7956-4-T

Air Force Avionics Laboratory
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TRANSMITTER IMPEDANCE CHARACTERISTICS FOR AIRBORNE SPECTRUM SIGNATURE

Interim Technical Report No. 4
1 January - 31 March 1967

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15 April 1967

Contract AF-33(615)-3454

Contract Monitor: K. W. Tomlinson AVWE

7956-4-T = RL-2169

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ABSTRACT

A simple linear model of a transmitter-transmission line - antenna system has been proposed in previous reports. However, experimental evidence has indicated that such a model is not adequate under those conditions where the antenna is not well matched ($<1.1:1$) to the characteristic impedance of the transmission line at the fundamental frequency. A new transmitter model, in which the power outputs at the harmonic frequencies are functions of the load impedance at the fundamental frequency, is proposed. Also included are experimental data to provide a comparison of an actual transmitter with the model. Finally, a new method for collecting harmonic output data is introduced.

I. INTRODUCTION

The statement of problem as set forth in the contract which provides for the present investigation is as follows:

(1) A determination of the power delivered to the antenna for "spectrum signature" purposes will require a measurement of the antenna impedance, transmission line characteristics, transmitter maximum power output, and transmitter output impedance at the fundamental, spurious and harmonic frequencies. The transmitter output impedance at the spurious and harmonic frequencies is not well understood and, therefore, requires further study. The prime payoff in this study will be better "spectrum signatures" for more accurate predictions of interference between systems.

(2) There is a requirement to verify the results of the earlier successful program, Contract AF 33(615)-2606 "Simplified Modeling Techniques for Avionic Antenna Pattern Signatures", with a mock-up of an aircraft transmitter system.

(3) The stated objective of the contract is: To conclude the development of "simplified" techniques for determining the RF spectrum signatures of flight vehicle electronics systems. To establish the validity of the techniques by comparing the results of data obtained by the "simplified" techniques with data obtained from tests employing a typical transmitter system in a mock-up.

(4) The present phase of the contract is concerned with developing a technique for the accurate prediction of the power output of a typical transmitter. The realization of such a technique requires a thorough knowledge of a transmitter's output as a function of the parameters most likely to vary in a practical situation at not only the fundamental frequency but the harmonic and spurious frequencies as well.

II. POWER TRANSFER ANALYSIS OF A NON-LINEAR SOURCE

The possibility of evaluating the power transfer characteristics of a transmitter-transmission line - antenna system with linear circuit elements has been introduced in previous reports. However, experiments with a military type RT-178/ARC-27 transceiver have revealed that the transmitter can be adequately represented by a linear model only when the fundamental frequency is terminated in a load that is well matched to the characteristic impedance of the transmission line. There are substantial deviations from a linear behavior when the fundamental is not well matched. Due to the magnitude of the deviations and the likelihood of a mis-matched condition at the fundamental for most systems of practical significance, the simple linear model is not adequate for accurate power transfer predictions without some modification.

2.1 The Development of a Non-Linear Model

The interharmonic effects that have been observed can be explained at least qualitatively in terms of non-linear mixing. For example, suppose that the power transferred to a load in a coaxial system consists of the fundamental component P_1 plus components harmonically related frequencies P_2, P_3, $P_4, \ldots P_n$. If the load is perfectly matched to the source at all frequencies present, all of the power will be absorbed by the load. Suppose now that the load is not perfectly matched. Some portion of each original component will be reflected back to the source. If the source is a vacuum tube amplifier (or any device characterized by a non-linear voltage current relationship) the reflected components will combine with the original components producing a new spectrum consisting of components at all possible sum, difference and harmonically related frequencies. The magnitudes of the new spectrum components will be a function of the circuit parameters and the magnitude and phase of each mixing component.

