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**THE UNIVERSITY OF MICHIGAN**  
**COLLEGE OF ENGINEERING**  
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**Radiation Laboratory**

VOR PARASITIC LOOP COUNTERPOISE SYSTEMS

Interim Report No. 1 (16 June - 1 October 1967)

By

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## I INTRODUCTION

This is the First Interim Report on Contract FA 67WA-1753, Project 330-004-05N "VOR Parasitic Loop Counterpoise System" and covers the period 16 June to 1 October 1967.

During this period we have done some theoretical and experimental investigation on the VOR parasitic loop counterpoise system. Some of the results obtained from the study are reported below. The results and conclusions of the present study should be considered as preliminary.

## II THEORETICAL STUDY

The VOR parasitic loop counterpoise system is shown schematically in Fig. 1. The Alford loop is the driven element. It is assumed that the parasitic loop is made of conducting wire of radius  $b$ . All the other parameters of the system are as shown in Fig. 1.

The radiation characteristics of the Alford loop and counterpoise system in the absence of the parasitic loop have been studied in great detail by Weston et al<sup>\*</sup>. The free space far field produced by an Alford loop carrying a current  $I = I_0 e^{-i\omega t}$  and in the presence of a large circular counterpoise ( $ka \gg 1$ ) is given by the following expression<sup>\*</sup>:

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<sup>\*</sup> Weston, V H., J E Clark and F. M Penar (1964), "New VOR Counterpoise System for Reduction of Siting Errors; Final Report" Conductron Corporation Report No. RD-64-47 (January).

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$$E_{\phi}^i \sim \eta I_o \left(\frac{ka}{2}\right)^2 \frac{e^{i(kR-\pi/4)}}{R} S^i(\theta), \quad \text{for } 0 < \theta < \pi \quad (1)$$

where

$$S^i(\theta) = \left\{ \frac{F(0)\sin\theta}{2} e^{-ikA\sin\theta} + \frac{\sin(\frac{\phi_1}{2}) |\cos\theta|}{\pi kr_1 \sin\theta} e^{ikr_1} L \right\} \quad (2)$$

$$L = e^{\frac{i(\frac{\pi}{2} - kA \sin\theta)}{\sqrt{1-\sin\theta}}} \left( \frac{\cos^{3/2}\phi_1 - \sin^{3/2}\theta}{\cos\phi_1 - \sin\theta} \right) - \frac{e^{ikA\sin\theta}}{\sqrt{1+\sin\theta}} \frac{\cos^{3/2}\phi_1}{\cos\phi_1 + \sin\theta}, \quad (3)$$

$$F(0) = e^{ikr_1 \sin(\theta-\phi_1)} \int_{-\infty}^{p_1} e^{i\pi t^2/2} dt - e^{ikr_1 \sin(\theta+\phi_1)} \int_{-\infty}^{p_2} e^{i\pi t^2/2} dt, \quad (4)$$

$$p_1 = 2 \left(\frac{kr_1}{\pi}\right)^{1/2} \cos\left(\frac{\phi_1 - \theta - \pi/2}{2}\right), \quad (5)$$

$$p_2 = 2 \left(\frac{kr_1}{\pi}\right)^{1/2} \cos\left(\frac{\phi_1 + \theta + \pi/2}{2}\right), \quad (6)$$

$$r_1^2 = A^2 + h^2, \quad (7)$$

$$\sin\phi_1 = \frac{h}{A}. \quad (8)$$

In the above equations  $k (= 2\pi/\lambda)$  and  $\eta$  are respectively the propagation constant and intrinsic impedance of free space. Computed values of  $S^i(\theta)$  at

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the frequency 100 MHz for  $A = 52'$  and for different values of  $h$  are given by Weston et al (1964) (see footnote, page 1).

The purpose of the present study is to see whether the parasitic loop can increase the gradient of the field produced by the system at the horizon and, in particular, whether a null can be produced in the field pattern at a desired **angle** below the horizon by suitably choosing the parasitic loop parameters  $b$ ,  $B$  and  $H$ .

The rigorous theoretical analysis of the field produced by the complete system is a complicated boundary value problem, and will not be attempted here. For the present we shall study approximately the field produced by the system only in the vicinity of the horizon. To simplify the problem we make the following approximations:

- 1) the parasitic loop is large, i. e.  $kB \gg 1$ ,
- 2) it is sufficiently large so that any coupling between the parasitic loop and the counterpoise may be neglected,
- 3) the parasitic loop is in the far zone of the Alford loop.

