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TRANSMITTER IMPEDANCE CHARACTERISTICS FOR AIRBORNE SPECTRUM SIGNATURE

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TABLE OF CONTENTS

ABSTRACT

Simplified expressions for the power transfer through a real (lossy) transmission line have been developed. These expressions are presented in two forms. The first is designed for an exact solution to the power transferred and requires accurate knowledge of the transmitter and source impedances, transmission line length, and transmission line loss. The second form is designed to yield the maximum and minimum power transfer and the probability that the power will exceed some intermediate point. This form requires only the transmitter and source VSWR and approximate transmission line length.

A computer program which solves for the spectrum output of a class C amplifier is presented. This is followed by an equivalent model of a high frequency triode which will allow the program to be extended to this type of device.

I. INTRODUCTION

The statement of problem as set forth in the contract which provides for the present investigation is as follows:

1) A determination of the power delivered to the antenna for "spectrum signature" purposes will require a measurement of the antenna impedance, transmission line characteristics, transmitter maximum power output, and transmitter output impedance at the fundamental, spurious and harmonic frequencies. The transmitter output impedance at the spurious and harmonic frequencies is not well understood and therefore, requires further study. The prime payoff in this study will be better "spectrum signatures" for more accurate predictions of interference between systems.

2) There is a requirement to verify the results of the earlier successful program, Contract AF 33(615)-2606 "simplified Modeling Techniques for Avionic Antenna Pattern Signatures", with a mock-up of an aircraft transmitter system.

3) The stated objective of the contract is: To conclude the development of "simplified"techniques for determining the RF spectrum signatures of flight vehicle electronics systems. To establish the validity of the techniques by comparing the results of data obtained by the "simplified" techniques with data obtained from tests employing a typical transmitter system in a mock-up.

4) The present phase of the contract is concerned with developing a technique for the accurate prediction of the power output of a typical transmitter. The realization of such a technique requires a thorough knowledge of a transmitter's output as a

function of the parameters most likely to vary in a practical situation at not only the fundamental frequency but the harmonic and spurious frequencies as well.

II. POWER TRANSFER THROUGH A LOSSY TRANSMISSION LINE

It has been previously shown that the power transferred from a transmitter to a load is a function of the transmission line length (DeHart, 1966). Expressions have been developed and a graph presented which solve the power transferred in terms of power available from the source, the source VSWR, load VSWR and transmission line electrical length. The solution was however for a lossless transmission line. In many practical situations, the transmission line may introduce substantial losses. The following portion of the text devotes itself to providing a solution to the power transfer problem for those instances when transmission line losses cannot be neglected.

The lossy line problem is not a new one and formulas are presently available in handbooks to handle this situation, however, they generally appear in an unusually cumbersome form. It is possible to simplify the problem considerably without sacrificing accuracy or generality by dividing the problem into two parts. In part 1 we consider the power transfer for line lengths of integer multiples of half wavelengths. In part 2, the line is allowed to assume any length by approximating the entire transmission line with the appropriate number of integral half wavelengths of lossy line connected to a short section $\left(< \frac{\lambda}{2} \right)$ of lossless transmission line.

The characteristic impedance of a transmission line with losses is given by,

$$Z_o = \sqrt{\frac{\bar{R} + j\omega\bar{L}}{\bar{G} + j\omega\bar{C}}} \quad . \tag{2.1}$$

In equation (2.1),

$\bar{R}$ = series resistance of the line per unit length

$\bar{L}$ = series inductance of the line per unit length

$\bar{G}$ = shunt conductance of the line per unit length

$\bar{C}$ = shunt capacitance of the line per unit length .

In the consideration of the loss in transmission lines, it is convenient to assume that the characteristic impedance of the line is real. The exact condition is,

$$\frac{\bar{R}}{\bar{L}} = \frac{\bar{G}}{\bar{C}} \quad . \tag{2.2}$$

The characteristic impedance, Z_o, is then

$$Z_o = R_o + j0 \tag{2.3}$$

and

$$R_o = \sqrt{\bar{L}/\bar{C}} \quad . \tag{2.4}$$

For most radio frequency lines the reactive component of the characteristic impedance will be very small so that equation (2.2) is sufficiently satisfied for acceptable accuracy in the computation of power transfer.

2.1 Power Transfer for Line Lengths of Integer Multiples of Half Wavelengths

An equivalent circuit for the general power transfer problem is shown in Fig. 2-1.

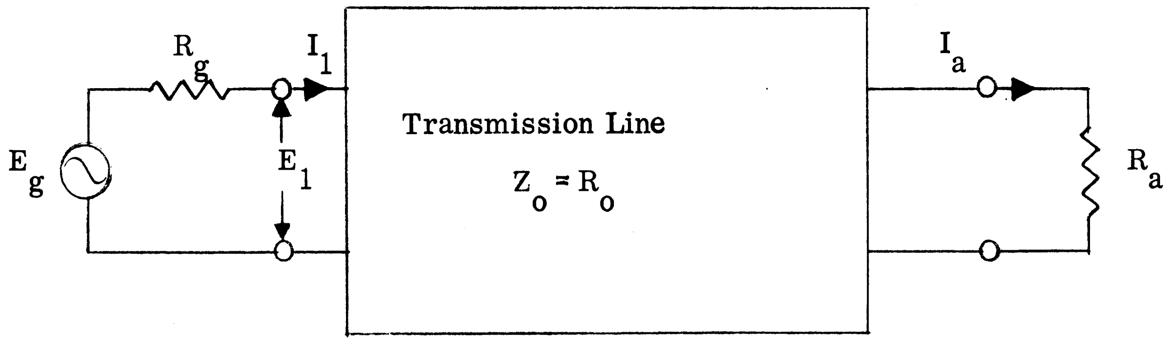

FIG. 2-1: Equivalent Circuit of the Power Transfer Problem

Let

R_a = resistance of the termination of the transmission line

R_o = characteristic impedance of the transmission line

R_g = internal resistance of the equivalent generator

E_g = internal voltage of the equivalent generator

I_a = current into the load resistance

I_1 = current into the transmission line

E_1 = voltage applied to the transmission line .

Note that the generator impedance and the load impedance have been chosen as real. No loss of generality has been sacrificed because an antenna (or generator) can be connected to an appropriate short length of lossless transmission line to transform any complex impedance to a pure resistance. In this discussion the transforming line will be assumed to be of such a length that $R_a > R_o$, $R_g > R_o$. In this case, let

$$\frac{R_a}{R_o} = r_a \tag{2.5}$$

$$\frac{R_g}{R_o} = r_g \tag{2.6}$$

in which r_a and r_g are the standing wave ratios for the antenna and generator impedances.

Consider first the power transfer characteristic of the transmission line alone.