One attempt to modify the linear model consisted of treating the voltages generated at the harmonic frequencies as a Taylor series expansion of the fundamental frequency voltage appearing across the series generator impedance. Let E_s be the fundamental frequency voltage generated, E_t be the output voltage at the terminals of the series impedance, and E_z be the voltage drop across the series impedance. Then if $\mathcal{E}$ is the sum of the voltages generated by non-linear mixing (see Figs. 2-1 and 2-2).

$$\mathcal{E} = \sum_n A_n \, E_z^{\,n} (\phi) \cos^n (\omega t + \beta) \tag{2.1}$$

where

$\mathcal{E}$ = sum of generated voltages

A_n = coefficient of the n'th Taylor series term

$E_z (\phi) \cos (\omega t + \beta)$ = voltage across non-linear impedance

ω = fundamental radian frequency

β = phase of E_z with respect to E_s

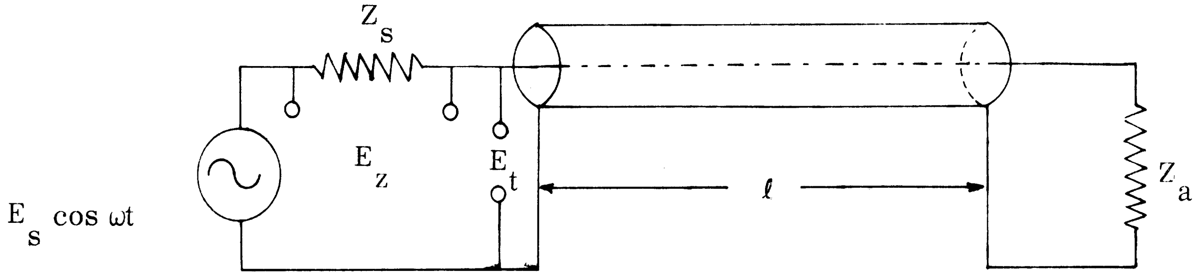

FIG. 2-1: Simple Transmitter Model

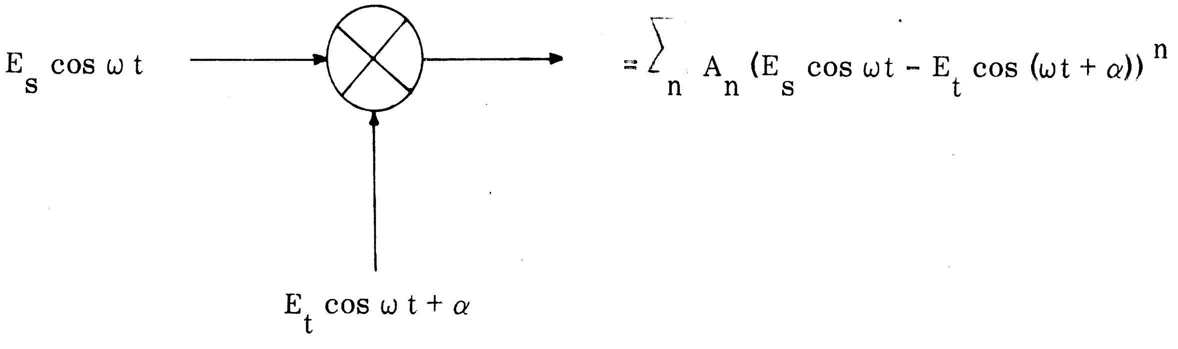

FIG. 2-2: Taylor Series Harmonic Mixer

$\phi = \frac{2\pi \ell}{\lambda g}$, ℓ = length of transmission line.