The first step is to evaluate the current induced by the Alford loop in the parasitic loop. Let the total current in the parasitic loop be

$$I_p = I_{p_0} e^{-i\omega t}. \quad (9)$$

It is now assumed that the current at any point in the parasitic loop is equal to that induced by a longitudinally polarized plane electromagnetic field on a conducting cylinder of radius  $b$  and of infinite length. The relevant component of the electromagnetic field in the present case is that produced by the Alford loop at the position of the parasitic loop. Under these conditions and assuming  $kb \ll 1$ , it can be shown that

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$$I_{p_0} = \frac{2\pi E_0}{i\eta k \left[ 0.577 + \ln\left(\frac{kb}{2}\right) - i\pi/2 \right]} \quad (10)$$

where  $E_0$  is the free space electric field produced by the Alford loop counterpoise system at the position of the parasitic loop. In order to find  $E_0$ , the diffraction effects produced by the counterpoise are neglected. Thus

$$E_0 = \eta \left(\frac{ka}{2}\right)^2 I_0 B \left[ \frac{e^{ikr_1}}{r_1^2} - \frac{e^{ikr_2}}{r_2^2} \right], \quad (11)$$

where

$$\left. \begin{aligned} r_1^2 &= B^2 + (H-h)^2 \\ r_2^2 &= B^2 + (H+h)^2 \end{aligned} \right\} . \quad (12)$$

Neglecting the diffraction effects of the counterpoise, the direct field produced by the parasitic loop in the vicinity of the horizon is

$$E_{\theta}^S \sim \eta I_0 \left(\frac{ka}{2}\right)^2 \frac{e^{i(kR-\pi/2)}}{R} S^S(\theta), \quad (13)$$

for  $\frac{\pi}{2} - \tan^{-1}(H/A+B) < \theta < \frac{\pi}{2} + \tan^{-1}(H/A+B)$ ,

where

$$S^S(\theta) = \frac{\pi (kB)^2 J_1(kB \sin\theta)}{0.577 + \ln\left(\frac{kb}{2}\right) - i\pi/2} \left[ \frac{e^{ikr_1}}{(kr_1)^2} - \frac{e^{ikr_2}}{(kr_2)^2} \right] e^{-i(kH \cos\theta + \pi/4)}, \quad (14)$$

where  $J_1$  is the first order Bessel function of the first kind.

Thus the complete far field pattern produced by the parasitic loop counterpoise system in the vicinity of the horizon is

$$S(\theta) = S^i(\theta) + S^S(\theta) \quad (15)$$

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## Numerical Results

Equation (14) has been computed for different values of  $b$ ,  $B$  and  $H$ . It has been found that for  $kb < 0.1$  the amplitude of  $S^S(\theta)$  is too small compared to  $|S^i(\theta)|$ , for all values of  $B$  and  $H$ , to produce a null in  $S(\theta)$  below the horizon. Figures 2 and 3 show some representative results of the computation. The values of  $S^i(\theta)$  are taken from Weston et al (1964) (reference on page 1).

In Fig. 3 the patterns produced by the parasitic loop counterpoise system  $|S(\theta)|$  in the vicinity of the horizon are shown for two different values of the parasitic loop parameters. Also shown in Fig. 3, is the pattern  $|S^i(\theta)|$  produced by the system in the absence of the parasitic loop. It can be seen that for  $B = 1\lambda$  the relative variation of  $|S(\theta)|$  is not appreciably different from that of  $|S^i(\theta)|$ . For  $B = 3\lambda$ , the field gradient in the horizon is increased and a minimum in the field appears in the direction  $4^\circ$  below the horizon. On the basis of the preliminary numerical studies it may be concluded that  $B$  must be larger than  $2\lambda$  in order that the parasitic loop may appreciably alter the field pattern below the horizon.

## III EXPERIMENTAL STUDIES

Measurements have been made at a frequency of 1080 MHz. In the present system  $h = 4.8''$  and  $A = 5.2'$ . A typical elevation pattern for the Alford loop and counterpoise system is shown in Fig. 4. The elevation pattern has been measured in different planes and the azimuthal symmetry of the system has been found to be satisfactory.

A parasitic loop counterpoise system has been built with the following dimensions:  $2B = 1.97'$ ,  $A = 5.2'$ ,  $h = 4.8''$  and  $H$  is variable from  $7.8''$  to  $19.8''$ .

A few elevation patterns of the above system have been measured for different values of  $H$ . In Figs. 5(a) - (c) are shown the patterns for  $H = 7.8''$ ,  $9.8''$  and  $17.8''$  respectively. These results do not show any significant alteration

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of the field below the horizon as compared with Fig. 6. This may be due to the low value of  $B$  used. However, the pattern is considerably altered in the high elevation angle directions.

## IV CONCLUSION

During the next period parasitic loops with larger diameter will be built and tested. The theoretical expression discussed will be corrected and modified by taking into account the appropriate diffraction effects neglected.

