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Radiation Laboratory

TRANSIENT RADIATION AND RECEPTION OF EM ENERGY

ANNUAL REPORT

by

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1. SUMMARY

The work in this period deals mainly with a thorough investigation of the structure of dyadic Green's functions and their eigen-function expansions. A shortcoming found in a previous work by the author of this report has been amended. A technical report entitled "Eigen-function Expansion of Dyadic Green's Functions" has been published and copies of that report have been submitted to the Foundation on July 25, 1973 [1].

The dyadic Green's functions pertaining to cavities have also been derived. The subject matter represents a part of the dissertation currently being written by P. Rozenfeld, a graduate student at The University of Michigan, under the supervision of C-T Tai.

2. POTENTIAL APPLICATIONS TO ENGINEERING AND TECHNOLOGY

The result of the research thus conducted represents so far the most extensive compilation of the dyadic Green's functions pertaining to waveguides, cavities and several scattering bodies. These functions are useful to people working in the area of electromagnetic pulse (EMP) sensors, and microwave circuits. The dyadic Green's function technique simplifies considerably the formulation of many electromagnetic boundary value problems.

3. DESCRIPTION OF THE RESEARCH

While much of the eigen-function expansions are discussed in the technical report [1], we would like to outline here the work on the structure of dyadic Green's functions not thoroughly covered in that report.

The classification of various dyadic Green's functions can best be presented by starting with Maxwell's equations in dyadic form. They are:

$$\nabla \times \bar{\bar{G}}_e = \bar{\bar{G}}_m \tag{1}$$

$$\nabla \times \bar{\bar{G}}_m = \bar{\bar{I}}\,\delta(\bar{R} - \bar{R}') + k^2 \bar{\bar{G}}_e \tag{2}$$

where we designate $\bar{\bar{G}}_e$ as the dyadic Green's function of electric type and $\bar{\bar{G}}_m$ as the dyadic Green's function of the magnetic type. They are solutions of the differential equations,

$$\nabla \times \nabla \times \bar{\bar{G}}_e - k^2 \bar{\bar{G}}_e = \bar{\bar{I}}\,\delta(\bar{R} - \bar{R}') \tag{3}$$

and

$$\nabla \times \nabla \times \bar{\bar{G}}_m - k^2 \bar{\bar{G}}_m = \nabla \times \left[\bar{\bar{I}}\,\delta(\bar{R} - \bar{R}')\right] \quad . \tag{4}$$

Another dyadic Green's function pertaining to the Hertzian vector potential function can be defined. It satisfies the differential equation

$$\nabla^2 \bar{\bar{G}}_A + k^2 \bar{\bar{G}}_A = -\bar{\bar{I}}\,\delta(\bar{R} - \bar{R}') \quad . \tag{5}$$

The relations between $\bar{\bar{G}}_e$, $\bar{\bar{G}}_m$ and $\bar{\bar{G}}_A$ are

$$\bar{\bar{G}}_m = \nabla \times \bar{\bar{G}}_A \tag{6}$$

$$\bar{\bar{G}}_e = \left(\bar{\bar{I}} + \frac{1}{k^2} \nabla\nabla\right) \cdot \bar{\bar{G}}_A \tag{7}$$

The function $\bar{\bar{G}}_A$ was used by Morse and Feshbach in their treatment of the eigen-function expansion of electromagnetic field; and the function $\bar{\bar{G}}_e$ was introduced by Levine and Schwinger in their work. The function $\bar{\bar{G}}_m$ never received much attention. In our recent research, we found that the function $\bar{\bar{G}}_m$ is much simpler to deal with as far as the eigen-function expansion is concerned. Since it is solenoidal only two sets of solenoidal vector wave functions are required to represent $\bar{\bar{G}}_m$. The technical report [1] gives a detailed derivation of the eigen-function expansions of various dyadic Green's functions based on the method of $\bar{\bar{G}}_m$.

4. (a) PERSONNEL

C-T Tai, the director of the project, has worked on the basic formulation which leads to the reclassification of the dyadic Green's functions and the derivation of the eigen-function expansion of these functions pertaining to waveguides, and scatterers.

P. Rozenfeld, a graduate student at The University of Michigan, has worked on the functions pertaining to cavities.

4. (b) PAPERS PUBLISHED AND IN PREPARATION

(1) C-T Tai, "On the Eigen-function Expansion of Dyadic Green's Functions," Proc. IEEE, Vol. 61, p. 480, April 1973 (Communication).

(2) C-T Tai, "The Structure of Dyadic Green's Functions and their Eigen-function Expansions", a paper submitted to the URSI/GA-S Symposium to be held in Atlanta on June 10-14, 1974.

(3) P. Rozenfeld, "Microwave Lens and Cavities", a Ph.D. dissertation to be submitted to the Department of Electrical and Computer Engineering, the University of Michigan in April, 1974.

5. OTHER ACTIVITIES

C-T Tai presented a series of lectures on dyadic Green's functions at The University of Washington, and at Harvard University in the Spring of 1973. The material is based essentially on the research conducted under this Grant.

REFERENCES

1 C-T Tai, "On the Eigen-function Expansion of Dyadic Green's Functions", Radiation Laboratory, The University of Michigan, Technical Report No. 011136-1-T (July, 1973).