Expanding this result for the first six terms,

$$\begin{aligned}
\mathcal{E} = &\left(\frac{1}{2} A_2 E_z^{\,2}(\phi) + \frac{3}{8} A_4 E_z^{\,4}(\phi) + \frac{5}{16} A_6 E_z^{\,6}(\phi)\right) \\
&+ \left(\frac{3}{4} A_3 E_z^{\,3}(\phi) + \frac{5}{8} A_5 E_z^{\,5}(\phi)\right)\cos(\omega t + \beta) \\
&+ \left(\frac{1}{2} A_2 E_z^{\,2}(\phi) + \frac{1}{2} A_4 E_z^{\,4}(\phi) + \frac{15}{32} A_6 E_z^{\,6}(\phi)\right)\cos 2(\omega t + \beta) \\
&+ \left(\frac{1}{4} A_3 E_z^{\,3}(\phi) + \frac{5}{16} A_5 E_z^{\,5}(\phi)\right)\cos 3(\omega t + \beta) \\
&+ \left(\frac{1}{8} A_4 E_z^{\,4}(\phi) + \frac{3}{16} A_6 E_z^{\,6}(\phi)\right)\cos 4(\omega t + \beta) \\
&+ \left(\frac{1}{16} A_5 E_z^{\,5}(\phi)\right)\cos 5(\omega t + \beta) \\
&+ \left(\frac{1}{32} A_6 E_z^{\,6}(\phi)\right)\cos 6(\omega t + \beta) \qquad (2.2)
\end{aligned}$$

Considering only the second and third harmonic components and neglecting all terms whose coefficients are A_4 or higher one obtains,

$$\mathcal{E}_2 = \frac{1}{2} A_2 E_z^{\,2}(\phi) \qquad (2.3a)$$

$$\mathcal{E}_3 = \frac{1}{4} A_3 E_z^{\,3}(\phi) \qquad (2.3b)$$

as a first approximation.

Consider $E_z(\phi)$.

$$E_z = E_s \cos \omega t - E_t \cos (\omega t + \alpha) \tag{2.4}$$

However,

$$E_t = \frac{E_s Z_t}{Z_s + Z_t} \tag{2.5a}$$

$$= |E_s| \left\{ \left[\frac{R_t (R_s + R_t) + X_t (X_s + X_t)}{(R_s + R_t)^2 + (X_s + X_t)^2} \right]^2 + \left[\frac{(R_s + R_t) X_t - (X_s + X_t) R_t}{(R_s + R_t)^2 + (X_s + X_t)^2} \right]^2 \right\}^{1/2} \angle \tan^{-1} \frac{(R_s + R_t) X_t - (X_s + X_t(R_t}{(R_s + R_t) R_t + (X_s + X_t) X_t} \tag{2.5b}$$

and

$$Z_t = \frac{R_a + j (X_a + \tan \phi)}{(1 - X_a) + j (R_a \tan \phi)} \tag{2.6}$$

where

$$Z_t = R_t + j X_t \tag{2.7a}$$

$$Z_a = R_a + j X_a \tag{2.7b}$$

2.2 A Comparison of Theoretical and Experimental Results

The difficulty of obtaining solutions for even the simplified forms of (2.3a) and (2.3b) are apparent. For this reason, the equations were solved with the aid of a small digital computer. Results corresponding to a constant mis-match at the fundamental frequency and a matched condition at the harmonic of interest are shown in Figs. 2-3 through 2-6. The data trends are similar for the second harmonic, however, the third harmonic output varies in a more radical fashion than the result predicted by (2.3b). It is well to note that the linear model would, of course, predict no variation of power output, since the load is matched to the transmission line at the frequency of interest.

2.3 Harmonic Rieke Diagrams

It is evident at this point, that the sheer magnitude of the number of frequencies coupled with a less than thorough knowledge of vacuum tube non-linearities has thus far hindered a precise, mathematical analysis of this problem. In order to gain additional insight into the behavior of the power output variations, a new measurement technique has been developed. The power variations at the harmonic frequencies are plotted on the Smith Chart as a function of the load impedance at the fundamental. Any pattern in the behavior will be evident, and will perhaps suggest fruitful approaches to provide a quantitative prediction method. In any case, the diagrams will be a graphic representation of the transmitter's behavior and will provide at least one method of power output predictions. The equipment arrangement utilized for plotting such diagrams is shown schematically in Figure 2-7.

III. CONCLUSIONS

The key to successfully predicting system interference lies in part with being able to successfully predict the radiated spectrum of the components of the system. Thus far, no predise analytical method for predicting the radiated spectrum exists. There are serious discrepancies in both the existing statistical and analytical models. However, the non-linear model presented in this report, while not yet providing an exact solution, is an advance inasmuch as it attempts to describe the inter-dependence of the transmitter parameters, which is vital to a successful analytical model.

