$$-\hat{\underline{i}}_\theta \frac{\partial A_{n-1}}{\partial \theta} \tag{18}$$

$$-\hat{\underline{i}}_\phi \frac{1}{\sin\theta} \frac{\partial A_{n-1}}{\partial \phi}$$

Interchanging the order of summation and integration it follows then, that expression (8) can be represented in the form

$$\underline{E}(\underline{x}) = \frac{e^{ikr}}{r} \sum_{n=0}^{\infty} \frac{\underline{E}_n}{r^n} \tag{19}$$

with

$$\underline{E}_n = -\frac{1}{4\pi} \int_S \left[i\omega\mu_o (\underline{n} \times \underline{H}) A_n + (\underline{n} \times \underline{E}) \times \underline{B}_n + (\underline{n} \cdot \underline{E}) \underline{B}_n\right] ds' \tag{20}$$

provided that $r > r'$, for all values of r' associated with the points of integration. This means that Wilcox's expansion is uniformly convergent outside the minimum sphere enclosing the body, and represents the scattered field <u>only</u> outside this sphere.

However, it is possible that expansion (19) may be uniformly convergent part way inside the minimum sphere enclosing the body, in which case it will represent there, the field produced by an equivalent source. The radius of convergence of expansion (19) determines the radius of the minimum sphere enclosing an equivalent source. If the scattered field is singular on sufficient portions of the surface of the body, then the radius of convergence of the expansion in this case will determine the radius of the minimum sphere enclosing the body. The fundamental question between the shape and material characteristics of the scattering body and the equivalent source distribution remains to be considered. What is required is the relationship between the radius of the minimum sphere enclosing an equivalent sources (determining the radius of convergence of expansion (19)), and the properties of the scattering body. This is being investigated.

It was shown that for a fixed origin, Wilcox's expansion represented the scattered field only outside the minimum sphere (with center at the origin) enclosing the body. By changing the origin of the coordinate system, one will get a new minimum sphere enclosing the body, outside of which Wilcox's expansion will represent the scattered field. Thus by considering a sequence of translations of the origin, a sequence of minimum spheres enclosing the body will be obtained, such that the envelope will be a convex shape enclosing the body. Thus on considering the sequence of minimum sphere, Wilcox's expansion will give the scattered field outside the convex envelope enclosing the body. Thus only for convex scattering shapes, can one obtain by Wilcox's expansion, the scattered field everywhere outside the body.

Since the expansion may be convergent in the interior of the convex envelope, additional information will be needed to determine the shape of the scattering body. For perfectly conducting bodies, one would have to look for the surface for which, the sum of the tangential components of the scattered and incident fields vanishes. A similar technique could be used for bodies whose material properties could be represented in terms of an impedance boundary condition of the form

$$\underline{E} - (\underline{E} \cdot \underline{n})\underline{n} = \eta \sqrt{\frac{\mu_o}{\epsilon_o}}\ \underline{n} \times \underline{H} \tag{21}$$

where $\underline{n}$ is the unit outward normal to the surface of the body, and η is a parameter depending upon material characteristics.

Thus it is seen that if the body is convex, its shape can be determined under the assumption of perfect conductivity, or impedance type boundary condition. The question of uniqueness has to be considered.

IV
THE CONICAL HARMONICS AND ALTERNATIVE APPROACH

For the case where the radius of convergence of expansion (19) is given by the radius of the minimum sphere enclosing the body, the question comes up as to how one can proceed beyond the mimimum sphere to portions of the scattering surface inside. As is seen from the set of recurrance relations (6), if the scattered field is known completely (phase and amplitude), only in some angular sector (such as $\theta_o < \theta < \theta_1$, $\phi_o < \phi < \phi_1$) one can obtain the scattered field everywhere in a conical region formed by this sector, outside the minimum sphere. The question then arises, can one obtain an analytic continuation of scattered field into the interior region bounded by the conical surface, the surface of the body and the minimum spherical surface, using conical harmonics?