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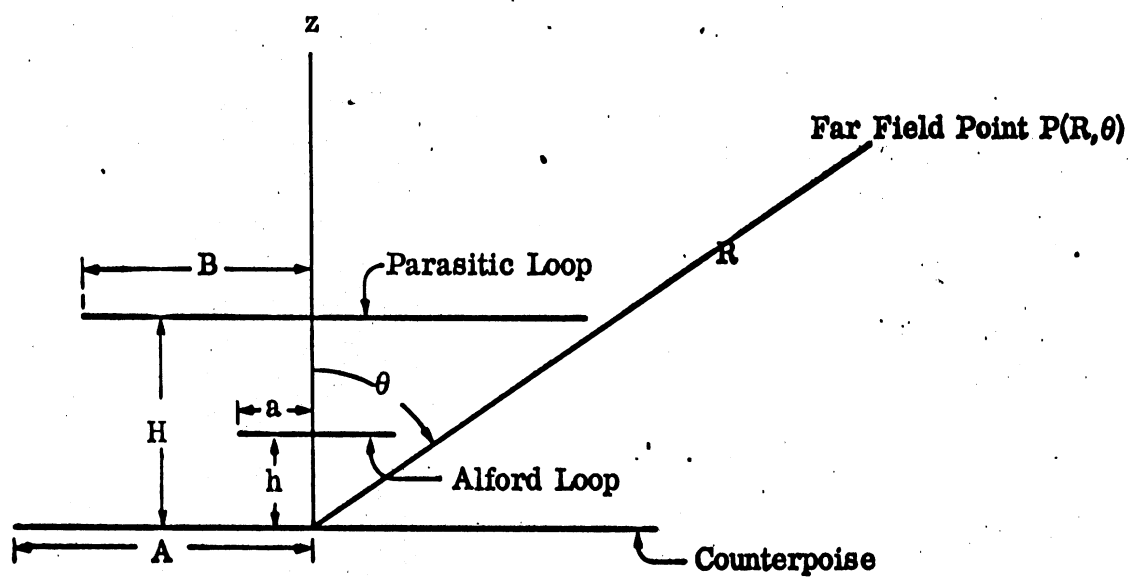
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**FIG. 1: GEOMETRIC REPRESENTATION OF THE VOR PARASITIC LOOP COUNTERPOISE SYSTEM.**



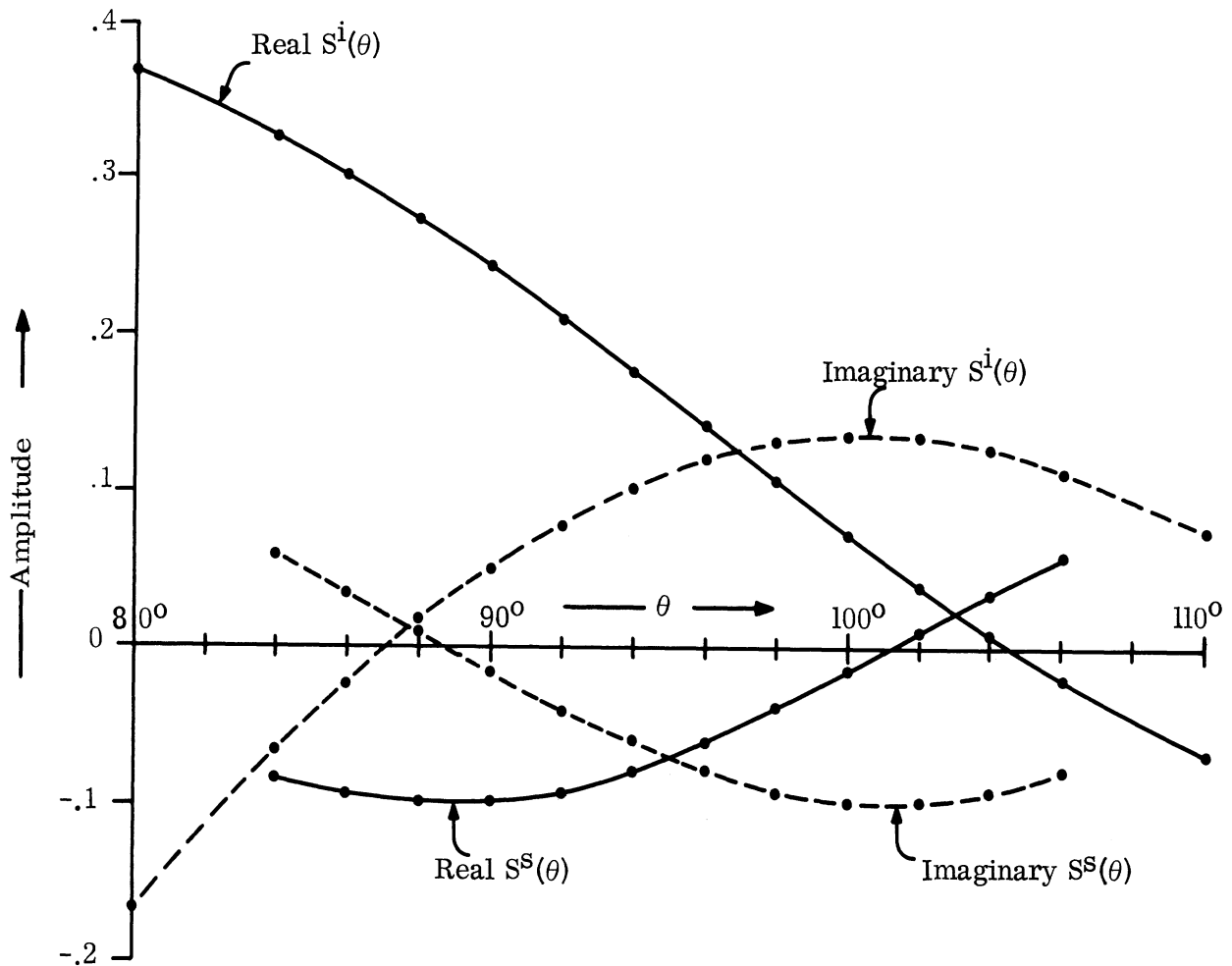


FIG. 2(a): THE REAL AND IMAGINARY PARTS OF THE DIFFERENT RADIATION PATTERN FUNCTIONS IN THE VICINITY OF THE HORIZON.  
 $kb=0.15$ ,  $kB=6.28$ ,  $kH=7.34$  and  $kh=2.554$ .