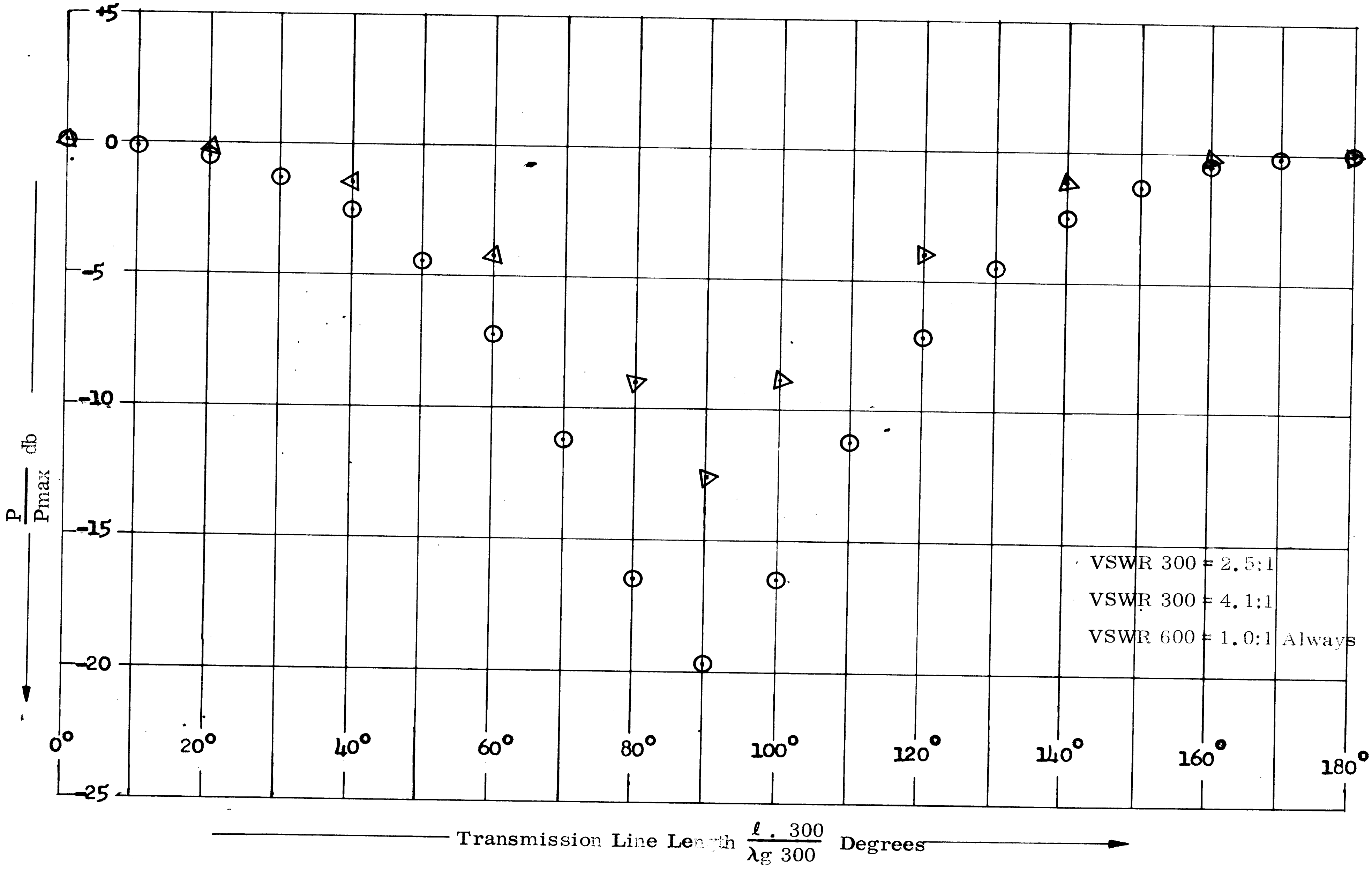

FIG. 2-3: Model Power Variation at the Second Harmonic as a Function of Transmission Line Length.