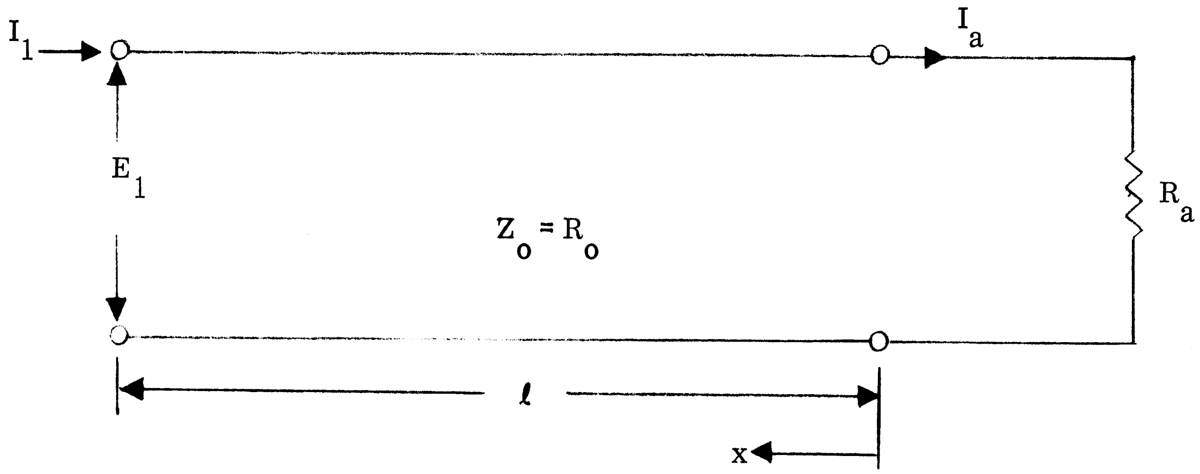

FIG. 2-2: Lossy Transmission Line and Load.

From the transmission line equations

$$I(x) = I_a \left[\cosh(\gamma x) + \frac{Z_a}{Z_o} \sinh(\gamma x) \right] \tag{2.7}$$

$$Z(x) = Z_o \left[\frac{Z_a \cosh(\gamma x) + Z_o \sinh(\gamma x)}{Z_o \cosh(\gamma x) + Z_a \sinh(\gamma x)} \right] . \tag{2.8}$$

In equations (2.7) and (2.8) γ = propagation constant for the transmission line. These equations represent our circuit in Fig. 2-2 for any line length x. If we restrict the values of x that we use to exact multiples of a half wavelength, then

$$\gamma x = \alpha x + j\beta x$$

$$= \alpha x + j\beta n\pi \tag{2.9}$$

and the hyperbolic functions are:

$$\cosh(\alpha x + j\beta n\pi) = \cosh \alpha x \tag{2.10}$$

$$\sinh(\alpha x + j\beta n\pi) = \sinh \alpha x \quad . \tag{2.11}$$

Under this condition equation (2.7) and (2.8) contain no complex quantities. Note that we can come within about 20 inches of the actual line length at 300 MHz without violating that condition. As indicated previously the additional line length necessary can be assumed without loss, though not without a transformer effect. This transformer effect was discussed in a previous paper (DeHart, 1966).

The power into the line at $x = \ell$ is

$$P_1 = I_1^2 R_1 \tag{2.12}$$

and the power delivered to the load resistance is

$$P_a = I_a^2 R_a . \tag{2.13}$$

The power transfer characteristic of the transmission line is the ratio of the power delivered to the load to the power input to the line,

$$\frac{P_a}{P_1} = \frac{I_a^2 R_a}{I_1^2 R_1} . \tag{2.14}$$

For this case $Z_o = R_o$, $Z_a = R_a$, and $\ell = \frac{n\lambda}{2}$ as previously discussed so that (2.7) and (2.8) become

$$Z(\ell) = R_1 = R_o \left[\frac{r_a A + B}{A + r_a B} \right] \tag{2.15}$$

and

$$I(\ell) = I_1 = I_a (A + r_a B) \tag{2.16}$$

where $A = \cosh \alpha x$, $B = \sinh \alpha x$ for convenience. Substituting equations (2.15) and (2.16) into (2.14)

$$\frac{P_a}{P_1} = \frac{r_a}{(r_a A + B)(A + r_a B)} . \qquad (2.17)$$

This result may be further simplified by rewriting the hyperbolic function in terms of exponentials

$$\cosh(\alpha x) = \frac{e^{\alpha x} + e^{-\alpha x}}{2} \qquad (2.18)$$

$$\sinh(\alpha x) = \frac{e^{\alpha x} - e^{-\alpha x}}{2} \qquad (2.19)$$

so that

$$\frac{P_a}{P_1} = \frac{4 r_a A_o / (r_a + 1)^2}{\left(1 - \rho_a^2 A_o^2\right)} \qquad (2.20)$$

where

$$A_o = e^{-2\alpha x} \qquad (2.21)$$

and ρ_a = the magnitude of the load voltage reflection coefficient $= \left(\frac{r_a - 1}{r_a + 1}\right)$.

Note that (2.20) includes the losses due both to line attenuation and load mismatch. When $R_a = R_o$, ρ_a becomes 0, r_a becomes 1 and (2.20) reduces to

$$\frac{P_a}{P_1} = A_o \quad \text{for } R_a = R_o . \tag{2.22}$$

Equation (2.20), while useful, requires that P_1, the power input to the line, be known before P_a can be found. A more practical expression would be one giving P_a in terms of the generator power delivered to a matched termination, (i.e. matched to the characteristic impedance of the transmission line, R_o). Referring to Fig. 2-1, the power input to the transmission line is

$$P_1 = \frac{E_g^{\,2} R_1}{\left(R_g + R_1\right)^2} = \frac{E_g^{\,2} r_1}{R_o\left(r_g + r_1\right)^2} \,. \tag{2.23}$$

If R_1 is replaced by a matched termination, R_o

$$P_o = \frac{E_g^{\,2} R_o}{\left(R_g + R_o\right)^2} = \frac{E_g^{\,2}}{R_o\left(r_g + 1\right)^2} \tag{2.24}$$

and

$$\frac{P_a}{P_o} = \frac{P_a}{P_1} \, \frac{P_1}{P_o} \,. \tag{2.25}$$

From (2.25), in order to solve for $\frac{P_a}{P_o}$ it is necessary only to find $\frac{P_1}{P_o}$ and combine this expression with (2.20).

$$\frac{P_1}{P_o} = \frac{r_1 \left(r_g + 1\right)^2}{\left(r_g + r_1\right)^2} . \tag{2.26}$$

Substituting (2.15) into (2.26) and simplifying one has

$$\frac{P_1}{P_o} = \frac{\left(r_a A + B\right)\left(A + r_a B\right)\left(r_g + 1\right)^2}{\left[A\left(r_g + r_a\right) + B\left(r_a r_g + 1\right)\right]^2} . \tag{2.27}$$