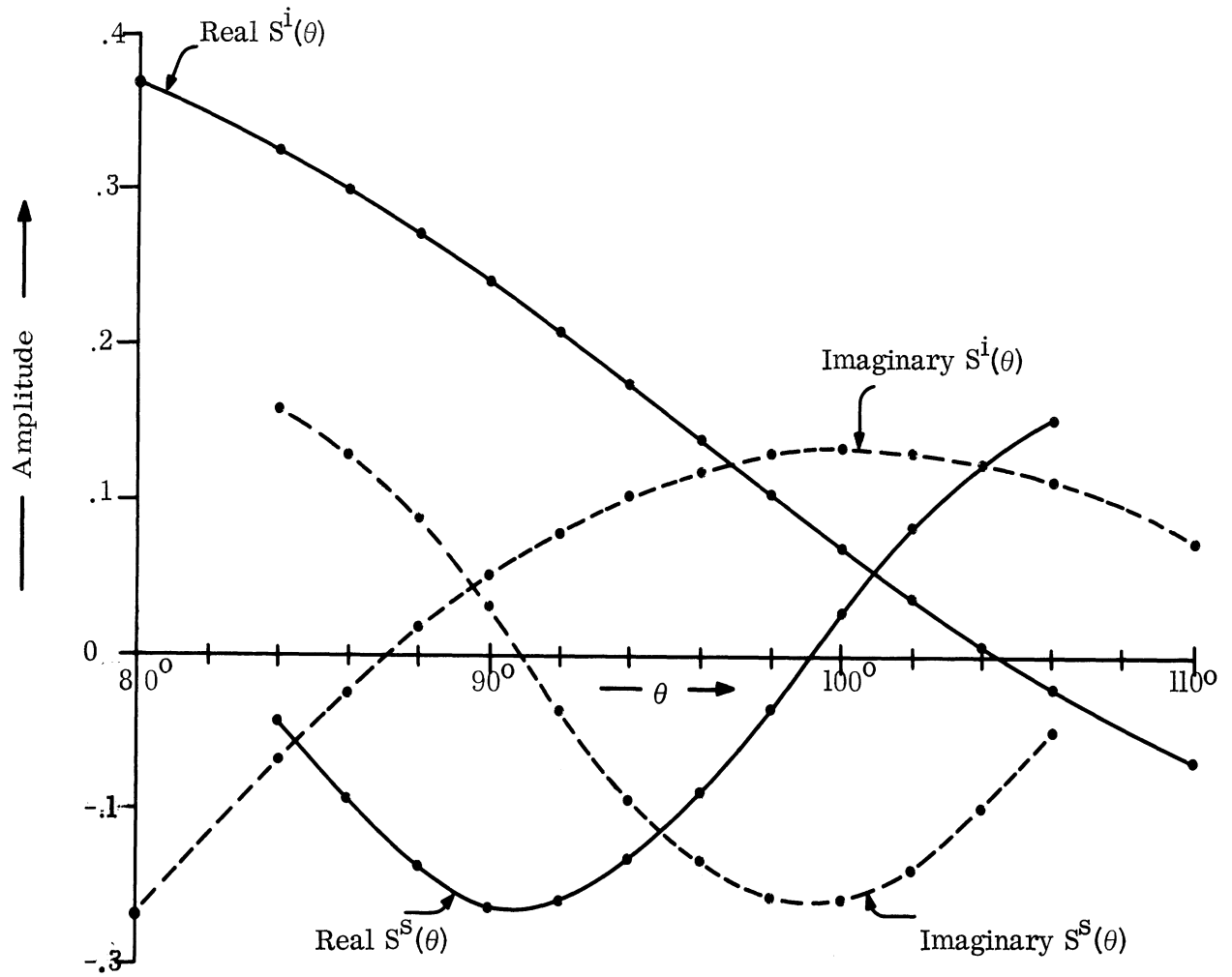


FIG. 2(b): THE REAL AND IMAGINARY PARTS OF THE DIFFERENT RADIATION PATTERN FUNCTIONS IN THE VICINITY OF THE HORIZON.  
 $kb=0.15$ ,  $kB=18.85$ ,  $kH=10.91$  and  $kh=2.554$ .