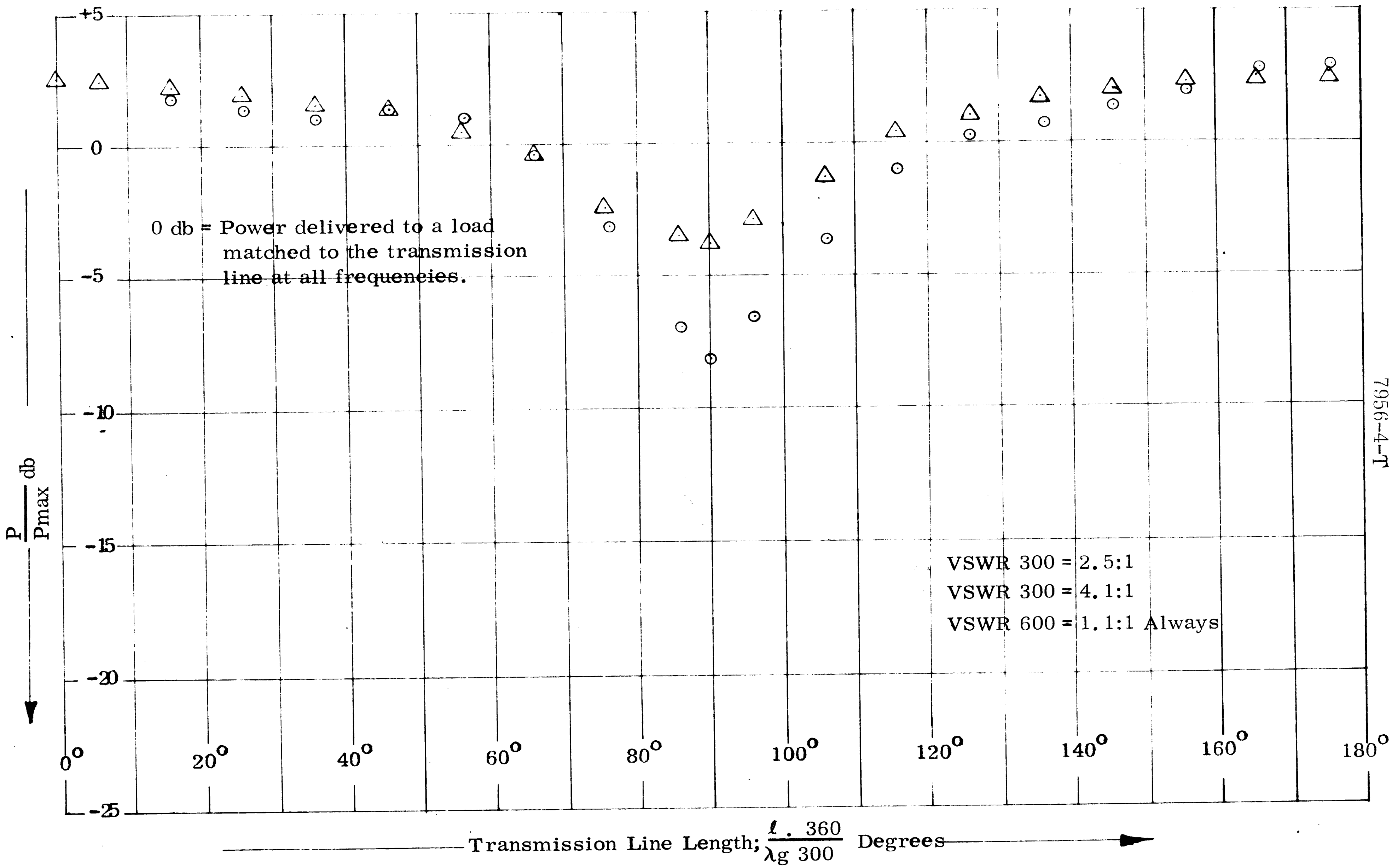

FIG. 2-4: Power Variations at 600 MHz as a Function of the Load Impedance at 300 MHz (fundamental).