Rewriting in terms of exponentials as before, (2.27) becomes

$$\frac{P_1}{P_o} = \frac{\left(1 + A_o \rho_a\right)\left(1 - A_o \rho_a\right)}{\left[1 - \rho_a \rho_g A_o\right]^2} \tag{2.28}$$

where again

$A_o = e^{-2\alpha x}$, the nominal line attenuation

ρ_a = the magnitude of the load voltage reflection coefficient,

$$\frac{r_a - 1}{r_a + 1}$$

ρ_g = the magnitude of the generator reflection coefficient,

$$\frac{r_g - 1}{r_g + 1} .$$

Setting $R_a = R_o$, ρ_a becomes 0 and

$$\frac{P_1}{P_o} = 1 \quad \text{for} \quad R_a = R_o \; . \tag{2.29}$$

To find the power transferred to the load in terms of load VSWR, generator VSWR transmission line length and generator power into a matched load, it is necessary to combine results (2.28) and (2.21) as indicated by (2.25) such that

$$\frac{P_a}{P_o} = \frac{4\, r_a\, A_o}{\left(r_a + 1\right)^2} \; \frac{1}{\left[1 - \rho_a \rho_g A_o\right]^2} \; , \tag{2.30}$$

or

$$P_a = \frac{4\, P_o\, r_a}{\left(r_a + 1\right)^2} \; \frac{A_o}{\left[1 - \rho_a \rho_g A_o\right]^2} \; . \tag{2.31}$$

It is interesting to consider the exponential factors in equation (2.31). The factor

$$\frac{A_o}{\left[1 - \rho_a \rho_g A_o\right]^2} = \frac{e^{-2\alpha x}}{\left[1 - \rho_a \rho_g e^{-2\alpha x}\right]^2}$$

$$= \frac{e^{2\alpha x}}{\left[e^{2\alpha x} - \rho_a \rho_g\right]^2}$$

varies linearly for changes in $\rho_a \rho_g$. That is, the curve of $\left[e^{2\alpha x} - \rho_a \rho_g\right]$ for

$\rho_a \rho_g = C_1$ is exactly the same curve as for $\rho_a \rho_g = C_2$ only with the origin shifted. Thus, the power transfer through a lossy line varies in the same fashion for changes in line length irrespective of source or generator VSWR.

A second important observation is that thus far the results derived have been valid only for line lengths of integer multiples of half wavelengths. However, R_1 (equation (2.15)) is real for line lengths which are integer multiples of quarter wavelengths. Choosing an odd multiple of a quarter wavelength, is equivalent to solving the same problem with $R_a < R_o$ or $R_g < R_o$. This causes the sign in the $(1-\rho_a \rho_g A_0)$ factor of (2.31) to change resulting in a minimum power transfer. The result is unchanged if both $R_a < R_o$ and $R_g < R_o$.

Figures 2-3 through 2-5 have been prepared to aid in solving (2.30) and (2.31). Letting

$$F_H = 10 \log \left[1 - \rho_a \rho_g A_o\right]^{-2} \quad (\mathrm{db})$$

$$F_\ell = 10 \log \left[1 + \rho_a \rho_g A_o\right]^{-2} \quad (\mathrm{db})$$

A'_o = nominal transmission line loss (db) (loss per foot x no. of feet) = $10 \log A_o$

$$A_1 = 10 \log \rho_a \rho_g \quad (\mathrm{db})$$

$$A_2 = 10 \log \; 4r/(r+1)^2 \; (\mathrm{db}),$$

(2.30) becomes,

$$\frac{P_a}{P_o} = \left(A'_o + A_2 + F_H\right) (\mathrm{db}) \quad \text{for } \ell = \frac{2 n \lambda}{4} \qquad (2.32)$$

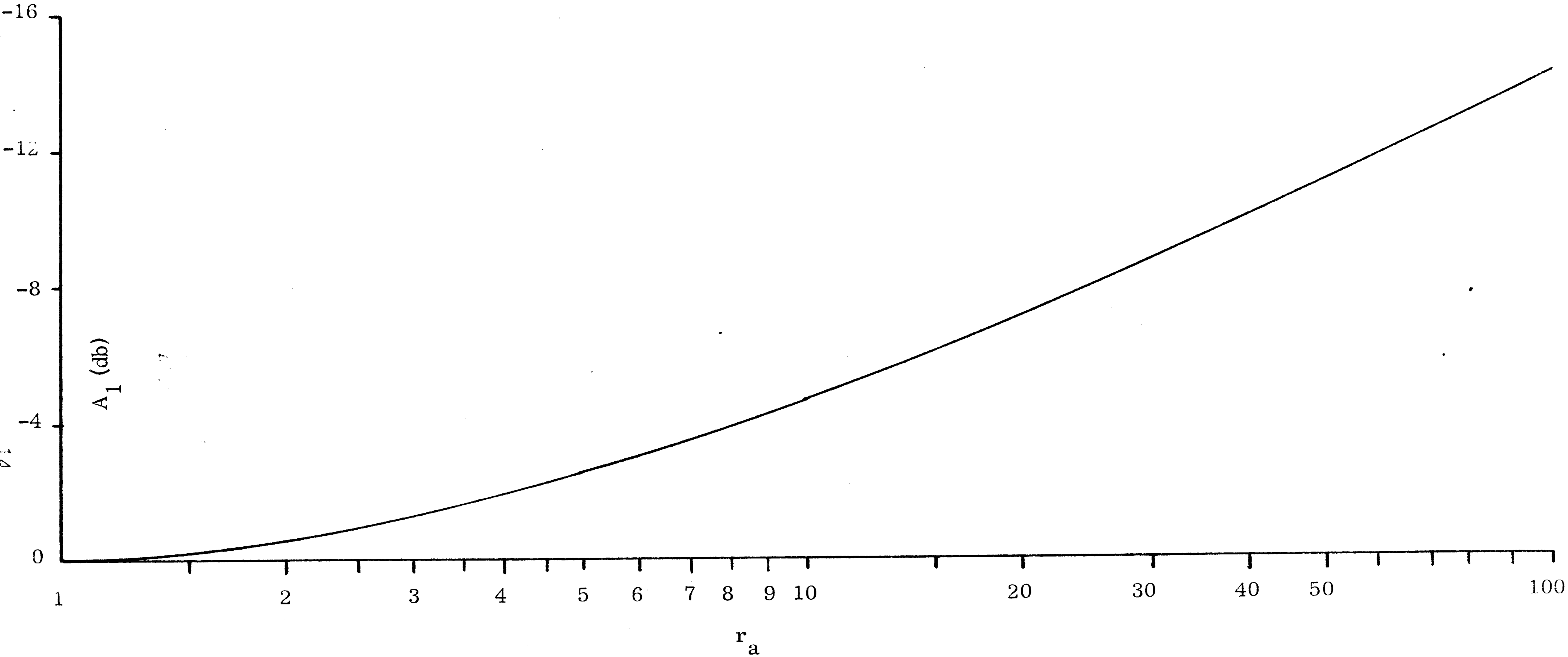

FIG. 2-3: A_1 as a function of r_a.