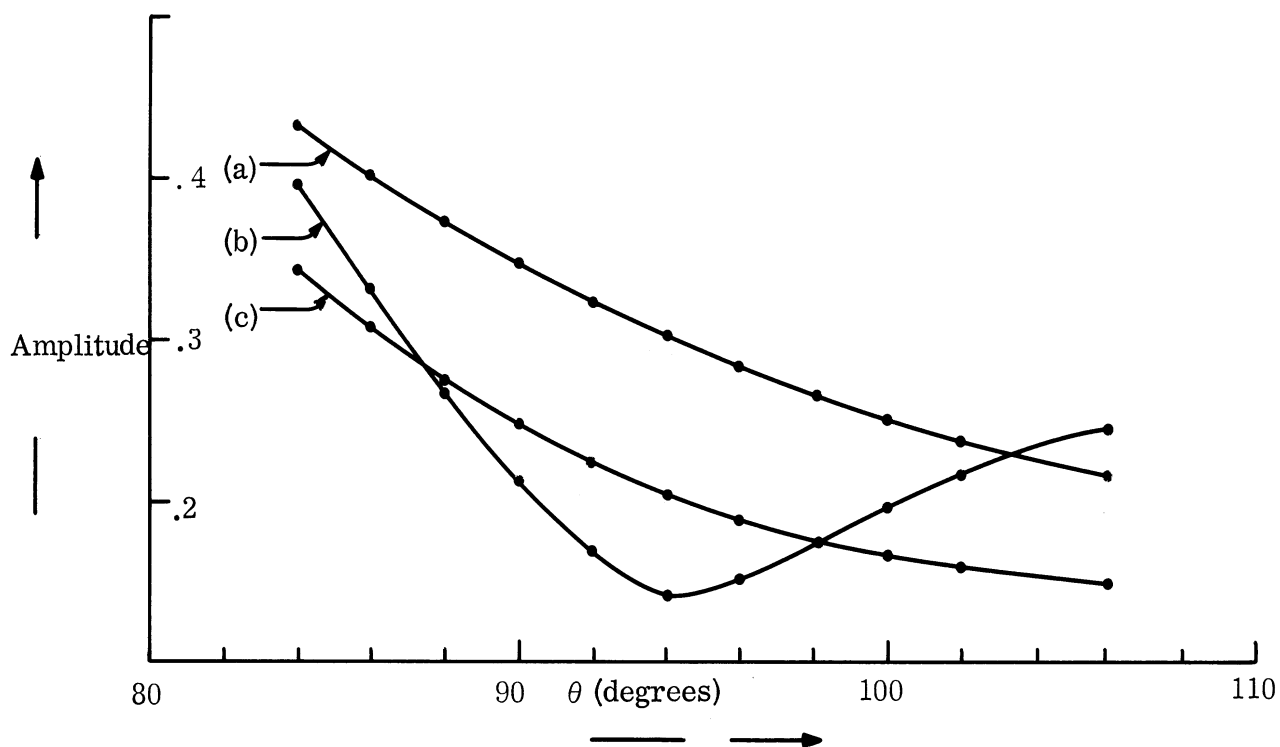


FIG. 3: THE FAR FIELD PATTERNS IN THE VICINITY OF THE HORIZON.  
 (a) Alford loop and the counterpoise only  $|S^i(\theta)|$  ; (b) Parasitic loop counterpoise system  $|S(\theta)|$  for  $k_B=6.28$ ,  $k_b=0.15$ ,  $k_H=7.34$  and  $k_h=2.554$ ;  
 (c) Parasitic loop counterpoise system  $|S(\theta)|$  for  $k_B=18.85$ ,  $k_b=0.15$ ,  $k_H=10.91$  and  $k_h=2.554$ .