Let C_1 be a sphere centered at the tip of the conical surface, with radius just greater than c min, the radius of concentric minimum sphere enclosing the body. Let C_2 be a concentric sphere with arbitrary radius r which is less or equal to c min. Further restriction will be placed on C_2, namely that it lie outside that portion of the scattering surface enclosed by the conical surface. Let S be the portion of the conical surface between C_1 and C_2. Let Σ_1 and Σ_2 be the portions of the spherical surfaces C_1 and C_2 contained in the conical section (see Fig. 1). Let $\underline{E}$, $\underline{H}$ be the scattered field and $\underline{E}_n$, $\underline{H}_n$ be the field components corresponding to the nth conical mode, the conical modes so chosen that their tangential components of $\underline{E}$ vanish on S and Σ_2. These modes will exhibit orthogonality properties. Using the Lorentz Lemma, it follows that

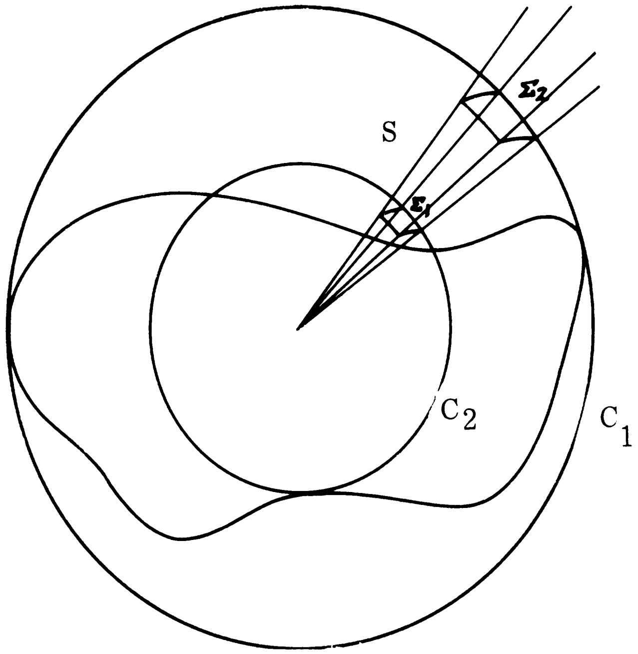

FIGURE 1

$$\int_{\Sigma_1+\Sigma_2+S} \underline{n}\cdot(\underline{E}\times\underline{H}_n - \underline{E}_n\times\underline{H})\, ds = 0$$

$$\text{i.e.} \int_{\Sigma_2} \underline{n}\cdot\underline{E}\times\underline{H}_n\, ds = -\int_S \underline{n}\cdot(\underline{E}\times\underline{H}_n)\, ds + \int_{\Sigma_1} \underline{n}\cdot(\underline{E}_n\times\underline{H}-\underline{E}\times\underline{H}_n)\, ds \tag{22}$$

By letting r vary from c min, we obtain a functional relationship in terms of r. If the right-hand side is known, then the component of $\underline{E}$ tangential to Σ_2 can be determined as a linear combination of the conical modes; and furthermore, by varying r, the coefficients are obtained as a function of r. Unfortunately, only the second integral on the right-hand side of the above equation is known explicitly. Thus, the conical mode approach cannot be used unless the scattered field $\underline{E}$ and $\underline{H}$ are normal to the surface S, in which case the first integral on the right-hand side of the equation vanishes. This requirement that $\underline{E}$ and $\underline{H}$ be normal to the conical surface S is very stringent. However, it can be achieved in the limit of vanishing wavelength, where geometric optics or ray theory approach can be used. If the generators of the conical section are along the rays, then $\underline{E}$, $\underline{H}$ will be normal to the conical surface. Thus, the conical mode approach is useful only in the limiting of vanishing wavelength and, in this case, is equivalent to the ray theory approach.

The above argument and other reasons, which will not be gone into here, suggest an optimum approach to the inverse scattering problem for large composite shapes should be as follows:

(1) From the far-field data, use Wilcox's recurrance relations to obtain the scattered field in the near-zone of the body. In practice, only the first two or three terms in the recurrance relations may be needed, since the scattered field expression will only be required in a region close enough to the body, so that the various components can be separated out as indicated below.