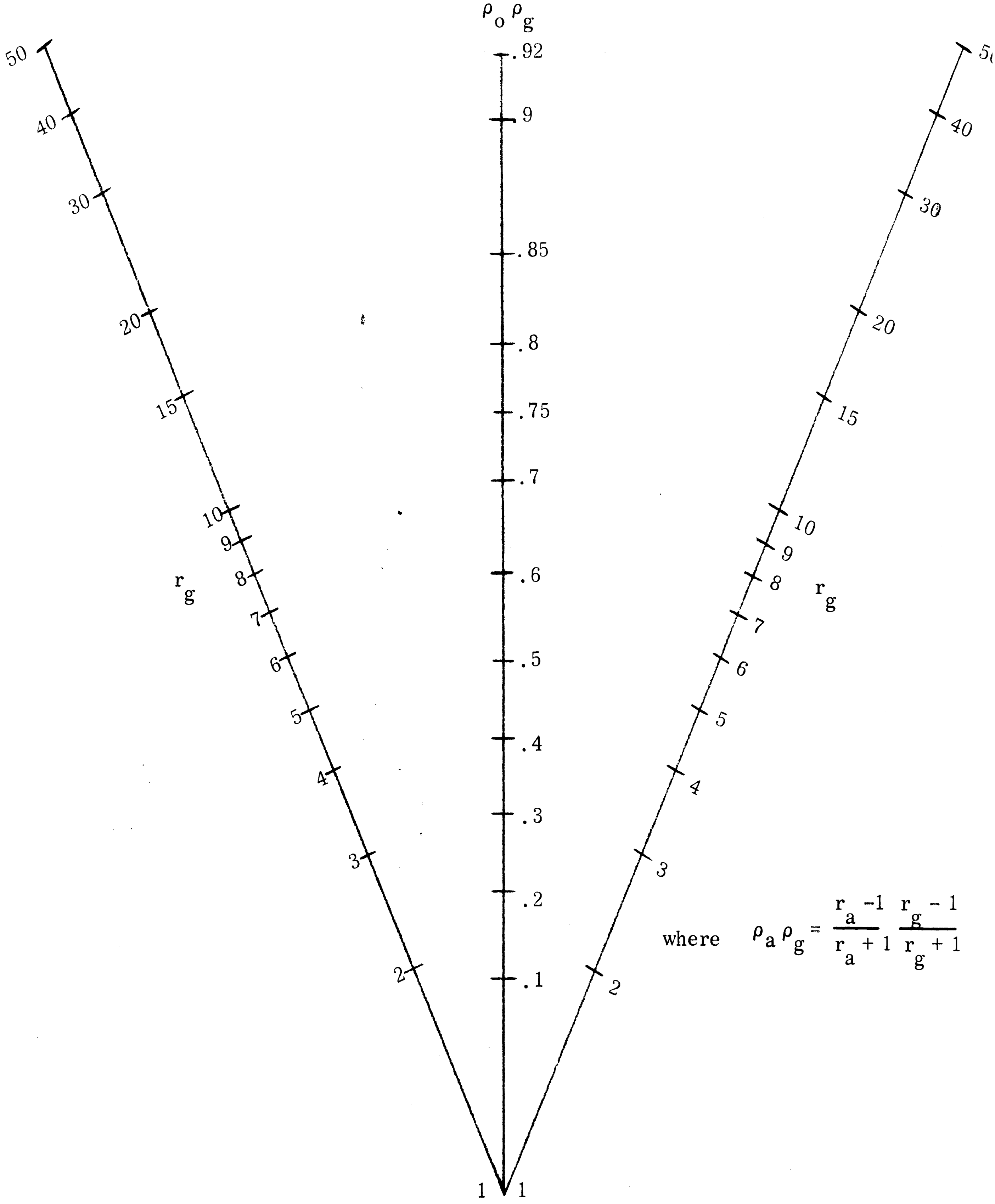

FIG. 2-4: $\rho_a \rho_g$ as a function of r_a and r_g.

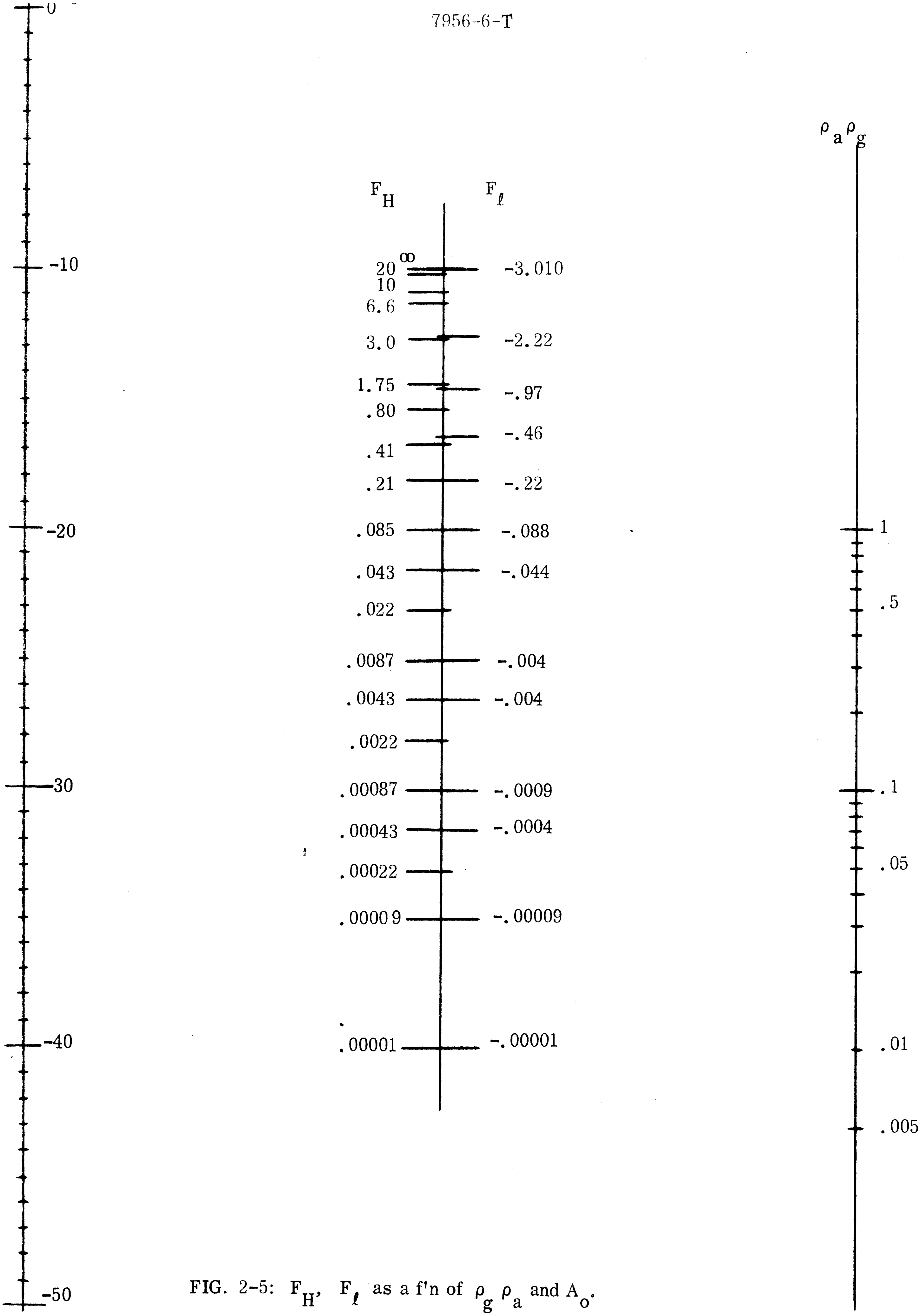

FIG. 2-5: F_H, F_ℓ as a f'n of ρ_g ρ_a and A_o.

$$\frac{P_a}{P_o} = \left(A'_o + A_2 + F_\ell\right)(db) \text{ for } \ell = \frac{(2n-1)\lambda}{4} \qquad (2.33)$$

One may now solve for $\frac{P_a}{P_o}$ as follows:

1) Enter r_a in Fig. 2-3 to find A_1.