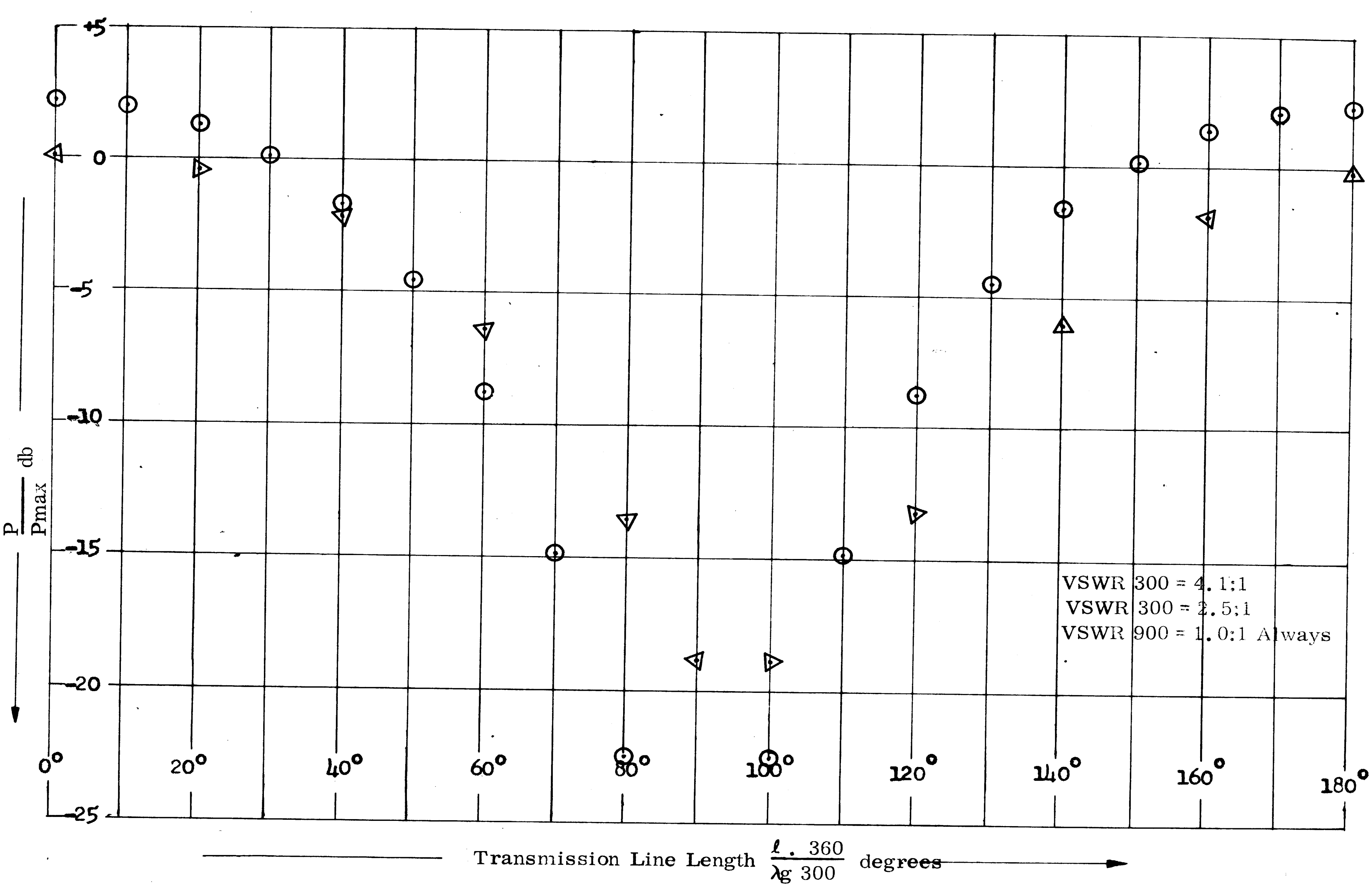

FIG. 2-5: Model Power Variation at the Third Harmonic as a Function of Transmission Line Length.