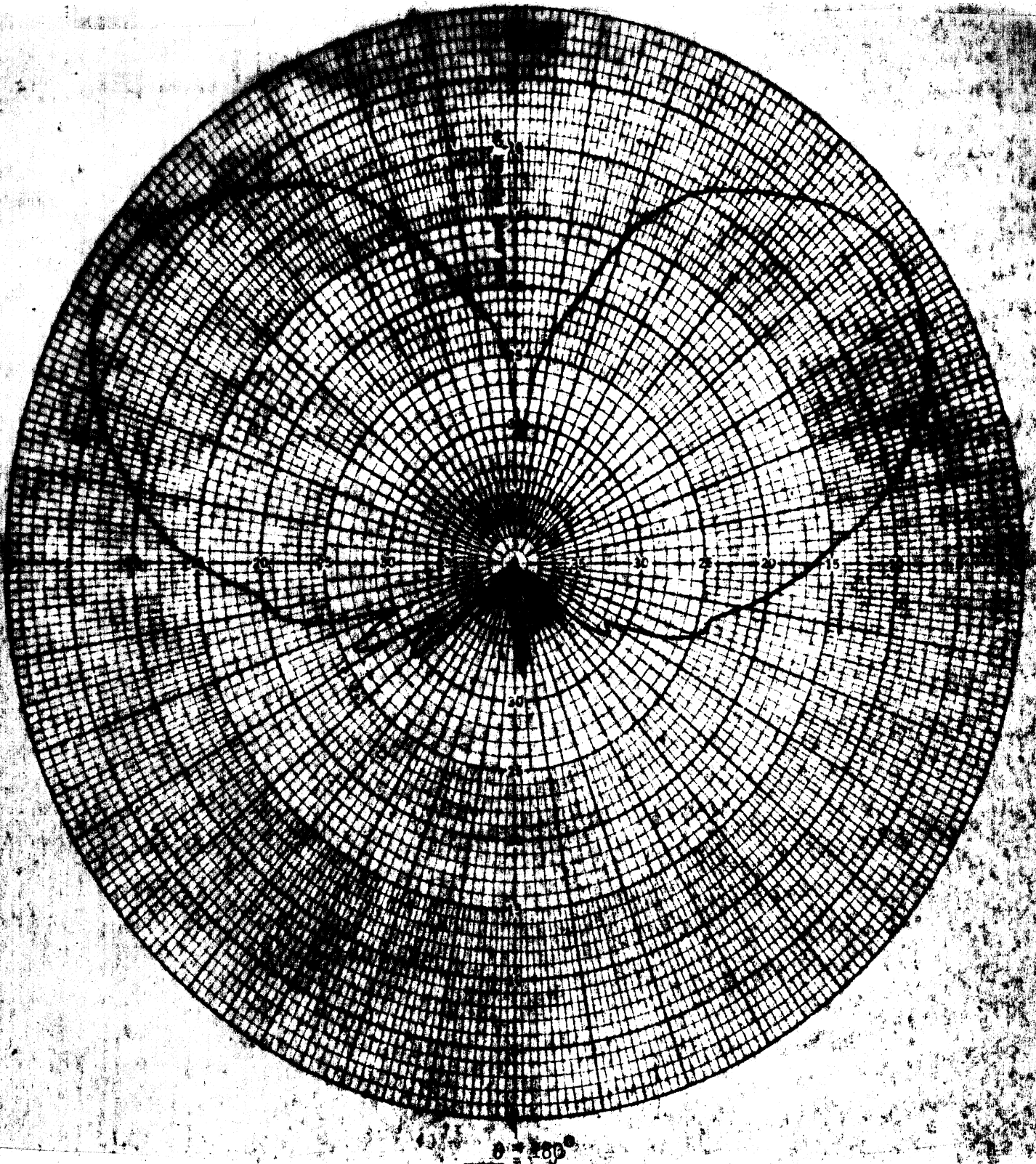


FIG. 4: THE MEASURED ELEVATION PLANE RADIATION PATTERN  
PRODUCED BY THE ALFORD LOOP COUNTERPOISE SYSTEM  
AT 1080 MHz.