2) Enter r_a and r_g in Fig. 2-4 to find $\rho_a \rho_g$.

3) Enter $\rho_a \rho_g$ and A'_o in Fig. 2-5 to find F_H and F_ℓ.

A'_o is given in standard tables of transmission line data. We have now all the necessary information to solve (2.32) and (2.33).

2.2 Power Transfer for any Length Lossy Line

The previous section derived an expression for the power transferred through a lossy line when the line length was an integer multiple of quarter wavelengths and the terminations were real. This section will generalize those results to include all of other physically realizable systems, the only constraint being that the total line length be of the order of magnitude of one wavelength or greater. An equivalent circuit of the generalized problem appears in Fig. 2-6. Referring to Fig. 2-6 the total length of transmission line ℓ_1 is divided into four lengths, ℓ_1, ℓ_2, ℓ_3, and ℓ_4. Length ℓ_1 transforms Z_g to R_g; length ℓ_4 transforms Z_a to R_a; ℓ_2 is the remaining integer number of quarter wavelengths of line, and ℓ_3 is what is left over. Since the maximum length of $(\ell_1 + \ell_4 + \ell_3) = \frac{3\lambda}{8}$ these lengths will be assumed lossless.

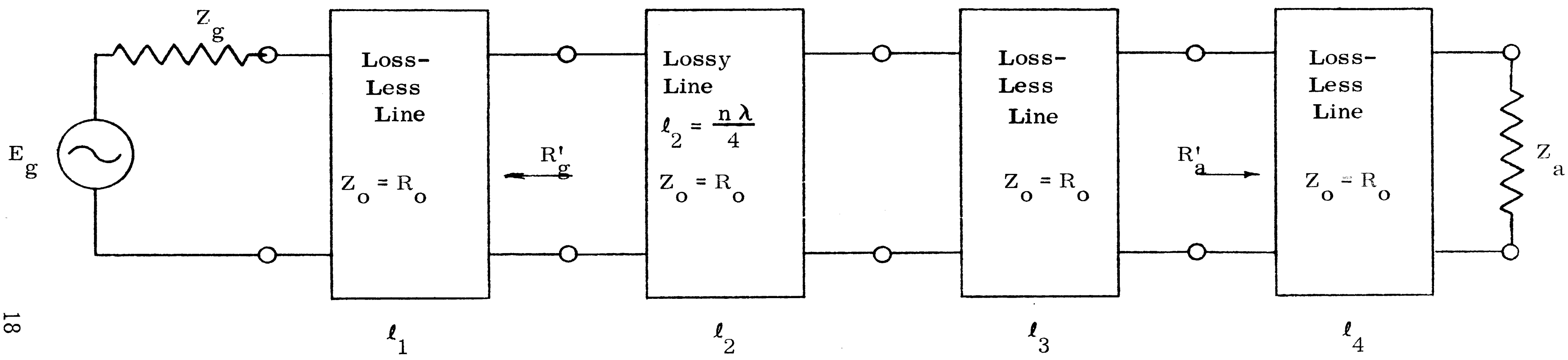

FIG. 2-6: **Equivalent Circuit of the General Power Transfer Problem.**

Assuming $\ell_3 = 0$ for the moment,

$$\frac{P_{a1}}{P_o} = A_o + A_1 + F_H \quad \text{for} \tag{2.34}$$

a) $R_a > R_o$, $R_g > R_o$, $\ell_2 = \frac{2\,n\,\lambda}{4}$

b) $R_a < R_o$, $R_g < R_o$, $\ell_2 = \frac{2\,n\,\lambda}{4}$

c) $R_a > R_o$, $R_g < R_o$, $\ell_2 = \frac{(2n-1)\,\lambda}{4}$

d) $R_a < R_o$, $R_g > R_o$, $\ell_2 = \frac{(2\,n-1)\,\lambda}{4}$

and

$$\frac{P_{a2}}{P_o} = A_o + A_1 + F_\ell \quad \text{for} \tag{2.35}$$

a) $R_a < R_o$, $R_g > R_o$, $\ell_2 = \frac{2\,n\,\lambda}{4}$

b) $R_a > R_o$, $R_g < R_o$, $\ell_2 = \frac{2\,n\,\lambda}{4}$

c) $R_a > R_o$, $R_g > R_o$, $\ell_2 = \frac{(2\,n-1)\,\lambda}{4}$

d) $R_a < R_o$, $R_g < R_o$, $\ell_2 = \frac{(2\,n-1)\,\lambda}{4}$.

The effect of length ℓ_3 will be to cause the power delivered to the local to vary between $(A'_o + A_1 + F_H)$ and $(A'_o + A_1 + F_\ell)$.

The ratio of the maximum to the minimum power available ≈

$$\frac{P_{a1}}{P_{a2}} \approx (F_H - F_\ell) \quad . \tag{2.36}$$

This ratio becomes α^2 in DeHart's expression (DeHart, 1966) so that

$$\frac{P_a}{P_{a1}} = \frac{1}{\alpha^2 \sin^2 \beta \ell_3 + \cos^2 \beta \ell_3} \tag{2.37}$$

for those conditions given under (2.34), and

$$\frac{P_a}{P_{a2}} \approx \frac{1}{\alpha^2 \cos^2 \beta \ell_3 + \sin^2 \beta \ell_3} \tag{2.38}$$

for those conditions given under (2.35), where $\alpha^2 \approx \log^{-1}\left[\left(F_H - F_\ell\right)/10\right]$.

Where an exact power level is not required, it may be sufficient to solve (2.32) and (2.33) to find the maxima and minima power levels available and use (2.37) or (2.38) to determine the probability that the power level will exceed some point between those levels given by (2.32) and (2.33).

III. COMPUTER ANALYSIS OF A CLASS C AMPLIFIER SPECTRUM

A program has been written which computes the output voltage of a class C amplifier as a function of both frequency and time. The time domain waveform is plotted by the computer and the frequency domain solution appears as the magnitudes of the coefficients a_n and b_n of a Fourier series where

$$E(t) = a_o + \sum_{n=1}^{I} a_n \cos n\omega t + \sum_{n=1}^{I} b_n \sin n\omega t . \qquad (3.1)$$

The necessary inputs include a piecewise linear device model, device input waveform, device static operating conditions (e.g., plate supply voltage, grid bias, etc.), and the complex load impedance as a function of frequency.

3.1 Spectrum I

A flow diagram of the spectrum analysis program appears in Fig. 3-1. The development of the iteration technique utilized in this program was presented in a previous report (J.E. Ferris, et al, 1967). This report will summarize that discussion and include a detailed description of Spectrum I.