(2) In the region (outside some sphere) for which the scattered field is now known, separate out the various field contributions. In other words, decompose the field in the following form

$$\underline{E} = \sum_n e^{ik\psi_n}\underline{E}_n \qquad \text{where} \quad \underline{\nabla}\psi_n \cdot \underline{E}_n = 0 \quad , \tag{23}$$

and $\underline{E}_n$ is, in most cases, a slowly varying function compared to the rapidly varying phase function ψ_n. Each of the components will arise from various contributors, such as the reflected wave, edge diffracted wave, creeping waves, and spherical waves arising from isolated perturbations, such as antennas, and thus each component will arise from different portions of the scatterer. The surfaces ψ_n correspond to the wave fronts and their orthogonal trajectories, the rays. However, a note of caution should be mentioned, in that there most likely will exist narrow angular sectors, transition regions, where the components may not be too easily separated. These transition regions separate scattering or diffraction regions such as would occur for example between the illuminated and shadow regions of geometric optics.

(3) Identify, if possible, each component as to type of return (i.e. reflected wave, creeping wave). This may be done by tracing back the individual wave points to the caustic surface (the envelope of the rays). The caustic surfaces corresponding to the reflected wave from convex and concave portions of the scatterer, will lie, respectively, inside or outside the body. The caustic surfaces for the creeping waves lie on the surface of the body. Caustic curves on the surface will occur for waves arising from wedge type singularities. The wave arising from a discrete scatterer, such as an antenna, will appear to come from a point.

(4) Depending upon the identification of the particular wave component, various methods may be used to identify the portions of the scatterer from which that wave originates. As an example, the curvature of the wave front associated with the reflected wave arising from a convex section of the surface, coupled with the assumption of perfect conductivity, should specify the convex section of the surface. (This introduces the additional problem of ascertaining the material characteristics of the body. Some techniques that may be used for this will be discussed below.) For the field component arising from an isolated singularity, it may be best to re-employ Wilcox's recurrance relations, where the origin of the coordinate system is taken to be at the scattering center obtained by tracing the rays back.

The above general approach has several advantages, among which are:

(a) Additional information, such as is given by the time dependent far field response, can be used to separate the various contributions to the scattered field directly.

(b) The Bistatic-Monostatic Theorem may be used when information is lacking, to get additional information on the amplitude of the reflected wave component.'

(c) If information of the scattered field is available only over a finite range of bistatic-angles, those portions of the scattering surface that contribute to the far zone scattered field in the observed angular region, can still be obtained.

(d) The technique is such that the degradation due to lack of various types of information with respect to the far-zone scattered field can be easily estimated.

The process indicated above essentially is a means to determine the various scattering centers. Associated with it, one must incorporate techniques to determine the material characteristics of the body. Some preliminary ideas on this subject are given as follows.

Some Means of Determining the Material Characteristics of a Scattering Body.

For a class of perfectly-conducting bodies, the monostatic-bistatic cross-section theorem stated as follows, is well known: In the limit of vanishing wavelength, the bistatic cross-section for transmitter direction $\underline{k}$ and receiver direction $\hat{\underline{n}}_o$ is equal to the monostatic cross-section for the transmitter-receiver direction $\underline{k} + \hat{\underline{n}}_o$ with $\underline{k} \neq \hat{\underline{n}}_o$ for bodies which are sufficiently smooth.

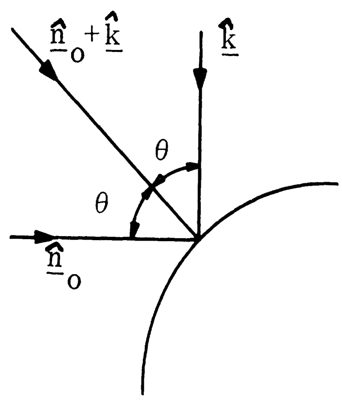

FIGURE 2

This theorem may be amended for non-perfectly conducting bodies, thus yielding information on the material characteristics of the body.