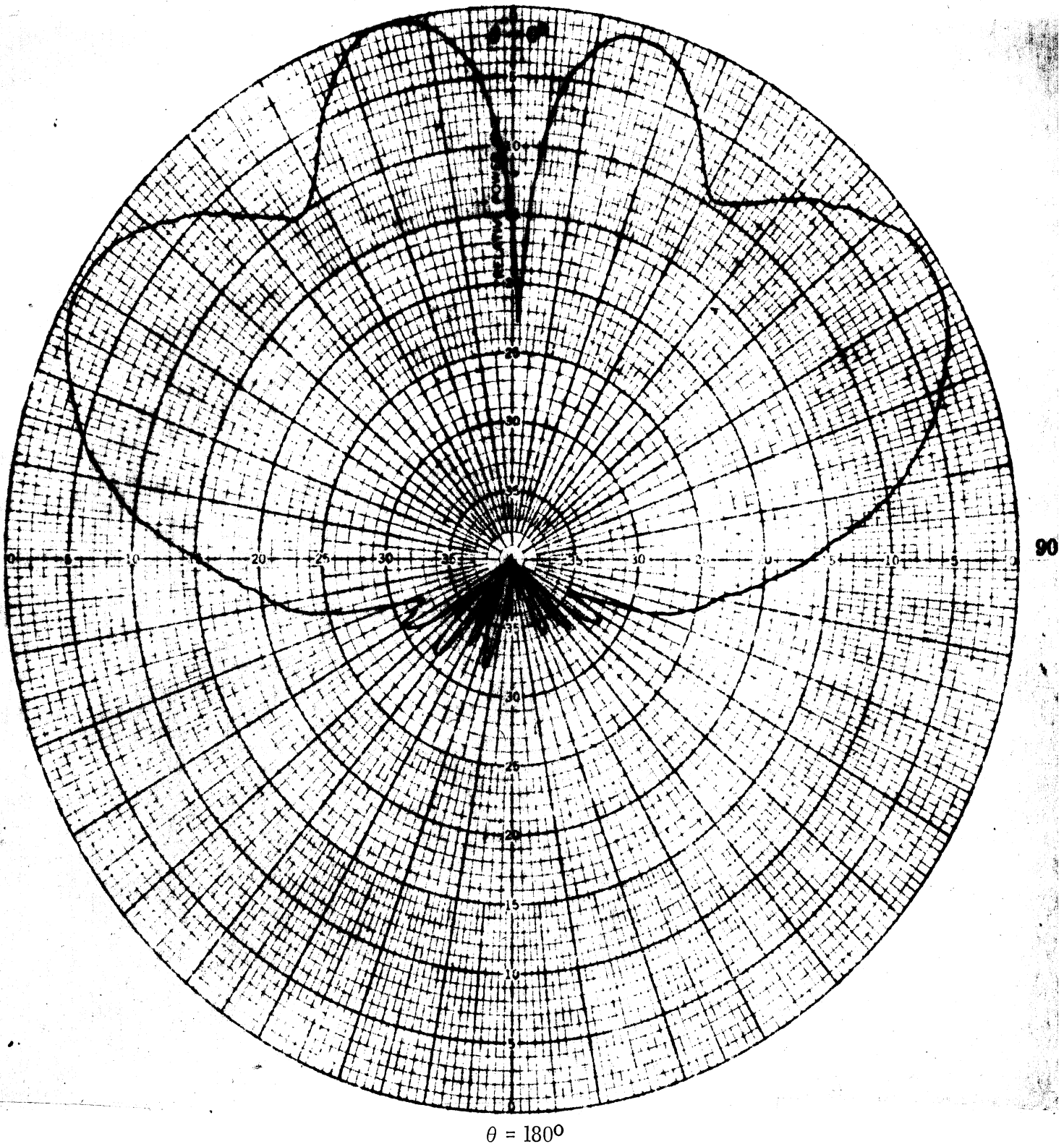


FIG. 5a: MEASURED ELEVATION PLANE RADIATION PATTERNS  
 PRODUCED BY THE PARASITIC LOOP COUNTERPOISE  
 SYSTEM AT 1080 MHz.  $B = 0.985'$ ,  $H = 7.8''$ ,  $h = 0.4'$ ,  
 $A = 5.2'$  and  $a = 0.105'$ .