Briefly, the output spectrum of the amplifier is found by calculating the frequency components of the plate current waveform and multiplying these components by the tube load impedance at the corresponding frequency. The device is modeled

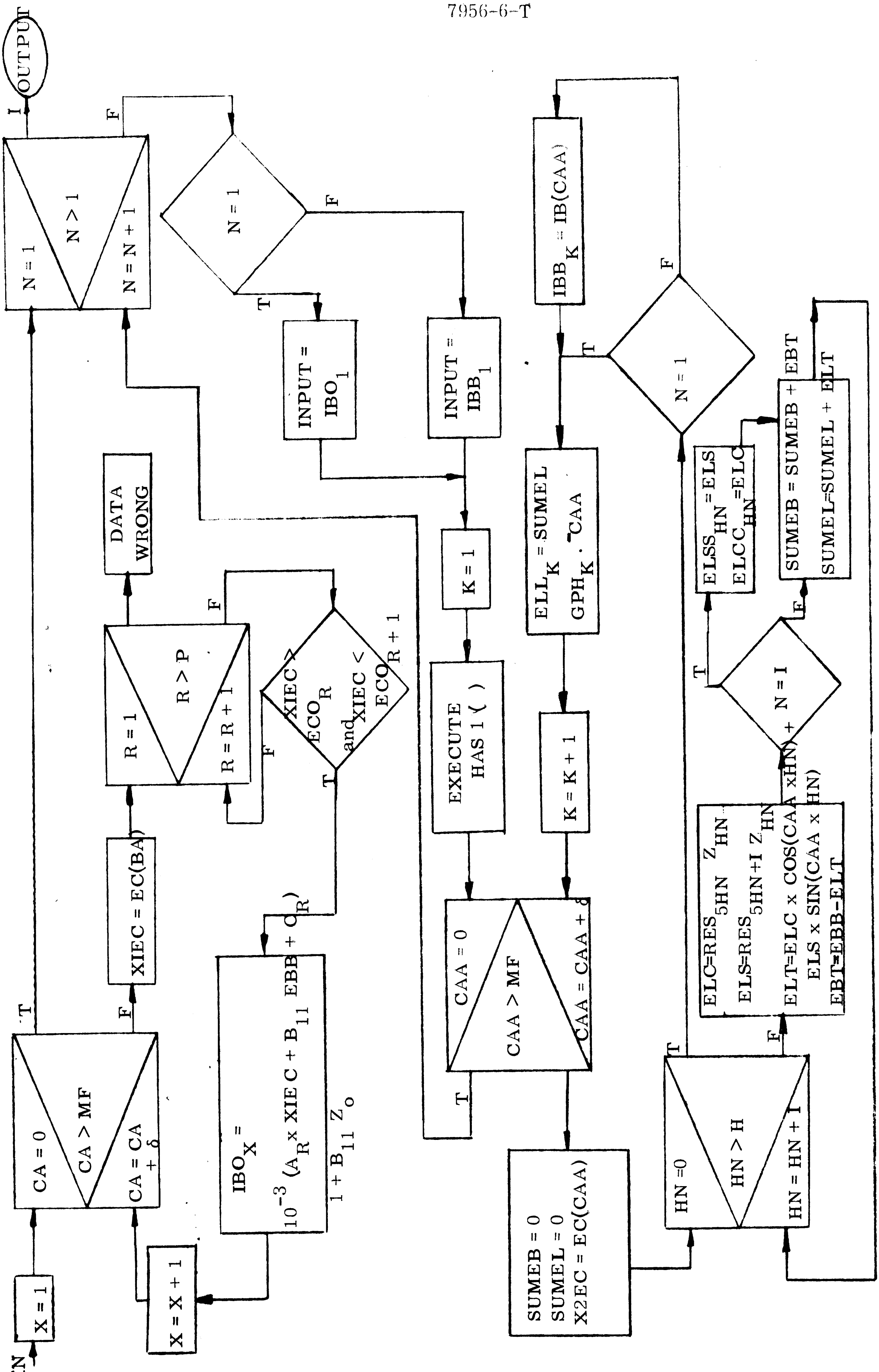

FIG. 3-1: Spectrum I

by a controlled current source as shown in Fig. 3-2.

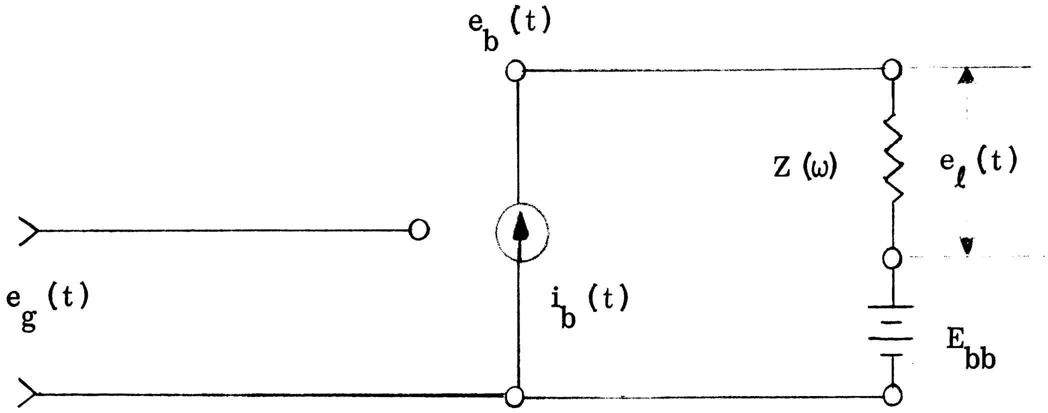

FIG. 3-2: Pentode Equivalent Circuit

One may now write

$$i_b(t) = A_r e_g(t) + B_{(r,s)} e_b(t) + C_r \tag{3.2}$$

where

$e_g(t)$ = instantaneous grid voltage

$e_b(t)$ = instantaneous plate voltage

$i_b(t)$ = instantaneous plate current

$e_\ell(t)$ = instantaneous load voltage

$Z_{(\omega)}$ = complex load impedance

E_{bb} = plate supply voltage

and A_r, C_r, and $B_{(r,s)}$ are constants, appropriately chosen over the region $e_{c(r-1)} < e_c \leq e_{c(r)}$ and $e_{b(s-1)} < e_b \leq e_{b(s)}$ to provide the best linear approximation to the tube static characteristics in that region.

Assuming that $e_g(t)$ is known, it is necessary only to find $e_b(t)$ in order to solve (3.2). From the model in Fig. 3-2,

$$e_b(t) = E_{bb} - e_\ell(t) = E_{bb} - i_b(t)\, Z(t) . \tag{3.3}$$

$Z(t)$, however, is not defined so that it is impossible to solve (3.3) directly. The iteration technique used in Spectrum I assumes temporarily that $Z(t)$ is defined and is equal to a constant Z'. Substituting this result into equation (3.2) and (3.3), one has

$$i_b(t) = \frac{A_r e_g(t) + B_{(r,s)} E_{bb} + C_r}{1 + B_{(r,s)} Z'} , \tag{3.4}$$

If $e_g(t)$ is known, (3.4) may be solved for $i_b(t)$. This provides a first approximation to the plate current waveform. This approximation may be improved by transforming i_b from the time domain to the frequency domain and solving

$$e_b(\omega) = E_{bb} - e_\ell(\omega) = E_{bb} - i_\ell(\omega)\, Z(\omega) . \tag{3.5}$$

Transforming $e_b(\omega)$ back to $e_b(t)$ allows an exact solution of (3.2) and a successively better approximation to $i_b(t)$. This process is continued until successive calculations of i_b (t) produce some arbitrarily small change in e_ℓ (ω).