Let 2θ be the angle formed by the vectors $\hat{\underline{n}}_o$ and $\underline{k}$ (i.e., the bistatic angle). Let the surface of the body be sufficiently smooth, and let its electrical properties be characterized by a voltage reflection coefficient R, which is a function of the angle of incidence (α) and polarization; i.e., $R = R_{\parallel}(\alpha)$ for polarization in plane of incidence and $R = R_{\perp}(\alpha)$ for polarization perpendicular to plane of incidence. Denote $\sigma_{\perp}(\underline{k}, \hat{\underline{n}}_o)$ as the bistatic cross section where both transmitting and receiving antennas are linearly polarized perpendicular to the plane formed by the vectors $(\underline{k}, \hat{\underline{n}}_o)$. Let $\sigma_{\parallel}(\underline{k}, \hat{\underline{n}}_o)$ denote the bistatic cross-section where both antennas are polarized parallel to the plane $(\underline{k}, \hat{\underline{n}}_o)$.

The bistatic cross-section $\sigma_{\perp}(\underline{k}, \hat{\underline{n}}_o)$ is a product of two factors, a geometrical factor depending upon the radii of curvature of the surface of the body, and a material factor depending upon the reflection coefficient. However, as implied by the monostatic-bistatic theorem given above, the geometrical factor for $\sigma_{\perp}(\underline{k}, \hat{\underline{n}}_o)$ and $\sigma_{\perp}(\underline{k} + \hat{\underline{n}}_o, \underline{k} + \hat{\underline{n}}_o)$ are the same. Thus, it follows that

$$\frac{\sigma_{\perp}(\underline{k}, \hat{\underline{n}}_o)}{\sigma_{\perp}(\underline{k} + \hat{\underline{n}}_o, \underline{k} + \hat{\underline{n}}_o)} = \left| \frac{R_{\perp}(\theta)}{R_{\perp}(0)} \right|^2 \tag{24}$$

Similarly, it follows that

$$\frac{\sigma_{\parallel}(\underline{k}, \hat{\underline{n}}_o)}{\sigma_{\parallel}(\underline{k} + \hat{\underline{n}}_o, \underline{k} + \hat{\underline{n}}_o)} = \left| \frac{R_{\parallel}(\theta)}{R_{\parallel}(0)} \right|^2 \tag{25}$$

Consider the case where the body is a lossy dielectric (non-magnetic) with relative permittivity given by $\epsilon = \epsilon' + i\epsilon''$. The ratio of the bistatic cross-section (bistatic angle 2θ) and the appropriate monostatic cross-section yield the ratio

$$\left|\frac{R_\perp(\theta)}{R_\perp(0)}\right| = \left|\frac{\cos\theta - \sqrt{\epsilon - \sin^2\theta}}{\cos\theta + \sqrt{\epsilon - \sin^2\theta}}\right|^2 \left|\frac{1+\sqrt{\epsilon}}{1-\sqrt{\epsilon}}\right|^2 = C^4 \tag{26}$$

where C is the constant determined from the measurements of the cross-sections. In particular, if the loss tangent is small $\epsilon'' << \epsilon'$ and $\epsilon' < 1$, then

$$C \sim \frac{\sqrt{\epsilon' - \sin^2\theta} - \cos\theta}{\sqrt{\epsilon'} - 1} \tag{27}$$

yielding

$$\sqrt{\epsilon'} = \frac{1 - 2C\cos\theta + C^2}{C^2 - 1} \tag{28}$$

Thus, the two measurements $\sigma_\perp(\hat{\underline{n}}_o + \underline{k}, \hat{\underline{n}}_o + \underline{k})$ and $\sigma_\perp(\underline{k}, \hat{\underline{n}}_o)$ yield the appropriate value for ϵ' in this case.

In general, it will not be known apriori whether the material composition of the body is non-magnetic or homogeneous. The body may be comprised of a perfectly conducting inner shell, coated with one or more layers of dielectric or magnetic materials, such as would occur with ablative coatings or absorber coatings. Thus, in this case, the bistatic cross-sections will have to be measured at more than one angle. This leads to the problem as follows. Given the ratio of the bistatic cross-section to the monostatic cross-section for a set of bistatic angles (i.e., given $|R_\perp(\theta)/R_\perp(0)|^2$ and $|R_{||}(\theta)/R_{||}(0)|^2$ for a set of θ) how much can be inferred about the material properties of the body. Consideration of this problem is being undertaken.

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