

**THE UNIVERSITY OF MICHIGAN**  
**COLLEGE OF ENGINEERING**  
**DEPARTMENT OF ELECTRICAL ENGINEERING**  
**Radiation Laboratory**

**ACOUSTIC AND ELECTROMAGNETIC SCATTERING  
PROBLEMS FOR LOW FREQUENCIES**

**First Annual Report**

**1 June 1966 - 1 June 1967**

**Ergun Ar**

**6 June 1967**



**NSF Grant: GP-6140**

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The following problems, which were listed in the Work Statement of the proposal for Grant GP-6140 have been successfully completed.

1. Low Frequency Scattering by Spheroids

The complete low frequency expansion of the field scattered when a scalar plane wave is incident in an arbitrary direction on a spheroid (prolate, oblate, or disc) has been obtained by Kleinman and Asvestas (1967). Both the Dirichlet and the Neumann boundary conditions are considered. The expressions for the field are valid everywhere in space for all values of the ratio of spheroid dimensions to wavelength within the radius of convergence of the low frequency expansion.

2. Scattering by a Torus

Solutions of low frequency scattering of acoustic and electromagnetic waves by a torus are derived by P. Laurin (1967). The solutions are in power series, in ascending powers of  $k$ , the wave number.

Since the method employed is such that the solution is constructed from the solution of the potential equation, Laplace's equation is solved in toroidal coordinates for both the Dirichlet and Neumann boundary conditions. Green's function is derived for the Dirichlet case and particular problems are solved in toroidal coordinates for the Neumann case. These results also have applications in fluid dynamics.

Two non-zero terms in the low frequency solution expansion are explicitly derived for the cases of acoustic scattering by soft and rigid tori. Two terms in the low frequency expansions for both the electric and the magnetic fields are derived for the scattering of a normally incident plane electromagnetic wave. For this case the torus is assumed to be perfectly conducting.

The far field is calculated for a small torus for the acoustic problem, with normal incidence on a soft torus, and compared with the known results for the corresponding problem of a sphere and of a disc. The radii of these bodies which give equivalent scattered far fields are calculated as a function of the radius of the torus.

2. Publications

The following publications have resulted from the support provided in whole or in part by Grant GP-6140.

Ar, Ergun (1965), " Low Frequency Scattering From an Ogive, " accepted for publication in the Quarterly of Applied Mathematics.

Ar, Ergun (1966), "On the Solution of Kleinman's Equation in a Banach Space, " Notices of American Mathematical Society, 13, No. 7 Issue No. 93. Presented to the Joint Meeting of the Societed Mathematica Mexicana and the American Mathematical Society, November 1966.

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Ar, Ergun and R. E. Kleinman (1966), "The Exterior Neumann Problem for Three-Dimensional Helmholtz Equation," Archives for Rational and Mechanical Analysis, 23, No. 3, pp. 218-236)

Laurin, Pushpamala (1967), "Scattering by a Torus" The University of Michigan Thesis, Physics Department.

Kleinman, R. E. (1967), "Low Frequency Solution of Electromagnetic Scattering Problems," Proceedings of the Symposium on Electromagnetic Wave Theory, Delft, The Netherlands, Pergamon Press, pp. 97-111.

Kleinman, R. E. and J. S. Asvestas (1967), "Scattering by Spheroids," The University of Michigan Technical Report No. 7133-5-T .

3. Students and Professional Staff Involved

Dr. T. B. A. Senior, Associate Head of the Radiation Laboratory has served as the Project Director of this Grant. He and Dr. R. E. Kleinman, Research Mathematician have served as consultants, at no cost to the grant, for different aspects of the investigation. Dr. Ergun Ar, Associate Research Mathematician is the Principal Investigator. Other contributors include Mrs. P. Laurin who has completed her PhD dissertation in Physics with support from the grant and a second graduate student, Mr. John S. Asvestas is working on a PhD thesis topic and is receiving support from the grant.

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