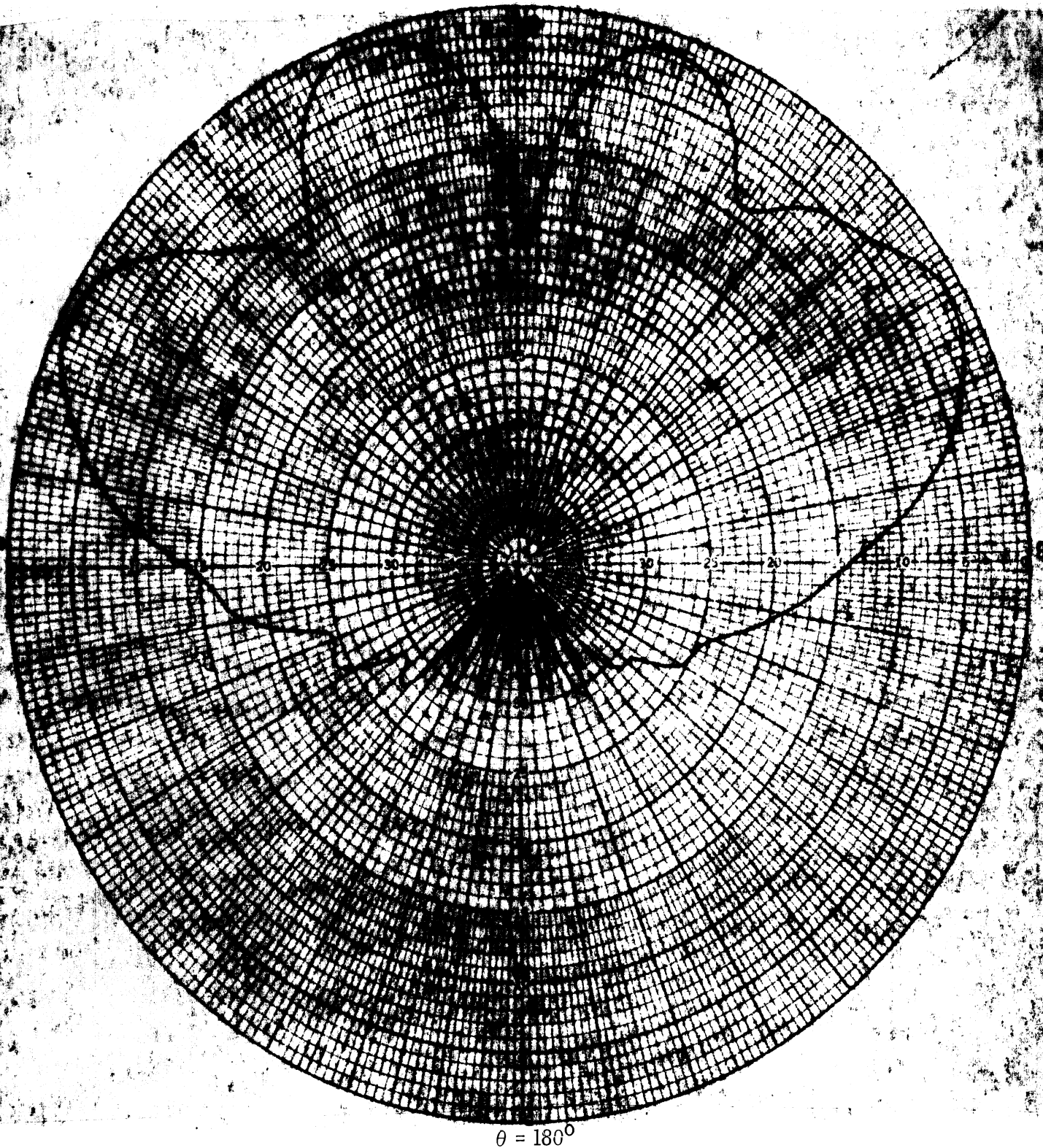


FIG. 5b: MEASURED ELEVATION PLANE RADIATION PATTERNS PRODUCED BY THE PARASITIC LOOP COUNTERPOISE SYSTEM AT 1080 MHz.  $B=0.985'$ ,  $H=9.8''$ ,  $h=0.4'$ ,  $A=5.2'$  and  $a=0.105'$ .



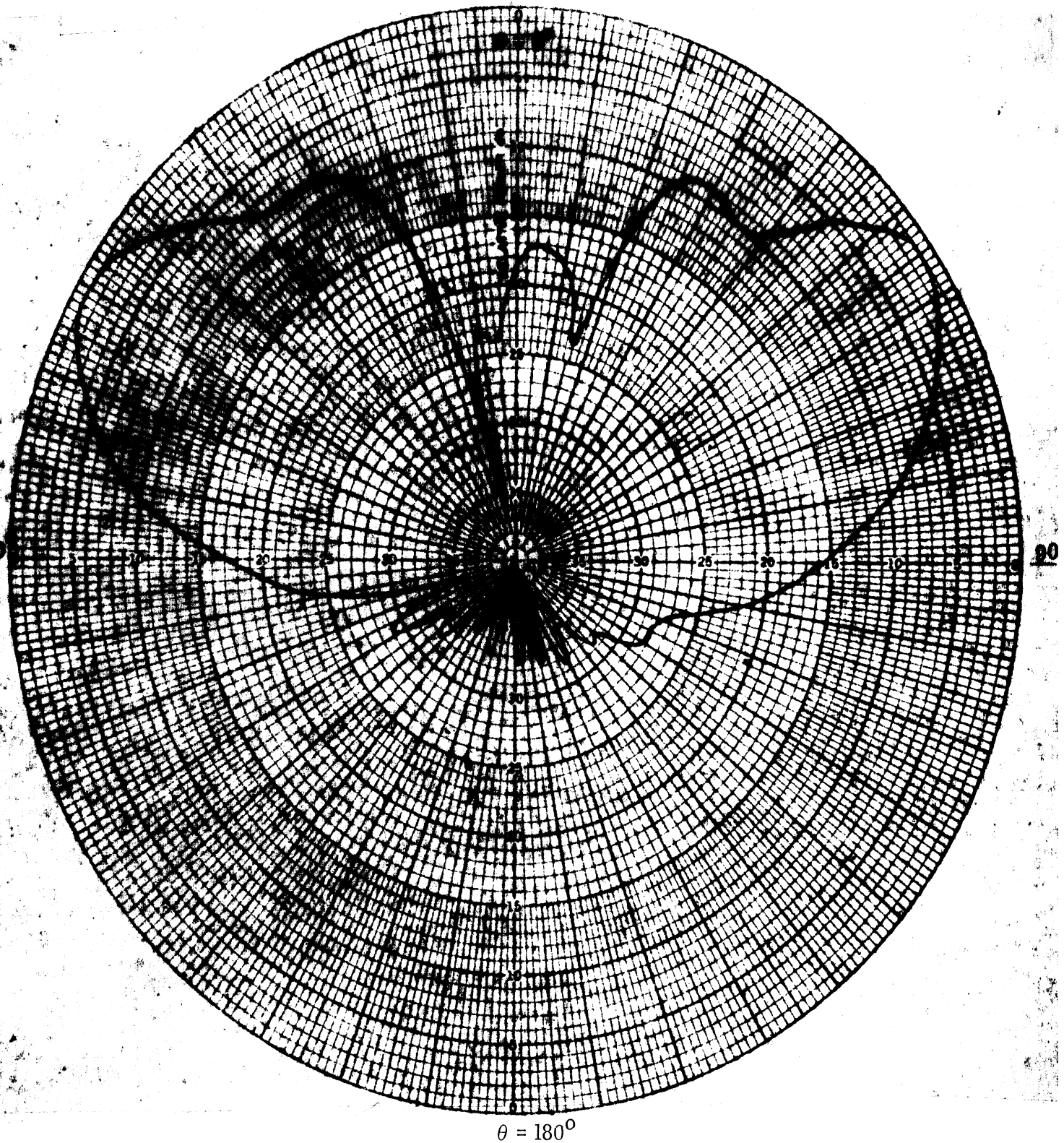


FIG. 5c: MEASURED ELEVATION PLANE RADIATION PATTERNS PRODUCED BY THE PARASITIC LOOP COUNTERPOISE SYSTEM AT 1080 MHz. B=0.985', H=11.8'', h=0.4', A=5.2' and a=0.105' .