THE UNIVERSITY OF MICHIGAN

COLLEGE OF ENGINEERING

DEPARTMENT OF ELECTRICAL ENGINEERING

Radiation Laboratory

ACOUSTIC AND ELECTROMAGNETIC SCATTERING PROBLEMS FOR LOW FREQUENCIES

Final Report
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By

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INTRODUCTION

In the final year of Grant GP-6140 attention was concentrated on the electromagnetic aspects of low frequency scattering in three space dimensions. Two types of problems were considered: i) scattering by perfectly conducting surfaces, and ii) scattering by dielectric or plasma coated perfectly conducting surfaces.

The problem of low frequency electromagnetic scattering by perfect conductors dates back to Lord Rayleigh (1897) who showed that the first term in the near field low frequency expansion of both electric and magnetic scattered fields can be obtained from the solution of standard potential problems. He also showed that the first terms in the near field expansions of the scattered fields determine the first term in the far field low frequency expansions. Rayleigh's technique was generalized by Stevenson (1953a, b) who showed that successive terms in the near field low frequency expansions of the scattered fields could be obtained from the solution of standard potential problems. Furthermore, he provided means of continuation of the near fields into the far field. Kleinman (1967a) has removed certain ambiguities in Stevenson's method and has shown (1967b) that a given number of terms in the near field determines an equal number of terms in the far field. The goal of the present investigation was to develop a method which would generate the low frequency expansions without having to solve 2m (m the number of terms in the low frequency expansion) boundary value problems, as is required by Stevenson's method, and which would give the scattered fields everywhere in space thus eliminating the task of continuation of the near fields into the far field. Details of the method are given below.

The second class of problems mentioned above (scattering by dielectric or plasma coated perfect conductors) was treated in the spirit of Stevenson's (1953a, b) method. Two types of coating were considered: i) lossy dielectric coating, and ii) plasma coating. In the first case a thorough investigation of the first near field (Rayleigh) terms within the coating and in the near field outside it was carried out. From these results the first far field term was determined. In the second case (which is of considerable practical interest) a preliminary investigation was made of the effect on the scattered fields of a plasma coating a perfectly conducting surface (Kleinman 1968a). Three types of plasma were considered: lossless plasma, lossy plasma with a collision frequency much less than the operating frequency, and lossy plasma with a collision frequency much greater than the operating frequency. The problem of finding the first (Rayleigh) term in the low frequency expansions of the fields, both in the plasma and outside it, was formulated and it was found that the determination of these quantities required the solution of both potential and eigenvalue problems. Work is still in progress.

THE PROBLEMS IN DETAIL

1. Iterative Solutions of Maxwell's Equations (Asvestas, 1968)

The problem of scattering of electromagnetic waves by a closed, bounded, smooth, perfectly conducting surface immersed in vacuum is considered and a method for determining the scattered electric and magnetic field vectors (solutions of the homogeneous Maxwell equations satisfying well known boundary conditions on the surface and the Silver-Müller radiation condition at infinity) everywhere exterior to the surface is presented. Specifically, two coupled integral equations are derived, one for each scattered field vector. The kernels of the equations are dyadic functions of position and can be derived from the solutions of standard interior and exterior potential problems. Once these dyadic kernels are determined for a particular surface geometry the integral equations can be solved by iteration for the wave number k being sufficiently small. Alternatively, the scattered fields in the integral equations may be expanded in a power series of the wave number k and recursion formulas can be found for the unknown coefficients in the expansions by equating equal powers of k. As a check, the method is applied to the problem of scattering of a plane electromagnetic wave by a perfectly conducting sphere. The first two terms in the low frequency expansions of the electric and magnetic scattered fields are found and are shown to be in complete agreement with known results.

Additional work on the problem is expected to be done in the near future. The tasks under consideration are:

i) A proof that the sequence of iterates converges to the exact solution for k sufficiently small.

ii) A non-trivial example, namely, scattering by a prolate spheroid. If, as was possible in the scalar case (Asvestas and Kleinman, 1967), recursion formulas can be found so that essentially all terms in the low frequency expansion can be found, this would be a significant step since the exact solution in terms of spheroidal wave functions is not known, save as a series whose coefficients must be found by inverting an infinite matrix.

iii) An attempt towards an alternate existence proof using the convergence of the sequence of iterates.

iv) An attempt to derive estimates of the radius of convergence of the low frequency expansion based on the norm of the operator and the function space in which convergence of the iterates is proven.

2. Rayleigh Scattering by Coated Conducting Objects (Kleinman, 1968b)

The problem of scattering by a plane electromagnetic wave incident on an arbitrarily shaped perfectly conducting object with a lossy dielectric coating is considered and it is shown that the electric field (the Rayleigh term in the near E field) vanishes in the coating while in the exterior it is the same as it would be if the coating were perfectly conducting. The corresponding static magnetic field is shown to be the same as it would be if the coating were absent. Furthermore it is demonstrated that, while the far field depends on both static near fields, the vector sum of the far fields at two opposite receiver positions (e.g. forward and backscattering direction) is precisely the same for the coated object as it would be if the coating were perfectly conducting whereas the vector difference of these two fields is the same for the coated object as it would be if the coating were absent.

PUBLICATIONS (June 67 - Sept 68)

The following publications have resulted from the support provided in whole or in part by NSF Grant GP-6140

Asvestas, J.S. (1968), "Iterative Solutions of Maxwell's Equations," Dissertation, The University of Michigan, Department of Electrical Engineering (81pp)

Kleinman, R.E. (1968a) "Rayleigh Scattering by Plasma Coated Perfect Conductors" The University of Michigan Radiation Laboratory Internal Memorandum No. 2500-331-M (23 August)

Kleinman, R.E. (1968b), "Rayleigh Scattering by Coated Conducting Objects," To appear in IEEE Trans. G-AP.

STUDENTS AND PROFESSIONAL STAFF INVOLVED

On 31 August 1967, Dr. T.B.A. Senior was replaced as Project Director by Dr. Ergun Ar. Dr. Senior and Dr. Kleinman have continued to serve as consultants at no cost to the grant, for different aspects of the investigation. Dr. John Asvestas completed his dissertation for the Ph.D degree in Electrical Engineering under the grant.

REFERENCES

Asvestas, J.S. "Iterative Solutions of Maxwell's Equations," The University of Michigan Thesis, Electrical Engineering Department.

Asvestas, J.S. and R.E. Kleinman (1967), "Low Frequency Scattering by Spheroids and Discs," The University of Michigan Radiation Laboratory Report 7133-5-T, AFCRL-TR-67-0360, AD 655782.

Kleinman, R.E. (1967a), "Low Frequency Solutions of Electromagnetic Scattering Problems," Electromagnetic Wave Theory Symposium, Delft, Pergamon Press, New York (pp. 891-905)

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