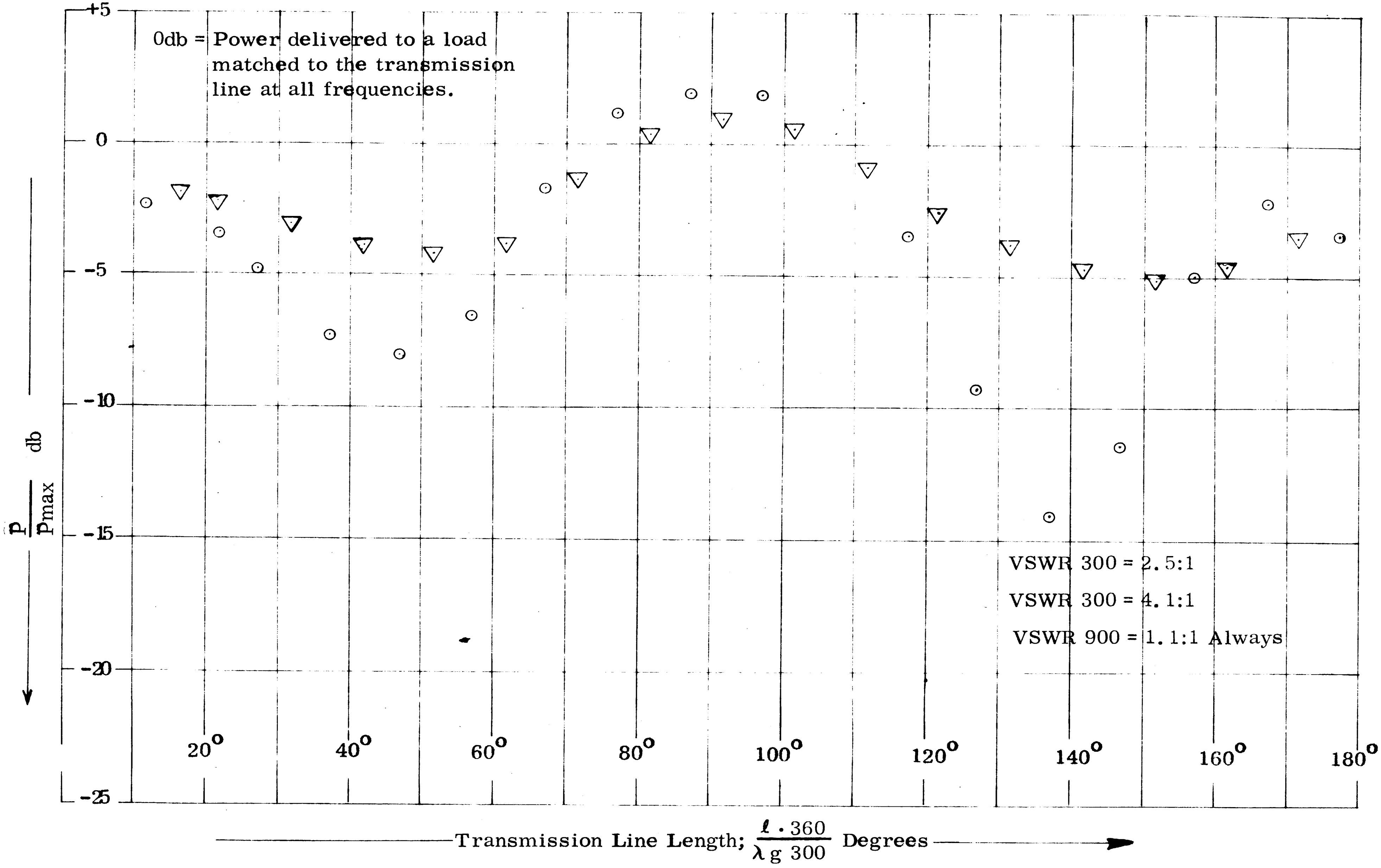

FIG. 2-6: Power Variations at 900 MHz as a Function of the Load Impedance at 300 MHz (fundamental).