Referring to Fig. 3-1 Spectrum I* accomplishes the above in the following manner: A variable, x, which counts the number of times the time variable, CA, has been incremented is set to 1. Next, CA is set initially to zero and CA is tested to determine if it is greater than the maximum time of interest, MF. If CA $\leq$ MF, X1EC is set to the current value of grid voltage, EC (CA), and X1EC is compared to ECO_R for $1 \leq R = P$. The ECO_R represent ranges of grid voltage for which the factors A_r and C_1 in (3.2) assume a particular value. If there is no ECO_R corresponding to the value of EC (CA), the loop will continue testing until R exceeds P, the number of ECO's available, and will cause an error flag to be printed out. Once the proper value of R has been established, the corresponding factors A_r and C_r are selected from the data and the approximation to the plate current waveform given in (3.4) is calculated for that value of CA, and stored in memory. CA is then incremented by S and the process continued until (CA + S) > MF at which time N, the number of times that the plate current waveform has been calculated, is set to 1. N is then tested to determine if it is greater than I, the number of times it is to be calculated, and if it is not, tested again to see if it is I.

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*See Table 3-1 for a complete input-output list.

When N = 1, INPUT, the time domain input to the Fourier series sub-routine, HAS 1, is set to IBO and HAS 1 is executed to find the coefficients a_n and b_n of the Fourier series given by (3.1). At this point, the time variable is renamed CAA and set to zero. EC (CAA) is calculated for the current value of CAA and renamed X2EC. The Fourier coefficients a_n (called $RES_{(5hn)}$ and b_n (called $RES_{(5hn + 1)}$ are multiplied by cos (HN) (CAA) and sin (HN) (CAA) respectively, Z (ω), and summed. This is repeated for all values of HN (harmonic number) up to the highest harmonic desired (H). The sum is then subtracted from the plate supply voltage, EBB, This result, EBT, is the plate voltage waveform at time CAA. Equation (3.2) is now solved by an internal function IBB for i_b at time CAA. This process is repeated for all CAA, $0 \leq$ CAA = MF, resulting in $i_b(t)$. $i_b(t)$ is then reentered into HAS 1, and cranked through the N loop, improving the approximation of $i_b(t)$ until, at least, N = 1. The last time through the loop, the Fourier coefficients are stored and printed out, and the final load voltage waveform as a function of time is plotted by the computer. It mus t be noted that this program was only recently completed and no results are currently available for publication.

While the example presented here computes the spectrum of a tetrode vacuum tube operating in the class C mode, the technique is very general and could be used with a variety of devices such as triodes, pentodes, and transistors, so long as the results are not effected by physical phenomena such as electron transit time, etc.

The next section devotes itself to the development of a corresponding model of a triode, taking into account the effect of inter-electrode capacitances.

TABLE 3-1

INPUT-OUTPUT LIST FOR SPECTRUM I

Inputs:

EC magnitude of the excitation voltage

EBB plate supply voltage

Z (ω) plate load impedance as a function of frequency

ECO (R) range of grid voltage for the R'th piece of the piecewise linear static characteristic model

EBO (R, S) range of plate voltage for the (R, S) piece of the piecewise linear static characteristic model

A (R), C (R), B (R, S) piecewise linear model coefficients

P, Q the range of R and S respectively

M F range of ωt (i.e., CA, CAA)

Delta increment of CA, CAA

I number of times iteration process is to be run

H maximum number of harmonics desired

Outputs:

E L (T) a plot of the load voltage waveform as a function of time

ELCC, ELSS magnitudes of the coefficients of the Fourier series of E L (T)

IV

EQUIVALENT CIRCUIT OF A CLASS C TRIODE

The spectrum analysis program presented in the previous section was developed around a piecewise linear model of a tetrode. All inter-electrode capacitances were neglected. These capacitances cannot be neglected in a high frequency triode. Fig 4-1 is an equivalent circuit of a triode including inter-electrode capacitances and driving source resistance.

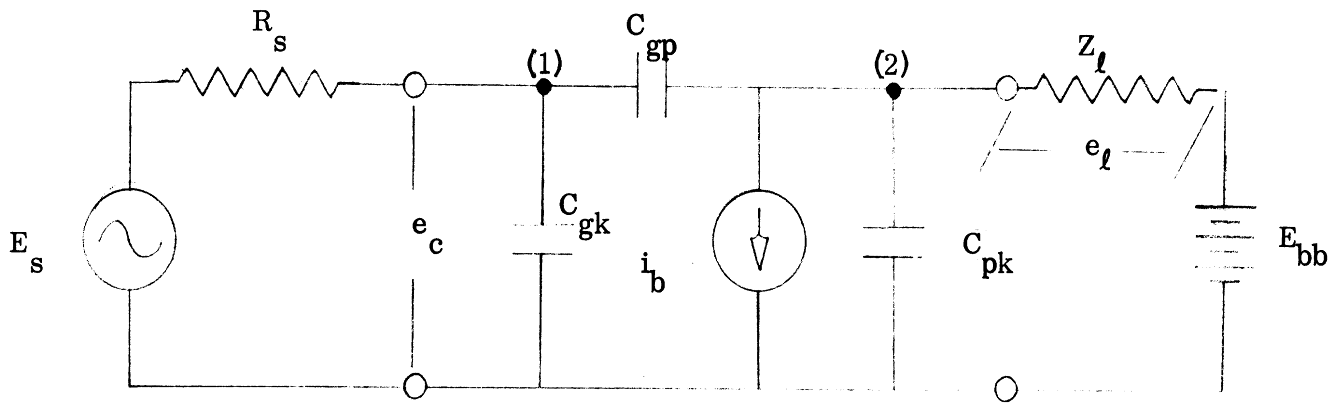

FIG. 4-1: Equivalent Circuit of a Triode

The instantaneous plate current is found by a piecewise linear approximation to the static characteristics as before.

$$i_b = A_r e_c + B_{(r,s)} e_b + C_r \; . \tag{4.1}$$

Again, an exact solution is not possible because the plate voltage is related to the plate current by impedances defined only as a function of frequency. Writing node voltage equations at nodes (1) and (2) in Fig. 4-1*,

At node 1,

$$0 = \frac{|\widehat{E}_s| - e_c(t)}{R_s} - e_c(t)\, j\omega C_{gk} + (e_b(t) - e_c(t))\, j\omega C_{gp} \, . \tag{4.2}$$

At node 2,

$$i_b(t) = (e_c(t) - e_b(t))\, j\omega C_{gp} - e_b(t)\, j\omega C_{pk} + (E_{bb} - e_b(t)) \frac{1}{Z_\ell(\omega)} \, . \tag{4.3}$$

The first step is to obtain an approximation to i_b. Perhaps the simplest approach is to neglect the inter-electrode capacitances and assume Z_ℓ is constant with frequency. Under these conditions,

$$e_b(t) = E_{bb} - i_b(t)\, Z'_\ell \tag{4.4a}$$

and

$$e_c(t) = \widehat{E}_s(t) \, . \tag{4.4b}$$

Thus (4.1) becomes

$$i_b(t) = \frac{A\,\widehat{E}_s(t) + BE_{bb} + C}{1 + B\,Z'_\ell} \, . \tag{4.5}$$

Once the first approximation to $i_b(t)$ has been found, i_b must be transformed from the time domain to the frequency domain,

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* See Table 4-1 for a list of symbols used in section 4.