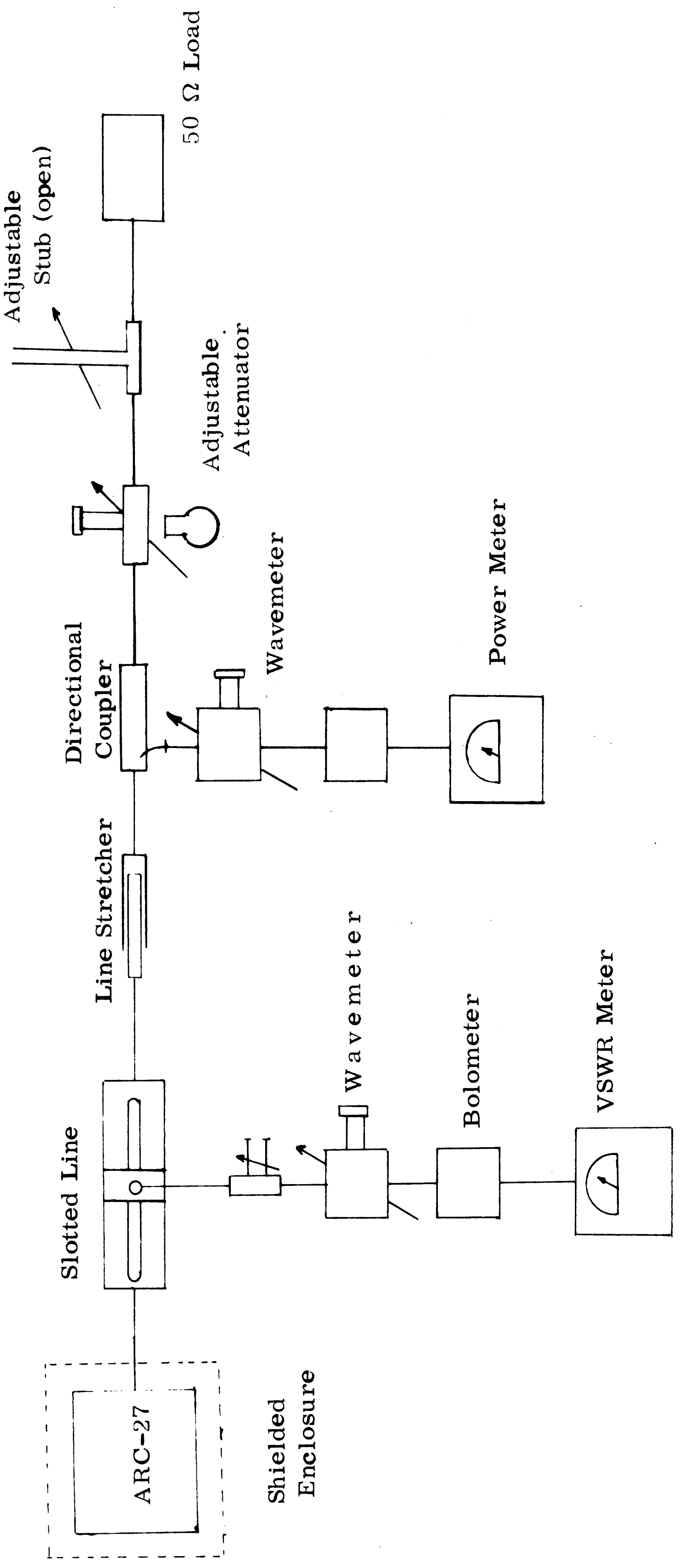

FIG. 2-7: Experimental "Harmonic Rieke Diagram" Equipment Arrangement