Let

$$i_b(t) = \overline{I_b} + \sum_{n=1}^{H} \left|\widehat{I}_p\right|_n \sin(n\omega t + \phi_n). \tag{4.6}$$

Rewriting (4.3) in the frequency domain and equating terms of the same frequency, (4.3) becomes

$$\widehat{I}_{p1} = \left(\widehat{E}_{c1} - \widehat{E}_{p1}\right) j\omega C_{gp} - \widehat{E}_{p1}\, j\omega C_{pk}$$

$$+ \left(E_{bb} - \widehat{E}_{p1}\right) \frac{1}{Z(\omega)}$$

$$\widehat{I}_{p2} = \left(\widehat{E}_{c2} - \widehat{E}_{p1}\right) j2\omega C_{gp} - \widehat{E}_{p1}\, j2\omega C_{pk}$$

$$+ \left(\widehat{E}_{bb} - \widehat{E}_{p1}\right) \frac{1}{Z(\omega_2)}$$

o

o

o

$$\widehat{I}_{pH} = \left(\widehat{E}_{cH} - \widehat{E}_{pH}\right) jH\omega C_{gp} - \widehat{E}_{pH}\, jH\omega C_{pk}$$

$$+ \left(\widehat{E}_{bb} - \widehat{E}_{pH}\right) \frac{1}{Z(\omega H)} \tag{4.7}$$

or

$$\widehat{E}_{pn} = \frac{\widehat{E}_{cn}\, jn\omega C_{gp} + E_{bb} \frac{1}{Z_o} - \widehat{I}_{pn}}{\frac{1}{Z(\omega_n)} + jn\omega(C_{gp} + C_{pk})} . \tag{4.8}$$

The term $\widehat{E}_{cn}\, jn\omega C_{gp}$ is the component of plate current fed back to the grid. This current is very much smaller than the total current $\widehat{I}_{pn}$ in (4.8) and may be neglected from this expression without appreciably affecting the result.

Thus,

$$\widehat{E}_{pn} = \frac{E_{bb}\frac{1}{Z_o} - \widehat{I}_{pn}}{\frac{1}{Z(\omega_n)} + jn\omega\,(C_{gp} + C_{pk})} \,. \tag{4.9}$$

Expressing (4.2) in the frequency domain in a manner similar to that described above,

$$0 = \frac{\widehat{E}_s - \widehat{E}_{cn}}{R_s} - \widehat{E}_{cn}\, jn\omega\, C_{gk} + \left(\widehat{E}_{pn} - \widehat{E}_{cn}\right) jn\omega\, C_{gp} \tag{4.10}$$

on solving for E_c

$$\widehat{E}_{cn} = \frac{\frac{\widehat{E}_s}{E_s} + \widehat{E}_{pn}\, j\,n\,\omega\, C_{gp}}{\frac{1}{R_s} + j\,n\,\omega\,(C_{gk} + C_{pk})} \,. \tag{4.11}$$

Combining (4.9) and (4.11),

$$\widehat{E}_{cn} = \frac{\frac{\widehat{E}_s}{R_s} + \frac{\frac{E_{bb}}{X_o} - \widehat{I}_{pn}}{\frac{1}{Z\,(n\omega} + j\,n\,\omega\,(C_{gp} + C_{pk})}}{\frac{1}{R_s} + j\,n\,\omega\,(C_{gk} + C_{gp})} \,. \tag{4.12}$$

Equations (4.9) and (4.12) are expressions for the plate and grid voltage in the frequency domain as a function of the plate current. Using a frequency domain transform of the time domain approximation given for the plate current as a function of time equation (4.5), (4.9) and (4.12) can be solved. Transforming these quantities back to the time domain (the inverse of (4.6)), (4.1) can be solved for the next successive approximation to $i_b(t)$. This process is repeated until $i_b(t)$ has been found to the required degree of accuracy.

TABLE 4-1

SYMBOLS USED IN SECTION IV

i_b	total instantaneous plate current
$\overline{I_b}$	average (DC) plate current
$\lvert\hat{I}_p\rvert_n$	magnitude of the n'th component of plate current
e_b	total instantaneous plate voltage
$\overline{E_b}$	average (DC) plate voltage
$\overline{E_{bb}}$	plate supply voltage
$\lvert\hat{E}_p\rvert_n$	magnitude of the n'th component of plate voltage
e_c	total instantaneous grid voltage
$\overline{E_c}$	average (DC) grid voltage
$\overline{E_{cc}}$	grid supply voltage
$\lvert\hat{E}_g\rvert_n$	magnitude of the n'th component of grid voltage
i_c	total instantaneous grid current
$\overline{I_c}$	average (DC) grid current
$\lvert\hat{I}_g\rvert_n$	magnitude of the n'th component of grid voltage

V. CONCLUSIONS

The text of this report is comprised of two phases of spectrum signature prediction. Section II predicts the output of the antenna once the transmitter behavior has been determined and Section III and IV discuss methods for predicting the behavior of a particular class of transmitter. If one is willing to assume that a given transmitter behaves in a linear fashion with respect to changes in the load impedance, the results in Section II have an immediate application to spectrum predictions.

No experimental evidence has been given to support any of the results. It is felt, however, that such evidence is necessary and experimental data should be gathered as soon as possible.

APPENDIX I

Errata Sheet for 7956-5-T

The following corrections should be made to the equations corresponding to the numbers given.

$$a_n = \frac{1}{\pi} \int_0^{2\pi} i_b(t) \cos(n\omega t)\, d\omega t$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} i_b(t) \sin(n\omega t)\, d\omega t \qquad (3.5)$$

$$i_b = A_r e_c + B_s e_b + C_r \quad \begin{cases} e_{c(r-1)} > e_c \geq e_{c(r)} \\ e_{b(s-1)} > e_b \geq e_b\ (s) \end{cases} \qquad (3.7)$$

$$i_b = 0 \qquad\qquad e_c \leq e_{co}$$

$$\left|\hat{E}_L\right|_n = \left|\hat{I}_p\right|_n Z_n \qquad (3.8)$$

$$i_b - B_s e_b = \bar{I}(1 - B_s R_o) + \sum_{n=1}^{\infty} \left|\hat{I}_p\right|_n \sin(n\omega t + \phi_n)(1 - B_s Z_n) = A_r e_c + C_r \qquad (3.9)$$

The screen grid and control grid connections are shown interchanged in Fig. 3-11, page 29.

REFERENCES

Ferris, J.E., W.R. DeHart, and W.B. Henry, (January, 1964) "Transmitter Impedance Characteristics for Airborne Spectrum Signature," Interim Technical Report No. 3, The University of Michigan Radiation Laboratory Report 7956-3-T, 14 pages.

Dehart, W.R. (1966), "Transmitter-to-Antenna Power Transfer Under Unmatched Conditions," IEEE Trans. EMC-8, No. 2, pp. 74-80.